Efficient Fiscal Spending by Supranational Unions

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We use a novel approach to address the question of whether a union of sovereign countries can efficiently raise and allocate a budget, even when members are purely self-interested and participation is voluntary. The main innovation of our model is to explore the link between budget contributions and allocation that arises when countries bargain over union outcomes. This link stems from the distribution of bargaining power being endogenously determined. Generically, it follows that unstructured bargaining gives an inefficient result. We find, however, that efficiency is achieved with fully homogenous countries, and when countries have similar incomes and the union budget is small. Moreover, some redistribution arises endogenously, even though nations are purely self-interested and not forced to participate in the union. A larger union budget, however, entails a trade-off between equality and efficiency. We also analyze alternative institutions and find that majority rule can improve efficiency if nations who prefer projects with high public good spillovers are endogenously selected to the majority coalition. Exogenous tax rules, such as the linear tax rule in the EU, which is designed to increase efficiency on the contribution margin, can also improve overall efficiency despite decreasing the efficiency of the allocation of funds.

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1 Motivation

Can a union of sovereign nations *efficiently* conduct the basic fiscal task of raising and allocating a budget? In light of the recent proposals to expand centralized fiscal spending in the European Union, and the heated political debate surrounding them, an answer to this fundamental question is of high interest.

The understanding of union-level fiscal spending is related to the topic of fiscal federalism.[2] In contrast to fiscal policy administered within the framework of a country, however, bargaining over fiscal outcomes at the supranational level is based on the implicit threat of veto. From a theoretical perspective, the voluntarily nature of supranational unions imply national participation constraints. In the EU, for example, any expansion of fiscal spending like the proposed growth pact must be decided unanimously, giving each nation veto power over its implementation. The importance of individual participation constraints has been analyzed by Alesina et al. (2005) for the case of policy harmonization at the supranational level. There, each individual country remains responsible for the implementation of a policy determined by majority rule. A centralized budget at the supranational level, however, opens up a larger set of allocative outcomes and introduces the possibility for redistributive spending.

We build a theoretical framework to analyze how bargaining affects centralized fiscal spending when self-interested nations voluntarily participate in a union. The motivation to form a union stems from a set of projects that benefit from centralized provision, modeled as a technology unavailable to each nation individually. Nations have heterogeneous preferences over these projects, but enjoy positive spillovers from all of them. As an example, one may think about EU structural fund resources being spent on infrastructure improvement: Every member country likely gains from an integrated transportation network; at the same time each country might prefer, all else equal, that spending is allocated to infrastructure projects within its borders.

To analyze how sovereign countries agree on contributions to the union budget as well as its allocation to union projects, a natural modeling choice is unstructured bargaining. Current EU centralized fiscal spending is largely comprised in the structural and cohesion funds, and both budget and allocation decisions for these funds are negotiated by national representatives behind closed doors - a process most closely approximated by the Nash bargaining mechanism we analyze. Importantly, any country can veto the allocation of any money to the EU structural fund, independent of their participation in other EU institutions[3].

The most important contribution of our paper is to show that in a supranational setup with voluntary participation the distribution of bargaining power arises endogenously from the countries' contributions, their national incomes, and the public good spillovers of their preferred projects. This distribution of power is implicit

[2]See for example the seminal contributions of Lockwood (2002) and Besley and Coate (2003).
[3]This has recently been highlighted by the UK's threat to veto the entire EU budget.

in the Nash bargaining solution, which is the outcome of unstructured bargaining between nations with equal ex-ante weights and the individual right to veto any allocation. We highlight this link between contributions and allocations through the implied bargaining position as the main source of inefficiency in the union's spending decision. This is the main innovation over the existing literature.[4]

Our setup allows us to explicitly track under which circumstances unstructured bargaining leads to inefficient outcomes. We find that in surprisingly many cases, the Nash bargaining solution does actually achieve efficiency: First, when nations are symmetric with regard to income and all projects have the same level of spillovers then the budget is raised and allocated efficiently. This holds regardless of countries' heterogeneous preferences over the projects and follows from the fact that the bargaining positions of symmetric agents are exactly equal. Thus, their agreement will include an equal split of the total surplus, which exactly coincides with the efficient allocation. Second, when utility is quasi-linear with respect to income and all countries have symmetric marginal utilities of income then the budget is also raised and allocated efficiently, even though the allocations are redistributive. This result obtains since with quasi-linear utility, the countries are able to utilize contributions as utility transfers. Lastly, we find that a large number of players will at least allocate (albeit not raise) the budget efficiently. This is due to the fact that, as the union grows larger, the bargaining power of countries with high spillover projects increases. For a very large union, unstructured bargaining allocates all funds to the projects with the highest level of spillovers, which is also the efficient allocation.

These circumstances, even though they describe special cases, are educational. As long as a union consists of relatively homogeneous countries, or its budget is small relative to national domestic consumption levels and income levels are comparable, efficiency is easily achievable. Generically, though, unstructured bargaining does lead to the budget being both raised and allocated inefficiently. In a number of numerical simulations, we analyze under which circumstances inefficiencies are severe, and which margin of efficiency (raising the budget or allocating it) is affected most.

We go on to show that redistribution arises endogenously and is sustainable as a bargaining outcome, despite all nations being self-interested. In our model, a union can consist of net-contributing and net-receiving countries, while maintaining voluntary participation. The allocation of the budget achieved by bargaining is crucially determined by the distribution of bargaining power, which in turn is a function of each country's outside option as well as both the contributions to the budget

[4] The allocation decisions of structural and cohesion funds provide evidence that intergovernmental bargaining does play a large role in existing EU fiscal programs: First, the guidelines for spending (even though labeled "impartial") are specifically designed so that each nation qualifies for spending (Hix and Høyland (2011) p. 233). In fact, each country within the EU also receives resources from the structural funds (European Commission (2012)). Second, controlling for need there is still large variability in spending between regions, suggesting that the bargaining power of individual nations is important to the final allocation (Bodenstein and Kemmerling (2011)). Additionally, contributions to the centralized budget are flexible and subject to bargaining given that "the member states maintain ex post control over every country's net transfer position" (Carrubba (1997), p. 473).

and the relative public good spillovers of the various projects. Therefore, countries which have access to projects with a high level of spillovers can end up receiving an allocation of the budget that is greater than their contribution to the joint funds. On the other hand, countries with low-spillover projects, especially when they have high income, may find it worthwhile to contribute more to increase their bargaining power and tilt the allocation in their favor.

When the correlation between a nation's income and the spillover effects from its preferred project is negative (as arguably is the case in the EU, where the most socially efficient projects are typically located in the poorer member states), the union is in principle able to achieve a level of redistribution that alleviates inequality between its members. However, precisely because of the link between contributions and allocations, there is an inherent efficiency-equity trade-off: The union cannot raise the contribution of any country without also increasing the allocation to its preferred project. A budget that will leave all union members equally well off is at the same time necessarily spent inefficiently. From a social welfare point of view, full redistribution, even if achievable, may not be desirable.

We also explore the potential of more complex institutional setups to improve efficiency results. First, we consider using an exogenous tax rule to fix contributions, for example raising funds with a linear tax. Because of the link between contributions and allocations, adjusting contributions will improve efficiency on either the budget or allocation margin, but will necessarily decrease efficiency on the other margin. We find that at the Nash bargaining solution, it is always weakly optimal to improve the budget margin instead of the allocation margin. This suggests that tying contributions to incomes can improve general efficiency.

We also consider majority rule and legislative bargaining as it breaks the link between contributions and allocations, at least for the countries in the minority. We show that majority rule can be welfare improving, but only if the countries with high spillover projects are endogenously chosen to form the majority. We find this to be the case if their relative contributions to the union budget are low enough. Therefore, in the case where income and spillovers are negatively correlated, majority rule and legislative bargaining can yield more efficient outcomes than unstructured bargaining.

Related literature

Our paper relates to three separate strands of the literature. Within the literature on federalism, Lockwood (2002) analyzes the decision of a federation to supply a district-level public good with global spillovers. Due to legislative bargaining, centralization can result in inefficiencies since the majority coalition will only take into account the spillover effects of projects in districts outside of the coalition, and not the direct benefit to the district. Similarly, Besley and Coate (2003) find that centralization can result in excessive public spending. Harstad (2007) considers the situation in which districts (or nations) have private information about their valuation of a public good and finds that a uniform federal (or union) policy mitigates the inefficiencies created by the private information. Our paper shows that centralized provision can result in the efficient outcome, both in terms of allocation and spending, as long as districts have similar incomes and the level of spending on public goods is small relative to domestic consumption.

Second is the literature on optimal decision rules. Aghion and Bolton (2003) and Harstad (2005) examine optimal majority rules in models of legislative bargaining where the legislature decides whether to provide a public good and determines a set of transfers between districts. Both find that districts (or nations) are willing to commit to a majority rule ex-ante given enough uncertainty regarding their ex-post preferences. In our model nations might be willing to commit to a majority rule, not due to ex-ante uncertainty, but because a majority rule serves as a commitment device to not fund low-spillover projects. A novel result is that all nations can be better off under the majority rule compared to autarky, even in the absence of uncertainty.

Perhaps the most related literature is a set of recent papers that model supranational governance as an intergovernmental process with voluntary participation by member nations. Alesina et al. (2005) model the provision of public goods by international unions, and compare the effect of uniform and non-uniform union policy on aggregate welfare and the equilibrium size of unions. Maggi and Morelli (2006) examine the optimal majority rule in a dynamic setting, where a single union project is repeated over time. If nations are patient enough, and are sufficiently uncertain about their future preferences, then the optimal majority rule can be supported even with voluntary participation.

The remainder of the paper is organized as follows: Section 2 introduces the basic setup, and is followed by the characterization of the relevant efficiency benchmarks in section 3. Then, section 4 analyzes the Nash bargaining solution, gives conditions under which efficiency is achievable, and discusses characteristics of the inefficiencies that generally result from the bargaining process. Section 5 shows that redistribution can arise endogenously in the unstructured bargaining setting, and derives a trade-off between equity and efficiency. In section 6, we proceed to analyze alternative institutions that may improve efficiency. We derive conditions under which majority decision rules can improve efficiency while being desirable for all bargaining parties and explore alternative contribution schemes like a linear tax. Section 7 concludes with a discussion of the results.

2 Setup

There are n nations, denoted with $i = 1, ..., n$, that may form a union. Each nation i is endowed with an income, y_i.

Technologies

Each country can either consume its income domestically (c_i) or contribute to a union-wide budget (x_i). Contributions to the union budget must satisfy the nation's individual budget constraint

$$c_i + x_i \leq y_i \quad \forall i. \tag{1}$$

Moreover, we require that no direct transfers toward domestic consumption can be made,[5] i.e. $x_i \geq 0$ for all i. Together the contributions form the union's budget

$$X = \sum_i x_i. \tag{2}$$

Forming a union allows the countries to implement a set of projects $\{g_i\}_{i=1}^n$. These joint projects produce according to a linear production function, so that the union wide budget constraint becomes

$$\sum_{i=1}^n g_i \leq X, \tag{3}$$

with $g_i \geq 0$ for all i. The union projects essentially produce public goods that can be enjoyed by all members of the union. We assume, however, that countries who do not join the union are excluded from enjoying its benefits.

Preferences

Each nation receives utility from domestic consumption as well as the union projects. Among the joint projects, each nation values one particular project the most, but may benefit from (positive) spillover effects from other projects:

$$U_i(c_i, g_1, ..., g_n) = u(c_i) + v(g_i + \sum_{j \neq i} \alpha_j g_j) \tag{4}$$

We assume $u(\cdot)$ and $v(\cdot)$ are continuously differentiable, strictly increasing and concave, and satisfy standard Inada conditions.[6] α_j denotes the spillover effect a country gains from the implementation of project g_j. It is restricted to $\alpha_j \in [0, 1)$. Thus, each project is valued most by the respective "home country," but produces weakly positive and symmetric spillovers for all other countries.[7] We restrict utility over

[5] In reality, it is unclear whether one nation can *directly* contribute to another's national income. Additionally, there is no clear mechanism by which utility can be directly transferred at the supranational level. It is conceivable that transfers are made by increasing centralized spending in a given nation, or by decreasing their contribution to the centralized budget. These are precisely the transfers that our model allows. That is, our model allows for transfers in the project and contributions dimensions instead of assuming that transfers can be made directly in the utility dimension.

[6] Specifically, we assume that $\lim_{x\to 0} u'(x) = \infty$, $\lim_{x\to 0} v'(x) = \infty$, $\lim_{x\to\infty} u'(x) = 0$, and $\lim_{x\to\infty} v'(x) = 0$.

[7] More generally, we could write the utility country j gains from being in the union as $v(\sum_i \alpha_{ij} g_i)$. In the analysis we restrict the spillover effects of each project g_i to be symmetric across all but one countries, i.e. $\alpha_{ij} = \alpha_i$ for all $j \neq i$, and for country i to strictly prefer project g_i over all others, i.e. $\alpha_{ii} = 1$. This restriction allows us to derive clean and intuitive expressions for the inefficiencies arising from bargaining. We point out during the analysis, when relaxing these constraints leads to additional interesting results.

consumption and participating in the union to be separable for tractability.

To economize on notation, we denote

$$u_i' = \frac{\partial u(c_i)}{\partial c_i}$$
$$v_i' = \frac{\partial v(g_i + \sum_{j \neq i} \alpha_j g_j)}{\partial (g_i + \sum_{j \neq i} \alpha_j g_j)}$$

3 Efficiency Benchmarks

A distinct feature of the supranational unions we seek to analyze is that nations participate voluntarily. The resulting participation constraints necessarily imply that any implemented union is a Pareto improvement over autarky. Given nations' heterogeneous preferences, however, it is not immediately obvious how to define efficiency. There exists no overarching authority with a unified objective that would justify an aggregate utility measure as the relevant metric. Rather, efficiency benchmarks have to be defined with respect to the stated goals of the intended union, a measure which all members have agreed on. In case of the EU structural funds, these are to support job creation, competitiveness, economic growth, improved quality of life and sustainable development, and to reduce social and economic disparities.

In the context of our model, we translate these objectives into two efficiency benchmarks: First, funds should be allocated to make the best use of all available technologies, i.e projects with higher spillovers should receive more funding. Second, contributions to the budget should be diverted where it is least costly in terms of forgone domestic consumption, i.e. nations with lower marginal utility of domestic consumption (i.e. higher income) should contribute the more to the joint budget.

We define these benchmarks for a *given* total budget X. That is, we do not include an efficiency benchmark that pins down the optimal budget size, but instead limit our analysis to a fixed budget. An aggregate utility measure as objective would imply an optimal budget.[8] However, since union-level spending is the only channel for inter-country redistribution available, it would be concerns regarding the redistribution of *income*, rather than gains from coordination, that determine it. Given that self-interested nations do not share such a redistributive objective, we instead define efficiency in terms of any given budget, where the size of X compared to the aggregate GDP in the potential union can be interpreted as a measure of importance of the intended union.

Note, however, that setting the total union budget exogenously does not mean that participation constraints are assumed to hold exogenously as well. In the bargaining process analyzed below, the outside option for every nation remains to withdraw from the union and consume all income domestically, regardless of whether the budget is determined exogenously or through a bargaining process. Therefore, the

[8]See Appendix A.5.

results we present in the following section all extend to the case where countries also bargain over the size of budget.[9]

Formally, the goal of diverting contributions where it is least costly in terms of forgone domestic consumption implies that the efficient contributions are the solution to the maximization problem:

$$\max_{\{x_i\}_{i=1,\dots,n}} \sum_{i=1}^{n} u(y_i - x_i) \tag{5}$$

$$s.t. \quad c_i + x_i \leq y_i \quad \forall i \tag{6}$$

$$\sum_i x_i = X \tag{7}$$

$$x_i \geq 0 \quad \forall i, \tag{8}$$

which gives the following efficiency benchmark for the budget:

Definition 1 (Budgetary Efficiency)
A set of contributions $\{x_i\}$ is called BUDGETARY EFFICIENT *if*

$$u'(y_i - x_i) = u'(y_j - x_j) \quad \forall i, j \text{ whenever } x_i, x_j > 0 \tag{9}$$

$$u'(y_i) \geq u'(y_j - x_j) \quad \forall i \text{ whenever } x_i = 0. \tag{10}$$

The goal of allocating funds where the union can make the best use of all available technologies implies that the efficient allocation is the solution to the analogous maximization problem:

$$\max_{\{g_i\}_{i=1,\dots,n}} \sum_{i=1}^{n} [v(g_i + \sum_{j \neq i} \alpha_j g_j)] \tag{11}$$

$$s.t. \quad \sum_i g_i \leq X \tag{12}$$

$$g_i \geq 0 \quad \forall i, \tag{13}$$

which gives the following efficiency benchmark for allocations:

Definition 2 (Allocative Efficiency)
A set of projects $\{g_i\}$ is called ALLOCATIVE EFFICIENT *if*

$$v'_i + \alpha_i \sum_{j \neq i} v'_j = v'_j + \alpha_j \sum_{i \neq j} v'_i \quad \forall i, j \text{ whenever } g_i, g_j > 0 \tag{14}$$

$$v'_i + \alpha_i \sum_{j \neq i} v'_j \geq v'_j + \alpha_j \sum_{i \neq j} v'_i \quad \forall i \text{ whenever } g_j = 0 \tag{15}$$

$$\sum_i g_i = X. \tag{16}$$

[9] In Appendix A, we give a detailed discussion on the justification of the assumptions made about the efficiency benchmark and how they impact our results.

Notice that because the total budget X is set exogenously, and since utility between the consumption good and public goods projects are separable, the two definitions of efficiency are not connected. Either benchmark could be reached without the other being satisfied. If both criteria are satisfied at the same time, the allocation coincides with the set of contributions and allocations that maximizes aggregate utility. Whenever the outcome departs from either benchmark, however, this is not the case. Treating the two margins separately implies that a priori we do not take a stand on which margin is more important (as letting X be determined endogenously would). In section 6, we use an aggregate utility measure to define the socially optimal trade-off between the two efficiency measures.

It is important to note that at the efficient allocation, there is no connection between what each specific country contributed to the budget and how much is allocated to its preferred project. However, when nations bargain, we will see that there is a link between their contributions and the allocation. Naturally, contributions influence the bargaining position of each nation. This is the source of inefficiencies at the heart of this paper. In what follows we will discuss how exactly the bargaining process between nations distorts the two efficiency margins and derive conditions under which the bargaining outcome achieves general efficiency:

Definition 3 (General Efficiency)
An allocation, i.e. a set of contributions and projects $\{x_i, g_i\}$ *is called* GENERALLY EFFICIENT *if it satisfies both budgetary and allocative efficiency.*

4 Nash Bargaining

In this section, we study an unstructured bargaining process. Unstructured bargaining is both the least complex institution for raising and allocating funds (from a political perspective), and is the institution most commonly used by the EU for fiscal spending programs. Formally, countries bargain à la Nash over the utility surplus created by the union. The Nash bargaining solution is tailored to situations where no specific institutions govern the bargaining process and each participant is a veto player, and is therefore the appropriate solution concept for our model.[10]

We assume that countries have equal ex-ante bargaining weights, so that the re-

[10] Additionally, the Nash bargaining solution can always be "rationalized" as an equilibrium of an analogous non-cooperative bargaining game (see e.g. Mas-Colell et al. (1995)).

sulting allocation solves the following problem:[11]

$$\max_{\{x_i,g_i\}_{i=1,\dots,n}} \prod_{i=1}^{n} [u(y_i - x_i) + v(g_i + \sum_{j\neq i} \alpha_j g_j) - u(y_i)] \tag{17}$$

$$s.t. \quad \sum_i g_i \leq X \tag{18}$$

$$\sum_i x_i = X, \tag{19}$$

as well as $x_i, g_i \geq 0$ for all i. The disagreement point is for all countries to revert to autarky and consume their individual income y_i. In our setup, this should be interpreted as each country having a full veto over setting up the union.[12] With the power to veto, participation constraints gain an important role in determining the bargaining outcome. No country can be worse off in the union than it would be under autarky. Moreover, the Nash bargaining solution reflects a compromise that weighs each player's payoff in the union against his outside option. The value of the union to each player, however, is endogenous to the specific set of contributions and allocations in question. Clearly, each country places a higher value on a compromise that, all else equal, lowers its own contribution. Concavity implies, though, that this motive is stronger for low-income nations. In addition, heterogeneous preferences over the union projects may render each specific allocation of funds more valuable for one country than another. The Nash bargaining solution takes all this into account - the distribution of bargaining power is endogenous.

A rich country, for example, is not ex-ante more powerful in the negotiations. The degree of bargaining power the richer country has depends on the specific compromise in question. If it is a net contributor to the union, and therefore has a higher outside option, then the Nash bargaining solution will also allocate a disproportionate amount of funds to its preferred (albeit possibly inefficient) project. Analyzing this channel that connects contributions to allocations in a bargaining setup is at the heart of this paper.

We analyze the problem as if nations were choosing both contributions and allocations simultaneously. Even if nations in reality sometimes bargain first over contributions and then separately over allocations, their outside option in the second stage remains to withdraw from the union and consume their contribution. Therefore, a two-step procedure does not break the link between contributions and the nations' bargaining positions in the second stage; it is easy to verify - assuming nations choose subgame-perfect strategies in the first stage - that the resulting

[11]The Nash bargaining solution assumes that agents bargain over a convex set of utility outcomes. As noted in Conley and Wilkie (1996), however, the set of utility outcomes is not generally convex when spillovers are present. In Appendix B.2, we prove that the Nash bargaining solution extends to the relevant non-convex sets.

[12]Regarding fiscal spending in the EU, this is a realistic assumption. It holds for the distribution of the Structural and Cohesion funds, as well as for the budget negotiations. Under other coalition-based approaches the fundamental link between contributions and allocations that we seek to analyze would be retained.

allocation is indifferent to whether the analysis is done in one or two steps. In this respect our setup differs significantly from the bargaining games analyzed by Harstad (2005), where contributions from the first stage are fixed in the second stage and create a hold-up effect that influences the incentive structure of the whole game.

We start by solving only a subpart of the full problem to illustrate the main source of inefficiency in subsection 4.1. Taking the set of contributions as exogenously given, but maintaining the outside options for nations to withdraw them, will illustrate the respective connection between budgetary contributions (x_i) and technological contributions (α_i) to the bargaining position.

We then proceed by formally analyzing the full bargaining setup over both contributions to the joint budget and its allocation to the union projects in subsection 4.2. For expositional simplicity, we present the main results for the special case of $n = 2$ countries. All formal proofs are done for a general number of countries and relegated to the appendix. Some interesting additional results do obtain when more than two countries bargain; we present them in subsection 4.3.

4.1 Bargaining over Allocations

Suppose that two countries, $i = a, b$, bargain only over allocating funds to the set of projects $\{g_a, g_b\}$, while their contributions $\{x_a, x_b\}$ to the union budget are fixed ex-ante. In this case, the bargaining problem simplifies to:

$$\max_{\{g_a, g_b\}} (S_a)(S_b) \tag{20}$$

$$s.t. \quad g_a + g_b \leq X \tag{21}$$

where S_a and S_b denote the surplus from forming a union $S_i = u(y_i - x_i) + v_i - u(y_i)$ for $i = a, b$ respectively.

Since Nash bargaining selects among the set of Pareto optimal points, constraint (21) is binding. The resulting maximization problem is concave, which allows us to use the first-order-conditions to implicitly solve for the equilibrium level of g_a and g_b:

$$v_a' S_b + \alpha_a v_b' S_a = v_b' S_a + \alpha_b v_a' S_b. \tag{22}$$

Equation (22) helps to illustrate some basic properties of the bargaining solution. It states that the allocation the two nations will compromise on will not equalize the marginal returns of the two union projects unless $S_a = S_b$ (a special case we discuss below). The Nash bargaining solution is the result of balancing the players' bargaining power, which depends on their outside options and thus their contributions. The allocation does tend toward equalizing marginal returns of the projects since that maximizes the total surplus, but weighs in the fact that *individual* surpluses change with it. In other words, it puts weight on a split of the total surplus that reflects the distribution of bargaining power. Recall from definition 2 that allocative efficiency requires

$$v_a' + \alpha_a v_b' = v_b' + \alpha_b v_a'. \tag{23}$$

Here, marginal returns are equalized without any regard of contributions x_i or income levels y_i. But since the outside option for each nation remains to consume its contribution domestically, both x_i and y_i enter in the Nash bargaining solution. This illustrates the basic point: Since outside options influence bargaining power, the bargaining process will generally distort efficiency.

From the primitives of the model it is not always obvious which player has the larger bargaining power and will tilt the allocation toward his preferred project. The bargaining position is influenced through two channels: First, it depends on the outside option, i.e. the utility gain from extra consumption. The same contribution x_i may be valued very differently if income levels y_i are different. Second, it depends on the utility $v(g_i + \alpha_j g_j)$ from setting up the union, which is not necessarily symmetric. Even if both projects were to receive the same funding, asymmetric spillovers $\alpha_i \neq \alpha_j$ would still render the allocation more valuable for one country than the other. While the first channel is fixed exogenously in this simple example, the relative bargaining positions of the two countries are still endogenous to the allocation of funds. Thus, the comparative statics of inefficiency with respect to either income or spillover differences are complicated. Note however that the surplus from joining the union as measured by S_i is an intuitive summary statistic of country i's bargaining position. The more a nation gains if the union were set up with a specific allocation of funds, the weaker is its bargaining position. Indeed, rewriting equation (22) reveals this connection:

$$\frac{(1-\alpha_b)v'_a}{(1-\alpha_a)v'_b} = \frac{S_a}{S_b}. \tag{24}$$

All else equal, a larger surplus for nation a, S_a, increases the right hand side. For (24) to be satisfied, the ratio v'_a/v'_b would have to be larger as well, which means that less funds would be allocated to nation a's preferred project g_a and more to g_b - a result of a's weaker bargaining position.

This sensitivity of the outcome to the distribution of bargaining power among the players generally distorts efficiency. Suppose countries are ex-ante completely symmetric, i.e. $y_a = y_b = y$ and $\alpha_a = \alpha_b = \alpha$. However, for some exogenous reason, their contributions to the joint budget differ such that $x_a > x_b$. The efficient allocation of the joint funds would be $g_a = g_b = g$, regardless of the difference in contributions. When bargaining, though, nation a's opportunity cost of setting up the union is higher (more forgone consumption). Then, were they to agree on the efficient allocation, nation a's surplus from the union would be lower:

$$S_a = u(y - x_a) + v((1+\alpha)g) - u(y) < u(y - x_b) + v((1+\alpha)g) - u(y) = S_b \tag{25}$$

Thus, country a's bargaining position is stronger, and it is able to skew the allocation toward its own preferred project g_a. The Nash bargaining solution that satisfies (24) would be such that $g_a > g_b$ and thus inefficiently allocate too much to g_a.

4.2 Joint Bargaining over Funds and Allocation

In section 4.1 we illustrated that the distribution of bargaining power is endogenous to the distribution of contributions and so bargaining distorts efficiency. When countries bargain at the same time over contributions, a second determinant of the bargaining positions is endogenous as well. We turn to this more general case now.

The allocation that solves the general Nash bargaining problem (17) through (19) for two countries is characterized by the following necessary conditions for optimality:

$$\frac{u_a'}{u_b'} = \frac{S_a}{S_b} \tag{26}$$

$$\frac{v_a'}{v_b'} = \frac{(1-\alpha_a)}{(1-\alpha_b)}\frac{S_a}{S_b} \tag{27}$$

$$g_a + g_b = X \tag{28}$$

$$x_a + x_b = X. \tag{29}$$

We first discuss two special cases when the Nash bargaining solution does achieve general efficiency. Their existence is remarkable, because they depict conditions under which a union of countries achieves an efficient allocation simply through unstructured bargaining. That is, under some conditions, simply sitting in a room and negotiating a compromise works at least as well as any other more structured institutional setup could.

Proposition 1 (Symmetry implies efficiency)
If countries are perfectly symmetric in domestic income and the spillovers they receive from each other's projects, i.e. $y_a = y_b$ and $\alpha_a = \alpha_b$, then, for any budget X, the Nash bargaining solution coincides with the generally efficient outcome.

Proof: See Appendix C.1. □

Since all countries have the same endowment, their opportunity costs of contributing to the joint budget are the same. Moreover, symmetric spillovers do not give one country a higher incentive to participate in the union than the other. Consequently, both countries have the exact same bargaining position. Thus, an equilibrium in the bargaining game must produce equal surpluses S_i for the two nations. At this particular point, the efficient allocation, too, produces the same surplus S_i for each nation, so that it coincides with the Nash bargaining solution. It is important to notice, though, that symmetric income and spillovers do not imply homogeneous preferences: Each nation still prefers its "own" project over the others. It is symmetry that leads to a perfectly uniform distribution of bargaining power in equilibrium (equal surpluses). Consequently, nations will agree on perfectly equal contributions and project funds which is also the efficient outcome.

Proposition 2 (Quasi-linearity implies efficiency)
If preferences are quasi-linear in domestic consumption, i.e. $U_i = c_i + v(g_i + \alpha_j g_j)$, then, for any budget X, the Nash bargaining solution coincides with one generally efficient outcome.

Proof: See Appendix C.2. □

Quasi-linear preferences reduce the effect of opportunity costs of funds on the bargaining position of each player to a simple linear relationship (since one unit of domestic consumption is valued the same at any income level). Effectively, bargaining simply sets the allocation that maximizes the total surplus and then contributions such that the utility surplus is split equally. Allocative efficiency is defined as maximizing the total return from the union projects, so the Nash bargaining solution in this case is *allocative* efficient. Moreover, in case of quasi-linear preferences, *budgetary* efficiency ($u'_a = u'_b$) is met regardless of the domestic consumption allocation. There is one additional degree of freedom, any allocation that achieves *allocative* efficiency is also *generally* efficient. Thus, the Nash bargaining solution is in the set of generally efficient allocations.

The case of quasi-linear preferences has the following standard interpretation: A quasi-linear utility function can be used to approximate an underlying, strictly concave, utility function when spending on a single good is small relative to overall consumption. Proposition 2 requires the additional condition that the slopes of the utility function over consumption are equal, since budgetary efficiency can only be achieved if the countries have the same marginal utilities of income. Arguably, this is an appropriate assumption if the countries in the union have similar income levels and are therefore at the same point on the underlying, strictly concave, utility function. Combined, the above implies the following interpretation of Proposition 2: If union spending is small relative to domestic consumption *and* the union is composed of countries with homogeneous income levels, then Nash bargaining will give efficiency.

Before analyzing the properties of the Nash bargaining solution more generally, we address corner solutions. If, for example, spillovers are very asymmetric, it may happen that it is efficient to fund only one of the projects. Equivalently, very asymmetric domestic incomes may call for the union activities being funded by the richest country exclusively. It turns out that the Nash bargaining solution can, in some cases, achieve these efficient corners[13] as well, even though neither conditions of Propositions 1 or 2 are satisfied.

Lemma 1 (Corners)
There exist Nash bargaining corner solutions that are budgetary and/or allocative efficient.

Proof: See appendix C.3. □

The lemma states that there can also be efficient "double corners" where the complete budget is provided by only one country *and* allocated to only one project. We do not consider this case to be particularly relevant or interesting,[14] and therefore

[13]It should be noted again, though, that efficient corners only arise because contributions and projects are restricted to be non-negative. At an efficient corner solution, marginal utilities are not equalized, as described in the efficiency definitions 1 and 2.

[14]There are two scenarios that constitute a "double corner": The first is when the same country

exclude it from the subsequent analysis.

Suppose from now on that $u(\cdot)$ is strictly concave.

Proposition 3 (Inefficiency from bargaining)
Generically, the generally efficient allocation cannot be supported as a Nash bargaining solution.

Proof: See appendix C.4. □

Efficiency requires the allocation of available resources to the highest return technologies. Ideally, marginal returns of all union projects are equalized and funds raised where it is least costly, without regard of each country's individual gain. When bargaining, however, each individual country is maximizing its own surplus only. How well it is able to push for its preferred allocation depends on its relative bargaining position toward the other players. The efficient allocation of funds, on the other hand, almost never implies a ratio of surpluses consistent with the distribution of bargaining power, because it doesn't take into account how differences in income or the spillover effects of projects benefit one country more than another.

Given the efficiency results derived above, the outcome of an unstructured bargaining process may sometimes not be very far from efficient. The purpose of the remainder of this section is to understand under which circumstances the inefficiencies are severe and which of the efficiency margins is typically distorted.

Suppose parameters are such that the efficient allocation is not a corner solution.

Corollary 1 (Both efficiency margins are distorted)
When the Nash bargaining outcome does not coincide with the generally efficient allocation, it distorts both budgetary and allocative efficiency.

Proof: See appendix C.5. □

In the commonly relevant case where the asymmetry in terms of income and spillovers of the proposed projects is not extreme, both margins of efficiency are distorted. This complicates the direct measurement of inefficiency and thus the comparison of different scenarios. In fact, the Nash bargaining solution is not monotone with respect to changes in asymmetry in either income or spillovers.

To nonetheless gain some intuition about the order of magnitude of inefficiencies, we

contributes *and* receives the complete budget. This case maps to a classic public good problem with positive supranational spillovers, union dynamics do not play a role. The second scenario has one country contribute the complete budget and another country receive all the funds, which intuitively resembles foreign aid or foreign direct investment. Both are not relevant in the context of a supranational union of sovereign countries that is the focus of our paper. There are however institutional examples where some countries contribute, but do not receive funding (e.g. the EU cohesion funds), and where some countries receive funding, but do not contribute (e.g. the World Bank). Such "single corners" are not excluded from our analysis, unless otherwise noted.

numerically explore different scenarios of parameter combinations for a simple example with log-preferences. Consider the case of two countries, $i = a, b$, and suppose their preferences over domestic consumption and union projects are of the following form:

$$U_i = log(c_i) + log(g_i + \alpha_j g_j) \quad \text{for } i = a, b$$

First, suppose spillovers are symmetric, i.e. $\alpha_a = \alpha_b$. Keeping aggregate income constant, we vary asymmetry in domestic incomes. Figure 1 shows the Nash bargaining outcome compared to the generally efficient solution. As country a's income increases, so does its outside option and thus its bargaining position relative to country b in equilibrium, leading to an inefficient outcome. The allocation of funds to the union projects (upper right panel) depicts this channel very clearly: While the efficient allocation is independent from the distribution of national incomes, the Nash bargaining solution reflects the changing distribution of power. Nation a is able to tilt the allocation more toward its own preferred project the higher its income. Moreover, it is able to negotiate a "discount" for its contribution. While x_a increases with y_a, country a pays less than would be budgetary efficient given its higher income. As a result of the inefficiencies introduced by the bargaining process, aggregate utility in the union declines as asymmetry grows. Notice, however, that the loss in aggregate utility is relatively small when asymmetry is small, but grows more than proportionally as the countries become more and more unequal.

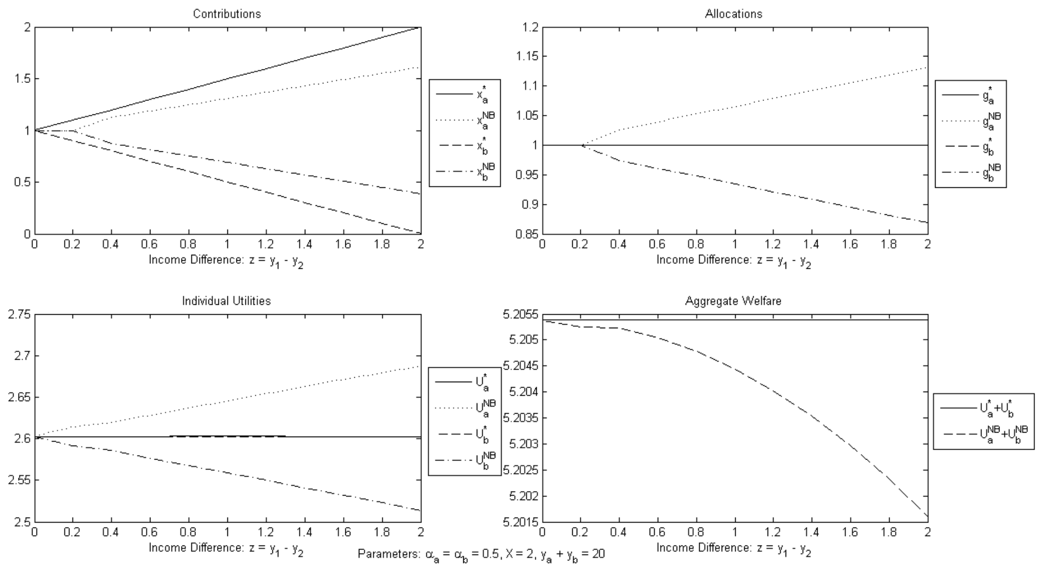

Figure 1: Asymmetry in income

Next, we present a similar experiment, keeping symmetric endowments, but varying the spillover effects of the projects. The change in technology has a direct impact on aggregate utility at the efficient solution. For a meaningful comparison between the efficient and the Nash bargaining solution, we vary spillovers such that aggregate utility remains constant.[15] Figure 2 shows similar results as before: Here, as

[15]Appendix C.7 explains the setup of this exercise in more detail.

α_b increases, funds should efficiently be re-allocated toward project g_b, while contributions should remain unchanged. At this efficient allocation, however, nation a's surplus would be smaller than nation b's,[16] leading to a increase in bargaining power of nation a relative to nation b. Consequently, the outcome of the bargaining process is again skewed in a's favor. The contributions of country a decrease away from the efficient level and at the same time the allocation of union funds tilts toward a's preferred project $g_a > g_a^*$. Again, the loss in aggregate welfare is small initially, but increasing as asymmetry grows.

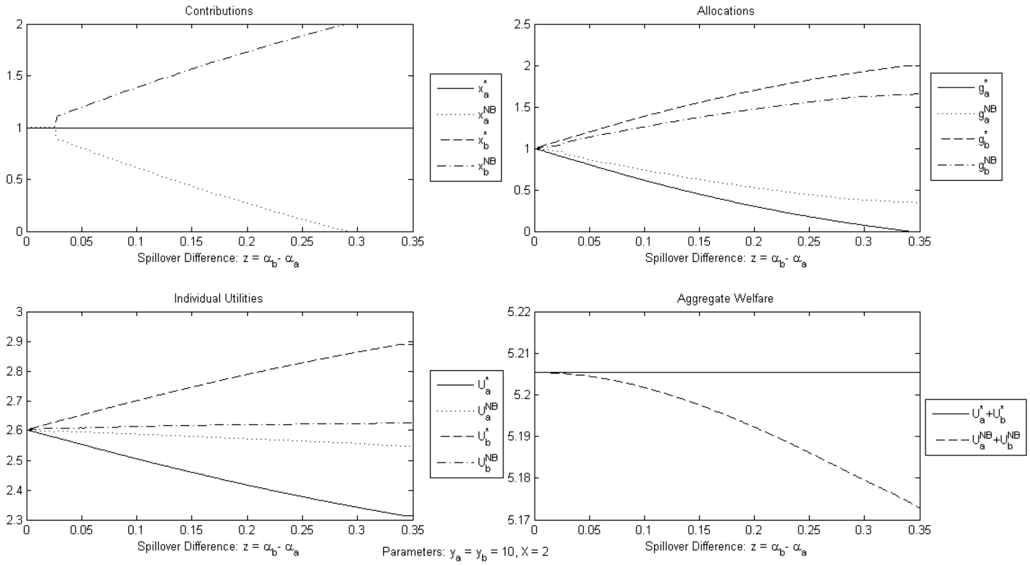

Figure 2: Asymmetry in spillovers

Finally, we compare two scenarios with asymmetry in both incomes and spillover effects. Again, as in the first experiment, we vary income inequality, while keeping aggregate income unchanged. However, here in the left column there is a negative correlation between income and spillovers, while in the right column, income and spillover effects are positively correlated.

Figure 3 shows that the allocation of the union budget to the projects is more efficient when income and spillovers are positively correlated. This is intuitive: The nation with the increase in income increases its bargaining power and is therefore able to tilt the distribution of funds toward its preferred project. When income and spillovers are positively correlated, this happens to be the efficient project. In the EU, the more relevant case, however, is that of a negative correlation, where the highest return projects lie in the poorest countries. The example thus highlights why the structural funds are allocated to seemingly inefficient projects in relatively rich member states.

[16] This follows since at the efficient allocation $v'_a(1-\alpha_b) = v'_b(1-\alpha_a)$, so that with $\alpha_a < \alpha_b$ we get $v_a < v_b$.

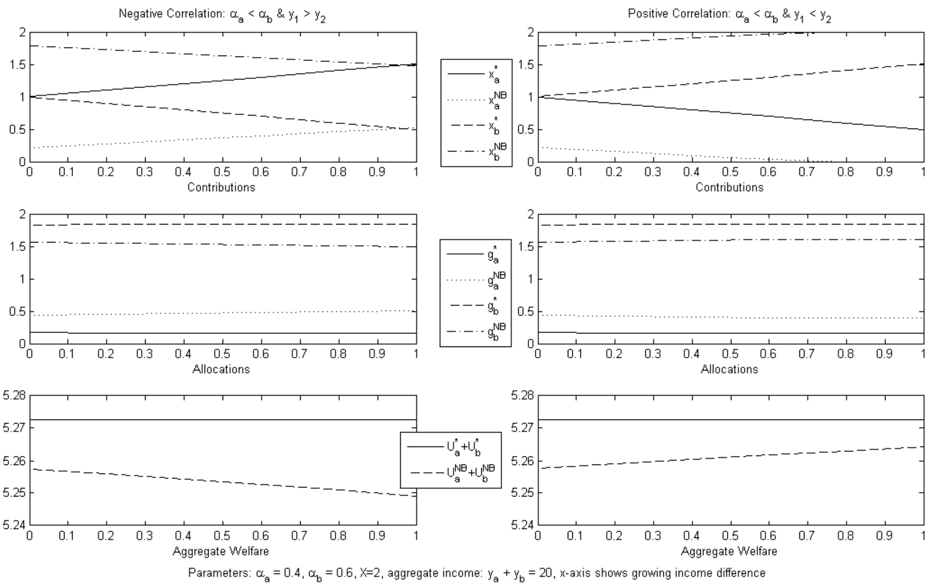

Figure 3: Asymmetry in spillovers

Generally, the overall distribution of bargaining power and resulting allocation in the Nash bargaining equilibrium is non-monotone in measures of asymmetry. It may in fact even happen that growing asymmetry has a positive effect on aggregate utility, as the lower right panel of figure 3 shows.

4.3 More than two countries

The Nash bargaining solution with $n > 2$ is characterized by:

$$\frac{u'_i}{u'_j} = \frac{S_i}{S_j} \qquad \forall i, j \tag{30}$$

$$v'_i(1-\alpha_j) = v'_j(1-\alpha_i)\frac{S_i}{S_j} - (\alpha_i - \alpha_j)\sum_{k \neq i,j} v'_k \frac{S_i}{S_k} \qquad \forall i, j \tag{31}$$

$$\sum_{i=1} g_i = X \tag{32}$$

$$\sum_{i=1} x_i = X \tag{33}$$

where again $S_i = u(y_i - x_i) + v_i - u(y_i)$ denotes the surplus generated for country $i = 1, ..., n$ respectively.

Note that equation (30) remains the same as in the two-country case (compare to equation (26)), implying that the relative split of utility surplus between country i and country j is still determined by the ratio of their marginal utilities from domestic consumption. Equation (31), which defines the relative allocation of funds between countries i and j, however, has the additional term $[(\alpha_i - \alpha_j)\sum_{k \neq i,j} v'_k \frac{S_i}{S_k}]$. This shows that the relative allocation of funds reflects the impact of country i's spillovers on all other countries, not just country j.

From these conditions it is clear that the results of Propositions 1 and 2 extend to the case of more than two countries, since efficiency is obtained under Nash bargaining when $S_i = S_j$ for all j, i.[17]

Expanding the analysis to a general number of countries allows us to study the impact of the size of the union on efficiency. First, we define a framework which allows us to formally characterize a "larger" union. Take Z to be an infinite ordered sequence of countries: $Z = \{(\alpha_1, x_1, y_1), (\alpha_2, x_2, y_2), ...\}$.

Definition 4 (Union Size)
A union of size n consists of a union of the first n countries in Z.

Then:

Proposition 4 (Efficient allocation in large unions)
For any sequence of countries, Z, there exists an N such that for any union of size $n \geq N$, unstructured bargaining yields ALLOCATIVE *efficiency.*

Proof: See Appendix C.6. □

Proposition 4 suggests that very large unions are better able to allocate funds and therefore closer to optimal. This argument, however, is driven by the fact that, in our model, there are no diminishing returns to spillovers. Therefore, the addition of a new country will always increase aggregate surplus. A more relevant way to characterize the result is that adding additional countries to the union increases the relative bargaining power of countries with high spillover projects. This effect is not dependent on the property of no diminishing returns.

This interpretation implies that, as the union grows, it will stop funding the most inefficient projects, i.e. those with the lowest level of spillovers. The result is supported by evidence from the European Union. After several rounds of expansion, the member countries have agreed on a modification of fiscal spending: A subset of the structural funds, the cohesion funds, can now only be spent in countries with a national income less than 90 percent of the median in the union. That is, allocation to relatively rich countries, where spending on transportation and the environment arguably has a smaller impact, is specifically excluded.

5 Endogenous Redistribution

In this section, we show that redistribution can arise endogenously in a union of sovereign nations who participate voluntarily - despite a lack of altruistic preferences, any uncertainty warranting an insurance mechanism, or a repeated game structure justifying mutual favors.

[17]See Appendix C for the formal proofs.

A social planner would naturally distribute resources from domestic consumption to the union projects without regard of each nation paying as much as it receives in funding to its preferred project. We show that without an overarching authority dictating the outcome, such resource redistribution is still sustainable. That a country might agree to be a net-contributor to the union budget is a simple consequence from the unstructured bargaining mechanism that links budget contributions to bargaining power. In fact, in the setup presented here, the Nash bargaining solution almost always redistributes resources between the members of the union:

Proposition 5 (Endogenous Redistribution)
Generically, $g_i \neq x_i$.

Proof: See Appendix D.1. □

The underlying cause for a nation to contribute more than it receives in resources is that a higher contribution, all else equal, improves its relative bargaining position. More forgone consumption decreases the surplus S_i, so that this nation would gain less from the setup of the union. Moreover, since the total budget X is fixed, an increase of one nation's contribution necessarily must lead to the decrease of another nation's x, which tilts the distribution of bargaining power even further. With more implicit bargaining power, a nation is able to tilt the allocation according to its own preferences, which might be especially lucrative if low spillovers of the own project render it otherwise inefficient.

Being a net-contributor, however, does not necessarily mean that the nation loses with respect to other union members. After all, the participation constraints ensure that every nation is better off in the union than under autarky. Rather than concentrating on resource redistribution, it is much more interesting to ask who gains the most under which circumstances. Again, what matters is a comparison of the surpluses S_i from joining the union. As we have seen before, comparative statics with respect to the underlying parameters are highly non-linear. To make some progress, we focus attention on the case where the participating nations have a negative correlation between income and spillovers of their most preferred projects, i.e. the highest return projects are located in the poor member countries of the union, as is arguably the case in the EU.

Proposition 6
If $y_a > y_b$ *and* $\alpha_a < \alpha_b$*, then* $S_a \leq S_b$.

Proof: See Appendix D.2. □.

The proposition states that the richer country always gains weakly less from the union than the poorer nation. Since the surplus for both countries must be positive, this is still not conclusive evidence for a change in inequality. Whether or not the difference between the nation's utility changes with respect to ex-ante inequality $u(y_a) - u(y_b)$, and in which direction, crucially depends on the size of the union budget X

Proposition 7 (Decreasing Inequality)
For any parameter combination such that $y_a > y_b$ and $\alpha_a < \alpha_b$, there exists a budget X such that $u(c_a) + v(g_a + \alpha_b g_b) - u(c_b) - v(g_b + \alpha_a g_a) < u(y_a) - u(y_b)$.

Proof: See Appendix D.3. □

This result shows that when a nation with high income but a low spillover project negotiates to form a union with a low-income but high-spillover nation, inequality of utility between the nations can be guaranteed to drop by choosing the right budget X. We have taken X as exogenously given because of the redistributive motive inherent in its choice.[18] From an ex-ante point of view, the choice of a total budget can be interpreted as the stated intent of the union members is to achieve a certain level of equality while still making every member better off than under autarky. The proof of Proposition 7 only states that there is always a budget small enough to decrease inequality. How much potential for reducing inequality through the union there is depends on parameters. Moreover, while increasing the budget X away from zero will initially decrease inequality between the nations, it also has a down-side: In the unstructured bargaining process, redistributive contributions influence the distribution of bargaining power, which has a crucial impact on the efficiency of allocating the funds to the union projects.

Proposition 8 (Equality-Efficiency Trade-off)
If $y_a > y_b$ and $\alpha_a < \alpha_b$, all else equal, increasing the net-contribution of nation a will result in a less efficient allocation of the joint funds.

Proof: See Appendix D.4. □

While letting the richer nation contribute more to the joint budget may seem desirable to achieve less inequality, it will give that country more bargaining power to tilt the allocation of funds to its own preferred, but relatively less valuable, project. Therefore, while a budget can be picked to alleviate the utility differences between the rich and poor country, this comes at the cost of an increasingly inefficient allocation of funds. In the bargaining process, since bargaining power is linked to contributions, redistribution arise endogenously. This, however, causes inefficient allocations at the same time. Therefore, when picking the total budget, policy makers must trade-off welfare gains from increased equality against efficiency losses.

Inefficiency occurs along two dimensions: budgetary contributions and allocation of funds. We present a numerical example to illustrate how initial income inequality determines which of these margins is affected most in the equity-efficiency trade-off. Figure 4 shows two scenarios in which a high income nation a and a low income nation b bargain over the union allocation. In both cases there is a negative correlation between income and spillovers; the right column shows a higher income asymmetry between the two countries.

We plot ex-ante vs. ex-post inequality in the first row panels. Here we see that a larger budget X leads to more equality. The two rows below show the simulta-

[18] See Appendix A.5 for a discussion of our results when X is chosen endogenously as well.

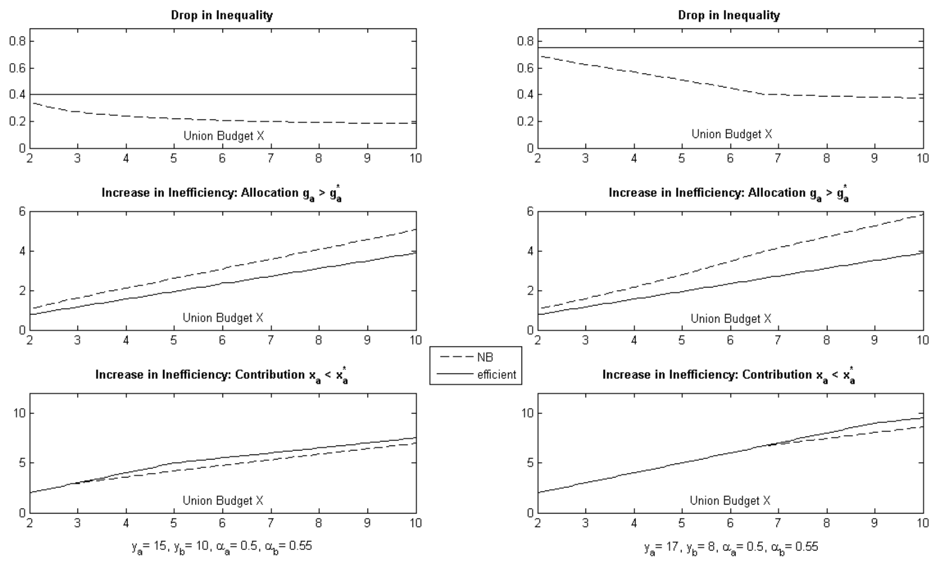

Figure 4: Ex-ante vs. ex-post inequality for varying budgets

neous increase in inefficiency: At higher levels of equality, more is allocated to the relatively inefficient project g_a, and nations a's contribution decreases further away from the budgetary efficient one. Consequently, the overall increase in equality comes at the cost of a decrease in aggregate welfare - this is the equality-efficiency trade-off.

Comparing the left and right column reveals that higher ex-ante inequality leads to more severe *allocative* inefficiency, but less *budgetary* inefficiency. Because preferences are concave, the high-income nation finds it cheaper to agree to a higher contribution and uses its higher implicit bargaining power to tilt the allocation of funds further to its own preferred project.

6 Alternative Institutions

While unstructured bargaining and unanimity remains the norm for current EU fiscal programs, there is no exogenous constraint that prevents unions from employing alternative institutions. In this section we detail when alternate institutions outperform Nash bargaining. In particular, we examine tax rules that determine contributions exogenously, and majority decision rules over the allocation of the budget. Again, we concentrate on the case in which spillovers and income are negatively correlated. The efficient solution in this case prescribes taxing the rich and spending funds in poor nations. As we saw in the previous section, the positive link between contributions and bargaining power allows for some resource redistribution from high to low income nations, but at the same time prevents the fully efficient use of the budget, so that there is potential room for improvement using instruments with a clear redistributive motive.

We would like to note that Nash bargaining is an "absorbing state," in the sense that it is Pareto efficient and member nations will therefore never unanimously approve a *switch* to an alternative institution.[19] Therefore, the discussion of alternative institutions is particularly relevant when considering new institutions that increase fiscal spending at the union level, such as the proposed growth pact. That is, if unstructured bargaining is used initially, even though a majority rule is more efficient and implementable relative to the status quo, the opportunity to adopt a more efficient institution is lost.

6.1 Exogenous Tax Rules

In this subsection, we consider the use of an exogenous tax rule to fix contributions, given unstructured bargaining in the allocation stage. For example, countries may restrict contributions to be a proportional share of national income (a linear tax). This approach is currently approximated by the EU, where national contributions to the general budget are roughly in proportion to GDP. To formally analyze the possibility of using fixed taxes to increase efficiency, we explore the set of constrained contributions which improve upon the Nash bargaining solution.

Specifically, we consider the problem of a hypothetical social planner who sets contributions x_a and x_b, subject to the constraint that countries bargain over allocations in the second stage. To facilitate the comparison between unstructured bargaining and constrained contributions, we label the Nash bargaining contributions x_i^{NB} and the constrained contributions x_i^c. Additionally, for this subsection we define g_i as a function of x_i, where $g_i(x_i)$ is the bargaining solution in the allocation stage (since $x_b = X - x_a$ we reduce the problem to considering x_a^c only).

Formally, the constrained social planner's problem is:

$$\max_{\{x_a^c, x_b^c\}} S_a + S_b \tag{34}$$

$$s.t. \quad X = x_a + x_b \tag{35}$$

$$\frac{v_a'}{v_b'} = \frac{(1-\alpha_a)}{(1-\alpha_b)} \frac{S_a}{S_b} \tag{36}$$

This maximization problem need not be concave, since $g_a(x_a)$ is not generally "well-behaved." Since $g_a(x_a)$ is continuous and smooth, however, we can still use the first order conditions to characterize local maxima.

The first order conditions give us the following condition:

$$u_a' - u_b' = [(1-\alpha_b)v_a' - (1-\alpha_a)v_b'] \frac{\partial g_a}{\partial x_a} \tag{37}$$

[19] Such switches are possible in the case of significant ex ante uncertainty (see Maggi and Morelli (2006)). In the case of international unions, however, uncertainty over medium-term relative income and project spillovers is likely quite low. This might be an explanation for why unanimity rule persists in the decision to allocate EU structural funds, even when many other decisions have transitioned to Ordinary Legislative Procedure (a qualified majority rule).

which states that the marginal cost of distorting budgetary efficiency is equal to the marginal benefit of moving toward the optimal allocation (or vice versa) at any maximum.

In comparison, note that Nash bargaining equates the ratios of the marginal returns:

$$\frac{u'_a}{u'_b} = \frac{(1-\alpha_b)v'_a}{(1-\alpha_a)v'_b} \tag{38}$$

Since equation 37 does not follow from 38, the Nash bargaining solution will not be generically equal to the constrained efficient solution.

Note that the Nash bargaining solution can only distort the efficiency margins in one of two ways: either $x_a^{NB} < x_a^*$ and $g_a^{NB} > g_a^*$, or $x_a^{NB} > x_a^*$ and $g_a^{NB} < g_a^*$. Since $\frac{\partial g_a}{\partial x_a}$ is positive, as x_a moves away from x_a^{NB} one efficiency margin will move towards parity, while the other will move away from parity. That is, when setting contributions, the social planner will choose to *either* improve allocative efficiency *or* improve budgetary efficiency, since improving one efficiency margin necessarily denigrates the other.

The following proposition characterizes which efficiency margin the social planner would prioritize at the Nash bargaining solution.

Proposition 9
At the Nash bargaining solution (x^{NB}) the distortion of budgetary efficiency outweighs the distortion of allocative efficiency. That is,

$$|u'_a - u'_b| \geq |(1-\alpha_b)v'_a - (1-\alpha_a)v'_b|\frac{\partial g_a}{\partial x_a}$$

Proof: See Appendix E.1. □

Proposition 9 allows us to characterize which motivation a social planner would prioritize at the Nash bargaining solution: raising funds efficiently or allocating the funds efficiently. Somewhat surprisingly, the distortion of the budget margin always dominates. That is, even in the case where incomes are comparable and the asymmetry in project spillovers is high, the distortion on the budget margin is higher than the distortion on the allocation margin.

Additionally, if the maximization problem is concave, a stronger result obtains. Take A to be the set of x_a^c that increase aggregate utility over the Nash bargaining solution; $A = \{x_a^c : S_a(x_a^c) + S_b(x_b^c) > S_a(x_a^{NB}) + S_b(x_b^{NB})\}$. If the maximization problem is concave, then A is an interval and x_a^{NB} is equal to either the sup or inf of A. Therefore, the location of the set A can be characterized by considering the marginal effect of changing x_a^c on aggregate utility at the Nash bargaining solution (x_a^{NB}). In this case, Proposition 9 implies that, relative to the Nash bargaining solution, the social planner will only set contributions to improve the distortion of the budget margin.

This implies that instituting measures designed to increase efficiency on the contributions margin, despite increasing the distortion of the allocation margin, is always weakly optimal.

6.2 Majority Rule and Legislative Bargaining

In this subsection, we examine the implementation of a majority rule and the allocative implications of legislative bargaining. We use a simple model of legislative bargaining similar to Harstad (2005): A formateur first forms a minimum winning coalition of q nations, where q is the number of nations needed to pass legislation, and then the coalition bargains over the allocation of the budget. Since the results we present hold independent of the identity of the formateur, we remain agnostic about the specific procedure used to select the formateur. We assume contributions are determined exogenously.[20]

When the required size of a winning coalition is equal to n, then this setup is identical to the Nash bargaining model. When the required size of a winning coalition is smaller than n, the legislative bargaining model differs from the previous analysis in two respects: One, the majority coalition bargains over the allocation of both their contributions and the contributions of the minority, and second, the composition of the set of nations that bargain over the utility surplus is endogenous. Unless stated otherwise, we assume a simple majority rule.

As illustrated above, the inefficiency of Nash bargaining stems from the link between contributions and allocations. Majority rule can break this link, at least for the countries in the minority. Therefore, majority rule can improve efficiency, but only if the "right" countries are endogenously chosen to form the majority. The formateur, however, will chose the majority coalition which maximizes their utility, with no regard for efficiency. Therefore, we first identify when the formateur will select countries with high spillover projects to the majority coalition.

Lemma 2 (Legislative Bargaining)
Take i, j with $\alpha_i > \alpha_j$. All else equal, the formateur prefers the majority coalition including i to the majority coalition including j if and only if $x_j - x_i > \underline{x}$.

Proof: See Appendix E.2. □

By Lemma 2 it is clear that the formateur will not generally choose the majority coalition that consists of the countries with the highest spillover projects. However, Lemma 2 does suggest that the formateur might prefer a majority coalition which contains high-spillover countries in the empirically relevant case where spillovers and contributions are *negatively* correlated (i.e. when the high spillover nations have low outside options). To explore this possibility further, we consider the following ex-

[20] The results are similar if contributions are determined by majority rule. However, in this case the minority is fully expropriated (they receive their outside option). Given that countries have perfect foresight as to who will be selected to the majority coalition, it is unlikely that majority rule over both contributions and allocations is implementable.

ample:

Example: Assume nations are one of two types $\{y_l, \alpha_l\}$ or $\{y_h, \alpha_h\}$, with $y_l < y_h$ and $\alpha_l > \alpha_h$. n_l nations have low incomes and n_h nations have high income. In this setting, Lemma 2 translates to the following result, which specifies when the high spillover countries will be chosen to the majority coalition:

Result 1
If X is greater than some $\underline{X}$ and smaller than some $\bar{X}$, then the set of $\{x_l, x_h\}$ for which the formateur will choose type l nations for the majority coalition is non-empty.

Proof: See Appendix E.3. □

Result 1 allows us to specify some simple situations when majority rule is more efficient than Nash bargaining. If the budget and contributions at the Nash bargaining solution satisfy the conditions of result 1, $n_l = N/2$, and efficiency prescribes a corner solution (i.e. allocative efficiency implies no funding for g_h), then majority rule is more efficient than Nash bargaining.

For a more concrete example, consider a union of three nations, two with $y_l = 100$ and $\alpha_l = 0.35$ and one with $y_h = 130$ and $\alpha_h = 0.3$. Additionally, $X = 10$, $x_l = 0$, $x_h = 10$ and $v(z) = u(z) = z^{\frac{1}{2}}$. In this example, a formateur of type l receives higher utility with the majority coalition $\{l, l\}$ than the coalition $\{l, h\}$. Additionally, legislative bargaining gives higher aggregate utility than Nash bargaining both when contributions are fixed at $x_l = 0$, $x_h = 10$, and when contributions are set by Nash bargaining.

By construction, these examples give the following proposition:

Proposition 10
The Nash bargaining solution is not generically more efficient than Majority Rule.

Proposition 10 illustrates that there are non-trivial subsets of the parameter space where adopting a majority rule can improve efficiency. We cannot, however, claim that legislative bargaining is more efficient than Nash bargaining in any general sense.

The reason for this is twofold: First, a necessary condition for legislative bargaining to be efficient is that the highest spillover nations are chosen to the majority coalition. As Lemma 2 and Result 1 demonstrate, this is not always the case, and will fail if high spillover countries also supply a relatively large proportion of the union budget. Second, since legislative bargaining allocates no funding to countries outside of the majority coalition, it must also be the case that it is relatively efficient to not fund the low spillover projects. While this is not true generally, as illustrated in the discussion of union size (Proposition 4), it will be true for large unions.

7 Discussion

In this paper, we have analyzed whether a union of sovereign nations can efficiently raise a centralized budget, and then efficiently allocate that budget over a set of joint public-good projects. A key element in our analysis is that nations retain the outside option of exiting the union and consuming their contribution to the budget. This leads to the distribution of bargaining power being endogenous to individual contributions to the budget and produces a link between contributions to and allocation of a joint budget that was previously unexplored.

We find that through this channel unstructured bargaining generally prevents both budgetary and allocative efficiency. However, if the potential members of the union are homogeneous with respect to their income and the social usefulness of the projects they propose to be implemented in the union, then inefficiencies are small. Moreover, inefficiencies are also negligible if the union budget is small relative to domestic consumption and member countries have similar incomes. As the asymmetries between member countries, or the importance of the union relative to domestic consumption grows, Nash bargaining leads to increasingly inefficient outcomes.

A dividing argument in the current debate over expanding fiscal spending at the union level in the EU centers around the redistributive nature of such activity. We do find that redistribution is sustainable even though the bargaining nations are purely self-interested. It is precisely because of the link between contributions and bargaining power that countries may agree to be net-contributors to the union budget. The level of redistribution inherent in the Nash bargaining solution depends crucially on the overall size of the budget the union intends to raise. We show that when there is a negative correlation between national income and spillovers from a country's preferred project (a scenario that fits most closely with EU reality) a budget can be picked such that inequality in terms of total utility between member nations is decreased compared to autarky. However, such an outcome necessarily implies that the net-gain from joining the union for high-income nations is lower (albeit still positive) than for low-income members. This in turn has consequences for the endogenous distribution of bargaining power: Richer nations are able to assert more power and receive an inefficient amount of funding for their own preferred projects. This trade-off between equality and efficiency implies that full redistribution is not necessarily socially desirable.

Currently, it is EU practice to raise the joint budget by a linear tax on national incomes. We find that such an exogenous tax rule may indeed improve overall efficiency when compared to a fully unstructured bargaining process. Specifically, such a rule improves efficiency on the contribution margin, forcing wealthier members to pay more. At the same time, however, when countries with high incomes are also the ones proposing the projects with the lowest social returns, this will necessarily decrease the efficiency of the allocation of funds. We show that the efficiency-equity trade-off inherent in the unstructured bargaining outcome will always distort the equity (budget) margin relatively more than the efficiency (allocation) margin. Therefore, an exogenous tax rule that shifts the contributions away from the bargaining outcome towards the efficient contributions will improve overall efficiency.

As the EU has expanded from the original six nations to the current 27, there has been a concurrent evolution of decision making rules. Specifically, the EU has transitioned from unanimity to a qualified majority rule in many areas of competency. We show that a majority rule can be welfare improving as long as the nations with high spillover projects are endogenously selected into the majority coalition. For the main fiscal activities of the EU, the CAP and structural funds, our analysis suggests that a majority decision rule could improve aggregate welfare as long the contributions to the budget by low income members are also relatively low.

The results of our model are remarkably suggestive of the reality of fiscal spending mechanisms of the European Union today. The explored link between contributing to the union budget and the resulting allocation through the endogenous distribution of bargaining power offers a new explanation as to why unstructured bargaining generally leads to inefficient outcomes and why redistribution in a union is sustainable even though participation is voluntary. Because of the simplicity of the model, we are able to highlight the trade-offs involved when employing an unstructured bargaining process between self-interested sovereign nations, and to scout out scenarios under which more structured institutions are likely to improve welfare.

Our setup allows another interesting route of research: studying the effect of centralized fiscal budgets on the incentives for union enlargement. Given a current set of member states and their composition in terms of income asymmetries, who would benefit most from adding an additional member? What are the effects of adding for example a small country to a union of otherwise fairly homogeneous members? Do certain institutional rules of decision making leave more room for strategic enlargements, and how can such rules be designed to share the generated surplus efficiently? The results from this ongoing work will provide further insights into current policy debates in the EU and other supranational unions.

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Appendix

A Discussion of the Efficiency Benchmark

In this appendix, we discuss our choice of the efficiency benchmark in more detail. A few of its characteristics have been chosen to ease exposition, possibly at the expense of direct applicability to reality. We discuss various assumptions here to convince the reader that our results hold also when we relax them. We suggest to read sections 4 and 5 before this appendix, as we refer to results derived there.

A.1 Maximizing the Sum of Utilities

Efficiency can only be defined with regard to achieving a certain objective. The choice of objective function technically is somewhat arbitrary. We chose our benchmarks to resonates with the EU's official goals regarding fiscal spending. In that, we specifically excluded a direct redistributive objective derived from a utilitarian aggregate welfare criterion. To achieve that, we take the total budget X as exogenously given. In fact, the underlying problem we derive the two efficiency benchmarks from differs from that of a utilitarian social planner *only* in that X is fixed: The two problems (5) through (8) and (11) through (13) can be combined to the following general problem:

$$\max_{\{c_i,g_i\}_{i=1,\dots,n}} \sum_{i=1}^{n}[u(c_i) + v(g_i + \sum_{j\neq i}\alpha_j g_j)] \tag{39}$$

$$s.t. \quad c_i + x_i \leq y_i \quad \forall i \tag{40}$$

$$\sum_i x_i = X \tag{41}$$

$$\sum_i g_i \leq X \tag{42}$$

$$x_i \geq 0 \quad \forall i \tag{43}$$

$$g_i \geq 0 \quad \forall i \tag{44}$$

From this problem, we derive our efficiency benchmarks *budgetary efficiency* (Definition 1) and *allocative efficiency* (Definition 2). They describe necessary and sufficient conditions for an allocation to be a solution of this problem.

A.2 Alternative Pareto Weights

One may argue that equal weights on the utility of each country are not the right benchmark. For example, weighing each nation by its population could be an alternative. Once one accepts the sum of utilities as a suitable metric for efficiency, picking a set of Pareto weights is arbitrary (and equal weights save notation).

One could change the objective function (39) to include general Pareto weights ϕ_i:

$$\max_{\{c_i,g_i\}_{i=1,\dots,n}} \sum_{i=1}^{n}\phi_i[u(c_i) + v(g_i + \sum_{j\neq i}\alpha_j g_j)], \tag{45}$$

where $\sum_i \phi_i = 1$. Then, the efficiency benchmarks would include ratios of the Pareto weights, so that equation (9) became

$$\phi_i u_i' = \phi_j u_j' \quad \forall i,j, \tag{46}$$

and equation (14) would change to

$$\phi_i v_i' + \alpha_i \sum_{j\neq i}\phi_j v_j' = \phi_j v_j' + \alpha_j \sum_{i\neq j}\phi_i v_i'. \tag{47}$$

Comparing these modified benchmarks to the Nash bargaining solution, specifically equations (30) and (31), reveals that for any Nash bargaining solution there exists a set of Pareto weights such that bargaining yields efficiency. However, the "right" Pareto weights would have to value nations according to both their income *and* their spillovers exactly such that they coincide with the implicit distribution of bargaining power. While Pareto weights that reflect income differences may be justifiable, there is no case for spillovers to play a role.

A.3 Binding Participation Constraints

Since we are analyzing a setup without a social planner who can dictate an allocation, nations will only join a union if it is *individually rational* (IR) for them to do so. They always have the outside option of simply consuming their endowment. Imposing these constraints

$$u(c_i) + v(g_i + \sum_{j \neq i} \alpha_j g_j) \geq u(y_i) \quad \forall i, \tag{48}$$

in the problem above, one might argue, yields a more relevant efficiency benchmark. With this additional set of constraints, it is not anymore true that the two problems (5) through (8) and (11) through (13) can be combined to the general problem (39) through (44). We show in this subsection why this is the case.

Given this complication, the efficiency benchmarks would have to be derived from the general problem (39) through (44) with the addition of the set of IR constraints. This, as we will see below, would simply yield an additional condition in each definition of efficiency benchmarks for the case that a participation constraint binds. We consider it unnecessary to keep track of the additional special case. If a participation constraint would bind at the efficient solution, the modified efficiency benchmark would change to *just* accommodate that. The Nash bargaining process, however, generally leaves all countries strictly better off, so that none of the IR constraints binds, so the Nash bargaining solution would be inefficient beyond what is called for by the binding IR constraint - all our results remain unchanged. Only the distance to efficiency in our numerical solutions might be smaller whenever an IR constraint would be violated at the benchmark.

For the sake of completeness, we show here how even one binding IR constraint distorts both budgetary and allocative efficiency. The resulting allocation would generally be a trade-off between decreasing the union budget and a distorted allocation of funds so that individual rationality is guaranteed. This trade-off is not trivial, and crucially depends on the functional form of preferences. Therefore we judge it to be impractical as a benchmark.

Suppose only one participation constraint, that of country k, binds at the first-best allocation. The first order conditions to this problem then are:

$$u_i' = \lambda \quad \forall i \neq k \tag{49}$$
$$u_k'(1+\mu) = \lambda \tag{50}$$
$$v_i' + \alpha_i \sum_{j \neq i} v_j' = \lambda \quad \forall i \neq k \tag{51}$$
$$v_k'(1+\mu) + \alpha_k \sum_{j \neq k} v_j' = \lambda \tag{52}$$

Since only one IR constraint binds, only one associated Lagrange multiplier is non-zero and enters the optimality conditions. Notice though that even this one additional constraint is enough to distort budgetary efficiency:

$$u_k' = \frac{1}{1+\mu} u_i' \tag{53}$$
$$\tag{54}$$

as well as allocative efficiency:

$$v_k'(1+\mu) + \alpha_k \sum_{j \neq k} v_j' = v_i' + \alpha_i \sum_{j \neq i} v_j' \tag{55}$$

Here the one binding participation constraint distorts the allocation in order to satisfy individual rationality.

A.4 Binding No-transfers and Technology Constraints

Unlike the IR constraints, we do consider the no-transfer constraints (43) as well as the technological constraints (44) important when defining a practicable benchmark. They are the source for corners both in the efficient allocation and the Nash bargaining solution. It is important to note that a binding no-transfer constraint will only cause a corner in contributions, while it is the technology constraint $g_i \geq 0$ that causes corners in allocations. We demonstrate the former here, the later follows trivially.

The first-best allocation, denoted with superscript FB is a solution to the following planning problem:

$$\max_{\{c_i, g_i\}_{i=1,\ldots,n}} \sum_{i=1}^{n} u(c_i) + v(g_i + \sum_{j \neq i} \alpha_j g_j) \tag{56}$$
$$s.t. \quad c_i + x_i \leq y_i \quad \forall i \tag{57}$$
$$\sum_i g_i \leq \sum_i x_i \tag{58}$$
$$x_i \geq 0 \quad \forall i \tag{59}$$

The first order conditions with respect to domestic consumption, contributions to the union budget, and union projects are respectively:

$$u'(c_i) = \lambda_i \quad \forall i \tag{60}$$

$$\lambda_i = \mu \quad \forall i \tag{61}$$

$$v_i' + \alpha_i \sum_{j \neq i} v_j' = \mu \quad \forall i, j \tag{62}$$

where λ_i and μ are the Lagrange multipliers associated with the individual budget constraints (57) and the union wide budget constraint (58) respectively. However, these conditions describe only interior solutions, i.e. are met only when constraints (59) are slack. Suppose one of these constraints is violated at the fully efficient first-best allocation, i.e. $y_i < c^{FB}$ for one country i. There is no choice but to implement a corner solution with $x_i = 0$ and $c_i = y_i < c_j = c^{FB}$. Notice however, that it will not distort allocative efficiency as a result. How a given budget is allocated between projects is entirely prescribed by conditions (62), which do not depend on the individual Lagrange multipliers λ_i. Allocative efficiency as defined in Definition 2 is never distorted at the first-best allocation. Only budgetary efficiency (definition 1) is distorted such that

$$u_j' = v_j' + \alpha_j \sum_{k \neq j} v_k' = v_i' + \alpha_i \sum_{k \neq i} v_k' < u_i' \tag{63}$$

From this analysis it is obvious to see that without preference separability between domestic consumption and union consumption, conditions (62) will generally depend on all λ_i, so that also allocative efficiency would be distorted. Intuitively, the allocation might be such that it indirectly transfers utility toward poor countries, since direct transfers are not allowed. We abstract from this channel.

A.5 Endogenous Budget

We have taken the overall size of the union's budget X as pre-determined. Letting it be determined by the bargaining process transforms the problem (39) through (44) into a true utilitarian planning problem - the efficient allocation would at the same time be the socially efficient one; any other allocation would trade-off budgetary against allocative efficiency to maximize aggregate welfare. To achieve the optimal trade-off, there is an efficient union budget. Here we show how an efficiency benchmark including the budget choice would be defined, and how it relates to the benchmarks we have chosen. It should be clear from this subsection that all our results extend to this case.

Suppose that the no-transfer constraints (43) and the technological constraints (44) do not bind (the corner analysis follows directly from the previous subsection and is omitted here). Then the remaining two constraints can be merged into one general feasibility constraint. Denote the solution to maximizing objective (39) subject only to feasibility $\sum(c_i + g_i) \leq \sum y_i$ with an asterisk for reference in the following definition of social efficiency:

Definition 5 (Social Efficiency)
An allocation $\{c_i, g_i\}$ is called SOCIALLY EFFICIENT *if*

$$u_i' = v_i' + \alpha_i \sum_{j \neq i} v_j' \quad \forall i \text{ whenever } y_i < c_i^* \tag{64}$$

$$v_i' + \alpha_i \sum_{j \neq i} v_j' = v_j' + \alpha_j \sum_{i \neq j} v_i' \quad \forall i, j \tag{65}$$

$$u_i' = u_j' \quad \forall i, j \text{ whenever } y_k > c_k^* \text{ for } k = i, j \tag{66}$$

$$c_i = y_i \quad \forall i \text{ whenever } y_i \leq c_i^*.$$

Like the generally efficient allocation that serves our benchmark, the socially efficient allocation makes the best use of all technologies available. The significant addition is condition (64), which pins down the optimal size of the union budget: Marginal utilities should be equalized across domestic and union consumption. This allocation may entail a great deal of redistribution between countries - a consequence of the aggregate welfare objective.

Extending bargaining to the size of the budget and extending our efficiency benchmark to include the size of the budget does not change our results regarding the efficiency of the Nash bargaining solution. In fact, we can add the result that the Nash bargaining solution achieves also the efficient budget in the cases of symmetry and quasi-linear preferences, but not generically (proof are analogous to the corresponding ones with a fixed budget).

B Nash Bargaining

This appendix discusses in more detail the characteristics of the Nash bargaining problem in the context of our model.

B.1 Full Veto

We assume that each nation has a full veto over the set-up of the union. This maps most closely to our main motivating example, the EU. In the EU, each country has veto power over the budget. Moreover, at least in the fiscal spending programs active to date, each EU member can veto any specific allocation of funds.

With this assumption, we do exclude the possibility that a subset of countries form a different union. The disagreement point is for all nations to revert to the ex-ante state. For the EU at its current state, this is the most relevant setup. Arguably, a break-up of the EU is not a costless alternative. Thus, the set of players to decide over an expansion of fiscal spending at the union level is fixed to include *all* current members.

Technically, this assumption allows us to avoid using more complex coalition-based approaches, which are highly sensitive to assumptions regarding the counter-factual compositions of post-breakup coalitions. Notice, though, that even such an approach would retain the fundamental link between contributions and allocations that we analyze in this paper, so that our results qualitatively extend to situations not covered under the full veto assumption.

B.2 Nash Bargaining over Non-Convex Sets

The Nash bargaining Problem asks how individuals "who have the opportunity to collaborate for mutual benefit" (Nash Jr (1950)) divide the utility gained through collaboration. Nash Jr (1950) proved that there exists a unique solution to this problem that is: independent of the cardinality of the utility functions, gives a Pareto optimal outcome, symmetric, and independent of irrelevant alternatives. Moreover, he shows that the bargaining solution maximizes the product of the individual utility surpluses from cooperation: this division of utilities is know as the Nash bargaining Solution.

Nash's original proof of the Nash bargaining Solution was limited to bargaining over a convex set of utility outcomes, S. A set of papers extend the Nash bargaining (NB) Problem to non-convex sets by imposing alternative axioms (Herrero (1989), Conley and Wilkie (1996), Zhou (1997)). This appendix takes an alternative approach: we show that the NB solution holds on a relevant set of non-convex sets utility outcomes. Specifically, we weaken the convexity constraint in Nash's seminal paper to the following:

Convexity Constraint:
The convex hull of the set of Pareto outcomes is in S.

The following Theorem shows that Nash's standard bargaining solution holds on this more general set of utility outcomes.

Theorem 1
There exists a unique set of utilities, $\{u_1^*, u_2^*\}$, that satisfy the Nash bargaining axioms. Moreover, $\{u_1^*, u_2^*\}$ is the unique maximum of $u_1 u_2$.

Proof: Since S is compact there exists a $\{\hat{u_1}, \hat{u_2}\}$ in S that maximizes $u_1 u_2$. Without loss of generality, we renormalize $\{\hat{u_1}, \hat{u_2}\}$ to $\{1, 1\}$. In Nash's original setup, where S is convex, two results would trivially follow:
Claim 1: $\{\hat{u_1}, \hat{u_2}\}$ is the unique maximum of $u_1 u_2$ in S.
Claim 2: There does not exist $\{u_1', u_2'\}$ in S such that $u_1' + u_2' > 2$.
Here we prove that claim 1 and claim 2 still hold given our weakened convexity constraint. We start with claim 2.

Proof of claim 2: First, we show that $\{1, 1\}$ is in the set of Pareto outcomes, which we label P. By contradiction, if $\{1, 1\}$ is not in P, then there is a point in S, $\{\check{u}_1, \check{u}_2\}$ with $\check{u}_1, \check{u}_2 \geq 1$, and either $\check{u}_1 > 1$ or $\check{u}_2 > 1$. Since $\check{u}_1 \check{u}_2 > 1$, $\{1, 1\}$ does not maximize $u_1 u_2$.

Next, note that if there exists a $\{u_1', u_2'\}$ in S such that $u_1' + u_2' > 2$, then there also exists $\{u_1'', u_2''\}$ such that $u_1'' + u_2'' > 2$ and $\{u_1'', u_2''\} \in P$: if $\{u_1', u_2'\} \notin P$, then there exists some exists $\{u_1'', u_2''\} \in P$ such that $u_1'' + u_2'' > u_1' + u_2' > 2$. Lastly, since $u_1'' + u_2'' > 2$ there is a convex combination of $\{u_1'', u_2''\}$ and $\{1, 1\}$ such that $u_1 u_2 > 1$. And since $\{u_1'', u_2''\}$ and $\{1, 1\}$ are both in P, and the convex hull of P is in S, this convex combination is also in S, which contradicts the fact that $\{1, 1\}$ maximizes $u_1 u_2$.

Proof of claim 1: Assume $\{u_1', u_2'\}$ maximizes $u_1 u_2$ on S, i.e. $u_1' u_2' = 1$. Claim 2 shows that $u_1' + u_2' \leq 2$. Substituting in for u_2' gives $u_1' + \frac{1}{u_1'} \leq 2$, which, after some algebra, gives $(u_1' - 1)^2 \leq 0$. This equation is only satisfied when $u_1' = 1$, which in turn implies that $u_2' = 1$.

This completes the proof; given Claim 1 and Claim 2 the result follows from Nash's original proof. □

Theorem 1 gives the following result, which proves that the Nash bargaining model hold in our model.

Corollary 2
The Nash bargaining Solution maximizes:

$$\max_{\{x_i, g_i\}_{i=1,\dots,n}} \prod_{i=1}^{n} [u(y_i - x_i) + v(g_i + \sum_{j \neq i} \alpha_j g_j) - u(y_i)] \tag{67}$$

$$s.t. \sum_i g_i = \sum_i x_i \tag{68}$$

Proof: The Pareto set is the set of g_i's such that $\sum_i g_i = \sum_i x_i$. Since $v(\cdot)$ is concave and $\partial v(g_i + \sum_{j \neq i} \alpha_j g_j)/\partial g_i > \partial v(g_i + \sum_{j \neq i} \alpha_j g_j)/\partial g_j$ for any combination of g_i's and for all $j \neq i$, the convex set of the Pareto set is contained within the set of utility outcomes that are achievable with budget $\sum_i x_i$. □

C Proofs of all results in section 4

We present all propositions and their proofs for the general case of n countries. The Nash bargaining solution for general n is characterized by:

$$\frac{u_i'}{u_j'} = \frac{S_i}{S_j} \qquad \forall i,j \tag{69}$$

$$v_i'(1-\alpha_j) = v_j'(1-\alpha_i)\frac{S_i}{S_j} - (\alpha_i - \alpha_j)\sum_{k \neq i,j} v_k' \frac{S_i}{S_k} \qquad \forall i,j \tag{70}$$

$$\sum_{i=1} g_i = X \tag{71}$$

$$\sum_{i=1} x_i = X \tag{72}$$

where again $S_i = u(y_i - x_i) + v_i - u(y_i)$ denotes the surplus generated for country $i = 1, ..., n$ respectively.

C.1 Symmetry (Proposition 1)

Proposition 11
If countries are perfectly symmetric in domestic income and the spillovers they receive from each other's projects, i.e. $y_i = y_j$ and $\alpha_i = \alpha_j$ for all i, j, then, for any intended budget X, the Nash bargaining solution coincides with the generally efficient allocation.

In case all n nations are perfectly symmetric, the definitions 1 and 2 imply efficient contributions and allocations $x_i = x_j = x$ and $g_i = g_j = g$ for all i, j, which exactly achieves $S_i = S_j$ for all i, j.

Then the equations in 1 and 2 are the same as 69 through 72 characterizing the Nash bargaining solution, so that the allocations are identical. □

C.2 Quasi-linear preferences (Proposition 2)

Proposition 12
If preferences are quasi-linear in domestic consumption, i.e. $U_i = c_i + v(g_i + \alpha_j g_j)$, then, for any intended budget X, the bargaining solution coincides with one generally efficient allocation.

With quasi-linear preferences, condition 69 reduces to $S_i = S_j$ for all i, j, so that in turn conditions 70 through 72 exactly coincide with the efficiency conditions in definitions 1 and 2. □

C.3 Corners (Lemma 1)

We simply proof this lemma by example. Suppose preferences are

$$U_i = log(c_i) + log(g_i + \alpha_j g_j) \quad i = a, b$$

and parameters $y_a = 10$, $y_b = 5$, $X = 2$, $\alpha_a = 0.9$, and $\alpha_b = 0.1$. Then, both the efficient allocation and the Nash bargaining solution specify $x_a = 2$, $x_b = 0$, $g_a = 2$, and $g_b = 0$. Examples with just a corner in contributions or allocations can be constructed similarly.

C.4 Generic inefficiency (Proposition 3)

Suppose that $u(\cdot)$ is strictly concave and that parameters are such that the solution is not a "double corner".

Proposition 13
Generically, the generally efficient allocation cannot be supported as a Nash bargaining solution.

Suppose the bargaining solution was efficient. Comparing equilibrium conditions (69) through (72) to the optimality conditions of the definitions (1) and (2) shows that the Nash bargaining solution is efficient if and only if the surpluses are exactly equal, i.e. $S_i = S_j$ for all countries. However, the efficient allocation implies $S_i \neq S_j$ almost always, which in turn implies that the bargaining solution is not efficient. We arrive at a contradiction. Exceptions may arise under specific combination of parameters where the efficient solution by coincidence exactly yields $S_i = S_j$. In any of these cases, changing any one of the parameters by $\epsilon \neq 0$ will again lead to $S_a \neq S_b$ and so to the Nash bargaining solution to not be generally efficient. □

C.5 Proof of Corollary 1

Corollary 3
When the Nash bargaining outcome does not coincide with the generally efficient allocation, it distorts both budgetary and allocative efficiency.

Proof: If $S_i \neq S_j$ for all i, j, then both budgetary efficiency and allocative efficiency are not satisfied. □

C.6 Proof of Proposition 4

The proof follows from the fact that, for a union large enough, both the efficiency condition and the Nash bargaining result specify that the whole budget be allocated to the project with the highest spillovers.

First, note that in the allocative efficiency condition (definition 2),

$$v_i'(1-\alpha_j) = v_j'(1-\alpha_i) - (\alpha_i - \alpha_j)\sum_{k\neq i,j} v_k' \quad \forall i, j,$$

the term $\sum_{k\neq i,j} v_k'$ is strictly increasing in n, and g_i is increasing in $\sum_{k\neq i,j} v_k'$ if $\alpha_i > \alpha_j$. Therefore, for an n large enough, allocative efficiency will require that $g_m = X$ for country m s.t. $\alpha_m = max\{\alpha_i\}_n$.

The same logic holds for the Nash bargaining solution:

$$v_i'(1-\alpha_j)\frac{S_j}{S_i} = v_j'(1-\alpha_i) - (\alpha_i - \alpha_j)\sum_{k\neq i,j} v_k'\frac{S_i}{S_k} \quad \forall i, j$$

□

C.7 Asymmetric spillovers: Description of the numerical experiment in section 4.2

We perform an exercise on increasing asymmetry. However, simply changing the spillovers α_i changes also the efficient solutions, and so hinders the comparison between welfare achieved by bargaining to the efficient outcome. Therefore we use the following routine to pick parameter combinations with increasing asymmetry, but constant efficient aggregate welfare.

Take $\alpha_a = \alpha$ and $\alpha_b = \alpha + z$. Fixing contributions at the efficient level (not influenced by α, only a function of incomes), take $W(\alpha) = \max_{g_a, g_b}(U_a + U_b)$; i.e. the efficient level of aggregate utility. Trivially, $W(\alpha)$ is strictly increasing in α, since aggregate utility increases as α increases even without re-optimizing g_a and g_b. Therefore, for a given budget and a given z, the α s.t. $W(\alpha) = Q$ (some constant), is unique.

This implies that, for any increasing sequence $\{z\}$, for a fixed Q and efficient contributions (x_a, x_b), there exists a unique corresponding sequence of $\{\alpha\}$, which we compute. Then, we compute the Nash bargaining solution for the parameters $\{X, \alpha, z\}$ and compare aggregate utility from Nash bargaining to Q.

D Proof of section 5

D.1 Proof of Proposition 5

Consider the Nash bargaining problem 17 through 19 with the additional constraint that there be no redistribution, i.e.

$$x_a - g_a = x_b - g_b \tag{73}$$

Setting up the Lagrangian and associating Lagrange multiplier μ to this extra constraint yields

$$\mu = \frac{1}{2}(u_b' S_a - u_a' S_b) \tag{74}$$

$$\mu = \frac{1}{2}(v_b'(1-\alpha_a) S_a - v_a'(1-\alpha_b) S_b) \tag{75}$$

For the multiplier to be zero, the functional forms and parameters would ahve to exactly line up so that both terms on the right hand sides are zero simultaneously exactly at the allocation where $x_a = g_a$ and $x_b = g_b$. This can only be true at points with zero mass. Whenever $\mu \neq 0$, the Nash bargaining solution without this constraint would actually violate it, i.e. have $x_i \neq g_i$.

D.2 Proof of Proposition 6

This proof proceeds by contradiction. Suppose $y_a > y_b$, $\alpha_a < \alpha_b$, and the Nash bargaining allocation is such that $S_a > S_b$. Condition (26) then implies that $u(c_a) < u(c_b)$. Moreover, condition (27) implies that $v(g_a + \alpha_b g_b) < v(g_b + \alpha_a g_a)$. These two together, however imply:

$$u(c_a) + v(g_a + \alpha_b g_b) < u(c_b)v(g_b + \alpha_a g_a) \tag{76}$$

$$u(c_a) + v(g_a + \alpha_b g_b) - u(y_a) < u(c_b)v(g_b + \alpha_a g_a) - u(y_b) \tag{77}$$

$$S_a < S_b \tag{78}$$

We arrive at a contradiction. Thus, at the Nash bargaining solution, when $y_a > y_b$ and $\alpha_a < \alpha_b$, it can only be that $S_a \leq S_b$.

D.3 Proof of Proposition 7

From proposition 6 we know that at the Nash bargaining solution, $S_a < S_b$, which means that $u(c_a) > u(c_b)$. Suppose the budget is very small $X = \epsilon > 0$. Then, by the Inada conditions, $v_i' \sim -\infty$, which means that $v_a \approx v_b$. Then

$$S_a < S_b \tag{79}$$

$$\rightarrow \quad u(c_a) - u(c_b) + v_a - v_b < u(y_a) - u(y_b) \tag{80}$$

$$\rightarrow \quad 0 < u(c_a) - u(c_b) < u(y_a) - u(y_b) \tag{81}$$

which is a decrease in inequality in terms of utility.

It should be clear from proposition 6, however, that inequality could also be reversed: Since the high-income nation always gains less than the low income nation, they could actually change place in the utility ranking.

D.4 Proof of Proposition 8

From Proposition 6 it is clear that at the Nash bargaining solution, $S_a \leq S_b$, which means that x_a is inefficiently low and g_a inefficiently high. Then, all else equal, increasing x_a will decrease S_a and increase S_b, giving even more relative bargaining weight to nation a, so that the allocation of funds will be tilted even more toward the inefficient project g_a.

E Proofs of Section 6

E.1 Proof of Proposition 9

First note that $u_a' > u_b'$ implies $(1-\alpha_b)v_a' > (1-\alpha_a)v_b'$ by equation 38. We will complete the proof for the case of $u_a' > u_b'$ (the opposite case is analogous), which implies that the desired result is:

$$u_a' - u_b' \geq (1-\alpha_b)v_a' - (1-\alpha_a)v_b' \frac{\partial g_a}{\partial x_a} \tag{82}$$

Using the Implicit Function Theorem we get:

$$\frac{\partial g_a}{\partial x_a} = \frac{v_a'(1-\alpha_b)u_b' + v_b'(1-\alpha_a)u_a'}{2v_a'(1-\alpha_b)v_b'(1-\alpha_a) - v_a''(1-\alpha_b)^2 S_b - v_b''(1-\alpha_a)^2 S_a} \equiv \frac{Q}{Z} \tag{83}$$

First, we plug in the solution for $\frac{\partial g_a}{\partial x_a}$ into equation 82 and divide both sides by u_a' to get:

$$1 - \frac{u_b'}{u_a'} \geq [(1-\alpha_b)v_a' - (1-\alpha_a)v_b'][(1-\alpha_b)v_a'\frac{u_b'}{u_a'} + (1-\alpha_a)v_b']Z^{-1} \tag{84}$$

We then use equation 38 to get:

$$1 \geq [(1-\alpha_b)v_a' - (1-\alpha_a)v_b'][2(1-\alpha_a)v_b']Z^{-1} + \frac{(1-\alpha_a)v_b'}{(1-\alpha_b)v_a'} \equiv Y \tag{85}$$

Since $(v_a''(1-\alpha_b)^2 S_b + v_b''(1-\alpha_a)^2 S_a)$ is non-positive,

$$Y \leq \frac{[(1-\alpha_b)v_a' - (1-\alpha_a)v_b'][2(1-\alpha_a)v_b']}{Z + (v_a''(1-\alpha_b)^2 S_b + v_b''(1-\alpha_a)^2 S_a)} + \frac{(1-\alpha_a)v_b'}{(1-\alpha_b)v_a'},$$

which is equivalent to:

$$Y \leq \frac{[(1-\alpha_b)v_a' - (1-\alpha_a)v_b'][2(1-\alpha_a)v_b']}{2v_a'(1-\alpha_b)v_b'(1-\alpha_a)} + \frac{(1-\alpha_a)v_b'}{(1-\alpha_b)v_a'}$$

After some algebra, this simplifies to:

$$Y \leq 1,$$

which gives the desired result.

E.2 Proof of Lemma 2

The result is complicated by the fact that the the formateur's utility may be non-monotonic in α_j. On one hand, the overall surplus of the majority coalition is increasing in α_j (income effect). On the other hand, the formateur's share of the surplus is decreasing in α_j (substitution effect). Depending upon the specific composition of α's, the substitution effect can dominate the income effect, in which case, all else equal, the formateur prefers to form a coalition of countries with low spillover projects.

The non-monotonicity of the formateur's utility in α_i can always be overcome, however, by the difference in contributions. A higher x_j implies a higher outside option, which increases j's bargaining power and decreases the share of the budget that is allocated to the formateur's project. Formally, Nash bargaining gives:

$$S_f = \frac{u'_f}{u'_j} S_j \tag{86}$$

which shows that S_f is strictly decreasing in x_j. Therefore, any decrease in the relative utility surplus of the formateur, due to a higher α_i, is outweighed by a large enough difference in x_j and x_i.

E.3 Proof of Result 1

Take d_i to be the utility value of the outside option; there exists some $(d_h - d_l) = \bar{d}$ large enough that the formateur's utility is higher if she includes type l countries in the majority coalition.

Therefore, the set of $\{x_l, x_h\}$ for which the formateur's utility is higher with type l nations in the majority coalition is non-empty as long as there exist x_l, x_h such that: $\sum_l x_l + \sum_h x_h = X$, and $(d_h - d_l) \geq \bar{d}$ large enough. Note that this will be satisfied as long as X is large enough that $(d_h - d_l) > \bar{d}$ for $x_l = 0$, $x_h = \frac{X}{n_h}$ if $\frac{X}{n_h} < y_h$, and X is small enough that $(d_h - d_l) > \bar{d}$ for $x_l = \frac{X - n_h y_h}{n_l}$, $x_h = y_h$ if $\frac{X}{n_h} \geq y_h$.