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Rational Bubbles and Economic Crises: A Quantitative Analysis

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Abstract

We extend the Bewley-Aiyagari-Huggett model by incorporating an incomplete stock market and a persistent income process. In this quantitative general equilibrium framework, non-fundamental asset values are both large and desirable for realistic parameter values. However, if expectations shift from one equilibrium to another, some markets may crash as others soar. In the presence of nominal assets and contracts, such movements can be highly detrimental. Our analysis is consistent with the view that some of the world's large recessions were caused by an avoidable failure of monetary and fiscal policy to prevent deflation in the aftermath of bursting asset price bubbles.

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1 Introduction

It is frequently suggested that bursting asset price bubbles precipitated the two major worldwide recessions during the last century as well as many other economic crises (e.g., Reinhart and Rogoff, 2009, Chapter 13). For the economics profession, this is an awkward hypothesis. Asset price bubbles play no role in the quantitative models that underpin policy-making in finance ministries and central banks. Instead, in responding to the recent global recession, we have relied on models in which the recession is caused by large shocks to preferences or technology.

In this paper, we develop a medium-sized quantitative macroeconomic model with a central role for rational asset price bubbles and their close cousins, rational Ponzi-values. The model suggests that sizable non-fundamental asset values are the rule rather than the exception, and it allows self-fulfilling changes in beliefs to propel such values from one asset class to another. In particular, a stock-market crash can coincide with a surge in bond prices. If public debt policy is passive, the outcome then depends on whether the debt is real or nominal. If public debt is real, the real interest rate drops.[1] If public debt is nominal, the price level drops. In the latter case, which is most relevant in practice, nominal wage stickiness may generate several years of unemployment.[2] Expansionary monetary policy in the form of temporary nominal interest-rate adjustments can only dampen the detrimental effects to a limited extent. On the other hand, fiscal policy in the form of a one-time expansion of public debt can prevent both unemployment and excessively low real interest rates.

The theory of rational asset price bubbles is controversial. It enjoyed a decade of great popularity among economic theorists in the 1980s. The primary graduate macroeconomics textbook at the time, Blanchard and Fischer (1989), devoted its longest chapter to the topic. But just as the theory of rational bubbles was about to become generally accepted, it was widely discarded. The influential empirical work of Abel et al. (1989) concluded that the return to capital investment had been robustly exceeding the rate of growth for long periods of time, thus contradicting the central assumption required for the existence of rational bubbles: $r \leq g$. Many theorists also became convinced that the theory was practically irrelevant. The degree of market incompleteness that would be required to produce $r < g$ was considered too great (e.g., Santos and Woodford, 1997), and numerical experiments suggested that rational bubbles wouldn't much matter for optimal policy anyway (Kehoe, Levine, Woodford, 1992). A decade later, LeRoy (2004, page 801) reluctantly concluded: "Within the neoclassical paradigm there is no obvious way to derail the chain of reasoning that excludes bubbles". Hence, while academic interest in bubbles did not entirely subside, the focus turned to irrational bubbles.[3]

[1] This is the case considered by Kocherlakota (2011).

[2] Nominal private debt contracts would be another source of problems, but we shall neglect these problems here.

[3] The debate about irrational bubbles remains polarized. For example, Shiller (2014) argues that irra-

But the evidence against rational bubbles is weaker than it might seem. The real return to holding short-maturity government debt has been lower than the growth rate in many countries and for long periods of time.[4] In other words, the return to such reasonably safe assets has historically been well below the growth rate. Moreover, Abel et al. (1989) effectively measure average returns to all investment, not the marginal returns that accrue to portfolio investors. In the presence of scale economies or financial frictions, the latter can be considerably smaller than the former (e.g., Woodford, 1990; Farhi and Tirole, 2012). Finally, when Geerolf (2013) revisits the analysis of Abel et al. (1989) with better data, he finds that the average real return to capital investment is considerably smaller than previously thought, and quite possibly below the rate of growth. In summary, we cannot discard the theory of rational bubbles on empirical grounds.

The paper makes three main contributions. First, we demonstrate that large non-fundamental asset values, in the form of rational bubbles and Ponzi-values, are theoretically likely. Specifically, we extend the dynastic general equilibrium model of incomplete markets developed by Bewley (1980, 1983, 1986), İmrohoroğlu (1989), Huggett (1993), and Aiyagari (1994) in two directions. First, we add to the model a stock market, in which agents may trade claims to all existing ideas, but not to future ideas. Thereby, we increase both asset supply and asset demand compared to the baseline model. Besides added realism, the presence of a stock market allows us to study bubble movements from one asset class to another, thus resuscitating the theory of crashes under rational expectations due to Tirole (1985). Second, we introduce an uninsurable labor income process that is persistent enough to emulate the life-cycle savings motive, creating additional demand for assets.[5] Under reasonable parameters, these two sources of uncertainty together with realistic borrowing constraints suffice to generate a real interest rate below the rate of growth, even after accounting for a large non-fundamental asset price component. That is, our two extensions suffice to resolve the "low risk-free rate" puzzle without assuming an implausibly incomplete asset market.[6]

A second contribution is conceptual. In our model with multiple private assets as

tional bubbles are prevalent and have great macroeconomic impact, whereas Fama (2014) argues that there is little or no evidence of irrational bubbles on such broad aggregates as the stock price index. Neither Fama nor Shiller mention rational bubbles. Scheinkman (2014) provides a recent survey of the literature on irrational bubbles.

[4]In the US since 1950, the average short-run real interest rate on government debt is around 1 percent (from 1985 it is 1.5 percent), whereas the average rate of productivity growth is 2 percent.

[5]The idea of introducing life-cycle savings motives into the dynastic framework is not new. For a quantitative investigation along these lines, see Castañeda, Díaz-Giménez and Ríos-Rull (2003).

[6]The hypothesis that precautionary saving could in principle account for the $r < g$ puzzle has been pursued for several decades; see in particular Aiyagari and Gertler (1991) and Huggett (1993). However, a typical quantitative result in this early literature is that one can shave little more than a percentage point off the complete markets interest rate, $g + r_s$ (where r_s is the subjective discount rate). More recently, in a model without durable assets, but with aggregate shocks and maximally tight borrowing limits, Krusell, Mukoyama, and Smith (2013) are able to reproduce the observed lower risk-free rate. However, this result is vulnerable to the critique of Santos and Woodford (1997) that asset markets are assumed to be unrealistically incomplete.

well as government bonds, it becomes clear that aggregate non-fundamental values are composed of both bubbles and Ponzi-values, and that their proportions are not rigidly determined. Thus, even as aggregate non-fundamental values remain constant, bubbles could migrate between different assets, or they could transform from bubbles to Ponzi-values and vice versa.[7]

Third, we emphasize the importance of nominal assets. Public debt is typically denominated in terms of money rather than in terms of output. Therefore, an unchecked increase in demand for public debt will drive down the price level rather than merely reducing the real rate of interest. With nominal price and wage flexibility, this is not a problem. A bursting bubble (on private assets) simply raises the price of existing public debt so as to keep the interest rate constant, and all other prices and wages might adjust to the new price level. But under nominal wage rigidity and passive public policy, the real value of fixed nominal wages goes up. According to the model, there are then two main ways to avoid large welfare losses. The first way involves maintained real interest rates, with public debt growing to accommodate the non-fundamental value that migrates from the private sector. The second way involves a maintained level of public debt and a firm commitment to permanently lower real interest rates.

Specifically, we study the properties of the following policies in dealing with the consequences of bursting bubbles.[8]

- *Interest rate policy.* (i) Any temporary (less than ten years) reduction of nominal interest rates has only limited effect on output, investment, and unemployment. It's main virtue is to free up public funds that would otherwise be used for interest payments. (ii) However, ceteris paribus, a credible commitment to a permanent decrease in nominal interest rates would counteract deflation and maintain the real level of public debt.

- *Public debt policy.* (i) A temporary large increase in the fiscal deficit, entailing a one-time increase in nominal public debt and a permanent increase in the real level of public debt, can perfectly counter the crisis. Indeed, under this policy response, a bursting bubble in the private sector is a blessing for many people, as it offers an opportunity to socialize the economy's non-fundamental value. (ii) To the contrary, a tight cap on the increase in nominal public debt will promote deflation and entail poor economic outcomes. (iii) Commitment to a permanent (but smaller) increase in the fiscal deficit will entail permanently lower real interest rate, and can sustain the real debt at the original level.

[7]Other work has touched on the close relationship between bubbles and debt; see in particular Hellwig and Lorenzoni (2009).

[8]We also briefly analyze so-called quantitative easing (QE) in the form of temporary asset purchases. There is an equilibrium in which QE affects neither the price level nor any real outcomes; it's not clear whether there are other equilibria.

Of course, at least since the Great Depression, generations of economists have called for deficit spending to mitigate large recessions. However, while supporting their prescription, we argue that a crucial part of their diagnosis is misleading. Keynes (1936) and his followers argue that depression arises because current aggregate demand is too small at prevailing prices. Faced with such shortfall in demand for current consumption relative to future consumption, authorities ought to increase their spending to plug the gap.[9] By contrast, we argue that several crises are caused by increased demand for one type of savings vehicle, government debt, relative to others (e.g., stocks or property). Thus, when the demand for debt goes up, authorities ought to increase the supply of bonds so as to prevent an undesired increase in real wages due to nominal rigidities. In other words, unemployment is *classical* in our model.

We hasten to add that a limitation of our analysis is that we do not consider the problem of strategic default. For some countries, strategic default risk limits the amount of public debt that can be sustained. In such countries, it might therefore seem desirable to prevent private bubbles from building up in the first place. But how might that be accomplished? Raising interest rates will only exacerbate the the problem; it requires either a larger private bubble or a larger public debt to support the high interest rate. While there cannot be private bubbles if r permanently exceeds g, the non-fundamental value can only be safely harbored in public debt if the threat of strategic default is small.[10] Perhaps economies with weak public administrations are doomed either to harbor domestic bubbles or to experience capital flight. We leave this question for future research.

The paper proceeds as follows. Section 2 provides some basic concepts and a rough statement of our argument against the backdrop of previous literature. Section 3 highlights the role of our key assumption – an incomplete stock market – within simple endowment economy model. Section 4 incorporates such a stock market into an otherwise standard calibrated model of a production economy with capital, labor, and profits. Section 5 studies the equilibrium magnitude of non-fundamental values in this quantitative model. Section 6 supposes that a bubble exists in the stock market and then for some reason migrates to the bond market, and goes on to ask how authorities should respond to bubble migration under different assumptions about nominal wage stickiness. For completeness, we add three sections on other dimensions of optimal policy. Section 7 studies optimal interest policy under our chosen parameters, provided authorities are free to choose any equilibrium. Section 8 investigates how policy should respond to durable changes in the growth rate or the technology turnover rate. Finally, section 9 indicates

[9] Keynesian economics comes in several flavors. See Tobin (1993) for a particularly accessible statement of the "old" Keynesian view; for references to "new" Keynesian analysis, see the next section. However, all flavors share the focus on how aggregate demand is determined by propensities to consume.

[10] Within an OLG model that builds on Tirole (1985), Galí (2014) discusses whether policy-makers should raise interest rates in order to dampen fluctuations in an aggregate bubble. Galí argues that the answer is negative, as the bubble will fluctuate more when interest rates are higher. Our current analysis does not study anticipated fluctuations of the aggregate bubble, but it is obviously consistent with Galí's view that the bubble grows faster when interest rates are higher.

some directions for future work.

2 Concepts and Related Literature

If all investors have rational expectations, an asset's non-fundamental value can only be positive if the asset's duration is indefinite. We therefore focus attention on models with an indefinite horizon.[11]

2.1 Ponzi-values and bubbles

We consider two types of non-fundamental values: Ponzi-values and bubbles. While these definitions will be standard, we find it useful to express them in the context and notation of the paper. To fix ideas, think about an infinitely small issuer of a financial asset, living in an economy that grows at rate g and with an equilibrium real interest rate of r. Suppose $r < g$.

Suppose first that the financial asset is real debt. A Ponzi-scheme is then broadly defined as a debt whose principal is expected never to be repaid by the borrower. A simple example is a perpetual government bond, such as the consolidated annuities (consols) sporadically issued by the British government. The British government has an option to redeem the bond at par, but that is not crucial: Even a finite maturity debt contract can be a Ponzi-scheme if the borrower is expected always to repay the debt's principal only by issuing an identical or larger amount of new debt. We define the *Ponzi-value* of a debt contract as the difference between the (market) price that lenders are willing to pay for it and the net present value of the sacrifices that the issuer must make in order to pay the agreed interest. (The narrowest definition of a Ponzi-scheme would insist that the issuer makes no sacrifice at all.)

If $r < g$, the issuer's sacrifices are zero along a balanced growth path, and the aggregate Ponzi-value of an economy is then the market value of those debts that are expected to be rolled over indefinitely. When the agreed interest rate is constant at r, the Ponzi-value of real debts is then simply equal to the face value of these debts.[12]

When $r < g$, issuers of indefinite debts potentially benefit immensely. For example, someone who issues new debt at the rate g can consume a fraction $g - r$ of the accumulated debt every period. In modern countries, most agents are legally precluded from rolling over net debts indefinitely. Gross debts should be matched by corresponding assets, and agents who are unable ever to serve their debt are considered bankrupt.[13] Thus,

[11] We prefer to use the word "indefinite" rather than "infinite", because the latter is sometimes misunderstood as eternal. Even a single tennis game is indefinite, because regardless of how many balls have been played in the game, there is a positive probability that the game will last at least one more ball.

[12] As we shall see later, nominal debts introduce an additional degree of freedom.

[13] Bankruptcy laws that preclude Ponzi-schemes are particularly stringent for financial intermediaries. There are often related laws that constrain the net indebtedness of local governments.

the value of the issuer's sacrifice equals the value to the debt holder. However, the central government, with its unique powers of taxation, usually does have the legal right to roll over debt indefinitely. Along a balanced growth path, the value of government debt is wholly non-fundamental if the $r < g$ and partially non-fundamental otherwise.

Suppose next that the financial asset is equity. Specifically, suppose the asset issuer owns a productive resource. The resource has a constant survival probability σ, and every period t that it survives it yields a dividend $D_t = (1+g)^t D_0$. Shares in this resource represent claims to analogous dividend streams $d_t = (1+g)^t d_0$. Finally, let us assume that $\sigma(1+g) < 1+r$, and that the idiosyncratic risk associated with the stock's survival can be perfectly diversified. Since investors then discount this future dividend stream at rate r, the *fundamental value* of a share is

$$\begin{aligned} f_t &= \frac{\sigma d_{t+1}}{1+r} + \frac{\sigma^2(1+g)d_{t+1}}{(1+r)^2} + \dots \\ &= \frac{\sigma d_{t+1}}{1+r-\sigma(1+g)}, \end{aligned} \tag{1}$$

where the last equality is a consequence of our assumption $\sigma(1+g) < 1+r$ (otherwise, the fundamental value would be infinite). The price of a share need not be equal to the fundamental value, however. Rather, the price p_t needs to satisfy the first-order difference equation

$$p_t = \frac{\sigma(p_{t+1} + d_{t+1})}{1+r}, \tag{2}$$

which, when $\sigma(1+g) < 1+r$, has the general solution

$$p_t = f_t + b_t, \tag{3}$$

with $\{b_t\}$ satisfying

$$b_t = \frac{b_{t+1}}{1+r},$$

where b_t is the stock's *rational bubble*. In other words, a rational bubble on a company's shares corresponds to the net present value of owning the share "at infinity," and (in expectation) it grows at the real rate of interest.

Notice that we could in principle have a bubble on a stock that yields zero fundamental value. By extension, the there could be rational bubbles on all kinds of unproductive assets. Rare stamps, paintings, and other expensive collectors' items are relevant examples.

The case of currency is special, as it might be seen either as an infinite debt that entails exactly zero interest (indeed, the monetary base is usually considered to be a component of public debt) or as a bubble.

2.2 Public debt and equilibrium interest rates

The theory of rational bubbles and Ponzi-values is intimately connected with the theory of monetary and fiscal policy. At the heart of the discussion lies the question of whether public policy, in a closed economy, can affect the long-run real interest rate.

Wicksell (1898, 1907) gives the first famous formulation of the view that long-run real interest rates are immune to interest rate policy. Wicksell's hypothesis goes under many names, including "the natural rate hypothesis," "the classical dichotomy" and "the Fisher hypothesis." The popularity of the natural rate hypothesis has waxed and waned. Keynes (1936) argues that the hypothesis will hold in the long run; Myrdal (1939) argues that it will not. Phelps (1967) and Friedman (1968) are strongly supportive of the hypothesis. Barro (1974) provides conditions under which the timing of taxes (and thus the magnitude of public debt) is irrelevant to the interest rate.

On the other hand, many economists have argued that financial markets are incomplete and that, as a consequence, there may large non-fundamental values in the form of bubbles or Ponzi-values. In this case, public policies will affect the interest rate: For seminal qualitative contributions to the theory of rational bubbles in an overlapping generations setting, see Allais (1947), Samuelson (1958), Diamond (1965), Wallace (1980), and Tirole (1985). In a dynastic setting, related results are due to Bewley (1980), Scheinkman and Weiss (1986), Kocherlakota (1992), and Santos and Woodford (1997). Hitherto, quantitative analysis has focused on the case of Ponzi-values only, as calibrations have implied $r > g$; see in particular Aiyagari and McGrattan (1998), Flodén (2001), and Heathcote (2005).[14]

However, the faith in models of incomplete markets has been repeatedly shaken by the suspicion that their results depend on unrealistic assumptions about the set of assets that can be traded (Brock, 1979; Scheinkman, 1980; Santos and Woodford, 1997), or that the policy impacts are quantitatively small (Kehoe, Levine, and Woodford, 1992). In particular, over the last few decades, many macroeconomists have adopted the "New Keynesian" view that interest rate policy has an impact on the real interest rate in the short run due to nominal price rigidities, whereas the natural rate hypothesis prevails in the long run (e.g., Clarida, Galí and Gertler,1999; Woodford, 2003).

2.3 Our central assumptions

In this paper, we extend the incomplete markets approach by adding a stock market to the market for physical capital. Thereby, we can directly confront the objections that the model's asset market is "too incomplete" and that policy will only have small long-run effects. Specifically, we argue that the turnover of publicly tradable firms is a major com-

[14] The close connection between public debt policy and interest rate policy in these models is well established in the literature: For an a textbook treatment of monetary analysis in dynastic incomplete market models, see Ljungqvist and Sargent (2004, Chapter 17).

ponent of asset market incompleteness, and that monetary and fiscal policies are bound to have a quantitatively significant impact on long-run real interest rates under plausible assumptions about this parameter.[15] Moreover, the introduction of a stock market enables us to study the optimal policy responses to bubbles and crashes.

The model economy is populated by a continuum of infinitely-lived heterogeneous agents facing rare but persistent idiosyncratic income shocks. Like many related models, ours is built on the assumption that these agents face limited insurance opportunities. However, we go further in two main respects. We admit more forms of insurance, and we offer a more extensive justification for the various contracting limitations. Structurally, the model resembles the heterogeneous-agents model of monetary analysis studied by Ljunqqvist and Sargent (2004, Chapter 17), which in turn synthesizes Bewley (1980, 1983, 1986), İmrohoroğlu (1989), Huggett (1993), and Aiyagari (1994). These previous models all impose strong and somewhat arbitrary limitations on the assets that agents are allowed to trade. This creates a well-known tension. On the one hand, realistic private financial contracts clearly depart from the ideal of a perfect Arrow-Debreu securities market, and the departures could in principle provide a rationale for asset price bubbles (e.g., Tirole, 1985, Weil, 1987, Gale, 1990, Woodford, 1990). On the other hand, there are seemingly plausible conditions under which rational asset price bubbles cannot logically exist; see Brock (1979), Scheinkman (1980) and especially Santos and Woodford (1997).[16] The contentious question is thus whether *plausible* deviations would create a demand for bubbles or Ponzi schemes.

We posit that the answer is affirmative, and that it relies primarily on two reasonable assumptions. First, as noted already, we assume that private borrowing for consumption by poor agents is constrained by their lack of collateral.[17] To be prepared for periods of low income, people therefore tend to engage in precautionary saving. Since private borrowing for consumption purposes is limited, so is the supply of corresponding financial assets. Precautionary saving will hence partly be in the form of other private assets, notably physical capital (or debt to fund such capital) and claims on intangible capital in the form of stocks.[18] Second, our central and somewhat more novel assumption is that the stock market is incomplete in the following way: *Agents cannot trade securities based on future projects that have not yet materialized.* Specifically, while there will always be new projects in the future, claims on them cannot be traded today.[19] As a case in point,

[15] Ultimately, empirical analysis has to decide the issue of policy neutrality. For evidence of non-zero long-run impact of monetary policy, see for example Garcia and Perron (1996); for a recent contribution and further references, see Bulkley and Giordani (2011).

[16] Some recent contributions along these lines recognize the objection by confining attention to economies with poorly developed asset markets; see, e.g., Wen (2014).

[17] On a closely related note, Kocherlakota (2007) suggests that the importance of public money is ultimately due to the Government's superior commitment abilities, which in turn relies on the power to tax.

[18] Unlike the large recent literature on liquidity shortage, such as Farhi and Tirole (2012), we assume for simplicity that the market for physical capital is perfect; we comment on this literature below.

[19] This assumption is particularly reminiscent of Tirole (1985, p 1508), who considers the case of succes-

consider Facebook, an internet company started by novices in 2003. When it became publicly traded in 2010, the company was worth around 40 billion US dollars. By contrast, in a complete asset market, well diversified investors would already have held financial claims on Facebook, and any other venture that the founder Mark Zuckerberg may have initiated, long before 2003.

A role of non-fundamental asset values is to increase the market value of financial assets and thereby helping agents smooth consumption in the face of long-run income fluctuations. In other words, our model emphasizes the same long-term savings motives as do, e.g., Samuelson (1958) and Tirole (1985), but without invoking intergenerational trading frictions. Instead of their OLG structure, with overlapping generations of agents, we devise an OLGA structure, with overlapping generations of assets. We show that for a broad range of parameters, there is an attractive equilibrium in which public debt is a bubble; that is, public debt may be positive, yet serving the debt does not require real resources at any time. In other words, the government never needs to run a surplus. This is true independently of whether the debt is in the form of currency or in the form of interest-bearing securities, such as bills or bonds.

The emergence and obsolescence of real assets is not merely a convenient modeling device, but a substantive assumption with well-defined empirical counterparts (e.g., Hobijn and Jovanovic, 2001). For the conceptually simple case in which all dividends grow at the same rate as overall productivity and the rate of asset destruction is the same as the rate of asset creation, we find that a conservative yearly rate of asset turnover of about two percent comfortably suffices to justify the public debt levels that we observe today.

Authorities affect the value of all assets both through the nominal interest rate on the public debt and through the rate of debt creation. Since there are two instruments at the authorities' disposal, the real interest rate is decoupled from the rate of inflation.[20] Other things equal, high public debt is an indication that the government has purposefully chosen a relatively high real interest rate, much as in Allais (1947) and Diamond (1965).

Lower real interest rates tend to be desirable for agents who are currently relatively poor, and undesirable for agents who are currently relatively rich. With low interest rates, seigniorage is larger, and with lump-sum transfers an increase in seigniorage redistributes resources from rich to poor. This argument is familiar from Scheinkman and Weiss (1986), for the case of a one-shot change in money supply, and Levine (1991) for the case of permanently low interest rates. However, Levine considers precautionary savings that guard against temporary shocks to the marginal utility of consumption, rather than against persistent shocks to income or wealth, and in this kind of setting Kehoe, Levine,

sive rent creation, and of Kiyotaki and Moore (2003) who consider successive generations of projects. By contrast, the model of Santos and Woodford (1997) assumes that claims on future projects are implicitly priced through assets that exist today.

[20]Hence, the notion of an inflation tax is somewhat misleading. The effective taxation of savers can be every bit as high with low interest rates and small money growth as with higher interest rates and high money growth.

and Woodford (1992) find that welfare effects are quantitatively small. Already in the endowment economy version of our model welfare effects are considerably larger. With production, welfare is even more sensitive to interest rate policy, due to the impact of interest rates on wages.[21]

Our quantitative results depend on strong savings motives, and for that purpose our dynastic model features a counterpart to retirement; the labor income process is designed to mimic an average work-life of 40 years followed by 15 years of no labor income. On the other hand, we mitigate the precautionary savings motive by assuming, realistically, that a government provides some insurance against the low-income states and also distributes uniform lump-sum transfers. These expenditures are funded through income taxation and through expansion of the public debt. Under our parameters, we find that the aggregate non-fundamental value is large, considerably larger than the observed public debt. Therefore, we conclude that the model admits rational bubbles in addition to the Ponzi value.

2.4 Other literature on non-fundamental asset values

During the last decades, much of the new work on bubbles has considered the role of financial frictions. There are two central ideas, occurring sometimes separately and sometimes jointly: First, bubbles on an entrepreneur's assets add to net wealth and thus help to support investment. Second, bubbles have higher liquidity than other assets and thus facilitate investment when financial constraints are particularly tight. Contributions to this literature include Woodford (1990), Holmström and Tirole (1998), Kiyotaki and Moore (2002, 2003), Farhi and Tirole (2012), Martín and Ventura (2011, 2012), and Wang and Wen (2012). We refer to these papers for a more detailed discussion and further references. Note that this literature implies that the demand for saving by entrepreneurs or firms is larger than we assume and that their supply of assets is more limited than we assume. Hence, if we were to take financial frictions into consideration, it would be even easier to support rational bubbles.

A notable difference between our approach and the financial frictions approach concerns the impact of a broad stock market crash. The financial frictions approach implies that there will be a recession due to the reduction of entrepreneurs' net wealth. By contrast, our approach suggests that there will be a major recession only if the bubble moves into the bond market and thereby affects the price level. If, for example, the bubble moves into the residential housing market, economic activity may be largely unaffected. In that way, our model allows a simple interpretation of the events in the decade 2000-2010: The dotcom crash had a minor impact on output because the bubble migrated to property markets. The joint crashes of property and stock markets in 2007-8 had a major impact

[21] For a detailed analysis of the large pecuniary externalities associated with precautionary savings in the Aiyagari-Huggett framework (albeit without considering bubbles), see See Dávila et al (2012).

on output because the bubble migrated to government bond markets (where it morphed into Ponzi-value).

Another more distantly related literature studies the role of money in facilitating transactions in the presence of matching frictions; see Williamson and Wright (2010) for a survey of this "new monetarism". Note that this literature is not about bubbles. As demonstrated by Tirole (1985), it is impossible for currency (money) to carry a rational bubble if it also provides transactions services; in the presence of transactions services, the value of currency is fundamental to the holder. On the other hand, this observation helps us with the above-mentioned quandary whether to classify currency as bubble or Ponzi-value; only the latter classification is robust to the provision of transactions services.

3 Simple model: An endowment economy

To clarify what we mean by an incomplete stock market, and indicate why the existence of such a market is consistent with large non-fundamental asset values, we first study a simple laissez-faire endowment economy.

Time is discrete, and the horizon is infinite. Period $t = 0$ refers to the current period. Periods $t = -\infty, ..., -1$ comprise the history and determine the "initial conditions" that characterize period 0. Periods $t = 1, ..., \infty$ comprise the future. There is a continuum of infinitely lived agents distributed along the unit interval.[22]

Preferences: Agents consume a homogeneous good and have identical preferences. Their utility function is of the form

$$U = \mathrm{E} \sum_{t=0}^{\infty} \beta^t u\left(c_t\right), \tag{4}$$

where c_t is consumption in period t, and $\beta \in (0, 1)$ is the subjective discount factor. We assume that the felicity function u is concave.

Technology: There are two kinds of productive assets. First, there is a continuum of length A of productive and tradable Lucas trees. In each period that a tree is alive, it yields an amount d of non-storable fruit. Survival is an i.i.d. process; each tree survives to the next period with probability $\sigma < 1$. Thus, a measure $(1-\sigma)A$ trees die each period. Likewise, a measure $(1-\sigma)A$ trees are born each period. Of the trees that emerged v years ago, there thus remain $(1-\sigma)\sigma^v A$. Each new tree is paired with a random agent, who initially has full ownership of this tree. For convenience, we assume that an agent can absorb at most one new tree in any period. Thus, in each period a fraction $1-\sigma$ of the agents receive a new tree. Let ε_t be an indicator variable, taking the value 1 if an agent gets one of the new trees in period t and 0 otherwise. Let $\varepsilon^t = \{\varepsilon_0, ..., \varepsilon_t\}$ denote the

[22] As usual, each infinitely lived agent could be seen as representing an altruistically linked dynasty.

partial sequence from period 0 up to period t and let $\mathcal{E}^t$, denote the corresponding set of all possible such sequences. Define probability measures $\gamma^t(x_0, \cdot) : \mathcal{E}^t \to [0,1]$, $t = 0, 1, ...$ where, for example, $\gamma^t(x_0, \varepsilon^t)$ is the probability of history ε^t given an agent's initial state x_0 specified below.

Second, there is a continuum of length 1 of infinitely lived non-tradable trees each yielding y units of fruit per period. Each agent owns one non-tradable tree; to distinguish them from the tradable trees, we refer to them as bushes from now on. (The role of the bushes is to keep consumption bounded strictly above zero and utilities bounded above negative infinity, which is convenient for computational purposes.)

There also exists a continuum of length M of an unproductive asset that has no intrinsic value but which is perfectly durable and can be traded. Whenever it turns out to have value in equilibrium, we will think of this asset as money. For convenience, we normalize $M = A = 1$.

Trade: At any time, agents can trade fruit and shares in existing trees in a frictionless market. In principle, they are also able to borrow against collateral (shares in trees), but not against the income from bushes.[23] Claims on trees that has not yet emerged cannot be traded.

While agents could in principle also write contracts contingent on future ownership of trees that have not yet emerged, our central assumption is that such contracts will not be worth writing. We find the assumption realistic, and the ensuing analysis does not hinge on its precise justification, but let us nonetheless briefly indicate one reason why it is difficult to contract on non-existing assets: Suppose that the paired agent privately observes the tree's emergence one period before other agents do so. The agent can choose whether to keep the emerging tree or covertly display and sell it to some other agents – potential buyers – before it becomes publicly visible. Suppose the seller has sold shares in own emerging trees, but some other agent has not. That agent can then buy the tree, pretend it emerged with him, and cash in the full market value of the tree in the next period. By contrast, the seller by waiting until the next period can only cash in the value of any retained shares. Thus, a coalition of agents will always find it profitable to deviate from any plan involving trade of claims on future trees. More precisely, any outcome in which a positive measure of agents share risk in a market for futures would fail to be coalition-proof, in the sense defined for this kind of private-information environment by Lacker and Weinberg (1993).

Prices: Since one of our aims is to endogenize the value of money, let the consumption good, fruit, be our *numeraire*. That is, the prices of other assets are expressed in terms of fruit.

We assume that all units of the unproductive asset are indistinguishable. There are no labels or other extraneous features that may be used to distinguish them. Thus, they

[23] This assumption can be justified more fundamentally by assuming that agents can hide such incomes. At any rate, our main results hold even if agents are allowed to have realistically negative asset holdings.

must be priced the same. The price of one unit of the unproductive asset at time t is denoted p_t^m.

While the productive assets are also intrinsically identical, in the sense that each tree yields the same expected future return, their age differs. We thus allow the price to differ across generations of trees, letting $p_{j,t}$ denote the price of a tree of generation $j \leq t$ at time t.[24] Allowing assets from different generations to be priced differently facilitates the study of price bubbles on productive assets. We do not admit different prices for assets within the same generation.[25]

Agents' wealth: Let m_t denote the quantity of unproductive asset that an agent possesses at the end of period t. Let $a_{j,t}$ denote the quantity of generation j assets that an agent possesses at the end of period t, and let

$$a_t = \{a_{j,t}\}_{j=-\infty}^{t-1}$$

denote an agent's holdings of all productive assets born in previous periods. As noted above, negative asset holdings are not feasible. Let $x_0 = (a_0, m_0, \varepsilon_0) \in X$ denote the initial state of an agent, where $X = \mathrm{R}_+^\infty \times \mathrm{R}_+ \times \{0,1\}$. Let $\mathcal{X} = \mathcal{R}^\infty \times \mathcal{R} \times \{0,1\}$ where $\mathcal{R}$ denote the Borel sets that are subsets of R_+.

Behavior: In period 0, given its initial state $x_{0,}$ each agent chooses consumption and savings for each possible sequence ε^t. Let $\phi_t : \mathcal{E}^t \to X_{am}$, $t = 0, 1, ...$ describe the savings plan, where $X_{am} = \mathrm{R}_+^\infty \times \mathrm{R}_+$ and $\phi_{a,j,t}(\varepsilon^t; x_0)$ denotes the value for $a_{j,t+1}$ that is chosen in period t if the history up to t is ε^t, conditional on the agent's initial state being x_0. Similarly $\phi_{m,t}(\varepsilon^t; x_0)$ denotes the value for m_{t+1}. Let $c_t : \mathcal{E}^t \to \mathrm{R}_+$ describe the associated plan for consumption.

At the beginning of period $t+1$, a fraction σ of the trees alive at t will have died; the remaining will yield a dividend d. An agent's budget constraint is therefore given by

$$c_t(\varepsilon^t; x_0) = y + p_t^m m_t + \sum_{j=-\infty}^{t-1} (p_{j,t} + d)\sigma a_{j,t} + (p_{t,t} + d)\,\varepsilon_t - \sum_{j=-\infty}^{t} p_{j,t} a_{j,t+1} - p_t^m m_{t+1}, \quad (5)$$

where $a_{j,t+1} = \phi_{a,j,t}(\varepsilon^t; x_0)$ and $m_{t+1} = \phi_{m,t}(\varepsilon^t; x_0)$. The agent's problem is thus to maximize expected discounted lifetime utility

$$\sum_{t=0}^{\infty} \sum_{\varepsilon^t \in \mathcal{E}^t} \beta^t u\left(c_t\left(\varepsilon^t; x_0\right)\right) \gamma^t(x_0, \varepsilon^t) \quad (6)$$

[24]It is not necessary that agents keep track of the assets' age, $t - j$. In order to sustain different prices for assets belonging to different generations, it suffices that agents recall the assets' price in the previous period.

[25]In principle, agents might know where individual assets originated and thus prices could condition on such information. On the other hand, private information and secret deals would work against such conditioning; see the analogous argument against futures contracts above.

through a set of choices $\phi_t(\varepsilon^t; x_0)$ for all t, subject to (5) and $\phi_t(\varepsilon^t; x_0) \in X_{am}$ and taking as given the sequences of prices $\left\{ \{p_{j,t}\}_{j=-\infty}^{\infty}, p_t^m \right\}_{t=0}^{\infty}$ and the initial state x_0.

Markets and distribution: The distribution of agents over the initial state is described by a measure $\kappa : \mathcal{X} \to [0,1]$. By integrating over κ, aggregate variables can be computed. Market clearing in financial markets implies that

$$M = \int_X m_0 d\kappa, \ t = 0, \tag{7}$$

$$\sigma^{t-j}(1-\sigma)A = \int_X a_{j,0} d\kappa, \ t = 0, \forall\, j = -\infty, ..., 0. \tag{8}$$

$$M = \int_X \sum_{\varepsilon^{t-1} \in \mathcal{E}^{t-1}} \phi_{m,t-1}(\varepsilon^{t-1}; x_0) \gamma^{t-1}(x_0, \varepsilon^{t-1}) d\kappa, \ \forall\, t > 0, \tag{9}$$

$$\sigma^{t-j}(1-\sigma)A = \int_X \sum_{\varepsilon^{t-1} \in \mathcal{E}^{t-1}} \phi_{a,j,t-1}(\varepsilon^{t-1}; x_0) \gamma^{t-1}(x_0, \varepsilon^{t-1}) d\kappa, \ \forall\, t > 0, \forall\, j \geq -\infty, \tag{10}$$

Similarly, market clearing in the market for fruit implies that

$$d + y = \int_X \sum_{\varepsilon^t \in \mathcal{E}^t} c_t(\varepsilon^t; x_0) \gamma^t(x_0, \varepsilon^t) d\kappa, \ \forall\, t \geq 0.$$

Equilibrium: An equilibrium comprises sequences of prices $\left\{ \{p_{j,t}\}_{j=-\infty}^{\infty}, p_t^m \right\}_{t=0}^{\infty}$ and sequences of decisions $\left\{ c_t(\varepsilon^t; x_0), \phi_t(\varepsilon^t; x_0) \right\}_{t=0}^{\infty}$ for all $x_0 \in X$ and $\varepsilon^t \in \mathcal{E}^t$, together with probability measures $\left\{ \gamma^t(x_0, z) \right\}_{t=0}^{\infty}$ for all $x_0 \in X$ and $z \in \mathcal{E}^t$, and a measure $\kappa(x)$ for all $x \in \mathcal{X}$ describing the initial distribution such that

1. the decision rules solve the agents' problem given prices and the initial state x_0;

2. all markets clear, and

3. the measure $\gamma^t(x_0, \varepsilon^t)$ satisfies $(i) \gamma^t(x_0, \{\varepsilon^{t-1}, 0\}) = \sigma \gamma^{t-1}(x_0, \varepsilon^{t-1})$, (ii) $\gamma^t(x_0, \{\varepsilon^{t-1}, 1\}) = (1-\sigma)\gamma^{t-1}(x_0, \varepsilon^{t-1})$ for all $x_0 \in X$, $t \geq 0$, and (iii) $\gamma^0(x_0, z) = 1$ if $\varepsilon_0 \in z \in \mathcal{E}^0$ and 0 otherwise.

3.1 Analysis

We will be mostly concerned with stationary equilibria, where returns to saving and the aggregate distribution of wealth are constant, but we shall also consider transitional dynamics.

Note that individual wealth may be highly variable even if the aggregate distribution is constant. Indeed, the agents will optimally engage in precautionary saving precisely because they are concerned about the negative consumption consequences of long strings of poor luck.

Let $R^a_{j,t} = \sigma(p_{j,t+1} + d)/p_{j,t}$ denote the return to holding a tree of generation j and $R^m_t = p^m_{t+1}/p^m_t$ denote the return to holding money. Since there is no aggregate uncertainty and all existing trees give fruit in any equilibrium, no-arbitrage implies that $R^a_{j,t} = R^a_t$. Moreover, if money is valued no-arbitrage implies that $R^m_t = R^a_t$. The stationary equilibrium hence come in three types. First, for all $\sigma \in (0,1]$ there exist a stationary *non-monetary* equilibrium without any price bubbles on trees, where $p^m = 0$, $\phi_m(\cdot; x_0) = 0$ for all $x_0 \in X$, and $p_j = p > 0$ such that $R^a_j = R^a$ for all j. Second, for all $\sigma \in [0, \bar{\sigma}]$ there exist a stationary *monetary* equilibrium, in which money is the only bubble, where $\bar{\sigma} < 1$, $p_j = \frac{\sigma d}{1-\sigma}$ for all j and $p^m > 0$ such that $R^a_j = R^a = R^m = 1$, and for all $\sigma \in (0, \bar{\sigma}]$ there also exist a stationary *non-monetary* equilibrium with price bubbles on trees, where $p^m = 0$, $\phi_m(\cdot; x_0) = 0$ for all $x_0 \in X$, and $p_{j,t+1} = (R^a/\sigma)\, p_{j,t} - d > 0$ where $R^a < 1$ for all j and t, and $p_{t,t} = p$. In the monetary equilibrium the value of the aggregate bubble is simply $p^m M$. In the non-monetary equilibrium with price bubbles on trees, the bubble on a new tree is $\left(p - \frac{\sigma d}{R^a - \sigma}\right)$ where $\frac{\sigma d}{R^a - \sigma}$ is the fundamental value of a tree. Over time the bubble on an individual tree grows at rate $(R^a/\sigma) - 1 > 0$. But since trees die at rate $(1-\sigma)$ the total bubble on a generation of trees falls at rate $R^a - 1 < 0$. This implies that the oldest existing tree carries the largest bubble in the economy, but that the youngest generation of trees carries the largest share of the aggregate bubble on all trees which is given by $(1-\sigma)A\left(p - \frac{\sigma d}{R^a - \sigma}\right) / (1 - R^a)$.

The absence of a complete set of Arrow-Debreu contingent claims markets is crucial for the existence of the monetary (bubble) equilibrium. To see this, note that if there was a market in period t where agents could buy and sell share in trees that do not exist in t but may exist in $t+1$, then a no-arbitrage condition would imply identical returns to a diversified portfolio of existing trees and to one of non-existing trees. The chance of receiving a new tree would then be insured away; there would be no consumption uncertainty, the equilibrium distribution of wealth would be degenerate, and the rate of return would be given by $1/\beta$. The equilibrium price of trees would then be $p_{j,t} = \frac{\sigma d}{1/\beta - \sigma}$ for all j, t.

In order to illustrate the conditions under which there will be bubble equilibria, and to study their welfare properties, let us now impose additional structure.

3.2 Parametrization

As we will be using many of the parameter values later, we discuss these choices now, even if the simple model itself is too rudimentary to be taken seriously for quantitative purposes. Let a period be one year, and let the felicity function take the CRRA-form

$$u(c) = \frac{c^{1-\mu}}{1-\mu}.$$

The discount factor β is set to 0.97. Kimball, Sahm and Shapiro (2009) provide some of the most recent estimates of the coefficient of relative risk aversion μ. Using survey results in the PSID they find that the average μ is 4.19 (see their Table 1). Since our main results are generally strengthened as μ increases, and we prefer to be conservative, we set $\mu = 3$.

The yield from trees is normalized to $d = 1$. Recall that bushes can not be traded. The yield from a bush can hence be thought of as the level of safety nets in the economy. We set it to $y = 0.1$ units of fruit each period. With average consumption equal to $d + y$, this is implies that the poorest agent in the economy, one with zero assets, can consume approximately 9 percent of average consumption. As a comparison, having an income of 9 percent of average income in the US would place an individual in approximately the bottom 5th percentile (see Díaz–Giménez, Glover and Ríos-Rull, 2011).[26]

The final parameter, σ, governs the rate of creation and destruction in the economy. Jovanovic and Rosseau (2005) find, using US data between 1885 and 2003, that the value of new firms is around 2 percent of total stock market value,with highs of above 10 percent in periods of rapid technological change such as the electrification era and the IT era. As the critera to be listed on the stock exchanges are quite stringent, these numbers can be thought of as a lower bound.[27] Even so, to be on the conservative side we set $\sigma = 0.98$ in our benchmark calibrations. But to illustrate the mechanisms at work we also consider values for σ in the range $[0.85, 1]$.

We compute the solution to the agent's problem using Carroll's endogenous grid method. Since the returns to money and a perfectly diversified portfolio of trees are equal in any equilibrium with valued money, agents are indifferent between holding trees and money in such equilibria. For simplicity we thus assume that all agents hold the same portfolio shares. We then compute the stationary distribution by approximating the invariant density function.

3.3 Monetary equilibria

Figure 1 summarizes the main results. It displays the equilibrium prices for yearly survival rates σ in the range $[0.85, 1]$ in three different classes of equilibria; (i) with money (but no other bubbles), (ii) with price bubbles on trees, (iii) without money and other

[26] Also, as of 2010, the maximum monthly allotment of food stamps for one person in the US is \$200 per month. According to the Bureau of Labor Statistics, average annual expenditure for single individual households was \$30,613 in 2011, which implies that food stamps provide a safety net of slightly less than 8 percent of average consumption.

[27] According to Caves (1997), who considers a broad sample of firms in eight different countries during the 1970s and early 1980s (using data assembled by John Cable and Joachim Schwalbach), average annual entry rates are 7.7 percent in the US and as high as 13 percent in Belgium. Average exit rates are very similar to entry rates. Using more recent data covering 24 countries, Bartelsman et al. (2004) find that gross turnover (exit plus entry rates) are between 20-25 percent across all firms. Among large firms, defined as those with at least 20 employees, exit and extry rates are around five percent. In the time series dimension, they find that between 15-20 percent of firms in industrial countries fail during the first two years, that across all countries 65 percent of firms survive the first four years while only 30-50 percent of all firms survive beyond seven years. This points to an average yearly rate of destruction/creation of 8 percent.

bubbles. The figure also indicates equilibrium prices under complete markets.

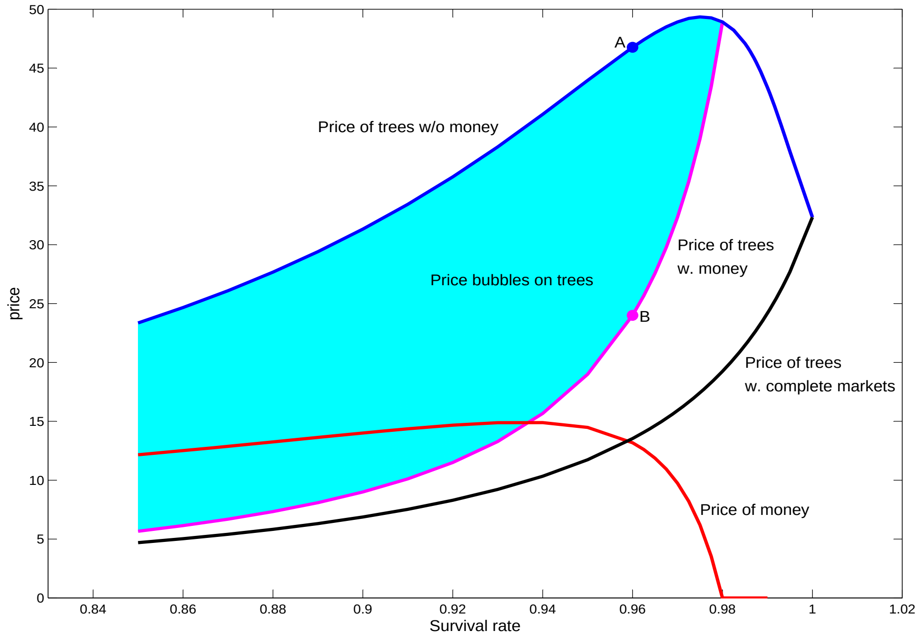

Figure 1: Asset prices as functions of the survival rate σ

With complete markets, the price of trees falls as their survival rate falls, since the discounted stream of dividends falls. With incomplete markets and no money, the price of trees first increases and then falls as the survival rate is reduced. The non-monotonicity reflects two competing effects. On the one hand, as the survival rate falls trees become less valuable since the expected dividends are smaller. On the other hand, without perfect insurance markets the demand for a store of value increases. For high survival probabilities the latter effect dominates. But as the survival rate decreases, the chance of receiving a new tree increases. The storage motive is reduced, and the price of trees falls. The strength of the storage motive depends on the relative size of the bush and and the dividends from trees; the larger the bush, the smaller is the storage motive. For a large enough bush, the non-monotonicity disappears.

A monetary equilibrium exists for all survival rates $\sigma < 0.98$. With valued money, the price of trees monotonically falls as the survival rate falls, just as under complete markets. This is natural; in both cases we are discounting a diminished stream of dividends at a constant interest rate (the interest rate $r = R^a - 1$ is always zero in the monetary economy, and $1/\beta - 1 = 0.03$ in the complete markets economy).

For all survival rates $\sigma < 0.98$ there also exist equilibria with price bubbles on trees. The shaded area in Figure 1 shows the price of new trees in these equilibria. For example, consider all equilibria on a vertical line between points A and B in the figure. A downward movement from A to B is associated with a lower price on new trees but a higher return to trees. This implies that the fundamental value of a new tree, $\frac{\sigma d}{R^a - \sigma}$, monotonically falls between A and B. The bubble value of a new tree however changes non-monotonically. In A there are no bubbles, and in B the bubble consist only of money. Hence, the bubble on new trees first increases and then decreases as we move from A to B. But even if the bubble on each new tree is smaller in the equilibria close to B, the return is higher, and the bubble on a generation of trees thus falls at a slower rate. Therefore, the aggregate bubble increases monotonically as we move from A to B and is largest in B where money is the only bubble. Finally, in terms of ex-ante average welfare these equilibria can easily be ranked; the larger the aggregate bubble the higher is average welfare. The larger are the bubbles, the higher is the return to saving, and the smaller is the consumption inequality.

Before leaving the endowment economy, let us make three additional qualitative observations.

First, if money would be useful for facilitating transactions, its value would be fundamental (Tirole, 1985). Moreover, the real interest rate would then exceed 0 if the amount of money remained fixed.

Second, instead of merely introducing a fixed amount of money at the beginning of time, the government could vary the money supply. It could print additional money every period, and it might use this revenue to fund a stream of lump-sum transfers. In this case (whether or not the money serves transaction purposes), the interest rate could drop below zero, opening up the possibility for money to coexist with bubbles on the productive assets. Alternatively, the government might reduce the money stock every period. It could use tax revenue to purchase money, thereby producing deflation and a larger positive interest rate.

Third, the model can readily be extended to admit growth, for example by letting the dividend d grow at rate $g > 0$. In this case, there can be bubbles on trees as long as $r < g$. In a stationary equilibrium, the real value of the money stock grows at rate g. If $r < g$, the government gets seigniorage every period even if it does not inflate. If $r > g$, the government must engage in taxation to cover the difference between r and g, but notice that with positive growth the net present value of these sacrifices is always smaller than the net present value of the government's debt.

4 The full model

We now add a number of realistic features to the model. On the production side, we let the homogeneous consumption good be produced by capital and labor. Moreover, we introduce an imperfectly competitive intermediate input goods sector, whose rents will be traded in the stock market. On the consumption side, the rents allow us to keep a similar source of wealth fluctuation as that considered in the endowment economy. In addition, we let people's labor productivity vary in such a way as to create a significant savings motive. Finally, we now replace money in the form of currency by one-period public debt that pays a nominal rate of interest, and we specify a system of redistributive taxation.

Let us begin by describing public debt and government policy: Each period, the government issues a single class of one-period nominal assets, M_t, that pay a gross return $1 + r_t^m$.[28] Government policy consists of setting the nominal interest rate (which we usually associate with monetary policy) and the growth rate, g_t^m, of the stock of government debt M_t (which we usually associate with fiscal policy). The government also imposes a proportional pension tax, τ_t^p, and potentially makes lump-sum payouts to all agents as well as some targeted transfers described below.

As before, time is discrete and the economy populated by a continuum of length 1 of heterogenous agents. In order to fit observed saving rates, and the associated demand for financial assets, we need to introduce some life-cycle or precautionary savings motive. We thus assume that agents "age" stochastically; they are either workers or retired. In each period a worker supplies an exogenous amount of one unit of labor services, whereas a retired agent receives a targeted lump-sum pension θ from the government. Age follows a first order Markov process.[29]

There is a single final good, which is used for both consumption and investment. The final goods sector is perfectly competitive. The final good is produced by a continuum A of imperfectly substitutable intermediate goods. Intermediate goods producers are all monopolies. Each monopoly owns an idea, that allows it to produce a specific intermediary good, by combining physical capital and labor. Ideas survives to the next period with probability $\sigma < 1$. If an idea becomes obsolete, the firm can no longer produce the intermediary good. Obsolete ideas are replaced with new ideas. Each new idea is paired with an agent who then becomes an entrepreneur and sets up an intermediate firm. All new firms are initially private companies, and the agent carries all the entrepreneurial risk associated with the death risk of the idea. Each period, each entrepreneur is enabled with probability σ_{12} to (costlessly) issue equity in their firm and transform it into a public company. Let A_1 denote the continuum of private companies and A_2 the continuum of public companies, where $A = A_1 + A_2$.

Each agent chooses a consumption-savings plan. There are three types of financial assets; bonds, equity and physical capital. Let m_t and k_t denote an agent's holding of bonds and physical capital respectively. We assume that investors do not keep track of companies' birth. At most, they remember the previous period's market price. Thus, to the extent that companies' price depend on the company's history, the dependence only

[28]We could admit multiple assets that differ in their duration. Such realistic extensions might be important in a model with aggregate shocks or government commitment problems, but would be immaterial here. Think of this model as as the "cashless limit" of such a richer model.

[29]Since agents are infinitely lived, stochastic aging is not equivalent to introducing missing markets between generations as in the overlapping generations framework.

involves price movements since the date they where listed. Henceforth, let $p_{j,t}$ denote the price at date t of a public company that was listed at date j. Similarly, let $a_{j,t}$ denote an agent's holdings at date t of a perfectly diversified portfolio of public companies listed at j and let $a_t = \{a_{j,t}\}_{j=-\infty}^{t-1}$.

We shall study equilibria in which the government follows a policy rule such that the aggregate stock of nominal debt grows at constant rate g^m;[30] $M_{t+1} = (1+g^m)M_t$, and where the revenue from changing the stock is used to finance lump-sum transfers (taxes); $\tau_t^m = p_t^m(M_{t+1} - (1+r^m)M_t)$. Substituting in the policy rule we have $\tau_t^m = (g^m - r^m)p_t^m M_t$. (It does not matter that the revenue is ear-marked for lump-sum as opposed to targeted transfers, but this accounting facilitates the specification of an actuarily fair social insurance system.)

The timing convention within a period is as follows. (i) Production takes place and income is distributed. (ii) A fraction σ_{12} of the entrepreneurs get the opportunity to make an IPO. (iii) Agents consume and save. (iv) New ideas are realized. (v) A fraction of the ideas become obsolete and the associated intermediary firms are destroyed.

Assume that the survival rate of private and public firms are $\sigma_1$and σ_2 respectively. The timing convention implies that the ratio of public to private firms are $A_2/A_1 = \sigma_2\sigma_{12}/(1-\sigma_2)$ and that $(1/\sigma_1 - 1 + \sigma_{12})\, A_1$ new ideas are realized each period.

Firms and production: The final goods sector is assumed to be competitive. Firms are characterized by the production function

$$Y_t = \left(\int_0^A (y_t^n)^\eta \, dn \right)^{1/\eta},$$

where y_t^n is an intermediary input and where $0 < \eta \leq 1$.

A final goods producer's problem is thus

$$\max_{y_t^n} \left(\int_0^A (y_t^n)^\eta \, dn \right)^{1/\eta} - \int_0^A p_t^y y_t^n dn,$$

where p_t^n is the price of intermediate good n. From the first-order condition is with respect to intermediary good n we define a downward sloping demand curve for intermediate good n as

$$p_t^y(y_t^n) \equiv \left(\frac{y_t^n}{Y_t} \right)^{\eta-1}.$$

The intermediate goods sector consist of a continuum A of monopolies. Private and public monopolies are identical in all dimensions except for their ownership structure. Each monopoly owns an idea that allows it to produce a specific intermediary good by

[30] By this assumption, public debt is a Ponzi-scheme, just as in Aiyagari and McGrattan (1998). However, there the debt is real, not nominal.

combining physical capital and labor using the technology

$$y_t^n = (k_t^n)^{\alpha} (\lambda_t h_t^n)^{1-\alpha},$$

where labor productivity $\lambda_t = (1+g)^t$ grows at an exogenous rate g. At each date t, each intermediary firm n maximizes profits

$$d_t^n = Y_t^{1-\eta} \left((k_t^n)^{\alpha} (\lambda_t h_t^n)^{1-\alpha} \right)^{\eta} - w_t h_t^n - r_t^k k_t^n$$

by renting capital k_t^n at price r_t^k and hiring labor h_t^n at price w_t.

We only consider symmetric equilibria where $y_t^n = y_t$ and thus in equilibrium $Y_t = A^{1/\eta} y_t$. From the first-order conditions,

$$\begin{aligned} r_t^k &= \eta \alpha N^{1/\eta - 1} k_t^{\alpha} (\lambda_t h_t)^{(1-\alpha)} k_t^{-1}, && (11) \\ w_t &= \eta (1-\alpha) N^{1/\eta - 1} k_t^{\alpha} (\lambda_t h_t)^{(1-\alpha)} h_t^{-1}, && (12) \end{aligned}$$

which implies that profits are given by

$$d_t = N^{1/\eta - 1} k_t^{\alpha} (\lambda_t h_t^n)^{(1-\alpha)} (1-\eta). \tag{13}$$

Note that the timing convention implies that the total supply of equity at the end of a period is $\bar{A}_2 = A_2 / \sigma_2$. Moreover, aggregate labor demand is $H_t = A h_t$, and aggregate demand for physical capital is $K_{t+1} = A k_{t+1}$.

Agents: The problem facing agents is similar to that in the endowment economy, so to save space we merely highlight the new features. The initial state of an agent is now given by $x_0 = (a_0, m_0, k_{0,} \varepsilon_0) \in X$, where $X = \mathrm{R}_+^{\infty} \times \mathrm{R}_+ \times \mathrm{R}_+ \times \{0,1\}^3$. Here $\varepsilon_t = \left(\varepsilon_t^a, \varepsilon_t^e, \varepsilon_t^i \right)$, where ε_t^a denotes whether an agent is of working-age ($\varepsilon_t^a = 1$) or retired ($\varepsilon_t^a = 0$), ε_t^e denotes whether an agent is an entrepreneur ($\varepsilon_t^e = 1$) or not ($\varepsilon_t^e = 0$), and ε_t^i denotes whether the agent received an opportunity to make an IPO ($\varepsilon i_t = 1$) or not ($\varepsilon_t^i = 0$). Let Γ denote the Markov transition matrix that describes how ε_t evolves over time. We assume that age is independent of entrepreneurship and IPO opportunities. Moreover, we assume that (i) an agent can at most have one idea, (ii) an entrepreneur who receives an opportunity to make an IPO can not receive a new idea in the same period, and (iii) a newborn firm can not enter the stock market. Thus,

$$\begin{aligned}
\Gamma\left(\left(\varepsilon^a_{t+1}=1,\cdot,\cdot\right) \mid \left(\varepsilon^a_t,=s\cdot,\cdot\right)\right) &= \Gamma_{s1} \text{ for } s=0,1;\\
\Gamma\left(\left(\varepsilon^a_{t+1}=0,\cdot,\cdot\right) \mid \left(\varepsilon^a_t,=s\cdot,\cdot\right)\right) &= 1-\Gamma_{s1} \text{ for } s=0,1;\\
\Gamma\left(\left(\cdot,\varepsilon^e_{t+1}=1,\varepsilon^i_{t+1}=0\right) \mid \left(\cdot,\varepsilon^e_t=0,\varepsilon^i_t=0\right)\right) &= \sigma_1\left(1/\sigma_1-1+\sigma_{12}\right)A_1/(1-A_1);\\
\Gamma\left(\left(\cdot,\varepsilon^e_{t+1}=1,\varepsilon^i_{t+1}=1\right) \mid \left(\cdot,\varepsilon^e_t=0,\varepsilon^i_t=0\right)\right) &= 0;\\
\Gamma\left(\left(\cdot,\varepsilon^e_{t+1}=1,\varepsilon^i_{t+1}=0\right) \mid \left(\cdot,\varepsilon^e_t=1,\varepsilon^i_t=0\right)\right) &= \sigma_1(1-\sigma_{12});\\
\Gamma\left(\left(\cdot,\varepsilon^e_{t+1}=1,\varepsilon^i_{t+1}=1\right) \mid \left(\cdot,\varepsilon^e_t=1,\varepsilon^i_t=0\right)\right) &= \sigma_1\sigma_{12};\\
\Gamma\left(\left(\cdot,\varepsilon^e_{t+1}=0,\varepsilon^i_{t+1}=0\right) \mid \left(\cdot,\varepsilon^e_t=1,\varepsilon^i_t=1\right)\right) &= 1;\\
\Gamma\left(\left(\cdot,\varepsilon^e_{t+1}=0,\varepsilon^i_{t+1}=1\right) \mid \left(\cdot,\cdot,\cdot\right)\right) &= 0.
\end{aligned}$$

This economy features exogenous growth of productivity and supply of bonds. To solve the agents' problems we therefore normalize real variables by the exogenous growth rate in productivity, and we normalize nominal variables by the growth rate of nominal bonds. Let variables with a tilde denote variables de-trended by productivity, e.g., $\widetilde{c}_t \equiv \frac{c_t}{\lambda_t}$. Moreover let $\bar{p}^m_t \equiv \frac{p^m_t(1+g^m)^t}{\lambda_t}$, $\bar{m}_t \equiv \frac{m_t}{(1+g^m)^t}$, and $\widetilde{\beta} = \beta(1+g)^{1-\mu}$. An agent's budget constraints is then given by

$$\begin{aligned}
\widetilde{c}_t(\varepsilon^t;x_0) &= \widetilde{y}_t + \widetilde{\tau}^m_t + \sum_{j=-\infty}^{t-1}(\widetilde{p}_{j,t}+\widetilde{d}_t)\sigma_2 a_{j,t} - \sum_{j=-\infty}^{t}\widetilde{p}_{j,t}a_{j,t+1} + \bar{p}^m_t\left((1+r^m_t)\bar{m}_t-(1+g^m)\bar{m}_{t+1}\right)\\
&\quad + (r_t+1-\delta)\widetilde{k}_t - (1+g)\widetilde{k}_{t+1} + \widetilde{d}_t\varepsilon^e_t + \widetilde{p}_t\varepsilon^i_t, \qquad (14)
\end{aligned}$$

for all all $\varepsilon^t \in \mathcal{E}^t$, $t=0,1,....$, where $a_{j,t+1}=\phi_{a,j,t}(\varepsilon^t;x_0)$, $\bar{m}_{t+1}=\phi_{m,t}(\varepsilon^t;x_0)$, $\widetilde{k}_{t+1}=\phi_{k,t}(\varepsilon^t;x_0)$, $\widetilde{y}_t=(1-\tau-\tau^p)\left(\widetilde{w}_t\varepsilon^a_t+(1-\varepsilon^a_t)\widetilde{\theta}_t\right)$, τ is a labor income tax, and δ is the depreciation rate for physical capital.

An agent's problem is thus to maximize expected discounted lifetime utility (6), with β replaced by $\widetilde{\beta}$, by a set of choices $\phi_t(\varepsilon^t;x_0)$ for all t, subject to (14) and $\phi_t(\varepsilon^t;x_0)\in X$, and taking as given the sequences for prices $\left\{\{\widetilde{p}_{j,t}\}_{j=-\infty}^{\infty},\bar{p}^m_t,r_t,\widetilde{w}_t\right\}_{t=0}^{\infty}$, transfers $\{\widetilde{\tau}^m_t\}_{t=0}^{\infty}$, pensions $\left\{\widetilde{\theta}_t\right\}_{t=0}^{\infty}$, tax rates $\{\tau,\tau^p\}$, as well as the transition matrix Γ and the initial state x_0.

Markets: Market clearing in financial markets is given by

$$\bar{M}_0 = \int_X m_0 d\kappa,\ t=0, \qquad (15)$$

$$\sigma_2^{t-j}(1-\sigma_2)\bar{A}_2 = \int_X a_{j,0}d\kappa(x_0),\ t=0,\forall\, j=-\infty,...,0. \qquad (16)$$

$$\widetilde{K}_0 = \int_X k_0 d\kappa,\ t=0, \qquad (17)$$

$$\bar{M}_t = \int_X \sum_{\varepsilon^{t-1} \in \mathcal{E}^{t-1}} \phi_{m,t-1}(\varepsilon^{t-1}; x_0) \gamma^{t-1}(x_0, \varepsilon^{t-1}) d\kappa, \ \forall\, t > 0, \tag{18}$$

$$\sigma_2^{t-j}(1-\sigma_2)\bar{A}_2 = \int_X \sum_{\varepsilon^{t-1} \in \mathcal{E}^{t-1}} \phi_{a,j,t-1}(\varepsilon^{t-1}; x_0) \gamma^{t-1}(x_0, \varepsilon^{t-1}) d\kappa, \ \forall\, t > 0, \forall\, j \geq -\infty, \tag{19}$$

$$\widetilde{K}_t = \int_X \sum_{\varepsilon^{t-1} \in \mathcal{E}^{t-1}} \phi_{k,t-1}(\varepsilon^{t-1}; x_0) \gamma^{t-1}(x_0, \varepsilon^{t-1}) d\kappa, \ \forall\, t > 0. \tag{20}$$

Similarly, goods market clearing implies that

$$\widetilde{Y}_t = \int_X \sum_{\varepsilon^t \in \mathcal{E}^t} c_t(\varepsilon^t; x_0) \gamma^t(x_0, \varepsilon^t) d\kappa + \widetilde{G}_t + (1+g)\widetilde{K}_{t+1} - (1-\delta)\widetilde{K}_t,$$

where G denotes government expenditures. Finally, labor market clearing implies that $H = 1$.

Government: To facilitate accounting, and without loss of generality of our results, we depict the government as running three separate budgets. Government expenditure is financed through a labor income tax

$$\widetilde{G}_t = \tau \widetilde{w}_t H, \tag{21}$$

public pensions is financed through a pension-tax

$$\int_X \sum_{\varepsilon^t \in \mathcal{E}^t, \varepsilon^a_t = 1} (1-\tau^p)\widetilde{\theta}_t \gamma^t(x_0, \varepsilon^t) d\kappa = \tau^p \widetilde{w}_t H, \tag{22}$$

and the lump-sum transfer is financed through the growth of nominal public debt net of interest payments

$$\widetilde{\tau}^m_t = (g^m - r^m)\, \widetilde{p}^m_t \bar{M}_t. \tag{23}$$

Equilibrium: An equilibrium comprises sequences of prices $\left\{ \{\widetilde{p}_{j,t}\}_{j=-\infty}^{\infty}, \bar{p}^m_t, r^k_t, \widetilde{w}_t \right\}_{t=0}^{\infty}$, dividends $\left\{ \widetilde{d}_t \right\}_{t=0}^{\infty}$, transfers $\{\widetilde{\tau}^m_t\}_{t=0}^{\infty}$, pensions $\left\{ \widetilde{\theta}_t \right\}_{t=0}^{\infty}$, government expenditure $\left\{ \widetilde{G}_t \right\}_{t=0}^{\infty}$, aggregate capital $\left\{ \widetilde{K}_t \right\}_{t=0}^{\infty}$, tax rates $\{\tau, \tau^p\}$, sequences of decisions $\{c_t(\varepsilon^t; x_0), \phi_t(\varepsilon^t; x_0)\}_{t=0}^{\infty}$ for all $x_0 \in X$ and for all $\varepsilon^t \in \mathcal{E}^t$, together with probability measures $\{\gamma^t(x_0, z)\}_{t=0}^{\infty}$ for all $x_0 \in X$ and for all $z \in \mathcal{E}^t$, and a measure $\kappa(x)$ for all $x \in \mathcal{X}$ describing the initial distribution, such that (i) the decision rules solve the agents' problem given prices and the initial state x_0, (ii) factor prices are given by (11)-(12), (iii) dividends are given by (13) (iv) all markets clear, (v) the government budget constraints (21)-(23) are satisfied, and (vi) the measure $\gamma^t(x_0, \varepsilon^t)$ is consistent with the transition matrix Γ.

Analysis: The same types of stationary equilibria that exist in the endowment model exist here. As before, they can be characterized by simple no-arbitrage conditions. Let the variables $\widetilde{R}^a_{j,t} = \sigma_2(\widetilde{p}^a_{j,t+1} + \widetilde{d})/\widetilde{p}^a_{j,t}$, $\widetilde{R}^k_t = \left(r^k_{t+1} + 1 - \delta\right)/(1+g)$, and $\widetilde{R}^m_t = \widetilde{p}^m_{t+1}(1 + r^m_t)/\left[\widetilde{p}^m_t(1+g^m)\right]$ denote the growth adjusted return to holding equity, physical capital and bonds respectively. If nominal bonds are valued, then no-arbitrage implies that $\widetilde{R}^a_{j,t} = \widetilde{R}^a_t = \widetilde{R_t}^k = \widetilde{R}^m_t \;\forall\; j$ and t, and if nominal bonds are not valued the last equality is replaced with a strict inequality. In a stationary equilibrium with nominal bonds, this implies that $\widetilde{R}^a = \widetilde{R}^k = \widetilde{R}^m = (1 + r^m)/(1 + g^m)$. Thus, the real return to holding equity, physical capital and bonds respectively is

$$R^a = R^k = R^m = (1 + g)\,\frac{1 + r^m}{1 + g^m}.$$

Note that the real interest rate depends not only on real growth, but also on the nominal interest rate and on the growth rate of nominal assets. In fact, as long as nominal bonds are valued, the government controls the long-run real interest rate in this economy. It does so either by using monetary policy (setting the nominal interest rate) or through fiscal policy (setting the rate of growth of nominal government debt, taxes and expenditures). What matters for the real interest rate is the difference between the nominal interest rate and the growth rate of nominal debt; the real interest rate can be every bit as high with a low nominal interest rate and a low growth rate in nominal debt as with higher nominal interest rates and higher growth rates of nominal debt.

Note also that since nominal bonds pay a nominal rate of interest, bond equilibria exist for real interest rates both below and above the real growth rate. In the former case, $g^m > r^m$ and the nominal debt is a pure Ponzi-contract; that is serving the public debt does not require real resources at any time. To the contrary, as can be seen from (23), the public debt generates revenue in every period. In the latter case, when $g^m < r^m$ and the real interest rate exceeds the economy's growth rate, debt takes the form of interest-bearing securities and is no longer a bubble. Serving the public debt requires real resources. However, only a part of the interest payments requires a real sacrifice; the rest can be financed through the issuance of new bonds. Public debt is hence still a Ponzi-contract which imposes a smaller cost on the debtor than the benefits obtained by the creditor.

Note further that equilibria with price bubbles on equity can only exist if the real interest is strictly below the real growth rate, just as in the endowment economy. Specifically, for a given $r < g$ there is a continuum of equilibria that differ in terms of the size of the bubble on equity and the bubble on public debt. We return to this issue in Section 6.

Finally, let x_t denote the aggregate demand for government debt, $x_t \equiv \bar{p}^m_t \bar{M}_t$, and let g^x_t denote the percentage change in x_t between $t-1$ and t. Then,

$$g^x_t = \frac{(1 + g^m)}{(1 + \pi_t)(1 + g)} - 1,$$

where $\pi_t \equiv p^m_{t-1}/p^m_t - 1$ denotes inflation. Thus, inflation is

$$\begin{aligned} \pi_t &= \frac{(1+g^m)}{(1+g^x_t)(1+g)} - 1 \\ &\approx g^m - g - g^x_t. \end{aligned} \tag{24}$$

In in a stationary equilibrium $g^x_t = 0$ and steady state inflation thus satisfies the the quantity theory;

$$\pi \approx g^m - g. \tag{25}$$

However, in the short run (along a transition path), changes in the demand for government debt will cause inflation to deviate from the quantity theory. It is also important to note that we here study equilibria in which the growth of the public debt is used to pay interest and to fund transfers. We do not claim that a similar quantity relationship would hold if bonds are being used to purchase stocks or capital. To the contrary, there are steady states in which the government owns a share of the capital, with the exact same price of bonds. The gross public debt is larger, but the net public debt is the same. In other words, a fiscal theory of the price level holds across these equilibria.

5 The Magnitude of the Non-fundamental

We are now ready to investigate how large the aggregate non-fundamental value will be in a calibrated steady state of our model. We aim to fit macro-figures for the United States in recent times, so we will be in a regime with $r < g$. More specifically, we think of the model as representing average US fundamentals during the last two decades.

Instead of guessing the location of non-fundamental asset values, we assume for now that the whole non-fundamental value takes the form of (Ponzi) public debt, which is thus the residual variable to be determined. Before displaying the computations, a caveat is in place. In interpreting our model's measure of non-fundamental value, we must keep in mind that at least three major factors are missing from the model: (i) Other productive assets, such as land and homes – their inclusion would reduce our Ponzi-value estimate; (ii) other unproductive assets that would serve as stores of value – their inclusion only affects the distribution of non-fundamental values across asset classes; (iii) aggregate uncertainty – with more uncertainty, market prices of risky assets would be lower in the model, so uncertainty would increase our Ponzi-value estimate. Finally, the model does not account for the fact that public debt is traded internationally. For example, a large fraction of US debt is held by foreigners.

Since agents in our model experience heterogeneous histories, wealth and income distribution are important targets for our calibration. In order to produce a suitable fraction of rich agents in our model, without introducing many different wage levels, our assumption about entrepreneurial incomes is central. Here, we set the fraction of the population that has received an idea that is still in use to 1.5 percent ($A_1 = 0.015$). As will become clear, this parameter choice contributes strongly to the model's ability to match the observed Gini-coefficient for wealth, which will be 0.82 – as it is in US data (see Díaz–Giménez, Glover and Ríos-Rull, 2011).

Regarding all remaining parameters we either use standard values from the literature or calibrate the parameters to match long-run averages in US data. The model period is one year. As before we set the discount factor β to 0.97, and the coefficient of relative risk aversion μ to 3. The price mark-up is set to 20 percent ($\eta = 5/6$), which is in the mid-range of estimates of US markups.[31] The resulting profit is the source of dividends, whose net present value constitutes the fundamental value of the stock. The capital share in production, $\alpha = .196$, is set to match a labor share in income of 2/3. The depreciation rate, $\delta = 0.06$, is set to match an investment to GDP ratio of 0.17 The exogenous growth rate of productivity, g, is set to 2 percent, to match the average growth rate in real GDP/hour between 1950 and 2010.[32].

We set the survival probability for public intermediate firms, σ_2, to 0.98 so that the value of new firms is 2 percent of total stock market value, matching the long-run average reported by Jovanovic and Rosseau (2005)). To be on the conservative side we also set the survival probability for private firms, σ_1, to 0.98; realistically lower values would amplify the effect of policy interventions. The fraction σ_{12} of entrepreneurs who get the opportunity to make an IPO is set so that the share of private firms is 90 percent. Interpreting private firms as "small" firms, this is the number reported by Bartelsman et al. (2004) for industrialized countries. Moreover, the latter two choices imply a capital-output ratio and a value of the stock market to GDP which are in line with US data.

According to Caselli and Feyrer (2007), the US capital-output ratio is 2.19 and according to data from the World Bank the stock market to GDP ratio in the Unites States has in the last twenty years varied between 0.6 and 1.6,[33] with an average of 1.09. In the model, these are 2.22 and 1.05 respectively.[34]

We interpret working-age and retirement as corresponding to ages 20-64 and 65-79 respectively, to match a retirement age of 65 and an expected longevity of 79 years. Hence, we set the transition probabilities between working-age and retirement so that agents spend on average 45 years working followed by 15 years in retirement. The pay-roll tax is set to 8.5 percent, so that Social Security payments are 4.2 percent of GDP, which is what Wallenius (2013) reports for the United States. The labor income tax, τ, is set to 26

[31]See for example Hall (1988), Basu and Fernald, (1997), Christopoulou and Vermeulen (2008), Martins, Scarpetta, and Pilat (1996) and Morrison (1992).

[32]Based on data from the Groningen Growth and Development Center (http://www.ggdc.net, the Total Economy Database, September 2011)

[33]The World Bank data refer to Stock Market Capitalization to GDP for United States, series id DDDM01USA156NWDB, World Bank, Global Financial Development (Not a Press Release), Not Seasonally Adjusted

[34]Note that here and henceforth we calculate the value of the stock market prior to the destruction of ideas. Ex post, the number is σ_2 times the ex ante value.

percent, implying a government expenditure of 19 percent of GDP.

Finally, regarding the policy parameters, recall that only the difference between r^m and g^m matters. Between 1950 and 2010, US CPI inflation has been on average 3.8 percent and the nominal interest on treasury bills has on average been 4.8 percent. Thus, the average real interest rate has been 1 percent since 1950. However, since the change in monetary policy in the early 1980's the real interest rates has been substantially higher for most of the time, and we have therefore chosen to match the post 1985 average of 1.5 percent. Specifically, we implement an nominal interest rate of 3.5 percent and a money growth rate of 4 percent, implying an inflation of 2 percent in the stationary equilibrium.

These policies imply a government debt to GDP ratio of 120 percent, which is considerably larger than the actual figure of US public debt, especially when we take into account the debt that is held by foreigners. Thus, with the caveats stated in the beginning of the section, we conclude that the model admits sizable bubbles on private assets in addition to the Ponzi-value contained in the public debt.

5.1 Policy opportunities

To get a rough sense of the long-run trade-offs that authorities face, we will now study the impact of changing the real interest in this economy. Of course, since the public debt is larger in the model than in reality, the exact numbers will be somewhat exaggerated.

Because the returns to a perfectly diversified portfolio of equity, physical capital and government debt are equalized, agents are indifferent between holding the different forms of assets. For simplicity we thus assume that all agents hold the same portfolio shares.[35]

Figure 2 displays, for the current parameters, the impact on output from changing the real interest rate. Note that there is a lower bound (denoted by a vertical line in the figures henceforth) to the real interest rate at about 0.65 percent. Below this level, bonds are not valued in equilibrium, and hence neither interest rate policy nor debt policy can affect the real interest rate. The relationship is virtually linear over the relevant range, and a one percentage point increase in the real interest rate entails almost a three percentage point decrease in output.

To better understand the importance of our main assumption, that there is a market for equity in existing firms but that agents cannot trade securities based on future projects that have not yet materialized, we consider two versions of our economy. In the first version, there is no market for equity (and hence no market for future projects). In the second, there is an equity market and claims on future projects are implicitly priced through assets that exist today.

We implement the first version – maximally incomplete stock markets – by assuming that all ideas die with certainty ($\sigma_1 = \sigma_2 = 0$). This is essentially the model econ-

[35]Clearly, this assumption would be inappropriate in a model with aggregate shocks.

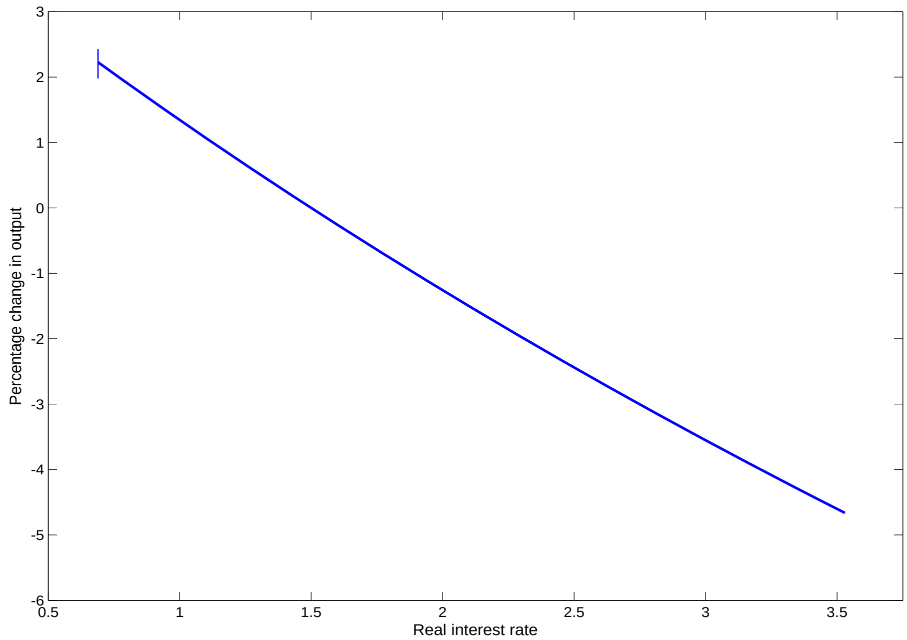

Figure 2: The impact of the real interest rate on real output

omy in Aiyagari and McGrattan (1998) and Flodén (2001) albeit with a slightly different production structure and process for idiosyncratic risk and with nominal instead of real government bonds. This economy is quite similar to our benchmark economy in several respects. There exist stationary bond equilibria with real interest rates both above and below the real growth rate, but the lowest real interest rate for which a bond equilibrium exists is higher than in the benchmark economy. The reason is that with no IPOs, entrepreneurs become less wealthy, and aggregate savings are smaller. If the real interest rate is 1.5 percent, the public debt to GDP ratio is 96 percent, compared to 120 percent in the benchmark economy. Furthermore, without IPO's wealth inequality is less stark; the Gini coefficient on wealth is 0.57, compared to 0.82 in the benchmark economy. While there are ways to recalibrate the idiosyncratic risk agents face in order for the model to better match the degree of inequality we observe in the data, e.g., using the approach in Castaneda et al. (2003), these perturbations do not come to grips with the objection of Santos and Woodford (1997) that the bubble regime only exists because there are severe limitations to the types of assets agents can trade. In the second version, we therefore add a market for equity and assume that there is no market incompleteness with respect to future projects.

We implement this second version – maximally complete stock markets – by assuming that ideas in private firms die with certainty ($\sigma_1 = 0$), while ideas in public firms survive with certainty ($\sigma_2 = 1$). Thus, no new firms enter ($\sigma_{12} = 0$) the stock market. The assumption is equivalent to assuming that ideas in public firms die with some probability but that all new public firms are off-spring of existing public firms. This is essentially the framework of Santos and Woodford (1997). In this economy, there is no stationary equilibrium in which the real interest rate is below the real growth rate, and hence there is no equilibrium in which there are price bubbles on trees. In fact, with this calibration, a stationary equilibrium with positive public debt only exists if the real interest rate is above 2.88 percent.

Compared to these two extreme regimes, our parameter choice of $\sigma = 0.98$ is not only more realistic; it also yields a better fit to the data.

6 A Stock Market Crash

As noted above, price bubbles on equity can exists in equilibrium if the real interest rate is below the real growth rate. In particular, for any $r < g$ there exists a continuum of stationary equilibria. Loosely speaking, these equilibria can be indexed by the real value of debt ($\widetilde{p}_t^m \bar{M}_t$), the associated lump-sum transfers ($\widetilde{\tau}_t^m$), the price of new public firms $\widetilde{p}_{t,t}$ and the asset distribution κ. For example, consider our benchmark calibration where policy is such that the real interest rate is 1.5 percent. Then one equilibrium (the one studied above) entails no price bubbles on trees and a public debt to GDP ratio of 1.2. But there also exist equilibria for any value of public debt in the range $0 - 120$ percent of GDP: The lower the public debt, the larger is the price of new public firms, and the larger is the total bubble on equity. A smaller public debt to GDP implies lower lump-sum transfers, which reduces publicly provided insurance. A higher price of new firms implies that entrepreneurs making an IPO become richer. Hence, comparing across stationary equilibria, agents with low savings are worse off the smaller the public debt and the larger the price bubble on equity, while the opposite is true for wealthy agents. From an ex-ante perspective and for a given real interest rate, average welfare[36] is thus higher the higher is public debt.[37] For example, ex ante steady state average welfare is 0.44 percent higher in the benchmark stationary equilibrium where the entire bubble is on public debt than in an equilibrium with a public debt to GDP ratio of 95 percent and a price bubble on equity of 26 percent of GDP.[38]

Many observers believe that the recent financial crisis, as well as Great Depression and several other crises, were caused by collapsing asset price bubbles. Let us now use our last example to illustrate how our model can capture such a crisis. In the equilib-

[36]Welfare is measured using a utilitarian social welfare function and expressed in terms of permanent changes in consumption. Some of our welfare analysis is conducted from an ex ante perspective – behind the veil of ignorance – comparing ex-ante average welfare across stationary equilibria, but we also investigate transitions from one steady state to another. We refer to the former as steady state welfare gains.

[37]In a more general model, there could be offsetting benefits from stock price bubbles. For example, investment might depend on the net worth of entrepreneurs or top managers.

[38]Note that since lump-sum transfers and the price of new public firms differ across equilibria, the size of aggregate saving and thus the aggregate bubble (on private equity and on public debt) also differ across equilibria even though the real interest rate is the same.

rium under consideration, the stock market value to GDP ratio is 1.31.[39] Suppose now that for some reason the price bubble on equity bursts, and suppose furthermore that policy is left unchanged. In the stock market crash, the value falls by 20 percent, back to the fundamental value of 1.05, as the bubble vanishes. As a consequence, the real value of government debt must increase from 0.95 to 1.2 as a fraction of GDP, reflecting the demand for assets that moves out of stocks and into bonds. This is our favored interpretation of the oft-repeated claim that "there is a scarcity of safe assets" during a financial crisis. Such an increase in the market value of government debt can come about in two very different ways. Either the authorities do nothing, and the economy experiences an immediate deflation of 19 percent.[40] Alternatively, authorities accommodate the increased demand for bonds by immediately increasing the supply of nominal bonds.

Suppose first the authorities do nothing. If all prices are completely flexible, the surprise deflation associated with the bubble bursting has only little effect on real aggregate variables. The reason is that the fall in the value of stocks are exactly offset by an increase in the real value of public debt, which implies that each agent's wealth is unchanged.[41] The only effects come from an increase in the lump-sum transfer due to a larger public debt and a fall in the price of new public firms. These effects turn out to be quantitatively small.

However, there is widespread concern that deflation has severe consequences. To address this concern, let us now incorporate one channel by which deflation may effect real activity: We assume that nominal wages cannot not be adjusted downwards.[42] More precisely, we impose the constraint $w_t/p_t^m \geq w_{t-1}/p_{t-1}^m$ for all $t \geq 0$. If prices fall, and nominal wages remain constant, real wages increase and firms' labor demand fall. Since all agents of working-age supply one unit of labor inelastically, there is excess supply of labor in equilibrium. Suppose for simplicity that the fall in employment is shared equally by all workers. As long as the growth rate of nominal debt, g^m, is larger than real productivity growth, g, there is inflation in the stationary equilibrium (see equation (25)). In our benchmark calibration, inflation is 2 percent. Hence nominal stickiness will cease to bind at some period, after which the economy will converge to the same stationary equilibrium as if nominal wages were flexible.

Figures 3 and 4 display the effect on real variables and financial prices following the burst of the bubble on equity for three different scenarios: (i) authorities do nothing and

[39]To put this in perspective, the US stock market value to GDP ratio was 1.61 in 2000, and in the years leading up to the recent crisis, the ratio gradually increased to 1.42 in 2007. Source: see footnote 33.

[40]As can be seen from equation (24) the implied deflation depends on our assumption of money growth, g^m, technological change, g, and the change in aggregate demand for government debt, g^x. Here, demand increases by 26 percent.

[41]Nominal government debt thus acts as a hedge against bubbles in the stock market. If the government had issued *real* one-period bonds instead, agents would not have been insured and rich agents would have experienced a significant loss of wealth.

[42]There is a well known spike at zero in the nominal wage change distribution, and this is accentuated at low inflation rates. For recent international evidence and further references, see Holden and Wulfsberg (2014). For explanations of such rigidity, see Bewley (1999).

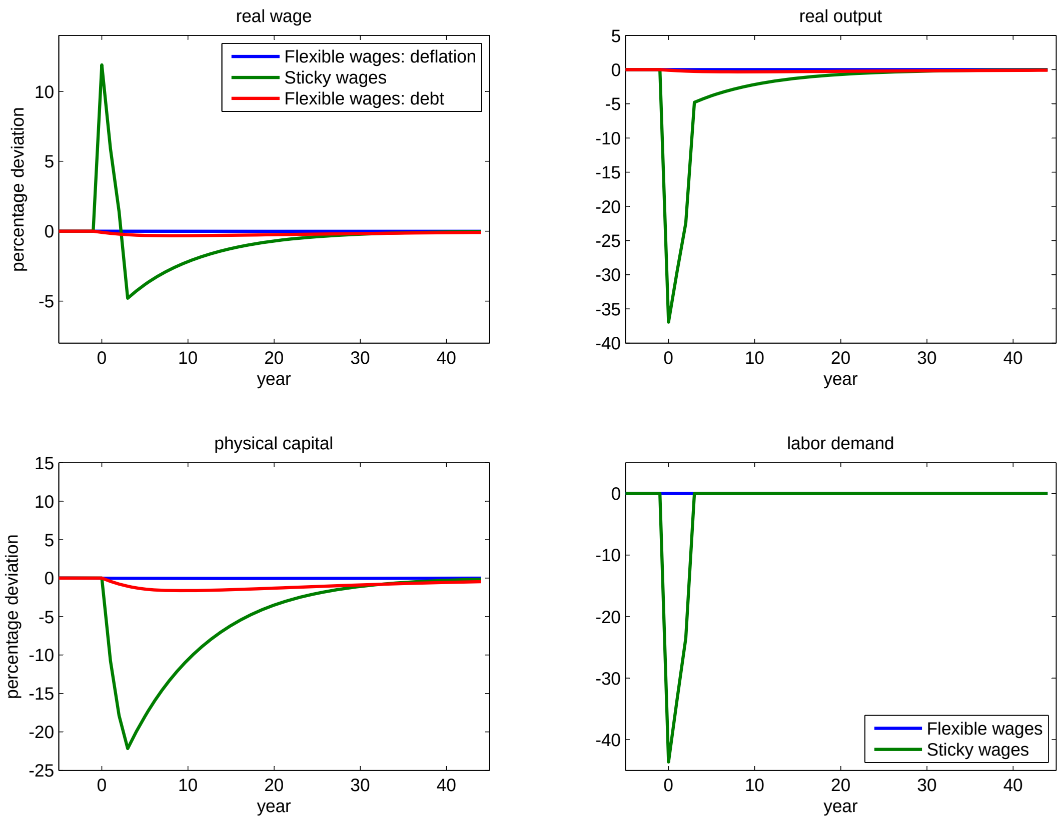

Figure 3: Impact of crash on real variables

nominal wages are flexible, (ii) authorities do nothing and nominal wages are downward sticky, and (iii) authorities accommodate the increased demand for bonds. As just discussed, in scenario (i) there is an immediate deflation of 19 percent, but there is essentially no effect on real aggregate variables. With downward sticky nominal wages, the effect is very different. Deflation causes real wages to increase, causing a fall in labor demand. The resulting fall in labor earnings reduces the demand for all forms of assets, mitigating the deflationary pressure. In this scenario, there is an immediate deflation of 9 percent, and the real wage increases by 12 percent. This leads to a more than 40 percent drop in labor demand, and a 35 percent fall in output. The fall in labor demand causes the real interest to fall by 2.5 percentage points and become negative. The reduced demand for assets reduces investment; the physical capital stock gradually falls by almost 25 percent. Also, the value of the stock market falls more than if wages were flexible; the immediate effect is 30 percent. That is, a significant decline in the fundamental value of the stocks adds to the initial 20 percentage points decline in the bubble value. The order of magnitude of the economic crisis under scenario (ii) thus resembles that of the Great Depression.

To understand what happens during the transition in scenario (ii), it is helpful to con-

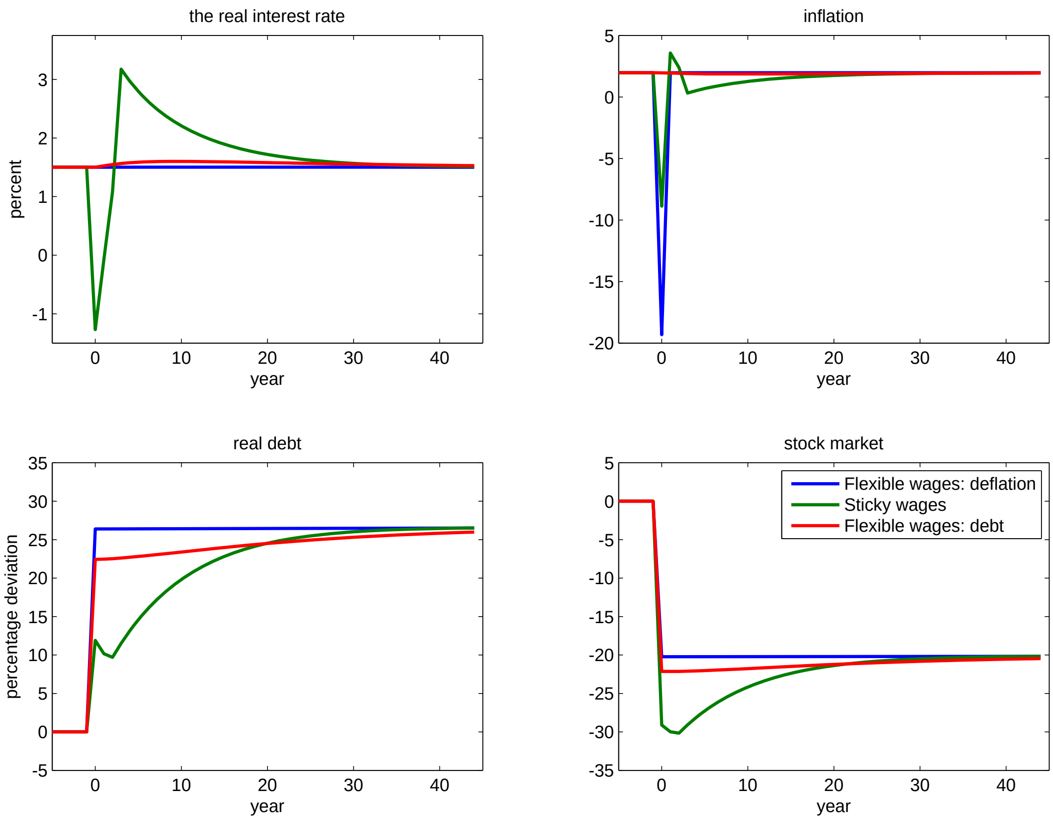

Figure 4: Impact of crash on prices

sider the evolution of government debt and of the real interest rate. As long as nominal stickiness binds, real wages are high, labor demand is depressed, and the real interest rate, $\widetilde{R}_t^m = \widetilde{p}_{t+1}^m(1+r_t^m)/\left[\widetilde{p}_t^m(1+g^m)\right]$, is below that in the stationary equilibrium. The low interest rate causes the real value of debt to fall, which in turn implies that inflation is above it long-run level (see equation 24). Over time, the economy must converge to the 'flexible wage' stationary equilibrium. Hence, the real value of debt must increase as the real interest rate converges to the real interest in the stationary equilibrium, $\widetilde{R} = (1+r_t^m)/(1+g^m)$, from above. Thus, when nominal stickiness ceases to bind, the real interest jumps up. At the same time, labor demand and output jump up and the real wage and inflation jump down. From then on, all variables converge monotonically towards their stationary values. But as the figures show, even though nominal stickiness only binds for three periods (years) in his case, it has much more long-lasting effects on the economy. The welfare implications are also huge; the average welfare loss for agents in the economy is 7.8 percent.

The figures also show what happens if fiscal policy accommodates the increased demand for bonds. The scenario we consider is one where accommodation occurs in the first period only. In all other periods policies follow the same path as before the crash.

Furthermore, we assume that accommodation targets an inflation rate in the first period of 2 percent (as in the long-run). Even though this policy implies that the labor market immediately clears, it is quite different from doing nothing in a world of flexible wages. As discussed above, doing nothing under full flexibility implies that the wealth an agent has is unaffected; the fall in stock prices is exactly offset by an increase in the real value of debt. Under the accommodation scenario, the increase in the value of real debt enables massive transfers. To be precise, the government can transfer 22 percent of GDP in the first period. Of course, the agents will not consume such a windfall immediately, but save most of it. This scenario however effectively redistributes from wealthy agents with high saving rates to poorer agents with lower saving rates and compared to scenario (i) aggregate saving falls. This implies that the demand for bonds goes up by less, the value of the stock market falls more, and investment in physical capital decreases. The transition that follows can be understood in the same way as that under scenario (ii). Even though the outcome in terms of aggregate real variables is in between that of scenario (i) and (ii), the welfare implications are very different. Accommodating the increased demand for public debt avoids deflation and it enables large transfers. This results in an average welfare gain of 4.3 percent, as compared to a small gain of 0.49 percent under scenario (i) or a huge loss of 7.8 percent in scenario (ii).[43]

Bad as it is, case (ii) is not the worst case scenario. Here, passive policy involves an unchanged rate of increase in nominal public debt. If the government were instead to respond to the crisis by reducing the rate of increase nominal debt, it would take more time for real wages to fall into line and for employment to recover.

Even in the very long run, when nominal rigidities would presumably dissolve, a tight nominal debt ceiling could have severe consequences. For example, if the growth in nominal government debt is set to zero, the long-run real interest rate will be $r^m + g$, which is above the original interest rate, and thus must entail a higher level of real debt.

The above logic shows that "nominal austerity" is bound to backfire. How, then, could authorities attain real austerity? If we define real austerity as a real ceiling for the public debt, the answer is plain: by committing to accelerate the issuance of nominal public debt. Permanently raising g^m entails higher inflation for a given nominal interest rate, and the reduction of the real interest rate serves to reduce the real value of public debt. Of course, the same objective could be attained through a permanent reduction of the nominal interest rate.[44] Whether this long-run outcome is desirable or not depends on the size of the original bubble. We know that it is not desirable for real interest rates to

[43] The average welfare gain could be even larger if the government aimed for higher inflation in the first period. For example, if the government issued debt targeting an inflation rate of 8 percent, the first period transfer would amount to 26 percent of GDP and the average welfare gain would be 5.2 percent.

[44] Krugman (1998) argues that such an extreme version of "forward guidance" of interest rates might not be credible. To address this concern, we would need to consider the political economy of monetary and fiscal policy. Before doing so, we should make more realistic assumptions about the sharing of the unemployment burden. If unemployment is unevenly distributed, it is conceivable that the lucky majority would oppose welfare-maximizing policies.

fall too far. Thus, if the original bubble was large, the new long-run outcome could also be significantly worse than the original equilibrium.

This example demonstrates the potency of expansionary debt policy, the dangers of nominally defined austerity, and the potential success of (indefinite) forward guidance. Could authorities instead have pursued a temporary expansionary interest rate policy with similar results? To probe this question, consider the effect of reducing the nominal interest rate from 3.5 percent to 0 percent in periods 1-4 (we assume that authorities cannot renege on the promise in period 0 , i.e., when the bubble bursts). Call this scenario (iv). Strikingly, what appears to be quite expansionary monetary policy is largely undone via price effects: The capital stock, labor demand, and output falls a bit less than under passive policy, but there is still a massive crisis. The reduction in the nominal interest rate does however enable larger lump-sum transfers; see equation (23). During the four periods, the government can now distribute a lump-sum transfers of 4 percent of GDP, compared to 0.6 percent in the stationary equilibrium. While some of this transfer is saved, which explains why the capital stocks falls a bit less than under passive policy, the main benefit is the redistribution towards poor agents that suffer from the income loss associated with the decline in labor demand. Increased consumption among these agents implies that the ex ante average welfare loss is halved (to 3.9 percent).

If scenario (ii) resembles the Great Depression, the crisis of 2008 is more akin to scenario (iv), but with considerable fiscal policy added, both through automatic stabilizers and discretionary spending. Still, from 2008 onwards, consumer prices have increased much less than the pre-crisis trend, and we have had significant increases in unemployment.

An additional intriguing part of crises management in recent years has been the use of so-called quantitative easing, where new public debt is being created either to purchase old debt (shorten the maturity of public debt) or to buy private debt and equity. These policies too can be analyzed with the help of the present model. One equilibrium, which is in line with the fiscal theory of the price level, involves no change to inflation or any real variables. The asset purchase merely temporarily reallocates assets across private and public balance sheets.[45]

7 Optimal interest rate policy

Depending on the level of the real interest rate, policy interventions either generate revenue or has to be funded through taxation. Since we assume that these taxes/transfers are lump-sum, policies involve redistribution. An interest rate above the growth rate im-

[45]While we cannot rule out that there are other equilibria, it is straightforward to prove that if quantitative easing moves the price level it impacts real variables too. For example, if the price level is proportional to gross public debt instead of net public debt, as suggested by a simple quantity theory, QE would also have significant effects on real interest rates (which would move inversely with gross debt).

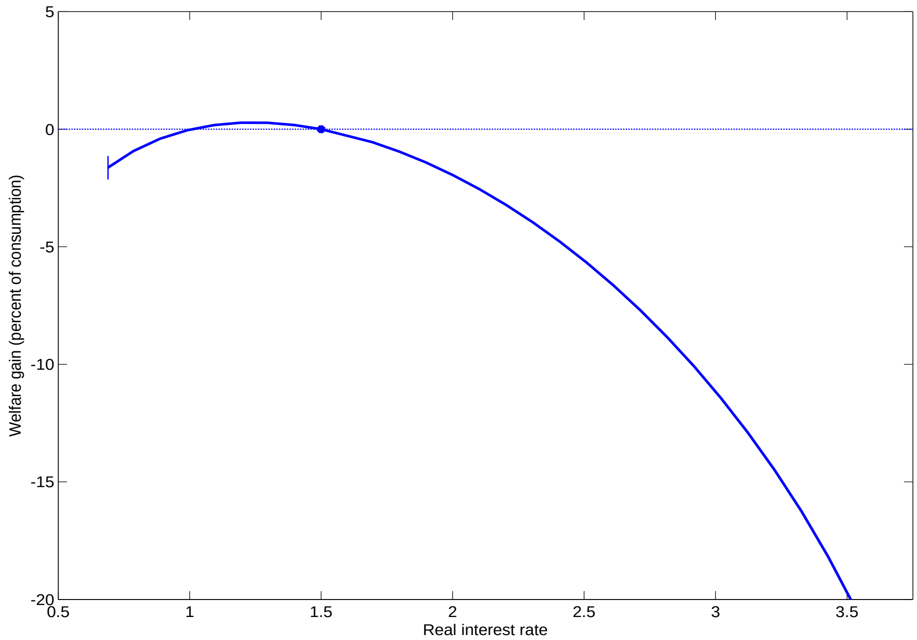

Figure 5: Relationship between the real interest rate and steady state welfare

plies a real cost of serving the public debt. A higher return to savings primarily benefits wealthy agents, and since taxation is lump-sum, a high real interest rate redistributes from poor to rich.

Figure 5 displays the average welfare effects of nominal interest rate policy, comparing the ex ante steady state welfare gain of moving from a stationary equilibrium with 1.5 percent real interest rate (denoted by a dot in the figures henceforth) to stationary equilibria with real interest rates between 0 and 3.5 percent. The figure shows that average steady state welfare is highest for a real interest rate of 1.25 percent, and that the estimated welfare gain of lowering the real interest rate by 25 basis points is 0.29 percent of permanent consumption. The figure also shows that increasing the interest rate towards or above the real rate of growth can be very costly. Setting the real interest rate equal to the real growth rate reduces welfare by almost 2 percent of permanent consumption, and further increases yield even larger losses. But again, we note that these figures are likely to be exaggerated.

Who gains and who loses from changes in the real interest rate? An overwhelming majority of 96 percent of all agents prefer a real interest rate of 1.25 percent compared to an real interest rate of 1.5 percent. Why is this? Reducing the real rate of interest distorts savings which primarily affects wealthy agents. Poor agents gain for two reasons. First, reducing the real interest rate by 25 basis points increases the capital stock, which increases GDP and real wages by 0.9 percent. Second, reducing the interest rate affects the lump-sum transfers agents receive in a non-trivial way. On the one hand, reducing the real interest rate reduces the real value of government bonds. Its ratio to GDP is 29 percent lower. On the other hand, reducing the the real interest rate increases seigniorage for a given amount of debt. The combined effect is that lump-sum transfers increase by almost 10 percent.

It is well known that steady state comparisons can be quite misleading when investigating policy reforms. How important is the transition between steady states for these results? To answer this question, we perform the following experiment. We assume that the economy is in a stationary equilibrium with a real interest rate of 1.5 percent, and that authorities unexpectedly make a once and for all change in policy, such that r^m is reduced by 25 basis points. We then solve for the transition to the new stationary equilibrium.[46] The immediate effect of reducing the nominal interest rate is to reduce the real value of debt by slightly more than 30 percent and to increase the price of equity by 18 percent. During the transition, the real interest rate gradually falls to 1.25 percent, while the real value of bonds and the price of equity gradually increase by 3 and 4 percentage points respectively.

What happens to welfare? The average welfare gain is now smaller, 0.09 percent of permanent consumption, and only 31 percent of agents gain. Winners comprise the two deciles of workers with lowest wealth and the 66 percent poorest retirees. It is also interesting to note that all entrepreneurs lose, even those that have just become entrepreneurs and have no previous wealth. The reason is that with a survival probability of 98 percent even these entrepreneurs quickly accumulate substantial wealth and are therefore primarily affected by the lower return to saving.

8 Comparative Statics I: Technology Turnover

Jovanovic and Rosseau (2005) document that the rate of creation and destruction in the United States increases a lot during periods of rapid technological change. For example during the electrification as well as during the IT-revolution it reached highs of 10 percent per year, as measured by the value of new firms entering the stock market. What are the consequences of increased turbulence for policy? To investigate this issue, we perform the following experiment. We assume that the economy is in a stationary equilibrium with a real interest rate of 1.5 percent, and that the rate of creation and destruction is the economy unexpectedly changes permanently from 2 percent to 5 percent ($\sigma_1 = \sigma_2 = 0.95$). Let us first consider what happens if policy is left unchanged (i.e., leaving r^m and g^m unchanged).

The immediate effect is that the price of equity falls by 63 percent as a result of the reduced longevity of firms. By contrast, the real value of debt increases by 106 percent.

[46]When solving for transitions, we keep tax rates fixed at initial levels, where expenditures (i.e., the left hand sides of equations 21-22) adjust period by period.

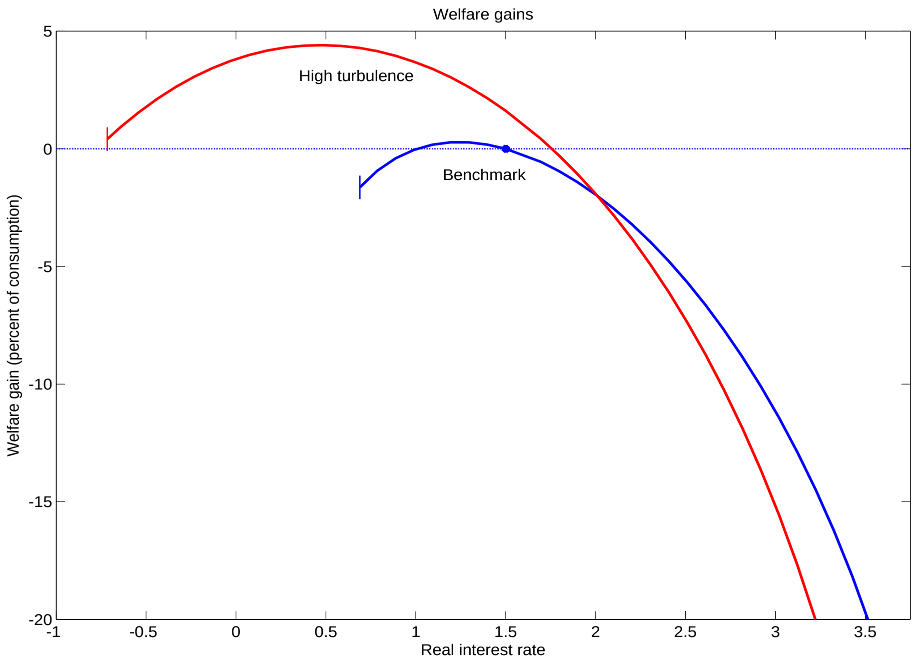

Figure 6: Real interest rate and steady state welfare for low and high turbulence.

During the transition, the real interest rate gradually increases from 0.87 percent back to 1.5 percent, entailing an additional fall in the price of equity by 3 percentage points, while the real value of bonds falls back to a level that is 81 percent above that in the benchmark economy.

What happens to welfare? All entrepreneurs lose from the increased pace of change. The survival probability of their firms is lower than before, and the possible gain from making an IPO falls due to the reduction of equity prices. But all others, constituting 98.5 percent of the population, are better off in the new environment. In particular, their probability of becoming an entrepreneur has increased. Average welfare (behind the veil of ignorance) increases by 2.88 percent.[47]

Let us now consider the effect of nominal interest rate or debt policy. Figure 6 depicts the relationship between the real interest rate and steady state welfare gains (relative the benchmark stationary equilibrium) before and after the increased turnover rate.

Comparing steady state welfare gains reveals that policy has more bite, and ought to be looser, in economies with faster change. In our example, reducing the real interest rate by a full 100 basis points to 0.5 percent yields the largest steady state welfare gains. Suppose that the government pursue such a policy. More specifically, assume that as soon as the rate of creation and destruction fell, policy is changed such that r^m is reduced by 100 basis points. This entails an average welfare gain of 4.32 percent. The large gains

[47]This result might be particularly sensitive to our assumption of perfect capital markets. In reality, there may be considerable temporary losses associated with moving capital from old to new firms.

are a result of an increase in real wages by 3.7 percent and a more than doubling of the lump-sum transfers (as a share of GDP they increase from 0.61 to 1.65 percent). The price of equity and the real value of bonds are 45 percent and 6.5 percent below that in the benchmark economy, respectively.

9 Comparative Statics II: Productivity Growth

Let us now instead consider what happens if real productivity growth unexpectedly and permanently falls from 2.0 to 1.5 percent. Again, let us first consider the case in which policy is unchanged. The scenario is qualitatively similar to that of an increase in turbulence. First, the price of equity falls (by 23 percent) and the real value of government bonds increases (by 126 percent). Subsequently, both the price of equity and the real values of bonds fall as demand for real investment slowly picks up (replacement investment to balance depreciation is required also at the new lower level) by 2 and 8 percentage points respectively. During the transition, the real interest rate gradually increases back to 1.5 percent.

In terms of welfare, all households lose as a consequence of lower real growth. However, policy can mitigate the loss associated with the fall in productivity growth. In terms of steady state comparisons, the best the government can do is to reduce the the real interest rate by 75 basis points to 0.75 percent. Figure 7 illustrates.

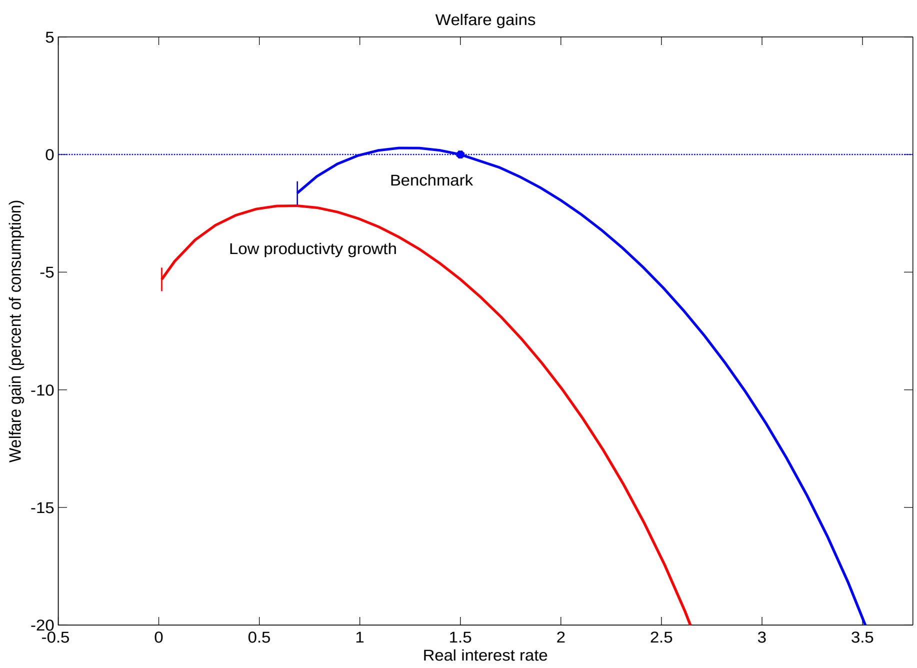

Figure 7: Real interest rate and steady state welfare before and after drop in g

Suppose, as above, that authorities immediately react when the growth rate change and reduce r^m by 75 basis points. The average welfare loss then goes down to 3.71 percent (from 6.82 percent). The reduction in the long run real interest entails an increase in the price of equity and in the real value of bonds of 24 percent and 19 percent respectively.

10 Final Remarks

We have argued that realistic market incompleteness in combination with realistic uninsurable income processes suffice to rationalize the low risk-free interest rates that we have seen over long periods in the past. Hence, the possibility of rational bubbles should not be discarded. The bursting of a rational bubble does not have to entail dire consequences, even if policy is unresponsive. If the bubble moves to another private asset, there will primarily be a shifting of wealth. If the bubble moves into real public debt, there will be low interest rates. However, if the bubble moves into nominal public debt, the price level will go down – unless public debt is increased aggressively. In the presence of nominal contracting, such deflation can have dreadful implications.

We see several potential avenues for future work. One avenue is empirical: Does our model actually help to explain the vast heterogeneity in macroeconomic outcomes following asset market crashes? For example, are depressions primarily triggered when the bond markets are involved and there is a drop in the price level? Does it really matter for the effects of fiscal policy whether public debt is nominal or real? Can the model be calibrated to fit the detailed patterns of historical events?

Other avenues are theoretical. Our model appears to suggest that it is always better to let non-fundamental values take the form of public debt then private bubbles. Since many will be reluctant to believe in the feasibility and desirability of large public debt, it is important to investigate the counterarguments. Will the temptation to default on a larger debt be irresistible under more realistic circumstances? Will the promise of rescue operations have undesirable incentive effects? A preliminary answer to the last question is that a bubble will be less likely to burst if investors expect expansionary responses, since the loss from not moving quickly enough into government bonds will be smaller – but the smaller likelihood of bursting in turn might encourage the bubble to form.

Many other theoretical extensions appear straightforward. For example, it would be conceptually easy to increase realism by introducing more assets, such as land and housing, to introduce nominal price rigidities, elastic labor supply, and distortionary taxation. Likewise, it seems quite feasible to extend the model to allow for financial frictions or to have several forms of public debt with different properties as means of payment – currency in addition to bonds.[48]

A more ambitious extension would be to introduce private financial intermediation.

[48]There is a significant literature on this issue, starting with Bryant and Wallace (1984). For a recent contribution that seeks to explain term premia of bonds with different duration, see Challe, Le Grand, and Ragot (2013). See also Chatterjee and Corbae (1992).

We conjecture that a realistic model of intermediation implies that intermediaries earn Ponzi-value, and a natural objective would be to assess the size and the social costs and benefits of non-fundamental values lodged in the financial sector.

To the extent that such private intermediation involves nominal contracts, deflation obviously has the potential to create havoc in debt markets. Pursuing this extension might thus admit a reformulation of Fisher's (1933) debt-deflation hypothesis.

On the more conceptual side, a shortcoming of our model is that we only consider totally unexpected movements from one equilibrium to another. One way to address this objection would be to introduce an explicit "sunspot" process. Then, the current analysis would presumably emerge as a limiting case. Another, more ambitious, extension is to introduce equilibrium refinements. Our current approach of considering all rational-expectations equilibria, and allowing authorities choose (more or less freely) one equilibrium in this set, may be overlooking other coordination mechanisms. For example, is it possible that coalitions of agents will find it easier or more attractive to introduce private bubbles in case interest rates become low? And just how close to the growth rate can the real interest rate go before investors in private bubbles get nervous? More generally, as for any model that admits multiple equilibria, it is desirable to understand when and how "animal spirits" are likely to set in. While problems of equilibrium selection are well known to be a challenging, there has been progress. For example, Morris and Shin (1998) provide a theory of equilibrium selection based on strategic risk (endogenous uncertainty) in economies with infinitely many agents. An great virtue of such an extension is that it would give rise to bubble risk premia in our setting.

Aggregate uncertainty, exogenous or endogenous, is a desirable feature not least because it might generate additional asset return differentials.[49] Studying exogenous aggregate income shocks, Krusell, Mukoyama, and Smith (2013) derive the magnitude of such return differentials, and show that they might be quite large, in a pure endowment economy without durable assets and with tight borrowing constraints.[50] However, absent bubbles, we conjecture that risk premia deriving from exogenous shocks would be quite small in our setting, which has extensive opportunities for saving in durable assets. On the other hand, with endogenous (i.e., bubble) risk, the risk premia could well be large. Only in the special case with full price and wage flexibility will government bonds serve as a perfect hedge in case a bubble moves from stocks to bonds; in the other cases we study, rich savers will lose when the bubble migrates to the bond market – either because of a depression caused by deflation or because authorities expands the supply of bonds in order to prevent such deflation. Indeed, we see undiversifiable bubble risk as being a natural candidate for some of the "rare disasters in stock markets that have been

[49]Of course, asset return differentials are essential to those models of economic crises that focus on shortage of safe assets; see e.g., Caballero (2006).

[50]The result may seem unsurprising in view of the lack of durable tradable assets, but in light of previous work by Krueger and Lustig (2010) the result was not obvious.

invoked to explain the equity premium and excess volatility puzzles (Rietz, 1988; Barro, 2006).

Bubble risk will naturally be largest for those assets whose bubble is larger relative to their fundamental value. In the stock market, these are the "value" stocks rather than the "growth" stocks. We thus think that analysis along these lines might contribute an explanation not only for the excess volatility puzzle and the equity premium puzzle, but also for the value premium.

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