Risk-averse firms in oligopoly

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Abstract

Does risk aversion lead to softer or fiercer competition? I show that, in general, the answer depends on whether firms set prices or quantities and if they face demand or cost uncertainty. For demand uncertainty, the risk-averse firm's best response price and quantity is lower than the corresponding risk-neutral strategy. For cost uncertainty, the best response price is higher but the best response quantity is lower. Hence, only for cost uncertainty is the expected price-cost margin unambiguously higher. It is shown that fixed costs reinforce the effects if firms have decreasing absolute risk aversion. I extend this to consider implications for strategic investment models and the importance of accumulated profits. Overall, the results emphasise that to empirically test strategic effects of risk-averse behaviour it is necessary to control for the type of uncertainty.

Keywords: Oligopoly; risk aversion; fixed costs; strategic investment.

JEL codes: D43; D81; L13; L21.

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1. Introduction

A standard assumption in oligopoly theory is that firms are risk-neutral. This implies that under uncertainty firms are maximising expected profits without any concern for risk. However, there are several reasons for why firms may act *as if* they were risk-averse. Non-diversified owners, liquidity constraints, costly financial distress and non-linear tax systems are some that are frequently invoked. And even if owners themselves wish to maximise expected profits, delegation of control to a risk-averse manager, whose payment is linked to firm performance, may cause the firm to behave in a risk-averse manner. Empirically, the reluctance to bear risk is evidenced by the extent of corporate hedging activity (see e.g. Géczy et al., 1997, Tufano, 1996, and Nance et al., 1993). In spite of this, surprisingly little work has focused on the effects of risk aversion on competition. In particular, there has been very little effort spent on trying to derive empirically testable predictions regarding the effects of risk aversion on competition.

Altering the assumption of risk-neutrality has several implications for the product market competition. Early works examined perfect competition and monopoly settings. Pioneering analyses by Baron (1970) and Sandmo (1971) show how increased uncertainty about price lowers the quantity produced in perfectly competitive markets. In a monopoly framework, Baron (1971) and Leland (1972) derived similar results. Some questions that been addressed in an oligopoly framework include: price leadership (Holthausen, 1979); information sharing (Hviid, 1989, and Kao and Hughes, 1993); effects of futures and forwards markets (Eldor and Zilcha, 1990, and Hughes and Kao, 1997); hedging strategies and investment (Froot et al., 1993); product differentiation (Tessiotore, 1994); locational choice (Mai et al., 1993); equilibrium market structure (Appelbaum and Katz, 1986, Asplund, 1999, and Haruna, 1996). The previous works have generally employed specific assumptions on the nature of competition and uncertainty (e.g. Cournot competition with demand uncertainty). It is widely accepted, however, that many predictions from oligopoly models are sensitive to the fine details (often very difficult, if not impossible, to observe in practice) and it therefore seems appropriate to consider whether some results can be empirically validated.

In this paper I present a simple, yet general, framework to study the strategy choice of risk-averse firms. Importantly, I consider combinations of the nature of competition and different types of uncertainty. Under the assumption of normally distributed profits and marginal profits, the first order condition states that the expected marginal operating profit should equal the absolute risk aversion times the covariance of profits and marginal profits. This makes first order conditions easy to interpret and useful for many applications where risk aversion plays a role. The key intuition behind the effects of risk aversion is that firms wish to be well adjusted in the bad realisations (where profits otherwise would be very low). For example, with demand uncertainty, profit is low when demand is low. And when demand is low, a low quantity or a low price performs relatively well. Hence, the best response prices and quantities are lower compared to those of a risk-neutral firm. On the other hand, with marginal cost uncertainty the risk-averse firm wishes to restrict output in realisations where costs are high. This corresponds to a low quantity or a high price. Hence, only for marginal cost uncertainty does risk aversion have an unambiguous positive effect on the expected price-cost margin. In contrast to previous works on risk-averse oligopolies I allow for decreasing absolute risk aversion. If the objective function has this property then fixed cost and accumulated profits influence the best response strategies. I extend this logic to cover two common types of oligopoly models. The paper concludes with remarks on empirical tests for strategic effects of risk aversion.

2. The model

Each firm i has a twice continuously differentiable concave von Neuman - Morgenstern expected utility function, $V^i = EU^i(\pi^i, f^i)$, where π is (net) operating profit, and $f > 0$ is fixed (sunk) cost.[1] The partial derivatives are $U^i_{\pi^i} > 0$, $U^i_{f^i} < 0$, $U^i_{\pi^i\pi^i} \leq 0$, $U^i_{f^i f^i} \geq 0$. It is

[1] This paper abstracts from capital market considerations, where the owner (or the manager) of the firm may have other risky assets. These risky assets can then be thought of as having some correlation with a market portfolio as in Bughin (1999), Harris (1986) and Tessiotore (1994). However, this could be incorporated into this framework by assuming that the utility is a function of the sum of firm profits and the return on a portfolio of other assets. Even if this is theoretically

assumed that the expected utility in equilibrium is greater than the reservation utility. Furthermore it is assumed that the utility functions are common knowledge.[2] To simplify the exposition I consider a market where two firms simultaneously choose their strategies, before uncertainty is realised. The operating profits of firm i is a continuously differentiable function of its strategy, s^i, and the strategy of the other firm, s^j. Differentiating the objective function, V^i, w.r.t. s^i yields the first order condition

$$V^i_{s^i} = EU^i_{\pi^i}\pi^i_{s^i} = 0. \tag{1}$$

Using the property $Cov(U^i_{\pi^i}, \pi^i_{s^i}) = EU^i_{\pi^i}\pi^i_{s^i} - EU^i_{\pi^i}E\pi^i_{s^i}$ (see any statistics textbook), (1) can be rewritten as

$$E\pi^i_{s^i} + \frac{Cov(U^i_{\pi^i}, \pi^i_{s^i})}{EU^i_{\pi^i}} = 0. \tag{2}$$

Some early works analysed the first order conditions in the form of (2). The denominator in the second term is positive. A negative (positive) covariance between marginal utility and marginal profit corresponds to a positive (negative) covariance between profits and marginal profits. Hence the expected marginal profit must be of opposite sign from the covariance to satisfy (2) (below I discuss the sign of the covariance term).

To derive more tractable first order conditions than previous works I further assume that π^i and $\pi^i_{s^i}$ are bivariate normally distributed

$$\pi^i, \pi^i_{s^i} \sim N(E\pi^i, E\pi^i_{s^i}, \sigma^2_{\pi^i}, \sigma^2_{\pi^i_{s^i}}, \rho^i), \tag{A1}$$

where the correlation coefficient is

$$\rho^i = \frac{Cov(\pi^i, \pi^i_{s^i})}{\sigma_{\pi^i}\sigma_{\pi^i_{s^i}}}. \tag{3}$$

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appealing, in practice it is likely to be difficult to assess the covariance between the payoffs of a strategy, and the return on a market portfolio.

[2] Even though it is unrealistic to believe that competitors exactly know each other's utility function, it is not unreasonable to assume that firms in concentrated markets have a clear idea about the risk attitude of their rivals. As mentioned in the introduction, some factors that can influence the objective function are observable to outsiders (for example, ownership structure, degree of diversification, and financial situation).

It is well known that the assumption of normality of payoffs gives a mean - variance model, since third and higher moments are zero.[3] However, under the assumption of normal distribution (and any other distribution with unbounded support) there is a positive probability that profits and marginal profits are either infinitely positive or negative. Nevertheless, even if the distribution is not exactly normal it may for practical purposes be a good approximation to the true distribution over the relevant range.[4] The conjecture is that the results derived in this paper would be qualitatively unaffected if other symmetric profit and marginal profit distributions were used, as explained after Proposition 1.

Under (A1) Steins lemma, proved by Rubinstein (1976), can be applied by which[5]

$$Cov(U^i_{\pi^i}, \pi^i_{s^i}) = EU^i_{\pi^i\pi^i} Cov(\pi^i, \pi^i_{s^i}) . \qquad (4)$$

The Arrow-Pratt measure of global absolute risk aversion is

$$R^i = -\frac{EU^i_{\pi^i\pi^i}}{EU^i_{\pi^i}} . \qquad (5)$$

Combining (2), (4), and (5) yields the first order conditions

$$V^i_{s^i} = E\pi^i_{s^i}(s^i, s^j) - R^i(\pi^i(s^i, s^j), f^i) Cov(\pi^i(s^i, s^j), \pi^i_{s^i}(s^i, s^j)) = 0 . \qquad (6)$$

To my knowledge, no previous works have stated the first order conditions as (6). In words, the first order condition states that the expected marginal operating profit should equal the absolute risk aversion times the covariance of profits and marginal profits. The Arrow-Pratt measure of absolute risk aversion is positive under the assumption of risk aversion (and zero

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[3] Mean - variance analysis may also be defended by quadratic utility functions. However, it has the undesirable property of increasing absolute risk aversion. In what follows I make use of decreasing absolute risk aversion which rule out quadratic utility functions.

[4] Normality of the density function is the standard defence of the mean - variance analysis in portfolio choice even though stock prices are truncated at zero, see e.g. Ingersoll (1987 p. 95-97) and Huang and Litzenberger (1988 p. 61-62). To numerically calculate the equilibrium strategies under the assumption that the distribution is approximately normal, it is necessary to truncate the distribution at some level.

[5] The theorem states that if x and y are bivariate normally distributed, and g(y) is at least once differentiable then $Cov(x,g(y))=Eg_y(y)Cov(x,y)$. See e.g. Huang and Litzenberger (1988 p. 101).

under risk-neutrality). Note that for R^i>0 the expected marginal profit has to be strictly positive to satisfy (6) when ρ^i>0, and strictly negative when ρ^i<0.

Let the best response function be $b^i(s^j)$, (i.e. $s^i = b^i(s^j)$ satisfies (6) for a given s^j) and s^{i*}, s^{j*} denote the unique pair of Nash strategies which satisfy the first order conditions (6) for both firms.[6] Differentiating (6) yields

$$b^i_{s^j}(s^j) = -\frac{V^i_{s^i s^j}}{V^i_{s^i s^i}}, \tag{7}$$

with $V^i_{s^i s^i} < 0$ from the second order condition and where

$$V^i_{s^i s^j} = E\pi^i_{s^i s^j} - R^i_{\pi^i} \pi^i_{s^j} Cov(\pi^i, \pi^i_{s^i}) - R^i Cov_{s^j}(\pi^i, \pi^i_{s^i}). \tag{8}$$

In the terminology of Bulow et al. (1985), s^i and s^j are strategic substitutes if best response functions are downward sloping, $V^i_{s^i s^j} < 0$, and strategic complements if best response functions are upward sloping, $V^i_{s^i s^j} > 0$. Informally, I refer to the strategies as 'quantities' and 'prices' for strategic substitutes and strategic complements, respectively.

3. Competition among risk-averse firms

3.1 The effects of risk aversion on prices and quantities

Before dealing with the duopoly case it is instructive to first consider the case of a monopolist who has to set a single price or quantity under uncertainty. Let s^{*RN} be the risk-neutral monopolist's strategy and s^{*RA} that of a risk-averse monopolist.

Proposition 0: (Baron 1971) *If a monopolist's Arrow-Pratt measure of absolute risk aversion is strictly positive, and profits and marginal profits are normally distributed then* $s^{*RN} > s^{*RA}$ *if* $\rho^i > 0$, *and* $s^{*RN} < s^{*RA}$ *if* $\rho^i < 0$.

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[6] To assure that a stable, unique equilibrium exists I assume that $V^i_{s^i s^i} V^j_{s^j s^j} - V^i_{s^i s^j} V^j_{s^j s^i} > 0$, (see Dixit, 1986). This condition can not be assumed to be met in general, but risk aversion makes the requirements for the existence of a stable equilibrium easier to satisfy as it makes the objective functions more concave.

Proof: If R=0 then the first order condition is $E\pi_s(s^{*RN}) = 0$, and the second order condition is $E\pi_{ss}(s^{*RN}) < 0$. If R>0 and ρ>0 then $E\pi_s(s^{*RA}) > 0$ which, by the second order condition, implies that s^{*RN}>s^{*RA}. If R>0 and ρ<0 then the expected marginal profit has to be negative at the optimum and thus s^{*RN}<s^{*RA}. *Q.E.D.*

If the covariance between profits and marginal profits is positive, then a quantity (price) setting risk-averse monopolist sets a lower quantity (price) than a risk-neutral counterpart. If the covariance is negative the risk-averse monopolist sets a higher price or a higher quantity. This is essentially the result of Baron (1971), but derived in a much simpler way. From this it is easy to generalise Proposition 0 to the duopoly case. Let θ^i be a parameter which influences firm i's absolute risk aversion (without changing the distribution of profits), and assume that it is increasing in θ^i, $R^i_{\theta^i} > 0$.

Proposition 1: *If firm i's Arrow-Pratt measure of absolute risk aversion is strictly positive, and profits and marginal profits are normally distributed, then* $ds^{i*}/d\theta^i < 0$ *if* ρ^i *> 0, and* $ds^{i*}/d\theta^i > 0$ *if* ρ^i *< 0. The sign of* $ds^{j*}/d\theta^i$ *is the same as* $ds^{i*}/d\theta^i$ *if* $V^i_{s^i s^j} > 0$ *and the opposite if* $V^i_{s^i s^j} < 0$.

Proof: See appendix. *Q.E.D.*

With a positive (negative) covariance between profits and marginal profits firm i's equilibrium strategy is decreasing (increasing) in its absolute risk aversion. Moreover, for any s^j, the expected marginal profit of firm i needs to be positive (negative) at the risk-averse best response strategy, denoted $b^{iRA}(s^j)$. Analogous to Proposition 0, this implies a lower (higher) best response strategy than that of a risk-neutral firm, $b^{iRN}(s^j)$. Given the sign of ρ^i, the effect on the other firm's Nash strategy is dependent on the slope of the best response functions. More precisely, $ds^{i*}/d\theta^i$ will have the same sign as $ds^{i*}/d\theta^i$ if $V^i_{i\ j} > 0$ and the opposite sign if $V^i_{s^i s^j} < 0$.

3.1.1 Illustration of the covariance between profits and marginal profits

Can anything be said about the sign of the covariance? As an illustration consider two profit functions, based on linear demand for differentiated products and constant marginal cost,

$$\pi^i(s^i, s^j) = s^i(a - b_1 s^i - b_2 s^j - c) = q^i(a - b_1 q^i - b_2 q^j - c) \qquad \text{(I1)}$$

$$\pi^i(s^i, s^j) = (s^i - c)(a - b_1 s^i + b_2 s^j) = (p^i - c)(a - b_1 p^i + b_2 p^j) \qquad \text{(I2)}$$

I1 is the quantity, *q*, version (strategic substitutes) and I2 the price, *p*, version (strategic complements). The marginal profits are

$$\pi^i_{s^i}(s^i, s^j) = a - 2b_1 s^i - b_2 s^j - c = a - 2b_1 q^i - b_2 q^j - c \qquad \text{(I3)}$$

$$\pi^i_{s^i}(s^i, s^j) = a - b_1(2s^i - c) + b_2 s^j = a - b_1(2p^i - c) + b_2 p^j \qquad \text{(I4)}$$

Now assume either a, b_1, b_2, or c is normally distributed. This implies that both the profit and marginal profit are normally distributed, as required by (A1). It is straightforward to calculate the covariance between the profit and the marginal profit for each of the possible combinations.

Table 1. Covariance between profit and marginal profit for demand and marginal cost uncertainty.

	$a \sim N(Ea, \sigma_a^2)$	$b_1 \sim N(Eb_1, \sigma_{b_1}^2)$	$b_2 \sim N(Eb_2, \sigma_{b_2}^2)$	$c \sim N(Ec, \sigma_c^2)$
Strategic Substitutes	$q^i \sigma_a^2 > 0$	$2(q^i)^3 \sigma_{b_1}^2 > 0$	$q^i (q^j)^2 \sigma_{b_2}^2 > 0$	$q^i \sigma_c^2 > 0$
Strategic Complements	$(p^i - c)\sigma_a^2 > 0$	$((p^i - c)^2 p^i + (p^i - c)(p^i)^2)\sigma_{b_1}^2 > 0$	$(p_i - c)(p^j)^2 \sigma_{b_2}^2 > 0$	$-b_1\left(a - b_1 p^i + b_2 p^j\right)\sigma_c^2 < 0$

In the demand uncertainty cases, (a, b_1, b_2), the covariances are positive for both strategic substitutes and complements. This implies that in realisations where the profits are high, marginal profits are high as well. This also holds for marginal cost uncertainty in strategic substitutes case. The only case where the covariance is negative is with cost uncertainty and strategic complements which is also the only case where the covariance is decreasing in the strategy.[7]

There is a simple intuition for the risk-averse firm's choice. The risk-averse firm puts relatively greater weight to the bad realisations where profits are low. Under demand uncertainty (positive covariance between profit and marginal profit), low quantities and low prices are optimal in bad realisations. Hence risk aversion shifts the best response functions downward in case of demand uncertainty. For cost uncertainty the sign of the covariance is dependent on whether firms set quantities (positive covariance) or prices (negative covariance). Thus the best response quantity is lower and best response price is higher. The reason is that it restricts the quantity the firm has to sell in high cost states. It follows that expected price-cost margins are higher under risk aversion and cost uncertainty, irrespective of whether firms set quantities or prices. Proposition 1 is illustrated in Figures 1A and 1B. The risk-neutral best response functions are lines, and risk-averse best response functions are dashed. Note that the iso-profit functions of firm i do not have extreme points at the risk-averse best response functions, (iso-utility functions would).

[FIGURES 1A AND 1B ABOUT HERE]

As indicated above, the conjecture is that the results generalise to other symmetric distributions of payoffs. If strategies have symmetric distributions of profits, conditional on the rival's strategy, the risk-averse firm will prefer strategies which do relatively well in the lower end of the distribution of realisations (and do relatively worse in the upper end). For other distributions and profit functions one needs to verify that for e.g. demand uncertainty a low price or a low quantity is the best choice in the worst realisation, which is a plausible property.

[7] Other forms of uncertainty in I1 and I2 are of course possible. Additive, normally distributed uncertainty results in zero covariance and is therefore irrelevant to strategy choice. Multiplicative, normally distributed uncertainty gives positive covariance.

3.2 Fixed costs

Fixed costs are irrelevant for the choice of strategy in a one shot game played by risk-neutral players. This, in general, does not hold for risk-averse players. Therefore one of the most interesting properties of risk aversion is that fixed costs matter for the strategy choice if the utility function displays decreasing absolute risk aversion. More fixed costs reduce firm's net wealth, which increases the absolute risk aversion. The effects on the equilibrium strategies are summarised in the following proposition.

Proposition 2: *If firm i's Arrow-Pratt measure of absolute risk aversion is strictly decreasing, and profits and marginal profits are normally distributed, then* $ds^{i*}/df^i < 0$ *if* $\rho^i > 0$*, and* $ds^{i*}/df^i > 0$ *if* $\rho^i < 0$*. The sign of* ds^{j*}/df^i *is the same as* ds^{i*}/df^i *if* $V^i_{s^i s^j} > 0$ *and the opposite if* $V^i_{s^i s^j} < 0$.

Proof: See appendix. *Q.E.D.*

For a positive covariance, an increase in firm i's fixed cost leads to a lower equilibrium strategy. The intuition is simple. If fixed costs are high, the bad realisations are even worse to a risk-averse firm. To reduce the impact of these the risk-averse firm lowers its quantity or price to be better adjusted in the bad outcomes. For a negative covariance firm i will increase its price to limit the effect of the worst outcomes. Similar results was derived in the early works of Baron (1970), Sandmo (1971) and Leland (1972) for perfectly competitive and monopoly environments. It is easy to extend Proposition 2 to cover two common classes of oligopoly models.

3.2.1 Strategic sunk cost investments

Proposition 2 have direct implications for the class of strategic investment which involves an initial fixed (sunk) outlay. The intuition is that fixed costs reduce firm wealth and thereby change its risk aversion in the future. As an illustration, in the quintessential strategic investment model of Dixit (1980) it is assumed that an incumbent firm can make a sunk investment in a cost reducing technology, and thereby commit to an output expansion which changes competition in a second stage. Assume that there is demand uncertainty in period 2 (ρ^i>0) and that firms compete in quantities, $V^i_{s^i s^j} < 0$. The only effect that the investment has for a risk-neutral firm is to increase the second period best response, due to the lower cost of production. To a risk-averse incumbent there is a counteracting effect from the reduction in wealth, which tends to lower the second period best response. Hence, the cost reducing investment is less attractive to a quantity setting risk-averse incumbent. On the other hand, if competition is in prices, $V^i_{s^i s^j} > 0$, rather than quantities, the two effects work in the same direction. The lower cost tends to reduce the prices, and demand uncertainty reinforces this effect. Therefore the risk-averse incumbent can achieve the same strategic effect by a smaller investment. Finally, note that with cost uncertainty, strategic investments are less effective both for both quantity and price competition. The reason is that higher fixed costs tend to lower quantities or raise prices and thereby counteract the cost reduction.

3.2.2 The relevance of accumulated profits

Just as fixed costs influence the risk-averse best response function so do past profits. To illustrate consider the case where two firms compete in two periods. Let firm 1 be risk-neutral and firm 2 risk-averse with decreasing absolute risk aversion. Denote the strategy of firm i at time t by s^{it}. For simplicity assume that there is only uncertainty in the second period with covariance ρ^{i2} and that the games firms play in the two periods are symmetric in the sense that the sign of $\pi^{it}_{s^{jt}}$ is the same in both periods.[8] The objective functions are then $\pi^{11}(s^{11}, s^{21}) + E\pi^{12}(s^{22}, s^{22})$ and $EU^2(\pi^{21}(s^{21}, s^{11}) + \pi^{22}(s^{22}, s^{12}))$. Only the risk-averse firm's choice of strategy in period 2 depends on the period 1 profit. In the second period, firm 2's first order condition is as (6) with the first period profit, π^{21}, included in R^2

[8] It is possible (but messy) to generalise the framework to two risk-averse firms. Moreover, by introducing uncertainty in both periods one needs to model the correlation of realisations across periods (for instance if demand shocks are positively autocorrelated).

$$V^{22}_{s^{22}} = E\pi^{22}_{s^{22}} - R^2(\pi^{21} + \pi^{22})Cov(\pi^{22}, \pi^{22}_{s^{22}}) = 0. \tag{9}$$

By assumption firm 2's risk aversion is decreasing in its first period profit, $R^2_{\pi^{21}} < 0$, such that the higher π^{21} the closer will it be to maximise second period expected profits. From proposition 2 it follows that if $\rho^{22}>0$ the best response strategy is increasing in the first period profit (it will, however, be below the risk-neutral since $R^2>0$). By the same token, with $\rho^{22}<0$ the best response strategy is decreasing in the first period profit (but remains above the risk-neutral).

Firm 1 may thus attempt to influence the risk-averse firm's first period profit to soften the second period competition. Firm 1's period one first order conditions is

$$V^{11}_{s^{11}} = \pi^{11}_{s^{11}} + E\pi^{12}_{s^{22}} s^{22}_{s^{11}} = 0. \tag{10}$$

The second term is the strategic effect. The sign of $s^{22}_{s^{11}}$ can be obtained by differentiating (9) w.r.t. s^{22} and s^{11}

$$b^{22}_{s^{11}}(s^{12}) = \frac{R^2_{\pi^{21}} \pi^{21}_{s^{11}} Cov(\pi^{22}, \pi^{22}_{s^{22}})}{V^{22}_{s^{22}s^{22}}}. \tag{11}$$

The denominator is negative by the SOC and $R^2_{\pi^{21}} < 0$ by assumption. Hence the sign of $s^{22}_{s^{11}}$ is that of $\pi^{21}_{s^{11}}\rho^{22}$. The game firms play in the two periods is symmetric, $\text{sign}(\pi^{21}_{s^{11}}) = \text{sign}(E\pi^{12}_{s^{22}})$, from what follows that the sign of strategic effect, $E\pi^{12}_{s^{22}} s^{22}_{s^{11}}$, is that of ρ^{22}. If $\rho^{22}>0$ then the first period best response function of firm 1, denoted $b^{11RNS}(s^{21})$, will be higher than without the strategic effect, $b^{11RN}(s^{21})$. This implies that under quantity competition firm 1 becomes more aggressive than otherwise in order to reduce the risk-averse firm's first period profit. The reason is that this induces the risk-averse firm to produce less in the second period which benefits firm 1. Conversely, under price competition firm 1 behaves less aggressive to avoid meeting a very risk-averse firm in the second period. If, on the other hand, $\rho^{22}<0$ the situation is reversed, $b^{11RN}(s^{21}) < b^{11RN}(s^{21})$ (firm 1 sets a low price in the first period to soften its rival in the next). It is straightforward to see that firm 2's first period best response function for ρ^{22}>0 with the strategic effect, $b^{21RAS}(s^{11})$ will be below its risk-neutral best response function without the strategic effect, $b^{21RN}(s^{11})$, and that if ρ^{22}<0 the situation is reversed.[9]

3.3 Firm specific risk and market risk

Now add an assumption on the Arrow-Pratt measure of absolute risk aversion to simplify the following exposition. The assumption is that R^i is convex and non-increasing. This is satisfied by most common utility functions, for example the HARA class with non-increasing absolute risk aversion.[10] Now consider a change in the covariance between profits and marginal profits of firm i, denoted $dCov^i$, which does not affect the expected profit and leaves the covariance of firm j unaffected. That is, it is a mean preserving change in the idiosyncratic risk of firm i.

Proposition 3: *If firm i's Arrow-Pratt measure of absolute risk aversion is positive, convex and non-increasing, and profits and marginal profits are normally distributed, then* $ds^{i*}/dCov^i < 0$. *The sign of* $ds^{j*}/dCov^i$ *is the same as* $ds^{i*}/dCov^i$ *if* $V^i_{s^i s^j} > 0$ *and the opposite if* $V^i_{s^i s^j} < 0$.

Proof: See appendix. *Q.E.D.*

If $\rho^i > 0$ an increase in covariance shifts the best response function further down, compared to the risk-neutral case. If $\rho^i < 0$ an increase in covariance implies lower risk, which shifts the best response function down towards the risk-neutral best response function.

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9 The insight that past profits matter for future competition is related to the model by Glazer (1994) where indebted firms compete in quantities in two periods. Limited liability of equity holders make them risk seeking, and more so the more outstanding debt there is. First period profits will determine the net debt in the second period. With quantity competition, it is shown that firms will reduce their quantities to give the competitor higher profits, which makes second period competition softer. Risk aversion has the opposite effect, the risk-neutral firm expands output to reduce the profits of the rival and thereby increases its second period risk aversion to soften competition.

10 The HARA class includes as special cases the negative exponential (with constant absolute risk aversion), quadratic and power utility functions. See e.g. Ingersoll (1987 p. 39-40).

The effect on the rival firm is that with strategic substitutes (complements) its equilibrium strategy increases (decreases).

Further, let *dCov* denote an equal change in the covariance of both firms, but which does not influence expected profits. This is referred to as a mean preserving change in market risk. The following proposition proves that the effect of a change in market risk on equilibrium strategies is unambiguous only for strategic complements.

Proposition 4: *If both firms' Arrow-Pratt measures of absolute risk aversion are positive, convex and non-increasing, and profits and marginal profits are normally distributed, then* $ds^{i*}/dCov<0$ if $V^i_{s^is^j}>0$. *For* $V^i_{s^is^j}<0$ *the sign of* $ds^{i*}/dCov$ *is that of* $V^j_{s^jCov}V^i_{s^is^j}-V^i_{s^iCov}V^j_{s^js^j}$.

Proof: See appendix. *Q.E.D.*

Increasing the covariance causes both firms' best response functions to shift down. Proposition 4 shows that for strategic complements, $V^i_{s^is^j}>0$ there is an unambiguous negative effect on the equilibrium strategies. The intuition is that increases in covariance causes each firm's best response function to shift down, to be better adjusted in the worst realisations. The clear result arises from that the best response to a lower price of the rival is to reduce price further. For strategic substitutes, $V^i_{s^is^j}<0$, the direction of the change in equilibrium strategies depends on the relative magnitude of the downward shifts. The special case of strategic substitutes and demand uncertainty has been previously analysed by Tessiotore (1994).

4. Empirical predictions and evidence

This paper has analysed competition between risk-averse oligopolists within a simple framework where profits and marginal profits are normally distributed. Most of the propositions follow directly from the first order condition; the expected marginal profits should equal the Arrow-Pratt measure of absolute risk aversion times the covariance between profits and marginal profits. Throughout I have stressed the intuition that risk-averse firms choose strategies that perform relatively well in realisations where profits are low. Under demand uncertainty, when demand is low a low quantity or a low price performs relatively well. Hence, the best response strategies are lower than under risk-neutrality. Increasing fixed costs, implying higher risk aversion, reinforce the effect. Under cost uncertainty, it is better to sell a low quantity in the realisations where marginal cost are high, which indicate that best response quantity is lower but best response price is higher than for a risk-neutral firm. Again more fixed costs reinforce this effect.

The above discussion shows that competition among risk-averse firms will be 'softer' or 'tougher' compared to the risk-neutral case, depending on whether they are assumed to set quantities or prices, and if uncertainty is primarily about the demand or cost conditions. Given the difficulty of determining whether competition is in 'quantities' or 'prices', no general prediction can be made regarding whether risk aversion tends to increase or reduce expected margins. However, if one knows that there is cost uncertainty then expected price-cost margins are unambiguously higher, and that this is reinforced by the importance of fixed costs. These are features that can be exploited in empirical testing. Empirical tests should then begin by distinguishing between firms or markets where uncertainty is primarily in input costs, and those that face significant demand uncertainty. A reasonable measure of cost uncertainty can be either the standard deviation of the most important input price, or the standard error from a time series estimation of the input price (to control for predictable changes in costs). To find a measure of demand uncertainty poses a potentially greater challenge. Lacking a measure of demand uncertainty it could be sufficient to identify firms or markets with little or no overall uncertainty as the control group.

To search for evidence of strategic effects of risk aversion it is useful to begin with inter-industry (Structure-Conduct-Performance) studies, despite their well-known limitations. First, there are only a few studies that have tested if some measure of risk (usually measured as standard deviation of historical profits) are correlated with profitability at the firm and industry level. In Schmalensee's (1989 p.973) survey five studies report a significant positive correlation, three were insignificant, and two found a negative correlation.[11] To my knowledge, no study has attempted to split the sample according to the nature of uncertainty. Next, one common finding in cross industry studies is that profitability is positively correlated with some measure of capital requirements in the industry, (see e.g. Schmalensee, 1989 p.978). This is often explained by that a large capital requirement is a proxy for a large minimum efficient scale, which may form an entry barrier. Risk aversion provides an alternative explanation - more fixed costs increase the risk aversion of firms. This tends to result in higher expected margins, except in case of demand uncertainty and price competition. A sharper test within this framework is to use a cross product of cost uncertainty and fixed costs, which is predicted to have a positive influence on margins.

There is also evidence from intra-industry and time series studies that can be interpreted in the light of risk aversion. In the paper it was shown that strategic investments could have additional effects if they increase fixed costs. The effectiveness of strategic investments turned out to be reduced under risk aversion, again with the price setting under demand uncertainty being the exception. This provides an explanation to why the empirical evidence on strategic investments is limited. For example, in Lieberman's (1987) seminal study of capacity investments in 38 chemical industries (where strategic investments should be effective given the magnitude of sunk costs) only in a handful of industries did firms appear to use capacity strategically. Next, it was shown that the intensity of competition is partly determined by firms' accumulated profits - firms being less willing to accept risks in bad times. Does this make competition softer or fiercer after a period of low profits? As noted above, little can be said in general as it depends on both the nature of competition and type of uncertainty. However, except for the demand uncertainty and price setting case, risk aversion makes competition more intense when the past profits have been low. And as long as there is only cost uncertainty, this will be the case. This effect introduces a counter cyclical tendency to price-cost margins over the business cycle. Clearly, there are several alternative mechanisms can produce the same pattern: temptation to deviate from implicitly collusive

[11] Note that the firms in the samples are large and likely diversified with operations in several industries (in some studies the sample is from Fortune 500). Such samples are clearly not ideal to trace effects of risk aversion on oligopolistic interaction.

arrangements in booms (Rotemberg and Saloner, 1986); inflows of new customers in high demand states (Bils, 1989); and liquidity constraints in recessions (Chevalier and Scharfstein, 1996, and Gottfries, 1991). Again, to empirically test whether risk aversion influences the intensity of competition over the business cycle it is necessary to control for the type of uncertainty.

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Appendix

Proof of Proposition 1.

Differentiating the first order conditions yields

$$V^i_{s^i s^i} ds^i + V^i_{s^i s^j} ds^j + V^i_{s_i \theta^i} d\theta^i = 0$$
$$V^j_{s^j s^i} ds^i + V^j_{s^j s^j} ds^j + V^j_{s^j \theta^i} d\theta^{i\ i} = 0 \quad .$$

The direct effect of a change in firm i's risk aversion is zero for firm j, so by Cramers rule

$$\frac{ds^i}{d\theta^i} = -\frac{V^i_{s^i\theta^i} V^j_{s^j s^j}}{DET} \text{ and } \frac{ds^j}{d\theta^i} = \frac{V^i_{s^i\theta^i} V^j_{s^j s^i}}{DET}.$$

Under the stability condition $DET = V^i_{s^i s^i} V^j_{s^j s^j} - V^i_{s^i s^j} V^j_{s^j s^i} > 0$ and by the second order condition $V^j_{s^j s^j} < 0$. The derivative of the first order condition w.r.t. θ^i is

$$V^i_{s^i\theta^i} = -R^i_{\theta^i}(\pi^i, f^i) Cov(\pi^i, \pi^i_{s^i}).$$

If ρ^j>0 (ρ^j<0) the covariance is positive (negative), and by assumption $R^i_{\theta^i} > 0$, and thus $ds^{i*}/d\theta^i < 0$ ($ds^{i*}/d\theta^i > 0$). The change in equilibrium strategy of firm j has the opposite sign of $V^j_{s^j s^i}$. *Q.E.D.*

Proof of Proposition 2

Differentiate the first order condition as in the proof of Proposition 1. The sign of ds^{i*}/df^i will be that of $V^i_{s^i f^i}$. The fixed costs of firm i enters only in its risk aversion term, and if the absolute risk aversion is decreasing then $R^i_{f^i} > 0$. The derivative is

$$V^i_{i f^i} = -R^i_{f^i}(\pi^i, f^i) Cov(\pi^i, \pi^i_{i}).$$

If ρ^j>0 (ρ^j<0) the covariance is positive (negative), then $ds^{i*}/df^i < 0$ ($ds^{i*}/df^i > 0$). The change in equilibrium strategy of firm j has the opposite sign of $V^j_{s^j s^i}$. *Q.E.D.*

Proof of Proposition 3

Differentiation of first order conditions shows that the sign of $ds^{i*}/dCov^{i}$ is that of $V^{i}_{s^{i}Cov^{i}}$. The derivative of the first order condition w.r.t. Cov^{i} is

$$V^{i}_{s^{i}Cov^{i}} = -R^{i} - R^{i}_{Cov^{i}}Cov^{i} .$$

R^{i} is positive by assumption. For $R^{i}_{Cov^{i}}Cov^{i}$ note that the Arrow Pratt measure is strictly convex (except for the limiting case of constant absolute risk aversion where $R^{i}_{Cov^{i}} = 0$). For $Cov^{i}>0$, increasing the covariance is an increase in risk, whereas for $Cov^{i}<0$ it is a risk reduction. In the first case it implies that the Arrow-Pratt measure is increasing by Jensen's inequality, since it is a mean preserving increase in risk. The sign of the second term is then positive and therefore $V^{i}_{s^{i}Cov^{i}} < 0$. For negative covariance, the Arrow-Pratt measure is decreasing, also by Jensen's inequality, since it is a mean preserving reduction in risk. The sign of the second term is again positive (it is the product of two negative numbers) and thus $V^{i}_{s^{i}Cov^{i}} < 0$. From this follows that $ds^{i*}/dCov^{i} < 0$ and the effect on $ds^{j*}/dCov^{i}$ is dependent on the sign of $V^{j}_{s^{j}s^{i}}$. *Q.E.D.*

Proof of Proposition 4

This is similar to the proof of Proposition 3. Differentiate the first order conditions,

$$\begin{aligned} V^{i}_{s^{i}s^{i}}ds^{i} + V^{i}_{s^{i}s^{j}}ds^{j} + V^{i}_{s^{i}Cov}dCov = 0 \\ V^{j}_{s^{j}s^{i}}ds^{i} + V^{j}_{s^{j}s^{j}}ds^{j} + V^{j}_{s^{j}Cov}dCov = 0 \end{aligned} .$$

By Cramers rule

$$\frac{ds^{i}}{dCov} = \frac{V^{j}_{s^{j}Cov}V^{i}_{s^{i}s^{j}} - V^{i}_{s^{i}Cov}V^{j}_{s^{j}s^{j}}}{DET} .$$

From the proof of Proposition 3 it is clear that the direct effect of a change in the covariance for each firm is $V^{i}_{s^{i}Cov} < 0$ and $V^{j}_{s^{j}Cov} < 0$. Given the sign of $V^{i}_{s^{i}s^{j}}$, the proposition follows from the second order condition $V^{j}_{s^{j}s^{j}} < 0$ and the stability condition $DET>0$. *Q.E.D.*