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Budget Deficits, Tax Risk and Consumption

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Abstract

This paper analyzes the effects of budget deficits on consumption when individual taxes are stochastic. It is shown that the co-movements between budget deficits and private consumption will depend on how risk averse individuals are. In the case of lump-sum taxes, it is sufficient to assume that individuals have a precautionary savings motive to obtain the result that consumption today will decrease with increased disposable income today. Furthermore, if we use a time separable iso-elastic utility function, the standard analysis of capital income risk predicts (precautionary) savings to increase with increased risk if the coefficient of relative risk aversion is greater than one. This is no longer sufficient when the risk is due to uncertain capital income taxes. In general, the coefficient must be greater than one to obtain precautionary savings in response to the greater risk implied by a budget deficit. The results in the paper are consistent with Ricardian equivalence only for some specific utility function, but not in general. However, in the same way, the results are consistent with standard ”Keynesian” models that display a positive relation between debt and private consumption only for certain utility functions, and could equally well generate the opposite result for individuals that are enough risk averse or prudent, without changing the expected value of government consumption. In other words, if future taxes are uncertain, increased disposable income in the present period will decrease present consumption, if households are prudent enough.

Keywords: budget deficits, tax risk, precautionary savings

JEL: D81, E21, H60

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1. INTRODUCTION

The effects of government budget deficits on private consumption is a central question for both macro economists and policy makers, the reason for this being that in recessions, stabilization policies have often aimed at increasing aggregate demand by increasing the households' current disposable income, without changing public spending, i.e. by creating a budget deficit. To obtain such a positive demand response to budget deficits, the public has, in general, to regard government bonds as net wealth. With Barro's famous [1974] paper, the debate on this issue (re)started, with the validity of the Ricardian equivalence proposition as the central theme. In short, the proposition states that private consumption will not change in response to changes in the timing of taxes, as long as government consumption remains unchanged.

There are of course theoretical models that support the equivalence proposition as well as models that create deviations from Ricardian equivalence. The focus in most models that create deviations from Ricardian equivalence has been on the potential wealth effect from postponing taxes. This is due to individuals regarding their holdings of government bonds as net wealth, i.e. they do not discount future tax payments one hundred per cent. However, the type of model, Ricardian or non-Ricardian, that best describes the real world can only be determined by empirical investigations.

There have been a number of papers trying to test the Ricardian proposition empirically, with mixed conclusions, see for example the studies in Yawitz and Meyer [1976], Kormendi [1983], Feldstein [1982] and Bernheim [1987], or for an overview Seater [1993]. When we use a stochastic model of Ricardian equivalence (which ought to be a natural starting point for an econometric study), one central issue is to sort out the effect current tax changes has on the expectations about future government consumption. This is vital, since the Ricardian prediction for postponing taxes is valid only for a given level of expected government consumption, so if a change in the timing of taxes leads to a revision of the expected value of government consumption, this will affect private consumption not because government bonds are perceived as net wealth but because the expected present value of government consumption has changed. An empirical study that explicitly handles this is Aschauer

[1985], while many of the earlier empirical studies neglect this signalling role that debt can play. Other papers that discuss the importance of future government policies are Drazen and Helpman [1990], Bertola and Drazen [1993] and Giavazzi and Pagano [1990].

Giavazzi and Pagano illustrate the important link between fiscal contractions and expectations about future levels of government consumption with an example where increased taxes (and thus reduced disposable income) lead to increased private consumption. They explain this phenomenon by stating that when the level of future government consumption is expected to fall in periods of fiscal contractions, households revise their expectations about future net income in a positive direction. This change in expected future net income will then increase present consumption, although present disposable income is reduced. This argument is of course fully consistent with standard Ricardian models.

However, Becker [1995a] finds that even when the expected level of future government consumption is unchanged, present consumption rises with increased taxes. This is not a pattern that can be explained in standard models that deal with the question of Ricardian equivalence. In this paper it is argued that we can explain this observation with households that care about risk, in combination with uncertainty about future tax payments and thus uncertain disposable income. In other words, households do not only consider changes in expected disposable income, as stressed in most of this literature, but they are also concerned about the spread of their disposable income.

The idea that households care about both the spread and the expected values is familiar from the literature that analyzes expected utility maximization, starting with von Neumann and Morgenstern [1944]. The more formalized analysis of risk aversion originates from Arrow [1963], Pratt [1964] and Arrow [1971], where the widely used measure of absolute risk aversion is defined. This measure describes how much an agent dislikes risk, but it does not tell us how agents change their consumption over time to hedge themselves against risk.

With Leland [1968] and Sandmo [1970], a formal treatment of the intertemporal consumption choice in connection with uncertainty is developed, and the concept of precautionary savings is introduced. In brief, the conclusion is that with increased uncertainty about future income, agents will save more if certain conditions of the utility function are fulfilled, and this type of saving in response to risk is labelled precautionary savings. Individuals that display a precautionary savings motive are said to be prudent, and in the same way that risk averse individuals' preferences have a positive measure of absolute risk aversion, prudent individuals have a positive measure of absolute prudence, see Kimball [1990].

To formalize the above discussion, in an atemporal model, individuals that have preferences that satisfy $-U''/U' > 0$ (i.e. the measure of absolute risk aversion is positive), where U' is the first derivative of the utility function, are said to be *risk averse*. In a two period model, individuals that have time separable utility and preferences over second period consumption that satisfy $-U'''/U'' > 0$ (i.e. the absolute measure of prudence is positive) are said to be *prudent*.

In the recent macro literature, the concept of precautionary savings has been used to explain observed deviations from Hall's [1978] formulation of the permanent income hypothesis. Hall's original paper implies that consumption can be described as a random walk, and that unexpected changes in income that are transitory should affect consumption relatively little, while changes in income that are permanent should give rise to larger changes in consumption. However, these predictions are based on a quadratic utility function, that displays the well known certainty equivalence outcome, and thus the precautionary savings motive is not present. If we instead allow preferences to display the precautionary savings motive, we are able to explain observed consumption patterns better than we can do with Hall's original formulation, see for example Zeldes [1989], Caballero [1990] and Weil [1993].

Most of the studies that analyze precautionary savings are concerned with risk in gross income or with general income risk, and do not incorporate uncertainty created by the government. However, the public sector determines a large part of households' disposable income, and the rules that determine taxes as well as transfers are subject to frequent revisions (or reforms to use the politically correct label). Thus it seems natural to investigate how risk created in connection with the tax system affects households, as a complement to the standard analysis of market risk.

One study that incorporates uncertainty about future taxes is Chan [1983]. He concludes that for some utility functions, we will get the result that increased taxes today can increase present consumption, without having changed the expected value of disposable income. Chan uses lump-sum taxes in his model, which we will also use in the first experiment below. The advantage of the present study is that we use a model similar to Sandmo [1970], which lets us explicitly formulate the conditions needed on the utility function to obtain this result. Not surprisingly, we need to assume that households' preferences are such that they display a precautionary savings motive to obtain the result that increased taxes today will increase present consumption.

The above result was achieved with lump-sum taxes, and thus the consumers cannot affect the tax base. In this paper, we also extend the analysis to the case of distortionary taxes. With distortionary taxes, the consumption choice affects the tax base, and thus we will need an endogenous response in tax rates to maintain the government's intertemporal budget balance. This formulation suggests from the start that we will get deviations from Ricardian equivalence, since we now use both distortionary and uncertain taxes. What the deviation will be is however an open question. In the paper it is shown that with strong enough risk aversion (or rather the implied precautionary savings motive), we will still get the result that increased taxes today will increase present consumption, without having changed the expected value of disposable income. We have thus presented a theoretical explanation of the empirical puzzle in Becker [1995a] for both lump-sum taxes and for a more realistic tax system with distortionary taxes.

2. A MODEL OF STOCHASTIC TAXES

Papers that describe how the consumption savings choice is affected by changes in general income and capital risk in two period models are, for example, Leland [1968] and Sandmo [1970], and Ormiston and Schlee [1992]. The two latter papers, among others, uses the concept of mean preserving spreads to define the changes in risk. The latter paper also provides conditions of the utility function for alternative distributional changes. In this specific model, we study an economy that exists for two periods, with one government sector, a large number of households, and the interest rate given by a world market. The government obeys an intertemporal budget constraint, and the households are expected utility maximizers. The methodology is to solve a two equation system for two unknowns, where the first equation is the government's budget constraint and the second equation is the (representative) household's first order condition.

The uncertainty we introduce here is, contrary to the optimal taxation literature, based on the assumption that the government could tax people differently although they have identical endowments and utility functions. This assumption requires some motivation, beside the purely technical aspect as a way to introduce uncertainty in the model. The most basic motivation is by simply observing past government actions and investigate if tax burdens carried by different individuals could be derived from their endowments and consumption patterns. In many cases this would probably require some imagination. Further, in the model used, we have aggregated both consumption and savings, but in reality people consume bundles of goods and have different assets in their portfolios, and, when making the consumption and investment decisions, the tax rules that will be used in the future are uncertain and potentially different for different assets and consumption goods. This would introduce uncertain net interest incomes.[1]

[1] One recent Swedish experience displaying the uncertain nature of the tax system is the "one-time" tax on private pension funds, not to mention all other constant revisions (or tax reforms) of the tax system. Furthermore, transition rules between different tax systems in Sweden have, for example, had the effect that some tax deductions have been worth between 20 and 250 per cent of the realized loss, depending on the composition of the remainder of the individual's capital income, as well as the timing of different portfolio decisions.

2.1 CHANGING THE TIMING OF LUMP-SUM TAXES

In this experiment, we assume that the government uses lump-sum taxes to finance its known consumption in both periods, that households have access to a capital market with a given interest rate, and that the planning horizon for the government and the households is the same. In a non-stochastic model, this would usually generate the Ricardian equivalence result, namely that changing the timing of taxes will not affect private consumption. In the present model, however, we introduce uncertainty about future taxes, but we keep the level of government consumption fixed. We will therefore not have deviations from Ricardian equivalence due to changed expectations about government consumption, which implies that the expected tax burden will be kept constant. This experiment is closely related to Sandmo's [1970] analysis of income risk, and to Chan's [1983] analysis of tax risk.

Government

The government obeys an intertemporal budget constraint according to

$$F^1 \equiv T_1 + \frac{T_2}{1+r} - G_1 - \frac{G_2}{1+r} = 0 \ , \tag{1}$$

where r is the interest rate, and G_1 and G_2 are government consumption in the first and second period, that are assumed to be deterministic, and thus known by households in the first period. First and second period aggregate lump-sum taxes, T_1 and T_2, are also certain, although second period individual taxes are uncertain in a way described below.

Households

We assume that we have n identical households that maximize expected utility according to

$$\begin{aligned} &\max_{c_1,c_2} \ E\left[U\left(c_1,c_2\right)\right] = \int_{\Pi} U\left(c_1,c_2\right) f(\pi) d\pi \\ &\text{s.t.} \quad c_1 = y_1 - \tau_1 - a_1 \\ &\qquad\ \ c_2 = y_2 - \tau_2 + (1+r)(y_1 - c_1 - \tau_1) \ , \end{aligned} \tag{2}$$

where $U(.)$ is a von Neumann-Morgenstern utility function, c_1 and c_2 are consumption in the first and second period, y_1 and y_2 are exogenous gross income in the first period and second period, τ_1 and τ_2 are lump-sum taxes in the first and second period, $a_1 \equiv y_1 - c_1 - \tau_1$ is savings and r is the given interest rate. The feature of the model that makes it stochastic is the specification of the tax system. Taxes in both periods are lump-sum, but first period taxes are certain and equal to $\tau_1 = T_1/n$, where T_1 is aggregate taxes, i.e. first period taxes are defined as a *given* proportion of total taxes determined by the known number of households. Second period taxes are instead assumed to be stochastic and equal to $\tau_2 = \pi T_2$, where π is a stochastic variable defined over the non-negative distribution Π with $E[\pi] = 1/n$. We then let each household make a draw of its share of taxes, π, from the probability distribution in the second period to get its second period tax payment. The household's expected share of total taxes over both periods is then simply the aggregate taxes divided by the number of households, while the realized tax can be smaller or greater than this. All the uncertainty in the model is due to the uncertain future taxes.

We also have to make sure that the realized taxes in the second period are such that the government's budget constraint is fulfilled. There are two alternative ways of handling this. First we can assume that all individuals except one make a draw from the distribution of taxes, and then the last individual pays the remaining tax necessary for the government to fulfill the budget constraint. An alternative interpretation is that there are sufficiently many households to make the law of large number apply, i.e. the realized mean of tax payments is arbitrarily close to the expected value of tax payments necessary to fulfill the budget constraint.

The first and second order conditions (FOC and SOC) for the maximization problem are

FOC: $$F^2 \equiv E[U_1 - (1+r)U_2] = 0 \qquad (3)$$

SOC: $$S \equiv E\left[U_{11} - 2(1+r)U_{12} + (1+r)^2 U_{22}\right] < 0 \ , \tag{4}$$

where subscripts on the utility function denote partial derivatives with respect to consumption in period one and two. The first order condition states that households equalize the expected marginal utility of consumption in the two periods, while the second order condition ensures that we have a maximum.

By using the household's FOC and the government's intertemporal budget constraint we get the following system of equations

$$\begin{cases} F^1 = 0 \\ F^2 = 0 \ . \end{cases} \tag{5}$$

If we differentiate the system with respect to first and second period taxes and first period private consumption and assume that second period taxes are exogenous and the other two variables are endogenous, we can solve for the partial derivatives describing how first period consumption and taxes will change in response to a change in second period taxes. (The derivation of the partial derivatives can be found in the appendix.) We first have the derivative describing how first period taxes have to change in order to fulfill the government's intertemporal budget constraint when second period taxes are changed. It is simply

$$\frac{\partial T_1}{\partial T_2} = -\frac{1}{1+r} \ , \tag{6}$$

which is of course an expression of the fact that second period taxes are discounted by the interest rate, so by increasing future taxes by one dollar, we can lower present taxes by $1/(1+r)$.

To describe how first period consumption will change in response to a change in second period taxes to fulfill the first order condition, we have the somewhat more complicated expression

$$\frac{\partial c_1}{\partial T_2} = \frac{1}{S} E\left[\left(U_{12} - (1+r)U_{22}\right)\left(\pi - \frac{1}{n}\right)\right] . \qquad (7)$$

This effect on first period consumption in response to a change in second period taxes is analogous to the effect obtained by Sandmo [1970] in the case of uncertain future income. From his analysis, we know that first period consumption will decrease with increased uncertainty about future income if households' preferences display decreasing temporal risk aversion. An equivalent assumption is that the household's utility function is such that the household has a precautionary savings motive, which is a widely used assumption that will be used also here.[2] The reason for the first period consumption to decrease with increased future taxes (and thus increased present disposable income) is here due to the fact that second period taxes are stochastic, while first period taxes are certain. Thus, by substituting taxes today for taxes in the future, individuals are exposed to a greater uncertainty and will engage in precautionary savings.

For a proof of the sign of the derivative between first period consumption and future taxes, see the appendix. Furthermore, we can note that it is not sufficient to assume that individuals are risk averse to determine how first period consumption changes in response to a changed timing of taxes, because risk aversion only determines how their utility will be affected and not how they will change their first period consumption when they are exposed to risk.

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[2]Decreasing temporal risk aversion implies that $U_{112} - U_{222} < 0$, which with time separable utility says that the third derivative with respect to second period consumption is positive. Furthermore, the coefficient of absolute prudence is defined as $\eta = -U_{222}/U_{22}$, and when this coefficient is positive, individuals display a precautionary savings motive. Since the coefficient of absolute prudence will be positive for a risk averse individual with a positive third derivative of the utility function, the assumption of decreasing temporal risk aversion is equivalent to the assumption that individuals have a precautionary savings motive if we assume a time separable utility function. We can note that in the case of time separable iso-elastic utility, assuming that individuals are risk averse also implies that the third derivative of the utility function will be positive.

We have now derived a deviation from Ricardian equivalence in a model where the households and the government have the same planning horizon, they have access to a perfect capital market and the government uses lump-sum taxes to finance a given amount of government consumption. The deviation from Ricardian equivalence is due to the stochastic nature of the tax system in combination with individuals that are not risk neutral. If we further use the assumption that households' preferences are such that they have a precautionary savings motive, we get the result that today's consumption will decrease with increased disposable income in the first period. This is contrary to most models that display deviations from Ricardian equivalence, in particular to a Keynesian model, where present consumption is always increasing with increasing present disposable income.

Also non-Ricardian models with an intertemporal utility maximizing framework usually get the Keynesian type of deviation from Ricardian equivalence, since most of them have assumptions that generate a positive wealth effect for households from postponing taxes, and neglect the stochastic elements involved when shifting taxes over time. One exception is the paper by Chan [1983], where a model similar to the one presented here is used to obtain the result that postponing taxes can lead to decreased consumption today. However, in his paper it is less straightforward to derive explicit conditions on the utility function needed to get this effect.

2.2 CHANGING THE TIMING OF CAPITAL INCOME TAXES

In this experiment, we use distortionary capital income taxes in the second period, rather than the lump-sum taxes used in the previous analysis. In the first period, we still use lump-sum taxes, but the interpretation of these taxes can be in terms of a tax on an exogenous income, where the income could be due to already installed capital. In other words, we can think of the tax system as a system taxing capital income in both periods, the difference being that in the first period the tax is non-distortionary and non-stochastic, while in the future it is both distortionary and stochastic. With distortionary taxes, we do not expect the Ricardian hypothesis to be true, even if we remove the stochastic features of the model. It is, however, still interesting to investigate what the deviations from the equivalence propositions are. In other words, the question is now how present consumption changes if we substitute certain, non-distortionary, taxes today for uncertain, distortionary, taxes tomorrow.

The tax system could be interpreted as if the government uses a standard range of tax rates on capital income, defined in such a way that for a balanced first period budget, the second period is also expected to balance. Further, if we assume that when the government runs a deficit it multiplies the tax schedule for the second period with a constant greater than one, and when it runs a surplus it uses a coefficient less than one. Thus, if the government runs a deficit, this has both the effect of raising the expected value of the second period interest income tax and increasing its spread, and hence we have introduced a very simple connection between high debt and increased risk.

Government

The government obeys the following intertemporal budget restriction.

$$\bar{g} = g_1 + \frac{g_2}{1+r} = \tau_1 + \frac{E(\tau_2)}{1+r} = \tau_1 + \frac{p\bar{\gamma} r a_1}{1+r} \quad , \tag{8}$$

where $\bar{g}$ is the net present value of the total financing requirement, due to the resource use, $g_{1,2}$ in periods one and two, r is the exogenously given interest rate, τ_1 is first period lump-sum taxes, a_1 is households' savings, and p is a policy parameter that multiplies the average tax rate on capital income, $\bar{\gamma}$. Rewrite the government's budget constraint and define

$$H^1 \equiv \tau_1 + p\bar{\gamma} r\left(y_1 - c_1 - \tau_1\right) / (1+r) - \bar{g} \ , \tag{9}$$

which will be the first equation in the system analyzed.

Households

Households are assumed to be rational economic units that all solve the following maximization problem

$$\max_{c_1,c_2} \; E\left[U\left(c_1,c_2\right)\right] = \int_{\Gamma} U\left(c_1,c_2\right) f\left(\gamma^i\right) d\gamma^i$$
$$\begin{aligned} \text{s.t.} \quad & c_1 = y_1 - \tau_1 - a_1 \\ & c_2 = (1+r-p\gamma^i r)(y_1 - c_1 - \tau_1) \quad , \end{aligned} \tag{10}$$

where U is a von Neumann-Morgenstern utility function, c_t is period t consumption, y_1 is exogenous income in the first period[3], $a_1 = y_1 - c_1 - \tau_1$ is savings between the first and second period, and γ^i is an individual tax rate on capital that is drawn in period two from the tax distribution Γ, with $E\left[\gamma^i\right] = \bar{\gamma}$. Since the stochastic tax rate is multiplied by p, we have that the realized tax rate is $p\gamma^i$, which is present in the household's budget constraint, while the expected tax rate is $p\bar{\gamma}$, which is present in the government's budget constraint. Note that with the present formulation of the tax system, increasing p is equivalent with increasing risk. The expectations operator is due to the uncertain capital income tax rate alone, since the interest rate is assumed to be known. Since we also assume that there is an infinite number of households, no individual household will affect the expected tax rate by its own consumption choice. Furthermore, the households are identical, so we analyze only a representative household, and all variables are in *per capita* terms. The first and second order conditions to this problem are

FOC: $$H^2 \equiv E\left[U_1 - R^i U_2\right] = 0 \tag{11}$$

SOC: $$S \equiv E\left[U_{11} - 2R^i U_{12} + R^{i^2} U_{22}\right] < 0 \;, \tag{12}$$

where $R^i = \left(1 + r - p\gamma^i r\right)$.

[3]Second period labor income is assumed to be zero, since otherwise it is not obvious that households will actually save, and then the use of a capital income tax is of little interest.

The second equation we use in our two equation system is the FOC, which gives us the following system of equations

$$\begin{cases} H^1 = 0 \\ H^2 = 0 \ . \end{cases} \tag{13}$$

If we differentiate the system with respect to the multiplicative tax parameter, p, first period taxes, τ_1, and first period private consumption, c_1, and assume that p is exogenous and the other two variables are endogenous, we can solve for the partial derivatives describing how first period consumption and taxes will change in response to a change in p. (The derivation of the partial derivatives can be found in the appendix.) We first have the derivative describing how first period taxes have to change in order to fulfill the government's intertemporal budget constraint when p is changed

$$\frac{\partial \tau_1}{\partial p} = -\frac{\frac{\bar{\gamma} r a_1}{1+r} S + \frac{p\bar{\gamma} r}{1+r} E\left[\gamma^i r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right]}{|J|} \quad , \tag{14}$$

where $\Psi \equiv R^i\left(U_{12} - R^i U_{22}\right)$, and $|J| = \frac{\bar{R}}{1+r} S - \frac{p\bar{\gamma} r}{1+r} E[\Psi] < 0$, with $\bar{R} \equiv 1 + r - p\bar{\gamma} r$. We can start by noting that the expression is much more complex than the corresponding expression derived when we shifted lump-sum taxes, and the reason for this is that we now have to take into account the changes in private consumption, since these affect the tax base. For the moment we will only note that if $E\left[\gamma^i r a_1 \Psi / R^i\right] \geq E\left[\gamma^i r U_2\right]$ the expression will be negative. This condition implies certain restrictions on the utility function, and thus for some utility functions the derivative can be positive.

If the expression is positive, this implies that an increased expected capital tax rate in combination with increased spread of individual tax rates, i.e. an increase in p, could have the effect of decreasing tax revenues in the second period, so that we would be to the right of the maximum on the Laffer curve. In the appendix it is shown that we will be to the right of the maximum on the Laffer curve if we use a time separable iso-elastic utility function with a sufficiently small coefficient of relative risk aversion (RRA). The intuition for this condition is best understood if we consider the limiting case where RRA approaches zero so that the individual is risk neutral. With a linear utility function and the net interest rate equal to the discount factor, it does not matter in which period to consume, and in the first period there is no tax to pay on consumption, while in the second period, the individual will have to pay tax on savings and thus on second period consumption. In that case, the capital income tax will be fully avoided by consuming all wealth in the first period, and there is then nothing left to tax in the second period.

Finally, the effect on first period private consumption is

$$\frac{\partial c_1}{\partial p} = -\frac{1}{|J|}\left(\frac{\bar{R}}{1+r} E\left[\gamma^i r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right] + \frac{\bar{\gamma} r a_1}{1+r} E[\Psi]\right). \tag{15}$$

To determine the sign of the derivative we start by rewriting the expression as

$$\frac{\partial c_1}{\partial p} = -\frac{r}{|J|(1+r)}\left(E\left[a_1\Psi\left(\bar{\gamma} - \frac{\bar{R}}{R^i}\gamma^i\right)\right] + \bar{R}E\left[\gamma^i U_2\right]\right). \tag{16}$$

The next step is to use $\Psi \equiv R^i\left(U_{12} - R^i U_{22}\right)$, and to add and subtract the term $R^i\bar{\gamma}U_2$ in the parenthesis. We then get

$$\frac{\partial c_1}{\partial p} = \frac{r}{|J|(1+r)} E\left[U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) - a_1\left(U_{12} - R^i U_{22}\right)\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) - \bar{\gamma}R^i U_2\right], \tag{17}$$

which could be analyzed in a way similar to Sandmo [1970] or Becker [1995b]. In this case we have three effects to consider, two that are standard when analyzing consumption choice with uncertain capital income (which are represented by the two first terms) and one due to the distortionary tax on capital income (represented by the last term). The two standard effects are analogous to the effects that Sandmo [1970] labels as the substitution and income effects, respectively.

The substitution effect (the first term) says that with increased uncertainty about the return on capital, second period consumption becomes riskier, and a risk averse individual will then prefer to consume in the risk free first period. However, the income effect (second term) goes in the other direction, and the reason for this is that the marginal utility of consumption in the second period will be very high if the individual has a bad realization that gives a low second period income, since we assume that the utility function is concave, i.e. individuals are risk averse. The income effect will hence make agents save more, i.e. consume less in the first period, to safeguard themselves against bad outcomes when risk increases.

A risk averse individual will thus have these two uncertainty effects to trade off. With a time separable utility function with iso-elastic utility in each period, we know from Sandmo [1970] that the strength of the income and substitution effects will depend on the coefficient of relative risk aversion (RRA), and with RRA equal to one these effects will cancel each other out. With RRA smaller than one the substitution effect will dominate, and with RRA greater than one the income effect will dominate. Ormiston and Schlee [1992] state a somewhat more natural condition, namely that the coefficient of relative prudence has to be greater(less) than two for the income(substitution) effect to dominate, and also develop conditions for other distributional changes. In the case of an iso-elastic utility function, it is straightforward to verify that if the coefficient of relative risk aversion is one, the coefficient of relative prudence is two.

In addition to the two ”standard” effects discussed, we here have an additional term. This is the last term, that describes how the average tax rate on capital makes the agent save less than would be the case if there were no tax on capital. With an increase in the riskiness, we also have the effect that the average tax rate on capital increases, which makes second period consumption more expensive and provides an incentive for consumption in the first period. This effect reinforces the standard substitution effect, and reduces the range where the income effect dominates.

In this case with a distortionary tax, the above condition on RRA will only determine the sign of the total expression in the case of a dominating substitution effect, since in that case the effect of the last term will reinforce the first effect. Since we know that the denominator is always negative, and that the numerator is positive in the case of a dominating substitution effect, we then have the result that the derivative is positive. In other words, with increased risk about the net return on capital, i.e. increased disposable income in the first period, households will consume more in the first period.

For the case of a ”dominating” income effect in the standard case, we can still get the result that first period consumption increases with increased risk here, if the last term is sufficiently large. In general, we can say that the income effect has to be stronger in this case than in the case without taxes to generate a negative derivative between first period consumption and the multiplicative tax parameter. This is due to the fact that the tax distribution is endogenous, and has to respond to changes in consumption to maintain the government’s budget balance. In other words, to make present consumption decrease with increased disposable income in the first period, it is no longer sufficient to assume that the coefficient of relative prudence is greater than two. In the following subsection, we will provide a sufficient condition on the coefficient of relative risk aversion to obtain the result that first period consumption decreases with a tax cut in the first period.

2.2.1 THE CASE OF TIME SEPARABLE ISO-ELASTIC UTILITY

In this section, a time separable iso-elastic utility function is used to derive a sufficient condition on the coefficient of relative risk aversion to obtain a negative derivative between first period consumption and the multiplicative tax parameter, i.e. that first period consumption will decrease in response to increased disposable income in the first period. Multiplying the first and second factor in (17) by -1 and cancel terms, we get the condition

$$E\left[a_1\left(U_{12} - R^i U_{22}\right)\left(R^i \bar{\gamma} - \bar{R}\gamma^i\right) + \bar{R}\gamma^i U_2\right] < 0 \quad , \tag{18}$$

to obtain the result that first period consumption will decrease in response to increased risk.

Using an iso-elastic utility function $U(c_1, c_2) = u(c_1) + \beta u(c_2)$ with $u(c) = c^{1-\alpha} / (1-\alpha)$, where α is the coefficient of relative risk aversion[4] and β is the time preference, the condition becomes

$$E\left[a_1 R^i \alpha\beta c_2^{-\alpha-1}\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) + \bar{R}\gamma^i \beta c_2^{-\alpha}\right] < 0 \ , \tag{19}$$

or, by using $c_2 = a_1 R^i$ and canceling the rate of time preference, equivalently

$$E\left[\alpha\bar{\gamma} R^i c_2^{-\alpha} + (1-\alpha)\bar{R}\gamma^i c_2^{-\alpha}\right] < 0 \ . \tag{20}$$

We know that the first term will always be positive, since the agents are assumed to be risk averse, i.e. $\alpha > 0$, we analyze taxes and not transfers, and finally the FOC implies that $R^i c_2^{-\alpha} > 0$.

The second term will be positive if $\alpha < 1$, since $Cov\left(\gamma^i, c_2^{-\alpha}\right) = E\left[\gamma^i c_2^{-\alpha}\right] - \bar{\gamma} E\left[c_2^{-\alpha}\right] > 0$ implies that $E\left[\gamma^i c_2^{-\alpha}\right] > 0$, and $E\left[\gamma^i c_2^{-\alpha}\right] > \bar{\gamma} E\left[c_2^{-\alpha}\right]$, which will be used later. Thus the expression is positive for $\alpha < 1$. In other words, for the case of a dominating substitution effect, we have demonstrated that first period consumption increases with increased tax risk, and thus increased disposable income in the first period.

The next question is to investigate whether there exists a coefficient of relative risk aversion that is large enough for the expression to be negative, implying that first period consumption decreases with a first period tax cut. Use $R^i \equiv 1 + r - p\gamma^i r$, and $\bar{R} \equiv 1 + r - p\bar{\gamma} r$ to rewrite the condition as

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[4]The coefficient of relative prudence for this utility function is $\alpha + 1$.

$$\alpha(1+r)\bar{\gamma}E\left[c_2^{-\alpha}\right]+\left(\bar{R}-\alpha(1+r)\right)E\left[\gamma^i c_2^{-\alpha}\right]<0\ , \tag{21}$$

where we note that the first term will always be positive, but the second term will be negative for large enough values on α. In general, it is hard to answer the question of how large α has to be to fulfill the condition, since we need to know the relation between $E\left[\gamma^i c_2^{-\alpha}\right]$ and $\bar{\gamma}E\left[c_2^{-\alpha}\right]$ to establish this. However, to make the condition a little more transparent in terms of α, start by writing the condition as

$$\alpha(1+r)+\left(\bar{R}-\alpha(1+r)\right)\Omega<0\ , \tag{22}$$

where $\Omega \equiv E\left[\gamma^i c_2^{-\alpha}\right] \big/ \left(\bar{\gamma}E\left[c_2^{-\alpha}\right]\right) > 1$. If we then solve for α, we get

$$\alpha > \frac{\bar{R}\Omega}{(1+r)(\Omega-1)}\ . \tag{23}$$

This condition on α only provides a hint to the exact condition on α, since Ω is a function of α. It thus still remains to see if this condition can actually be fulfilled, and we will now use a numerical example to show that it can actually be fulfilled. Start by assuming that $\alpha = 2$, $r = 1$, $p = 1$, and that the tax distribution is two-valued, with $\gamma^i = 0 \text{ or } 1$ with equal probabilities. We further assume that savings, a_1, which is an endogenous variable, is equal to one. This is really an implicit assumption about the first period income and the level of government consumption. However, to make this an explicit assumption about the net income level, we would have to derive explicit solutions to the problem, which is complicated, and not necessary for demonstrating that the condition can be fulfilled. With this parametrization, we get that $\Omega = 1.6$ and that the left hand side in (23) is equal to 1.2, and since the calculation was done for $\alpha = 2$, we see that the condition is fulfilled. In other words, we have constructed an example where first period consumption decreases when the risk and average tax are increased, i.e. first period taxes are reduced.

3. DISCUSSION AND EXTENSIONS

This paper uses a simple two period framework with a government sector and a large number of households. A two-equation system is formulated to analyze the effects of different levels of risk, which are positively related to the level of public debt. The first equation is the government's budget constraint and the second is the household's first order condition for optimal consumption. We then analyze the effects on private consumption from changing the timing of taxes. Two different taxes are analyzed, first lump-sum taxes in the Ricardian tradition, and then the more realistic case of capital income taxes that are distortionary.

In the case of lump-sum taxes it is sufficient to assume that households' preferences display a precautionary savings motive to get the result that increased disposable income in the first period due to shifting taxes to the second period implies that households consume less in the first period. This is due to the assumption that taxes today are certain, while taxes in the future are uncertain. This result is analogous to Sandmo's [1970] result on income risk, and his mean preserving spread analysis uses the same assumptions about the utility function as the present analysis of an intertemporal budget preserving spread to show that present consumption will be reduced if risk is increased, i.e. first period taxes are reduced in the present model.

In the case of distortionary capital income taxes, we can still obtain the result that present consumption decreases with increased current disposable income, i.e. with increased uncertainty about future taxes. However, individuals must be more prudent in this analysis of an intertemporal budget preserving spread to obtain this result than they have to be in Sandmo's [1970] analogous mean preserving spread analysis of capital income risk. The paper can thus explain the empirical puzzle found in Becker [1995a], both for the case of non-distortionary and the case of distortionary taxes, if we make the appropriate assumptions on the utility function.

The analysis has provided a mechanism that links the level of risk to the government's budget deficit, and the condition on the utility function needed for this mechanism to provide the result that private consumption today is *reduced* in response to current tax cuts. However, depending on how prudent households are, the case of precautionary savings in response to tax cuts as well as a ”Keynesian” demand injection could be generated by the model, although the reason for the demand injection would be far from the standard Keynesian explanations. It is worth noting that households now care about both the expected value and the spread of their tax payments, and that we do not alter the present value of government consumption. So the goods news for the advocate of active stabilization policy through changes of the government’s financing policy is that it could actually change aggregate demand without changing government consumption. The bad news, however, is that one has to know the degree of risk aversion of households to know if aggregate demand increases or decreases in response to a tax cut, as well as how households perceive the risk induced by postponing taxes.

A justification for this type of tax schedule that lies outside this framework is to think of the government as the controller of inflation, and that the government uses a tax scheme based on nominal incomes rather than real. In this case, the way of raising extra tax revenues in case of a deficit is to create inflation, and thereby increase the real incomes as well as the spread of tax payments. The multiplicative tax system used here could potentially be regarded as a short-cut for the inflation story. It is interesting to think of this as a potential explanation for the excess burden of an inflation tax that we often think exists but has been little accounted for in the literature. An area of future research is to include a monetary side explicitly in the model, to combine the income side of creating inflation, with the cost in terms of increased uncertainty.

Finally, in this paper, gross returns on capital were assumed to be known. This implies that we neglect the potential of the tax system to act as an insurance of gross returns. The reason for making this simplifying assumption was to have a model of pure government uncertainty, to see how this alters the standard analysis of capital income uncertainty. However, if we want to come closer to the real world, it is reasonable to make also gross returns stochastic. We would then have to take into consideration the covariance between the gross returns and the tax rate. If there is a positive correlation between the return and the tax rate, this could potentially offset some of the precautionary savings created by the tax system in the present model.

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5. APPENDIX

5.1 LIST OF VARIABLES

θ	additive tax parameter
p	multiplicative tax parameter for comparative statics exercises
γ^i	the stochastic tax variable, the superscript denotes that it is individual specific
	$E[\gamma^i] = \gamma$, (non-stochastic) is what the government uses, since there is a large (enough) number of households
g_t	government consumption in period t = 1,2
τ_1	first period lump-sum tax
U	von Neumann-Morgenstern utility function
U_{ij}	partial derivative with respect to elements i, j
c_t	private consumption in period t = 1,2
y_t	income in period t = 1,2
r	interest rate given from world market
a_1	household savings ($a_1 \equiv y_1 - c_1 - \tau_1$)
$f(x)$	density function of variable x defined over X

5.2 DERIVATION OF DERIVATIVES

The analysis of both lump-sum taxes and capital income taxes below starts from the points in the main text where the equation systems have been formulated.

5.2.1 LUMP-SUM TAXES

Totally differentiate the system of equations in (5) to get

$$\begin{bmatrix} \frac{\partial F^1}{\partial T_1} & \frac{\partial F^1}{\partial c_1} & \frac{\partial F^1}{\partial T_2} \\ \frac{\partial F^2}{\partial T_1} & \frac{\partial F^2}{\partial c_1} & \frac{\partial F^2}{\partial T_2} \end{bmatrix} \begin{bmatrix} dT_1 \\ dc_1 \\ dT_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad . \tag{24}$$

The Jacobian matrix for the system is

$$\begin{bmatrix} \frac{\partial F^1}{\partial T_1} & \frac{\partial F^1}{\partial c_1} & \frac{\partial F^1}{\partial T_2} \\ \frac{\partial F^2}{\partial T_1} & \frac{\partial F^2}{\partial c_1} & \frac{\partial F^2}{\partial T_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{1}{1+r} \\ \frac{1+r}{n} E\left[(1+r)U_{22} - U_{12}\right] & S & E\left[\pi\left((1+r)U_{22} - U_{12}\right)\right] \end{bmatrix} . \tag{25}$$

To use the implicit function theorem, the endogenous variable Jacobian determinant has to be non zero, or

$$|J| = S \neq 0, \tag{26}$$

which is fulfilled since S is the left hand side of the second order condition and thus a negative number. Divide the system by ∂T_2 to get

$$\begin{bmatrix} \frac{\partial F^1}{\partial T_1} & \frac{\partial F^1}{\partial c_1} \\ \frac{\partial F^2}{\partial T_1} & \frac{\partial F^2}{\partial c_1} \end{bmatrix} \begin{bmatrix} \frac{\partial T_1}{\partial T_2} \\ \frac{\partial c_1}{\partial T_2} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F^1}{\partial T_2} \\ -\frac{\partial F^2}{\partial T_2} \end{bmatrix} . \tag{27}$$

Use Cramer's rule to get the partial derivatives of interest. The derivative between first and second period taxes is

$$\frac{\partial T_1}{\partial T_2} = \frac{\begin{vmatrix} -\frac{\partial F^1}{\partial T_2} & \frac{\partial F^1}{\partial c_1} \\ -\frac{\partial F^2}{\partial T_2} & \frac{\partial F^2}{\partial c_1} \end{vmatrix}}{|J|} = -\frac{\frac{\partial F^1}{\partial T_2}\frac{\partial F^2}{\partial c_1} - \frac{\partial F^2}{\partial T_2}\frac{\partial F^1}{\partial c_1}}{|J|} , \tag{28}$$

which is equivalent to

$$\frac{\partial T_1}{\partial T_2} = -\frac{1}{1+r} . \tag{29}$$

The derivative between first period consumption and second period taxes is

$$\frac{\partial c_1}{\partial T_2} = \frac{\begin{vmatrix} \dfrac{\partial F^1}{\partial T_1} & -\dfrac{\partial F^1}{\partial T_2} \\ \dfrac{\partial F^2}{\partial T_1} & -\dfrac{\partial F^2}{\partial T_2} \end{vmatrix}}{|J|} \,, \tag{30}$$

or equivalently

$$\frac{\partial c_1}{\partial T_2} = \frac{1}{S} E\left[\left(U_{12} - (1+r)U_{22} \right) \left(\pi - \frac{1}{n} \right) \right]. \tag{31}$$

To determine the sign of this expression, we start by noting that the denominator is the SOC from a maximization problem, and is thus negative. Decreasing temporal risk aversion then implies that the numerator is positive, so that $\partial c_1 / \partial T_2 < 0$.

Before presenting the proof of this statement, which is an adoption of the proof in Sandmo [1970] , we will evaluate another derivative, namely $\partial c_1 / \partial y_1$. This will provide a condition that later will help us to determine the sign of $\partial c_1 / \partial T_2$. To obtain $\partial c_1 / \partial y_1$, we use implicit differentiation of the FOC to obtain

$$\frac{\partial c_1}{\partial y_1} = -\frac{\partial F^2}{\partial y_1} \Bigg/ \frac{\partial F^2}{\partial c_1} = -\frac{E\left[U_{12} - (1+r)U_{22} \right](1+r)}{S} \quad . \tag{32}$$

It seems reasonable to assume that this derivative is positive, thus implying that $E\left[U_{12} - (1+r)U_{22} \right] > 0$, which is a condition that will be used in the following analysis.

A proof that decreasing temporal risk aversion implies $\partial c_1 / \partial T_2 < 0$ will now be stated. Start by totally differentiating the absolute risk aversion function, $A = -U_{22} / U_2$, which yields

$$dA = \frac{\partial A}{\partial c_1} dc_1 + \frac{\partial A}{\partial c_2} dc_2 \quad . \tag{33}$$

Along the budget line we have $dc_1 = -\dfrac{1}{(1+r)} dc_2$, which we substitute into the differential and divide by dc_2 to obtain

$$\frac{dA}{dc_2} = -\frac{1}{1+r}\frac{\partial A}{\partial c_1} + \frac{\partial A}{\partial c_2} < 0 \ , \tag{34}$$

where the sign is determined by the assumption of *decreasing temporal risk aversion*. Continuity assumptions let us write

$$\frac{\partial A}{\partial c_1} \equiv \frac{\partial}{\partial c_1}\left(-\frac{U_{22}}{U_2}\right) = \frac{\partial}{\partial c_2}\left(-\frac{U_{12}}{U_2}\right), \tag{35}$$

which is substituted into the inequality, so that we get the condition

$$\frac{\partial}{\partial c_2}\left(\frac{U_{12} - (1+r)U_{22}}{U_2}\right) < 0 \ . \tag{36}$$

In the next step this condition implied by decreasing temporal risk aversion is used to determine the sign of $\partial c_1 / \partial T_2$.

Start by defining the second period consumption for tax payments equal to the expected value $\bar{c}_2 = y_2 + (1+r)y_1 - (1+r)c_1 - T_2 / n - T_1(1+r)/n$. By using the intertemporal budget restriction this could be written as $c_2 = \bar{c}_2 - \pi T_2 + T_2 / n$. Decreasing temporal risk aversion could now be used to write

$$\frac{U_{12} - (1+r)U_{22}}{U_2} \geq \left\{\frac{U_{12} - (1+r)U_{22}}{U_2}\right\}_{\pi = 1/n} \quad \text{if } \pi \geq 1/n \ , \tag{37}$$

where the right hand side is deterministic. Furthermore, $U_2(\pi - 1/n) \geq 0$ if $\pi \geq 1/n$. Multiplying both sides with this expression gives

$$\left(U_{12} - (1+r)U_{22}\right)(\pi - 1/n) \geq \left\{\frac{U_{12} - (1+r)U_{22}}{U_2}\right\}_{\pi = 1/n} \mathrm{U}_2(\pi - 1/n) \quad \text{if } \pi \geq 1/n \ , \tag{38}$$

which we take expectations of to get

$$E\left(U_{12} - (1+r)U_{22}\right)(\pi - 1/n) \geq \left\{\frac{U_{12} - (1+r)U_{22}}{U_2}\right\}_{\pi = 1/n} \mathrm{E}\left[\mathrm{U}_2(\pi - 1/n)\right] \quad \text{if } \pi \geq 1/n \ . \tag{39}$$

We want to show that the left hand side is positive, but a sufficient condition for this is that the RHS is positive, which is what we will show. The first part of the expression is positive due to the assumption that $\partial c_1 / \partial y_1 > 0$. To show that $E[U_2(\pi - 1/n)] > 0$, we start by noting that $U_2 \geq \{U_2\}_{\pi = 1/n}$ if $\pi \geq 1/n$, since $U_{22}<0$. Obviously, $\pi - 1/n \geq 0$ if $\pi \geq 1/n$, and thus $U_2(\pi - 1/n) \geq \{U_2\}_{\pi=1/n}(\pi - 1/n)$.

Taking expectations yields $E[U_2(\pi - 1/n)] \geq \{U_2\}_{\pi = 1/n} E[\pi - 1/n] = 0$, which then implies that the LHS > 0 in (39). We have therefore proved that $\partial c_1 / \partial T_2 < 0$, if we assume decreasing temporal risk aversion.

5.2.2 Capital income taxes

Totally differentiating the system (13) gives

$$\begin{bmatrix} \dfrac{\partial H^1}{\partial \tau_1} & \dfrac{\partial H^1}{\partial c_1} & \dfrac{\partial H^1}{\partial p} \\ \dfrac{\partial H^2}{\partial \tau_1} & \dfrac{\partial H^2}{\partial c_1} & \dfrac{\partial H^2}{\partial p} \end{bmatrix} \begin{bmatrix} d\tau_1 \\ dc_1 \\ dp \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{40}$$

The Jacobian matrix for the system is

$$\begin{bmatrix} \dfrac{\partial H^1}{\partial \tau_1} & \dfrac{\partial H^1}{\partial c_1} & \dfrac{\partial H^1}{\partial p} \\ \dfrac{\partial H^2}{\partial \tau_1} & \dfrac{\partial H^2}{\partial c_1} & \dfrac{\partial H^2}{\partial p} \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{p\bar{\gamma} r}{1+r} & -\dfrac{p\bar{\gamma} r}{1+r} & \dfrac{\bar{\gamma} r a_1}{1+r} \\ -E[\Psi] & S & E\left[\gamma^i r U_2 - \dfrac{\gamma^i r a_1}{R^i}\Psi\right] \end{bmatrix} , \tag{41}$$

where $\Psi \equiv R^i(U_{12} - R^i U_{22})$, and $a_1 \equiv y_1 - c_1 - \tau_1$. In order to use the implicit function theorem we have to check that the Jacobian determinant of the endogenous variables in this problem is non-zero. Or

$$|J| = \begin{vmatrix} \frac{\partial H^1}{\partial \tau_1} & \frac{\partial H^1}{\partial c_1} \\ \frac{\partial H^2}{\partial \tau_1} & \frac{\partial H^2}{\partial c_1} \end{vmatrix} = \begin{vmatrix} \frac{\bar{R}}{1+r} & -\frac{p\bar{\gamma}_r r}{1+r} \\ -E[\Psi] & S \end{vmatrix} \neq 0 \ , \tag{42}$$

where $\bar{R} = 1 + r - p\bar{\gamma}_r r$. This is equivalent to

$$|J| = \frac{\bar{R}}{1+r} S - \frac{p\bar{\gamma}_r r}{1+r} E[\Psi] \neq 0 \ . \tag{43}$$

The expression will be negative if E[Ψ] ≥ 0, which we usually assume, since this implies that $\partial c_1 / \partial y_1 > 0$. This connection between E[Ψ] ≥ 0 and $\partial c_1 / \partial y_1 > 0$ was shown above.

We then proceed by dividing both equations in the differentiated system with ∂p to obtain the following system

$$\begin{bmatrix} \frac{\partial H^1}{\partial \tau_1} & \frac{\partial H^1}{\partial c_1} \\ \frac{\partial H^2}{\partial \tau_1} & \frac{\partial H^2}{\partial c_1} \end{bmatrix} \begin{bmatrix} \frac{\partial \tau_1}{\partial p} \\ \frac{\partial c_1}{\partial p} \end{bmatrix} = \begin{bmatrix} -\frac{\partial H^1}{\partial p} \\ -\frac{\partial H^2}{\partial p} \end{bmatrix} . \tag{44}$$

From this we can solve for the derivatives of interest by using Cramer's rule. For the government's budget to be satisfied, first period lump-sum taxes have to change according to

$$\frac{\partial \tau_1}{\partial p} = \frac{\begin{vmatrix} -\frac{\partial H^1}{\partial p} & \frac{\partial H^1}{\partial c_1} \\ -\frac{\partial H^2}{\partial p} & \frac{\partial H^2}{\partial c_1} \end{vmatrix}}{|J|} = -\frac{\frac{\partial H^1}{\partial p} \frac{\partial H^2}{\partial c_1} - \frac{\partial H^2}{\partial p} \frac{\partial H^1}{\partial c_1}}{|J|} \ . \tag{45}$$

This is equivalent to

$$\frac{\partial \tau_1}{\partial p} = -\frac{\frac{\bar{\gamma} r a_1}{1+r} S + \frac{p\bar{\gamma} r}{1+r} E\left[\gamma^i r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right]}{|J|} . \tag{46}$$

Finally, the effect on first period private consumption is

$$\frac{\partial c_1}{\partial p} = \frac{\begin{vmatrix} \frac{\partial H^1}{\partial \tau_1} & -\frac{\partial H^1}{\partial p} \\ \frac{\partial H^2}{\partial \tau_1} & -\frac{\partial H^2}{\partial p} \end{vmatrix}}{|J|} , \tag{47}$$

or equivalently

$$\frac{\partial c_1}{\partial p} = -\frac{1}{|J|}\left(\frac{\bar{R}}{1+r} E\left[\gamma^i r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right] + \frac{\bar{\gamma} r a_1}{1+r} E[\Psi]\right) . \tag{48}$$

5.3 PROOF OF SIGN DETERMINATION

Below a proof is constructed to verify the sign of the first term of the numerator in the derivative between p and c_1. What we want to determine the sign of is

$$E\left[\underbrace{U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)}_{-}\underbrace{-a_1\left(U_{12} - R^i U_{22}\right)\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)}_{+}\right] . \tag{49}$$

The first term we will show is negative, and the second term is positive if we assume decreasing temporal risk aversion. The proof is constructed along the lines of Sandmo's [1970] proof in the case of capital income risk (see the appendix of that paper). Starting with the first factor, we have that

$$U_2 < \{U_2\}_{\bar{\gamma}} \ \text{ if } \ \gamma^i < \bar{\gamma} \ , \tag{50}$$

since we assume that individuals are risk averse, so that $U_{22} < 0$, and c_2 increases with lower realized tax rates. We now multiply both sides of the inequality with $\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)$ to get

$$U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) < \{U_2\}_{\bar{\gamma}}\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) \ \text{ if } \ \gamma^i < \bar{\gamma} \ . \tag{51}$$

Taking expectations on both sides, we obtain

$$E\left[U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)\right] < \{U_2\}_{\bar{\gamma}} E\left[R^i\bar{\gamma} - \bar{R}\gamma^i\right] \ \text{ if } \ \gamma^i < \bar{\gamma} \ . \tag{52}$$

To see that the right hand side is zero, write out the expression as

$$E\left[R^i\bar{\gamma} - \bar{R}\gamma^i\right] = \bar{\gamma}E\left[1 + r - p\gamma^i r\right] - \left(1 + r - p\bar{\gamma}r\right)E\left[\gamma^i\right] = 0 \ . \tag{53}$$

We have thus shown that the right hand side is zero, so the left hand side must be negative.

To show that the second term in (49) is positive, we assume that the utility function displays decreasing temporal risk aversion, or equivalently that $\left(U_{12} - R^iU_{22}\right)/U_2$ is decreasing in c_2. We can then write

$$\frac{U_{12} - R^iU_{22}}{U_2} < \left\{\frac{U_{12} - R^iU_{22}}{U_2}\right\}_{\bar{\gamma}} \ \text{ if } \ \gamma^i < \bar{\gamma} \ . \tag{54}$$

Further, we have that

$$U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right) > 0 \quad \text{if} \;\; \gamma^i < \bar{\gamma} \quad . \tag{55}$$

Multiplying both sides of (54) with this expression and taking expectations we get

$$E\left[\left(U_{12} - R^i U_{22}\right)\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)\right] < \underbrace{\left\{\frac{U_{12} - R^i U_{22}}{U_2}\right\}_{\bar{\gamma}}}_{+} \quad \underbrace{E\left[U_2\left(R^i\bar{\gamma} - \bar{R}\gamma^i\right)\right]}_{-} \;\; \text{if} \;\; \gamma^i < \bar{\gamma} \;\; . \tag{56}$$

To show that the left hand side is negative, it is sufficient to show that the right hand side is negative. We know that the factor in braces is positive, and in the above proof we have showed that the last factor is negative; we can thus conclude that the right hand side is negative. To summarize, we have proved that under the assumptions of risk aversion and decreasing temporal risk aversion, the first term in (49) is negative and the second term is positive.

5.4 CONDITION FOR $\partial\tau_1/\partial p > 0$

To be to the right of the maximum of the Laffer curve, we have the condition that

$$\frac{\partial\tau_1}{\partial p} = -\frac{\frac{\bar{\gamma} r a_1}{1+r} S + \frac{p\bar{\gamma} r}{1+r} E\left[\gamma^i r\left(U_2 - \frac{a_1}{R^i}\Psi\right)\right]}{|J|} > 0 \; . \tag{57}$$

Since the denominator is negative, the numerator has to be positive in order to make the expression positive. By using the definitions of S and Ψ we get

$$\frac{\bar{\gamma} r}{1+r} E\left[a_1\left(U_{11} - 2R^i U_{12} + R^{i2} U_{22}\right) + p\gamma^i r\left(U_2 - a_1\left(U_{12} - R^i U_{22}\right)\right)\right] > 0 \; . \tag{58}$$

Using a time separable iso-elastic utility function (c.f. section 2.2.1) and $c_2 = a_1 R^i$ this becomes

$$-a_1\alpha c_1^{-\alpha-1} + E\left[\left(p\gamma^{i} r\beta - \alpha\beta(1+r)\right)c_2^{-\alpha}\right] > 0 \;. \tag{59}$$

Letting α approach zero, the condition becomes $p\bar{\gamma}r\beta > 0$, which will always be fulfilled if the return on capital is positive and we study taxes rather than transfers, since consumption in the first period increases with smaller α and thus $1/c_1$ goes to a positive number less than infinity, implying that the first term goes to zero.