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Abstract

This paper analyzes the effects on present consumption of budget deficits under different assumptions regarding demographics. In the first part, birth and death rates are deterministic, and in the second part, birth rates are assumed to be stochastic. In the case of a deterministic population size, an increase in public debt raises present consumption, if the (deterministic) birth rate is greater than zero, while with a zero birth rate we obtain debt neutrality. This is consistent with the results in Blanchard [1985] and Buiter [1988]. However, for the case of stochastic birth rates, it is shown that we can obtain the result that present consumption will *decrease* when public debt is increased, both when we have a zero expected birth rate, and when the expected population size is assumed to be constant, so that the expected birth rate is positive and equal to the death rate. The explanation is that with an uncertain birth rate, the future tax base is uncertain, which makes per capita taxes uncertain in the future. Shifting taxes to the future thus implies greater uncertainty about future net income, and induces precautionary savings.

Keywords: tax base uncertainty, Ricardian equivalence, precautionary savings, demographics

JEL: D81, H60, E21, J10

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1. INTRODUCTION

The Ricardian equivalence proposition states that households will not change their consumption path in response to changed timing of taxes, provided the level of government consumption is kept unchanged. This implies that households do not regard their holdings of government bonds as net wealth, and that in this context government deficits do not affect the real economy, see for example Barro [1974]. The model assumptions underlying the Ricardian debt neutrality are quite a few; access to a perfect capital market, the use of lump-sum taxes, and ”infinite” planning horizons, to mention some of the most noted and questioned. In this paper, we will focus on the assumption of infinite planning horizons, where infinite planning horizons is really a collective label for the assumptions that the households and the government have the same planning horizon, and that there are no new entrants into the economy that will share future tax burdens. Stated differently, if we fix the government’s planning horizon, Ricardian models usually assume that both the death and birth rates for households during the planning horizon are equal to zero, although it is sufficient to assume that the birth rate is equal to zero to obtain debt neutrality in a deterministic world with an annuity market.

In some influential papers on budget deficits and consumption, the focus of attention has also been on the planning horizon of individuals and population growth, see for example Barro [1974], Blanchard [1985], Weil [1989], Weil [1987a], and Buiter [1988]. The main point is that it is not, in fact, the length of the planning horizon of individuals but rather the new entrants to the economy that create deviations from Ricardian equivalence. The important aspect of demographics in these models is that the tax base in future periods is affected by the assumptions on death and birth rates, but the level of government consumption is fixed. This implies, for example, that the tax burden for a particular individual will be reduced if there are new tax payers born into the economy. Consequently, if the timing of taxes is changed, so that more of the tax burden is shifted to periods where the tax base is larger and/or different than today, this will have a positive wealth effect for the current generations. (As in the standard Ricardian analysis, perfect capital markets, lump-sum taxes, and an exogenously given level of government consumption are assumed). In Buiter [1988], productivity growth is added as a factor that creates additional room for the government to postpone taxes, and levy them on both later and more productive generations.

In the papers discussed above, there is only room for positive changes in the future. Either the aggregate economy can become more productive, or the tax base can be extended. Furthermore, the total population size in future periods is known, since both death and birth rates are deterministic, which implies that the tax base and thus the size of lump-sum taxes in the future are known. However, in the face of real world budget deficits today, it is not always obvious that households expect the future to be more rewarding than the past in terms of tax base and productivity. One interpretation would be to say that the size of the population is stochastic and that it could actually fall, so that there might actually be less people in the future to share the tax burden. This case has been analyzed to the extent that deaths are allowed, while the birth rate is set to zero. However, if we widen our interpretation of the birth rate to include migration, we have the possibility of people leaving the economy in the second period, and thus making the ”birth” rate negative. In this paper we will allow a negative birth rate for this reason. This implies that the tax base can deteriorate both because agents can die and because they can leave the country.

A decreasing population is not something that we have experienced to a great extent, but if we instead of population think in terms of the tax base, which is basically people earning income, it seems more in line with the unemployment experiences over the last decades. The uncertainty could thus be derived from uncertainty about future employment levels. Furthermore, productivity could be stochastic and subject to negative shocks as well as positive, or in the case of a small open economy, the terms of trade could be stochastic and a source of uncertain future purchasing power. In the following analysis, this uncertainty will be labeled population size uncertainty to adhere to the Blanchard [1985] framework. However, this should only be regarded as a collective label for the types of uncertainty discussed here, and an alternative label could be *tax base uncertainty*. Finally, if we ignore these factors, the case of decreasing population due to emigration seems to be an important subject in the EU, where people are free to work (and be taxed) in any member country. Potentially, countries that have high taxes compared to the other countries without providing better public services, production possibilities or other benefits, will probably face a risk of a decreasing population due to emigration.

From the literature dealing with intertemporal choice under uncertainty, we know that agents change present consumption not only in response to changes in expected values of income, but also in response to changes in the risk (or ”spread”), see for example Leland [1968], Sandmo [1970], Ormiston and Schlee [1992] for comparative statics results, Kimball [1990], for a formalization of the theory of precautionary savings and prudence, and Zeldes [1989] and Caballero [1990] for the importance of precautionary savings for explaining observed consumption patterns. In the present paper, we introduce risk in connection with future lump-sum taxes, since the tax base is stochastic, which in turn implies that the net income in the future is stochastic. By shifting more or less taxes to the future, the risk that the agent is facing changes. If we make the standard assumption that agents are risk averse, postponing taxes will not only have (potentially) the standard wealth effect on private consumption, but also an effect due to changed income risk in the future. The risk in this paper is an aggregate tax risk in the sense that all households have the same realization of the stochastic variable. This can be compared to Becker’s [1995] analysis of individual tax risk, where instead each household experiences a specific realization of the uncertain tax.

There are of course other aspects of the tax system that will not be explicitly considered in this paper. For instance, in other parts of the tax literature, the tax system is regarded as an insurance of uncertain gross income, see Stiglitz [1969] and Varian [1980] for a general discussion, Barsky et al. [1986] for a discussion in connection to Ricardian equivalence, and Smith [1982] for a discussion of intergenerational redistribution and uncertain population size. The potential insurance aspect of a tax system is however not the only role taxes will play in an uncertain world, but in some cases the tax system in itself will create additional uncertainty. To focus on this latter aspect of the tax system and abstract from the insurance aspect, we will in this paper assume that gross returns are certain.

The purpose of this paper is to first summarize the effects of budget deficits on consumption when the death and birth rates are known, implying that individuals might not know if they live for one or two periods, but in aggregate the size of the second period population is known. This is the standard framework that is used in the papers described above. Put in another way, the life time of a particular individual is allowed to be stochastic, but second period population is deterministic. In the second part of the paper, a stochastic second period population size is modeled, by introducing a stochastic birth rate. The assumption of stochastic birth rates implies that the future lump-sum tax becomes stochastic, and there is thus a reason for prudent households to engage in precautionary savings when taxes are postponed.

2. DETERMINISTIC POPULATION SIZE

In this section, we will analyze the effect of budget deficits on present consumption in the cases of deterministically increasing, decreasing, and constant future population. It is important to make the distinction between the aggregate and individual levels here, since although individuals do not know for certain if they are alive in the next period, they still know what the future population size is. Put differently, we know the number of individuals that will be alive for sure, but we do not know who the survivors will be. In this way the model is deterministic, although it is stochastic for the individual household. The stochastic nature is, however, of a particularly simple kind from the problem solver’s viewpoint, since it affects the discount factor in a deterministic way, and has no impact on future (lump-sum) income or taxes. The framework used is an economy that exists for two periods, with one government sector that obeys an intertemporal budget constraint, an infinite number of identical households that are expected utility maximizers, and a deterministic interest rate given from the world market. For the original analysis of a deterministic population size in continuous time, see Yaari [1965] and Blanchard [1985], and for extensions see, for example, Buiter [1988].

Population and insurance

In period one population is N_1 and in period two population is $N_2 = (1+q-\pi)N_1$, where π is the (deterministic) death rate, which is greater than zero. q is the (deterministic) ”birth” rate, which is allowed to be negative as long as $1+q-\pi > 0$, since second period population cannot be negative.

Since individuals face a constant probability of dying, and we assume that the population is large, there is room for an insurance policy that will cost the individual his savings, a_1, if he/she dies, and pay a bonus of $a_1\pi^*$, where $\pi^* \equiv (1+r)\pi/1-\pi$, if the individual survives to the second period, see Yaari [1965] or Blanchard [1985] for this type of self-financed insurance arrangement. Because we are using a model in discrete time, we have to adjust the factor multiplying a_1 to account for the fact that there are less people alive in the second period to share the return from the insurance company, and that the savings earn interest. Thus we get π^* instead of simply π, which is the premium received at every moment in a continuous time model. In this way there will be no unintentional bequests in the economy, and we will later assume that individuals are not altruistic, so there will be no motive for intentional bequests. We could also note that this is a policy that would be provided by a perfect competition market of insurance where the insurance company has no transaction costs.[1] In other words, $1+r+\pi^*$ will be the return on savings for the surviving individuals, due to the insurance arrangement, while the government only gets (or pays) $1+r$. However, as we will see in the following analysis, this difference in the return on savings will not in itself imply that public debt is net wealth to the households, since there is a counteracting effect in terms of the reduction in tax base implied by a positive death rate.

Government

The government obeys a budget constraint in the first and second period according to

[1]Adding transaction costs that are a fraction of the payments would of course reduce the bonus accordingly, but would not affect the present analysis in a significant way.

$$\begin{aligned} T_1 &= G_1 - D_1 \\ T_2 &= G_2 + (1+r)D_1 \ , \end{aligned} \tag{1}$$

which states that first period aggregate taxes are equal to first period aggregate consumption minus aggregate debt, and in the second period, aggregate taxes must cover both consumption and debt repayment plus interest on the debt. Define per capita values of a variable X as $x_t \equiv X_t / N_t$. To make notation slightly easier, second period government consumption is set to zero. The per capita equivalent of the above budget restrictions then becomes

$$\begin{aligned} \tau_1 &= g_1 - d_1 \\ \tau_2 &= \frac{1+r}{1+q-\pi} d_1 \ , \end{aligned} \tag{2}$$

where of course second period per capita tax payments fall with increasing(decreasing) birth(death) rate.

Individuals

The identical individuals are expected utility maximizers that solve the problem

$$\begin{aligned} &\max_{c_1, c_2} E\left[U(c_1, c_2)\right] = U(c_1) + \frac{1-\pi}{1+\delta} U(c_2) \\ &\text{s.t.} \qquad c_1 = y_1 - \tau_1 - a_1 \\ &\qquad\qquad c_2 = y_2 + (1 + r + \pi^*) a_1 - \tau_2 \ , \end{aligned} \tag{3}$$

where U(.) is a von Neumann-Morgenstern utility function, y_t is exogenous income in period *t*, c_t is consumption in period *t*, τ_t is lump-sum taxes in period *t*, savings from period one to two is denoted by a_1, r is the deterministic and exogenous interest rate, δ is the subjective discount factor, and, finally, π is the probability of death between period one and two. Two things to note are the factor $1-\pi$ in front of second period utility, which is due to the probability of dying, and the appearance of $a_1\pi^*$ in the budget restriction, which is due to the insurance arrangement. Finally, to reduce notation slightly, the exogenous income in the second period is assumed to be zero in the following analysis, since this assumption does not alter any of the results.

By using the per capita version of the government's budget restrictions we can write second period consumption as

$$c_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \left((1+r+\pi^*) - \frac{1+r}{1+q-\pi}\right)d_1 \ . \tag{4}$$

We can here make the observation that the level of government debt affects the individual, since it does not cancel out in the budget restriction of the individual when we substitute in the government's budget restriction for arbitrary choices of death and birth rates. The first term in the parenthesis displays the fact that individuals can save the period one tax cut when debt is increased by one unit, while the second term represents the (per capita) tax that has to be paid in period two to repay one unit of debt. The coefficient on public debt describes the wealth effect for the household from creation of public debt in the first period, which we will use in the following analysis.

The next step is to solve for optimal consumption. The first and second order conditions (FOC and SOC) for the individual's maximization in (3) are

FOC:
$$U'(c_1) - \frac{(1-\pi)(1+r+\pi^*)}{1+\delta}U'(c_2) = 0 \tag{5}$$

SOC:
$$S \equiv U''(c_1) + \frac{(1-\pi)(1+r+\pi^*)^2}{1+\delta}U''(c_2) < 0 . \tag{6}$$

We now ask the question how an increased public debt affects first period consumption by implicit differentiation of the FOC. This yields

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{\underbrace{S}_{-}}\underbrace{U''(c_2)}_{-}\underbrace{\left(1+r+\pi^* - \frac{1+r}{1+q-\pi}\right)}_{?} , \tag{7}$$

where $\kappa \equiv (1-\pi)(1+r+\pi^*)/(1+\delta) > 0$. We note that the first two factors are both negative, if we assume that the problem has a maximum and that individuals are risk averse, implying that $U''(c_2) < 0$. To determine the sign of the last factor, we must know the relationship between death and birth rates. There are three main cases of interest: constant population, decreasing population and increasing population. In the following analysis, we will note that the results will always be determined by the last factor, i.e. by the coefficient on public debt. (N.B.: the cases are all deterministic, so it is at this stage not an issue in what population regime an individual lives in.)

Case 1: Constant population

In this case, either no one enters or leaves the economy, or agents enter and leave the economy at the same rate. As will be demonstrated, it is not irrelevant for debt policy what the reason is for a constant population. From the literature on Ricardian equivalence, see for example Blanchard and Fisher [1989], we know that infinite horizon models without population growth differ from the model in Blanchard [1985], where agents die and are born at the same rate, with respect to the effects of debt policy.

a: $\pi = q = 0$ *(constant population, no deaths or births)*

This is the case of standard Ricardian models. By inspection of either second period consumption above or by inspection of the derivative between consumption and debt, it is clear that the timing of taxes/debt level has no effect on individuals. In this case, individuals cannot evade taxes in the second period by sharing them with new entrants to the economy.

b: $\pi = q > 0$ *(constant population, with equal death and birth rates)*

In the case of equal death and birth rates, second period consumption will be

$$c_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \pi^* d_1 , \tag{8}$$

where the coefficient on debt is simply the savings premium from the life insurance, which is always positive, so increasing public debt will make the current generation wealthier. The explanation is that the individuals of the current generation that survive to the second period have new taxpayers in the future to share the tax burden with.

To investigate how first period consumption changes in response to increased public debt, we have the derivative

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2)\pi^* > 0 \ . \tag{9}$$

The first two factors are negative and the last is positive, so the expression is positive. In other words, individuals raise their present consumption in response to higher public debt. This is in line with the result generated in Blanchard’s [1985] paper of finite horizons. The explanation is that since individuals only care about their own consumption, they become wealthier when some of the tax burden is levied on ”their” children. Furthermore, for the individuals to maximize their utility, a fraction of the additional wealth will be consumed in the first period, and the remainder in the second period.

Case 2: Decreasing population

This case is perhaps not the most noted when analyzing the effects of debt policy, although Buiter [1988] shows that in the case of a positive death rate and a zero birth rate, we will obtain debt neutrality. However, Buiter’s case is only one case where we have a decreasing population, and the other cases that generate a decreasing population will be discussed below. The reasons for not emphasizing the case of decreasing population might have been obvious from historical observations of growing population, but in many parts of the industrialized world, and with the interpretation of employed population as the tax payers, this case seems rather relevant. In other words, today it is no longer obvious that the tax base cannot decline as well as increase, and this case is the case of declining tax base.

a: $\pi > 0$ *and* $q = 0$ *(decreasing population, deaths but no births)*

Start with consumption in period two, which is now

$$c_2 = (1 + r + \pi\ *)\left(y_1 - c_1 - g_1\right)\ , \tag{10}$$

where we see that debt does not enter the expression determining consumption in the second period, i.e. the debt coefficient in (4) is zero. This is consistent with Buiter [1988], who concludes that a zero birth rate is sufficient to make the wealth effect equal to zero in a continuous time model of the Yaari-Blanchard type, (see Yaari [1965] and Blanchard [1985]).

The derivative between first period consumption and debt is now

$$\frac{\partial\, c_1}{\partial\, d_1} = \frac{\kappa}{S} U''\left(c_2\right) \cdot 0 = 0\ , \tag{11}$$

which again is a statement of debt neutrality when the birth rate is zero. In other words, since debt does not affect the level of wealth, it will not affect the consumption decision in a deterministic world. This is the clearest example of that the assumption of finite horizons alone does not generate deviations from Ricardian equivalence. At first it might appear puzzling, since the return on savings for the households is larger than the interest the government has to pay on its outstanding debt. Therefore, we might think that the households will be better off if they save between periods rather than the government, i.e. introducing public debt would be regarded as net wealth by the households. The reason that debt is not net wealth is, of course, that the tax base deteriorates between the periods when we have a positive death rate. Another way of analyzing the effects of debt in this particular case where $q = 0$ is to rewrite the debt coefficient in (4) according to

$$1 + r + \pi * - \frac{1+r}{1-\pi} = \frac{1+r}{1-\pi} - \frac{1+r}{1-\pi} = 0\ , \tag{12}$$

where the first term is again what the households get if they save a dollar's tax cut to the second period, and the second term is what they have to pay in extra tax in the second period. We can now see that the difference in returns on savings is completely wiped out when the government's savings rate is adjusted for the decreasing population. In other words, the government has a different return on savings at the aggregate level, but not at the per capita level, which is the key to understanding this result and Buiter's [1988] statement that a zero birth rate is a necessary condition for debt neutrality in a continuous time model.

b: $\pi > q > 0$ *(decreasing population, death rate higher than birth rate)*

Second period consumption is

$$c_2 = (1 + r + \pi^*)(y_1 - c_1 - g_1) + \left(\frac{q(1 + r + \pi^*)}{1 + q - \pi}\right) d_1 \ . \tag{13}$$

The coefficient on debt is positive, which implies that the current generation becomes wealthier if public debt is created. In this case the derivative between first period consumption and debt is

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2) \left(\frac{q(1 + r + \pi^*)}{1 + q - \pi}\right) > 0 \ . \tag{14}$$

In other words, with increased public debt, first period consumption will increase, since households regard their holdings of government bonds as net wealth. Again, this is a result of the new entrants in the second period, who will share the tax burden with the individuals from the current generation that survive to the next period.

c: $\pi = 0, q < 0$ *(decreasing population, no deaths but negative "birth" rate, i.e. emigration)*

Here we let the "birth" rate be negative, which implies that people leave the country in the second period. Second period consumption becomes

$$c_2 = (1 + r + \pi^*)(y_1 - c_1 - g_1) + \left(\frac{q(1 + r)}{1 + q}\right) d_1 \ . \tag{15}$$

The coefficient on debt is now negative, which implies that debt creation in the first period represents negative wealth to the surviving households that remain in the country. The derivative between first period consumption and debt is

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2)\left(\frac{q(1+r)}{1+q}\right) < 0 \ . \tag{16}$$

Since the derivative is negative, first period consumption will be reduced in response to increased public debt, which is of course a consequence of public debt being net debt rather than net wealth for the households.

d: $\pi > 0, q < 0$ *(decreasing population, deaths and negative "birth" rate, i.e. emigration)*

Second period consumption is now

$$c_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \left(\frac{q(1+r+\pi^*)}{1+q-\pi}\right) d_1 \ . \tag{17}$$

The coefficient on debt is still negative, which implies that public debt is negative wealth to the households. The derivative between consumption and debt is

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2)\left(\frac{q(1+r+\pi^*)}{1+q-\pi}\right) < 0 \ . \tag{18}$$

The derivative tells us that first period consumption will decrease when public debt is created. We can also note that the coefficient on debt is a larger negative number than it is in case 2c, i.e. with negative birth rates, public debt represents more net debt when death rates are positive than when the death rate is equal to zero.

Case 3: Increasing population

This is the usual way of modifying the assumption of a constant population. Again, this can be motivated by historical observations of the population size, but with the interpretation of population as the equivalent of work-force, it is not totally obvious that this is the most relevant case in many economies today.

a: $\pi = 0$ *and* $q > 0$ *(increasing population, no deaths but births)*

With no deaths and positive birth rates, second period consumption can be written as

$$c_2 = (1+r)(y_1 - c_1 - g_1) + \frac{(1+r)q}{1+q} d_1 \ . \tag{19}$$

In this case it is straightforward to see that the debt coefficient is positive, and thus that increased debt is regarded as net wealth.

To analyze how first period consumption changes in response to an increase in debt, we have the derivative

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2) \frac{(1+r)q}{1+q} > 0 \ , \tag{20}$$

with $\kappa \equiv (1+r)/(1+\delta)$ since death rates are set to zero. The first and second factors are both negative, while the last factor is positive, implying that first period consumption increases with increases in the debt level. In other words, it here becomes obvious that we do not need the assumption of finite horizons to have the result that public debt is net wealth to the households, which in turn makes households consume more today when public debt is created.

b: $0 < \pi < q$ *(increasing population, death rate lower than birth rate)*

If we add positive death rates, the expression for second period consumption can be written as

$$c_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \left(\frac{q(1+r+\pi^*)}{1+q-\pi} \right) d_1 \ , \tag{21}$$

which gives a positive coefficient on debt, and thus implies that public debt is net wealth to the households alive in the first period.

The derivative between first period consumption and debt is now

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} U''(c_2)\left(\frac{q(1+r+\pi *)}{1+q-\pi}\right) > 0 \ , \tag{22}$$

where the first two factors are negative and the last is positive, implying a positive derivative. In other words, postponing taxes to period two increases consumption in period one. In this case we can note that the assumption of positive death rates makes the coefficient on public debt larger than in the case with increasing population without deaths, i.e. although a positive death rate in isolation does not make public debt net wealth, it reinforces the wealth effect of public debt when we have a birth rate that is non-zero.

To summarize the results of deterministic death and birth rates, we start by noting that a constant population will generate the Ricardian result if both death and birth rates are equal to zero, but else a positive relation between present consumption and budget deficits/the level of debt. When the population is increasing, we get the result that debt always represents net wealth to the households alive in the first period, and that first period consumption thus increases with higher debt levels. In the case of decreasing population, we get the result that individuals will consume less in the first period when public debt is increased, if ”birth” rates are negative, i.e. some people emigrate.

In some discussions on the validity of Ricardian equivalence, one could get the impression that a positive death rate alone would make individuals regard government bonds as net wealth. However, as pointed out by Buiter [1988] in a continuous time model, and here in a model in discrete time, it is the assumption about birth rates that is central for the question of debt neutrality. We note that it is the sign of the birth rate that determines how first period consumption will change when debt changes, and with a positive/zero/negative birth rate we will get a positive/zero/negative wealth effect from the creation of public debt, which in turn determines how first period consumption will change. The existence of a positive death rate is, however, relevant when determining the magnitude of the wealth effect, although it is irrelevant for determining its sign. We can note that in the case where we have a positive birth rate, introducing a positive death rate makes the debt coefficient greater (see *3a* and *b*), while in the case where we have a negative birth rate and introduce a positive death rate, the debt coefficient becomes smaller (more negative). In other words, the existence of a positive death rate creates a leverage effect on the wealth effect (positive or negative) created by a non-zero birth rate.

3. STOCHASTIC POPULATION SIZE

With deterministic death and birth rates, second period population is deterministic, and thus also the tax base. This implies that the per capita tax in the future is known in the first period. The question in this section is what happens if the population size in the future is stochastic, which implies that the tax base is uncertain and thus also the *per capita* tax in the future. There are several ways of making second period population size stochastic, basically both death and birth rates could be stochastic, or we could make one rate stochastic and the other deterministic. In order to keep the analysis as simple as possible, we will introduce a stochastic birth rate together with a deterministic death rate, which makes the population size in the second period stochastic. In this way, the individual still knows the return on capital and by how much to discount future utility, but the second period *per capita* lump-sum tax, and thus disposable income, becomes stochastic. This uncertain tax will be equal for all households, so we are analyzing an aggregate tax risk. In Becker [1995], tax risk and budget deficits were analyzed under the assumption that individuals have different realizations of their second period tax payment, i.e. that paper analyzed individual rather than aggregate tax risk.

3.1 THE MODEL

There are still three sectors in the economy, one providing insurance, one government sector, that can finance its budget with either taxes or public debt creation, and finally, one household sector maximizing expected utility.

Population and insurance

Population and insurance are defined as in the previous section, with the vital distinction that in period two, population is now $\tilde{N}_2 = (1-\pi+\tilde{q})N_1$, where π is the (deterministic) death rate, and $\tilde{q}$ is the stochastic birth rate. Since the birth rate is stochastic, so is second period population size, (thus the ”tilde” over these variables, and other variables that are stochastic). The insurance system will be arranged as previously, and this is the first instance where the choice of having a deterministic death rate matters, since otherwise the return on savings would be stochastic under a self financed insurance system.

Government

The government still obeys a budget constraint in the first and second period according to

$$\begin{aligned} \mathrm{T}_1 &= G_1 - D_1 \\ \mathrm{T}_2 &= G_2 + (1+r)D_1 \ , \end{aligned} \tag{23}$$

with the per capita equivalent of the above budget restrictions now being

$$\begin{aligned} \tau_1 &= g_1 - d_1 \\ \tilde{\tau}_2 &= \frac{1+r}{1+\tilde{q}-\pi} d_1 \ . \end{aligned} \tag{24}$$

Second period *per capita* tax payments fall with increasing(decreasing) birth(death) rate, and are now stochastic due to the uncertain birth rate.

Individuals

The identical individuals are expected utility maximizers with time separable utility that solve the problem

$$\max_{c_1,c_2} E\left[U(c_1,c_2)\right] = U(c_1) + \frac{1-\pi}{1+\delta} E\left[U(\tilde{c}_2)\right]$$
$$\text{s.t.} \qquad c_1 = y_1 - \tau_1 - a_1 \qquad (25)$$
$$\tilde{c}_2 = (1 + r + \pi^*)a_1 - \tilde{\tau}_2 \ .$$

The variables are defined as before, the only difference being that variables with a "tilde" are stochastic. This is the other instance where letting the death rate still be deterministic matters, since we can now move the factor $1-\pi$ outside the expectations operator. By using the *per capita* version of the government's budget restrictions we can write second period consumption as

$$\tilde{c}_2 = (1 + r + \pi^*)(y_1 - c_1 - g_1) + \left((1 + r + \pi^*) - \frac{1+r}{1+\tilde{q}-\pi}\right)d_1 \ . \qquad (26)$$

Still the level of government debt affects the individual, since it does not cancel out in the budget restriction of the individual when we substitute in the government's budget restriction, for arbitrary values of death, birth and interest rates. We could also note that in this case of stochastic population, it is the presence of second period taxes through the debt level that creates the uncertainty about future consumption after conditioning on survival. Put differently, with deterministic death and birth rates, an individual knew how much taxes he had to pay if he survived. In the presence of stochastic birth rates, however, he also needs to know the realized birth rate to calculate the *per capita* tax and thus second period consumption. Finally, we note that the coefficient on debt is not linear in the realized birth rate, which has consequences that will be discussed below.

The first question to ask is how the coefficient on debt, or the wealth effect, is affected by having a stochastic birth rate rather than a deterministic. We know that in the deterministic model, the reason for the wealth effect is that individuals will regard their bond holdings as net wealth if they can levy future tax payments on new entrants to the economy, while they will experience a negative wealth effect if the birth rate is negative. In addition to this, with a positive death rate, agents have a higher return on their savings than the government, due to the life insurance system, which creates an additional leverage effect from the creation of public debt.

At first, it might seem natural to assume that the wealth effect will not be affected by having a stochastic birth rate rather than a deterministic. This is, however, not the case, since when we introduce uncertainty about the birth rate in the present model, we will not only make the debt coefficient stochastic, but we also get a smaller *expected* value for the debt coefficient compared to the value it would have if the expected birth rate were deterministic. The reason is that the coefficient on debt that determines the wealth effect is non-linear in $\tilde{q}$. In Figure 1, a plot of the debt coefficient, over a range of birth rates such that the expected value of the birth rate is equal to the death rate, is displayed.

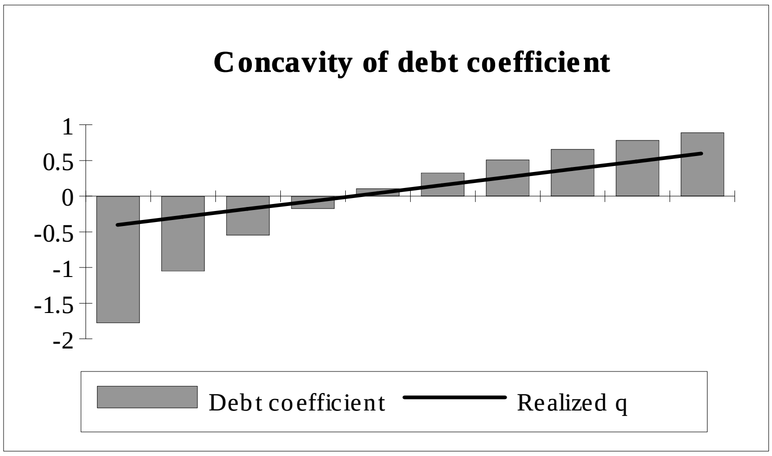

Figure 1. *The size of the debt coefficient for different values of the birth rate, with the expected birth rate equal to the death rate.*

The question is then what this picture implies for the expected debt coefficient, i.e. the debt coefficient implied by a postulated birth rate distribution. In general, we cannot determine the sign of the coefficient without making assumptions about the probability distribution of the birth rate, and the magnitude of the death and interest rates, i.e. a positive expected birth rate is not a sufficient condition for a positive wealth effect. If we in this example assume that the birth rate is uniformly distributed over the range, we could simply add the bars together and divide by the number of

”observations” or birth rate classes to obtain the expected debt coefficient. The values of the expected debt coefficient for different values of the interest rate and the death rate are displayed in Table 1.

The numbers in the table are generated by assuming that birth rates are uniformly distributed, with a distribution starting at zero and with an expected value equal to the death rate. In the case of a deterministic birth rate equal to the death rate, the debt coefficients will be higher than the expected debt coefficients, (compare Table 1 and 2).

Expected debt coefficients

	p		
r	*0.1*	*0.5*	*0.9*
0.5	0.16	1.31	11.8
1	0.21	1.75	15.7
2	0.32	2.62	23.6

Table 1. *Expected debt coefficients for different values of the interest rate, r, and death rate, p, assuming that the birth rate distribution is uniform, starting at zero, with ten possible values, and with the expected birth rate equal to the death rate.*

Deterministic debt coefficient

	p		
r	*0.1*	*0.5*	*0.9*
0.5	0.17	1.5	13.5
1	0.22	2.0	18.0
2	0.33	3.0	27.0

Table 2. *Deterministic debt coefficient, death rate equal to birth rate. In this case (1b) we know that the debt coefficient is equal to p*.*

We can note that although the expected birth rate in the distribution of birth rates is equal to the death rate, the implied expected debt coefficient is smaller than it is when the birth rate is assumed to be a constant equal to its expected value, as in the case of a deterministic population size. As we mentioned above, the reason for this, perhaps non-intuitive, behavior of the debt coefficient is that it is a concave function of the birth rate (see Figure 1). This entails that the *expected* debt coefficient will be smaller than the deterministic coefficient. Furthermore, if we allow for negative birth rates, i.e. individuals leaving the country in period two, we can obtain a *negative* expected debt coefficient, although the expected birth rate is equal to the death rate. If, for example, the interest rate is equal to one and the death rate is equal to 0.5 and we use a distribution of birth rates that starts at -0.4, but has an expected value of 0.5, this will produce a debt coefficient of -0.25. The cases of negative expected debt coefficients are perhaps not the cases we are most interested in, but the point is that the standard deterministic debt coefficient will in some cases give a substantial overvaluation of the wealth effect if birth rates are stochastic. We can, however, note that if we have an expected birth rate that is equal to zero, the associated debt coefficient will always be negative, due to the concavity. This is a feature of the expected debt coefficient that will be used below.

The next step is to solve for optimal consumption. The first and second order conditions for the individual's maximization in (25) are

FOC: $$U'(c_1) - \frac{(1-\pi)(1+r+\pi^*)}{1+\delta} E\left[U'(\tilde{c}_2)\right] = 0 \tag{27}$$

SOC: $$S \equiv U''(c_1) + \frac{(1-\pi)(1+r+\pi^*)^2}{1+\delta} E\left[U''(\tilde{c}_2)\right] < 0. \tag{28}$$

We could now ask the question how an increased public debt affects first period consumption by implicitly differentiating the FOC, which now yields

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S} E\left[U''(\tilde{c}_2)\left(\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi}\right)\right]. \tag{29}$$

where $\kappa \equiv (1-\pi)(1+r+\pi^*)/(1+\delta)$. Compared to the deterministic case, we now have an expectations operator in front of the last two factors, and thus the sign of this derivative could not be determined as easily as in the deterministic case. However, a way to handle this problem is to adopt a similar strategy to the one in Sandmo's [1970] analysis of income risk, which we will do below. We can start by noting that the first factor is always negative and that the derivative thus has the opposite sign of the factor inside the expectations operator.

To determine the sign of the factor inside the expectations operator and thus how first period consumption will change in response to an increase in public debt, start by writing second period consumption as

$$\tilde{c}_2 = (1+r+\pi^*)(y_1 - c_1 - g_1) + \left(\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi}\right) d_1 \ . \qquad (30)$$

We know that if the realized birth rate is equal to zero, the debt coefficient will be zero. If the realized birth rate is positive(negative) we know that the debt coefficient is positive(negative). From this define

$$c^* \equiv (1+r+\pi^*)(y_1 - c_1 - g_1) \ , \qquad (31)$$

which will be the consumption in the second period if a zero birth rate is the realization. We also know that second period consumption will be larger than c^* if $\tilde{q} > 0$ when $d_1 > 0$.

Start by assuming that $U'''(\cdot) > 0$, which is the standard assumption if we want individuals to have a precautionary savings motive, see for example Leland [1968], Sandmo [1970] and Kimball [1990]. This gives us

$$U''(\tilde{c}_2) \geq U''(c^*) \quad \text{if } \tilde{q} \geq 0 \ . \qquad (32)$$

Multiplying both sides with the debt coefficient, we get

$$U''(\tilde{c}_2)\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi} \geq U''(c^*)\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi} \quad \text{if } \tilde{q} \geq 0 \ . \qquad (33)$$

Taking expectations on both sides yields

$$E\left\{U''(\tilde{c}_2)\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi}\right\} \geq U''(c^*)E\left\{\frac{\tilde{q}(1+r+\pi^*)}{1+\tilde{q}-\pi}\right\} \quad \text{if } \tilde{q} \geq 0 \ . \qquad (34)$$

To show that the left hand side is positive (non-negative), which implies that first period consumption decreases when debt increases, it is sufficient to show that the right hand side is positive (non-negative). The first factor is obviously negative if we assume that individuals are risk averse, while the second factor in principle could have any sign. At this stage we need to know what the expected debt coefficient is to determine the sign. Clearly, if the expected debt coefficient is non-positive, the right hand side will be non-negative, and thus the left hand side must be non-negative. One example where we know that the expected debt coefficient is non-positive, from the above discussion on the wealth effect, is when the expected birth rate is zero.[2]

This implies that the sign of the derivative between first period consumption and debt is negative when the expected birth rate is zero, so that if debt increases, present consumption decreases. In other words, if we modify Buiter's [1988] zero birth rate condition to a condition that states that the *expected* birth rate is zero, we get precautionary savings in response to increased debt, instead of debt neutrality, which would be the result in the Yaari-Blanchard model with deterministic population size.

The above condition of a zero expected birth rate is sufficient but not necessary to obtain a negative derivative between present consumption and public debt, and below we will illustrate that we can obtain a negative derivative between public debt and present consumption also when we have a constant expected population. In Section 2, it was shown that with deterministic population size the derivative is positive, i.e. with increased public debt first period consumption rises. This is also the result obtained by Blanchard [1985] in a continuous time model with a constant population. There are two reasons why we can obtain the opposite result with a stochastic population size. First, the debt coefficient is concave here, and secondly, debt creation affects the

[2]An alternative way of deriving a condition on the expected debt coefficient that generates a negative derivative between first period consumption and public debt is as follows. Define $\theta \equiv \tilde{q}(1+r+\pi^*)/(1+\tilde{q}-\pi)$. We know that $Cov\left(U''(\tilde{c}_2)\theta\right) = E\left[U''(\tilde{c}_2)\theta\right] - E\left[U''(\tilde{c}_2)\right]E[\theta] > 0$ if $U''' > 0$. Furthermore, to show that $E\left[U''(\tilde{c}_2)\theta\right] > 0$, which is the condition for a negative derivative between consumption and debt, we note that this will always be true from the covariance expression if $E[\theta] \le 0$, since then the last term is negative, and thus the first term has to be positive to make the covariance positive.

distribution (”spread”) of the future uncertain income, and thus induces precautionary savings if households are prudent.

To create an example where we have a constant expected population and still a negative derivative between debt and first period consumption, we assume the following two-valued probability distribution for the birth rate

$$\tilde{q} = \begin{cases} -x & \text{with probability } 1\text{-}\lambda \\ ((1-\lambda)x+\pi)/\lambda & \text{with probability } \lambda \end{cases} , \tag{35}$$

which implies that $E[\tilde{q}] = \pi$, and $Var[\tilde{q}] = (1-\lambda)(x+\pi)^2/\lambda$, so that the variance increases with smaller values of λ and with larger values of x and π. If we use this probability distribution to evaluate the derivative, we get

$$\frac{\partial c_1}{\partial d_1} = \frac{\kappa}{S}\left\{ -U''(c_{2l})\frac{(1-\lambda)x(1+r+\pi^*)}{1-x-\pi} - U''(c_{2h})\frac{-(x(1-\lambda)+\pi)(1+r+\pi^*)}{1+(x(1-\lambda)+\pi)/\lambda-\pi} \right\} , \tag{36}$$

where c_{2l} is the consumption in period two when we have a negative realization of the birth rate, and c_{2h} is when we have a positive realization of the birth rate, and thus $-U''(c_{2l}) > -U''(c_{2h})$ if $U'''(\cdot) > 0$, i.e. households are prudent. We also note that the first factor is negative, since $S < 0$. To obtain a negative derivative between first period consumption and debt, the factor in braces must be positive. A sufficient, but not necessary, condition for this can be obtained by assuming that $-U''(c_{2l}) = -U''(c_{2h})$, which yields the following condition for a negative derivative between debt and first period consumption

$$\frac{(1-\lambda)x(1+r+\pi^*)}{1-x-\pi} - \frac{(x(1-\lambda)+\pi)(1+r+\pi^*)}{1+(x(1-\lambda)+\pi)/\lambda-\pi} > 0 , \tag{37}$$

which is simply to say that the expected debt coefficient is negative.

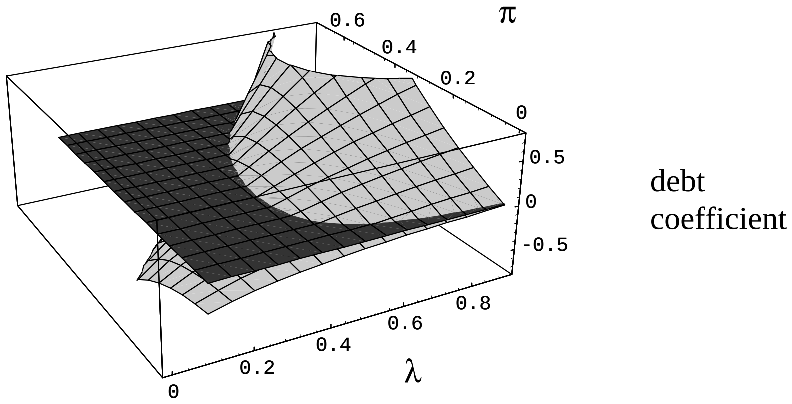

Figure 2. *Sufficient condition on the death rate and spread parameter of the birth rate distribution to determine the sign of the derivative between first period consumption and debt. The lighter surface below the darker implies a negative expected debt coefficient and thus a negative derivative, i.e. higher debt reduces first period consumption.*

In Figure 2, the expected debt coefficient (lighter surface) is plotted together with the surface representing a zero debt coefficient (darker surface). Where the lighter surface is below the darker, the expected debt coefficient is negative, and thus the above condition is fulfilled. The implication of this is again that we have a sufficient, but not necessary, condition for making first period consumption decrease with higher public debt. This will in general be the case for a small value of λ, which implies a high variance in the birth rate distribution. Furthermore, we note that the condition is more easily met when the death rate is either very high or very low.

As we have noted, the above condition is actually a condition that states that the expected debt coefficient is negative, so in a way we have at this stage analyzed the wealth aspect of shifting taxes over time, and not really the effect that is due to the perceived uncertainty. The precautionary savings effect, on the other hand, depends on the preferences that we assume, and we note that the above condition did not involve the magnitude of the precautionary savings motive, i.e. the size of the coefficient of relative prudence. However, since $-U''(c_{2l}) > -U''(c_{2h})$ if consumers are prudent, we know that we can set the parameters such that the debt coefficient is zero, and still make (36) negative, and with a stronger precautionary savings effect, the debt coefficient can be positive (or the spread of the birth rate distribution smaller), without making households consume more in the first period when the level of public debt is increased. This is obviously a result that depends on assumptions of the degree of prudence rather than solely the wealth effect from debt policy. We have thus demonstrated that first period consumption can decrease when debt is increased also in the case where the expected population is constant, i.e. the expected value version of the Blanchard [1985] case, without relying on a negative wealth effect.

4. SUMMARY AND CONCLUSIONS

The paper first discussed the effects of debt creation when the population size is deterministic. In general, increased debt will generate an increased consumption in the first period, due to a wealth effect for the agents alive in that period. However, for negative ”birth” rates, i.e. emigration, the outcome is reversed, so that first period consumption is reduced in response to an increased debt.

In the second part of the paper, a model of stochastic population size was formulated to show that present consumption can *decrease* in response to public debt creation. To start with, we showed that if we use the assumption that the expected birth rate is zero, i.e. the expected value analogue of Buiter’s [1990] condition of zero birth rate, we now get the result that increased debt reduces first period consumption, although Buiter’s conclusion is that of debt neutrality. We also showed that the precautionary savings motive can make first period consumption decrease when debt is increased, even when the expected birth rate is equal to the death rate, if we assume that the variance in the birth rate is high. Furthermore, we can note that when the derivative is actually positive, we still have a precautionary savings effect, which reduces the consumption increase in the first period when debt is increased compared to the deterministic case. In the stochastic case, most of a (potential) positive wealth effect from debt will be consumed in the second period, which can be viewed as a modification of the deterministic case, originally analyzed in Blanchard [1985], where the wealth effect is evenly consumed over all periods. We also noted that if we allow for bad enough realizations of the birth rate, the expected wealth effect itself will become negative due to the concavity of the debt coefficient, although the population is expected to be constant. In other words, if a stabilization policy aims at increasing present private consumption by debt creation, this paper not only suggests that the effect on present consumption will be substantially smaller than the effect predicted by a deterministic model, but also that the effect can be negative.

The analysis of stochastic birth rates used the assumption that the ”birth” rate could be negative. This is motivated in the paper by allowing a fraction of the population to emigrate in the second period. In the present analysis, the fraction that leaves the country is not dependent on the level of public debt. An area of future research is to connect the debt level to the emigration rate, by investigating a two-country model where people are free to move between countries. It seems quite plausible that a high level of public debt in one country will make people more inclined to emigrate to the other country in the second period (i.e. we get a higher probability of negative ”birth” rates), since they would then avoid the higher tax in the future. However, if the public sector in the high debt country engages in activities that improve the income in the second period (or rather consumption possibilities), for instance by investing in pension funds or infra-structure, we will have an effect that counteracts the high tax. In such a model, we can discuss both the uncertainty effects of debt policy and ”fiscal federalism” questions. An interesting question is to connect this analysis to the opening of borders within the EU.

Furthermore, in the present model, the interest rate was given from the world market. If we instead would like to determine the return on savings endogenously from the marginal product of capital, we note that this, in general, depends on the size of the labor force. In this model, it would be natural to interpret the young generation, i.e. new entrants, as the labor force in the second period. How would that affect the present analysis? If we use the standard assumption of decreasing marginal product on capital for a given number of workers, we realize that for a small realized birth rate, the labor force will be smaller, and thus the return on savings is reduced. At the same time, the per capita tax will be relatively high. In other words, in this case, the second period tax will not be an insurance against market risk, but instead increase the variation in second period net income.

Finally, the present analysis can be viewed as an aggregate tax risk analogue of Becker's [1995] analysis of individual tax risk. The essential feature of both these papers is that taxes in the future are stochastic, and if individuals display a precautionary savings motive, this can make present private consumption decrease with increased public debt, i.e. when the present taxes are shifted to the future, households engage in precautionary savings. This stochastic feature of debt policy is often ignored, but in case fiscal policy aims at affecting aggregate demand in the economy, it is vital to understand that households do not only respond to changes in present values but also to changes in the risk they perceive.

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