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Optimal deterrence and inducement of takeovers: An analysis of poison pills and dilution

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Abstract

This paper presents a theoretical alternative to the commonly held belief that poison pills affect shareholder wealth negatively. Specifically, the paper models how ex ante shareholder wealth can be maximized with contractual provisions that resemble poison pill plans and, reversely, voluntary dilution à la Grossman and Hart (1980) by allowing an optimal choice of takeover probabilities and premia. The model's predicitions are consistent with recent empirical evidence [Comment and Schwert (1995)]. The paper shows that, under optimal employment of the proposed provisions, the comparative statics on takeover probabilities and premia differ partially from those proposed in Shleifer and Vishny (1986). As an extension, an analysis of the wealth effects of changes in the control threshold, as implied by, e.g., supermajority rules and a mandatory bid rule, is conducted.

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1. Introduction

This paper combines the insights of two well-known papers in the literature on corporate finance – Grossman and Hart (1980) and Shleifer and Vishny (1986) – to explore how contracts featuring takeover-triggered wealth transfers affect takeover premia and the probabilities of takeovers. The contractual provisions proposed in the paper will serve either as takeover inducement or as takeover defense and have borrowed features from the concept of voluntary dilution introduced by Grossman and Hart (1980) and from real-life antitakeover provisions in the form of poison pills.

Grossman and Hart (1980) display the potential free-rider problem present in takeover attempts on widely held firms. Small shareholders have incentives to free-ride on the potential acquirer's improvement on the production plan by demanding a bid premium that at least equals the expected value improvement. This would undermine the bidder's profit potential to the extent a takeover will not take place. Grossman and Hart propose, as a resolution of this free-rider problem, that the acquirer should be permitted to expropriate some of the target's assets. The shareholders would consequently be excluded from some of the benefits of not tendering and would require a lower reservation price, thereby increasing the likelihood that the acquirer will make a profitable takeover bid. Shareholders would thus voluntarily agree to this takeover-triggered expropriation of target firm assets in order to benefit from the elimination of a free-rider dilemma. In the terminology of Grossman and Hart, this transfer of assets is a *voluntary dilution* of the shareholders' property rights.

Shleifer and Vishny (1986) demonstrate the importance of large initial shareholdings to increase the probability of a takeover bid. A bidder with a prebid stake in the firm will possibly make sufficient takeover gains on his toehold to induce him to bid, despite the possibility of a loss on the purchased shares. Hence, the potential free-rider externality is potentially internalized, without exclusionary devices, by a bidder with a large initial stake in the firm by the fact that he is the largest consumer of the public good.[1,2] Although the presence of a large shareholder may suffice to induce a positive takeover probability, not all potentially value improving bids will be internalized by the existence of bidder toeholds. Therefore, the small shareholders may benefit from attempts to increase the takeover probability further by a voluntary-dilution provision. It will be difficult for an insufficiently large shareholder to considerably increase his stake by pre-takeover trading –not the least because of the existing disclosure rules – without revealing the fact that a value improvement has been found and thereby increasing the market price of the shares, possibly to the extent that the expected takeover gain is altogether lost.[3,4] Voluntary dilution has the effect of lowering the takeover premium demanded by the shareholders, while increasing the probability of a takeover. If the takeover premium could be reduced so that a shift in the probability of takeover is incurred from zero to a positive level, this will have a positive effect on the shareholders' wealth as long as the takeover premium is not negative.[5] Similarly, positive wealth effects may, for specific levels of dilution, also occur when takeovers have nonzero probability even without dilution.

Poison pill defense imply payment streams in the opposite direction to those of voluntary-dilution schemes. Folklore suggests that poison pills are adopted by managers primarily to protect their private benefits of control. To the extent that poison pills result in absolute deterrence of value-improving takeovers, such provisions are

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[1] The role of toeholds has been further explored by e.g., Jegadeesh and Chowdry (1989), Ravid and Spiegel (1992), and Burkart (1996).

[2] Other takeover-inducing mechanisms have been suggested in the literature. For instance, Bagnoli and Lipman (1988) show how the free-rider problem can be overcome in a model with a finite number of owners, where some shareholder will be pivotal. Private benefits have served as takeover-inducing incentives in models by e.g., Grossman and Hart (1988). Bradley (1980) and Bradley and Kim (1984) focus on front-loaded two-tier bids. The bidder offers to pay the front-end price for a controlling interest in the firm. If he takes over, then the minority stockholders are forced to sell their shares for the back-end price. As long as the back-end price is less than the value of a share under the bidder's management , this is a form of exclusion.

[3] See e.g., Shleifer and Vishny (1986), pp. 474-477.

[4] Under the Williams Act, a 13D disclosure form must be filed with the Securities and Exchange Commission by anyone accumulating more than 5% of the firm's shares. The filing must be made within 10 days after the acquisition. The 13D form will contain the disclosure of the general intent of the purchase, name and background of each acquiring individual, or of any individuals who control the acquiring corporation. If there is material change relative the initially filed information such as sale or purchase of shares, an amended schedule 13D must be filed.

[5] The pressure to accept negative takeover premia has been theoretically analyzed by e.g., Bebchuk (1987, 1989). Because of free-rider mechanisms, atomistic shareholders have private incentives to accept a takeover bid that equals the firm's expected value under the new regime, even if the expected post-takeover firm value is lower than the firm's present worth. The theory thus implies that prices below current firm value are possible. The empirical evidence, however, suggests that shareholders tend to receive positive premiums relative to current firm value.

associated with negative effects on shareholder wealth. However, by analogy to the situation where shareholders gain from trading off takeover premiums for increased takeover probability through voluntary dilution, the opposite trade off implied by setting up a poison pill plan may be beneficial to the firm's owners. By requiring an acquirer to insert a specific amount into the firm conditional on a successful bid, the takeover premium can be increased while the ex ante takeover probability is reduced to an optimal level.

This paper explores the tradeoff between the takeover premium and the probability of takeovers associated with the adoption of voluntary dilution and poison pills. In this, the paper extends the analysis and interconnects some of the insights presented in Grossman and Hart (1980), Shleifer and Vishny (1986), and Hirshleifer and Titman (1990). The paper generalizes the Grossman and Hart voluntary dilution concept to include negative dilution. The wealth-redistributive feature is then added into a modified version of the Shleifer and Vishny takeover model, and a set of equilibrium conditions is derived. Hirshleifer and Titman (1990) analyze optimal bidder strategies and ultimately extend their analysis to include exogenous dilution and takeover defense. In contrast, this paper endogenizes the level of dilution and poison pill defense while adopting the perspective of the small shareholders.

In terms of methodology, a simple triangular distribution assumption is employed with the tractable properties of providing a flavor of realism as well as simple calculations. Along with the derivation of an optimal dilution amount (positive or negative), a further analysis of the equilibrium properties in optimum is conducted. The results include the extension of some of Shleifer-Vishny's (1986) propositions. For example, it is shown that the dynamics under optimal dilution are such that the comparative statics on the takeover probability and the ex post takeover premium are partially different.

The presented model provides a theoretical alternative to the explanation that poison pills are adopted primarily to protect managers' private benefits. The model's prediction that poison pill adoptions should result in increased takeover premiums *ex ante* receives empirical support by a recent study by Comment and Schwert (1995). The model also makes empirical predictions about the characteristics of firms that are likely to adopt poison pills. The results are consistent with the empirical observation that firms tend to adopt poison pills when the likelihood of a takeover is unusually high, and with the findings that poison pill adoptions are associated with increased takeover premiums.

In addition, the analysis yields the result that shareholders uniformly lose by increases in the threshold for control (as implied by, e.g., the adoption of supermajority rules or a mandatory bid rule) under the condition that optimal dilution can be maintained. In a specific analysis of the adoption of the *mandatory bid rule* for the case of exogenous dilution, the paper obtains precise conditions for when the rule is in the shareholders' interest and when it is not.

The paper discusses the empirical support for the model. It appears that, while there is a multitude of studies of poison pill adoptions, there exists little evidence concerning explicit dilution contracts. The paper discusses the reasons for this discrepancy. Ultimately, the paper informally discusses why real-life poison pills have the form of security issues rather than corporate charter amendments.

The paper is organized as follows. In the next section, the model framework is presented, leading up to a formula for optimal dilution. Section 3 examines the model's equilibrium properties under dilution-optimizing as well as for fixed dilution amounts. The results are compared with the results of Shleifer and Vishny. In section 4, the effects of changes in the control threshold in general and the implementation of the mandatory bid rule in particular is analyzed. Section 5 presents some sensitivity analysis, while Section 6 discusses the empirical support for the model. Section 7 concludes the paper.

2. The model

The model framework has borrowed its basic characteristics from Shleifer and Vishny (1986). Consider a firm where a single riskneutral outside owner, L, initially holds a fraction e of the firm. L's toehold is insufficient to generate control of the firm; $e < \alpha$ where α is the threshold fraction of voting equity needed to obtain control.[6] The remaining $(1-e)$ portion of the shares is owned by a fringe of atomistic shareholders. The large shareholder, L, is a potential bidder for a control position in the firm. Conditional on his achieving control, L has the capacity to change the value of the firm by an amount Z. Z is henceforth referred to as "the value improvement" in the text.

At the time of a takeover bid, Z will be known by L, but is unknown (stochastic) to the shareholders. L incurs a fixed cost, c, if he decides to make a takeover bid.[7,8] The takeover costs, the size of L's toehold, and the distribution of the value improvement, $F(Z)$, are assumed to be common knowledge. Let π denote the takeover premium offered to the shareholders in the event of a bid. The takeover premium is defined as the bid price less the status quo value of the firm.

2.1. Contract specification

For the sake of argument, consider an amendment to the corporate charter stating that, contingent on the success of a takeover bid, an amount of δ dollars is to be transferred from the firm's assets to the successful bidder. Provided that δ is a positive amount, the charter provision is equivalent to "voluntary dilution" as defined by Grossman and Hart. However, there are no restrictions on specifying a negative δ. A negative δ implies that a successful bidder would be required to insert the amount into the firm. In terms of conditional payment streams, this requirement is equivalent to a takeover defense strategy commonly known as a poison pill. Despite the fact that a negative δ does not

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[6] The control threshold can be thought of as 50% as implied by the simple majority rule operational in most corporations.
[7] For simplicity, the bidder's research intensity is left out of this model version.
[8] For all relevant cases, it is assumed that $c < Z_{max}$.

have a “dilution-interpretation”, I will, for simplicity, sometimes refer to δ as “the dilution amount” in the text.[9]

2.2. Equilibrium

In order to acquire control of the firm, *L* needs to add a fraction, $\alpha - e$, of the firm’s equity to his initial holding, *e*. A necessary condition for *L* to make a takeover bid for these shares is that it provides him with a nonnegative profit. For any set of parameters, this can be written as $\alpha Z - (\alpha - e)\pi + (1-\alpha)\delta - c \geq 0$. Define Z_c as the minimum value improvement that ensures the bidder of a nonnegative takeover profit. We can write

$$Z_c = \frac{c + (\alpha - e)\pi - (1-\alpha)\delta}{\alpha}. \tag{1}$$

The shareholders will chose to tender their shares only if the takeover premium at least equals the expected value improvement less the dilution amount. The shareholders’ best assessment of this expected value will be formed conditional on the fact that a sufficient value improvement has been found by the bidder. Consequently, a necessary condition for shareholders to tender is

$$\pi \geq E\left[Z - \delta \middle| Z \geq Z_c\right]. \tag{2}$$

The bidder likes to take over at the lowest price possible, and it is assumed that an equilibrium will be established at lowest price that satisfies shareholders’ acceptance

[9] The practical implementation of dilution and poison pills can have other forms than amendments to the corporate charter. Grossman and Hart give several examples of specific methods of voluntary dilution: (i) the bidder could be secured of a large salary, or (ii) the bidder could be allowed to issue a number of new stocks to himself, or (iii) the bidder could be permitted to sell some of the firm’s assets or output. The practical implementation of poison pills usually takes one of the following five forms: (i) flip-over rights plans, (ii) ownership flip-in plans, (iii) back-end rights plans, (iv) preferred stock plans, and (v) voting plans. In Section 6, it is discussed why the contracts are more likely to be designed as security issues than as charter amendments.

condition (2). Under this equilibrium, the weak inequality in (2) can be substituted by an equality.[10] Thus

$$\pi = E\left[Z - \delta \middle| Z \geq Z_c\right]. \qquad (2)'$$

As recognized by Shleifer and Vishny, other pure-strategy sequential equilibria, all involving larger than the minimum acceptable premium, are possible. However, Shleifer and Vishny make a strong case for the minimum bid equilibrium, by demonstrating its uniqueness as one which is supported by credible out-of-equilibrium beliefs in the sense of Grossman and Perry (1984a).[11]

2.3. Distribution assumption

Suppose that the probability density function for the bidder's value improvements is linearly decreasing in the size of the value improvement, and has support on a bounded interval [Z_{min}, Z_{max}]. The assumption of a monotonically decreasing density function makes economic sense to the extent that we believe that small value improvements are more probable than large ones. There is no restriction on the sign of the lower bound, which means a generalization visavi Shleifer and Vishny who assume strictly positive value improvements.[12] The assumption about linearity just simplifies calculations. Hence, I assume a "triangular" density function with the tractable properties of providing a reasonably realistic representation of the distribution of potential value improvements, and, at the same time, facilitating straightforward calculations. We can write

[10] Hirshleifer and Titman (1990) model an environment where uncertainty about the shareholders' personal costs and benefits of tendering implies some probability of bid failure. This uncertainty results in a mixed-strategy equilibrium rather than a pure strategy equilibrium.
[11] A detailed account of this is given in Shleifer and Vishny (1986), pp. 467-468.
[12] In contrast with Shleifer and Vishny (1986), the existence of inferior bidders is not ruled out *a priori*, since dilution may possibly induce takeovers by less efficient acquirers.

$$f(Z) = \frac{2(Z_{\max} - Z)}{(Z_{\max} - Z_{\min})^2}, \tag{3}$$

where $f(Z)$ denotes the probability density function for Z. The conditional expectation of the shareholders' takeover gain can be written

$$E[Z - \delta | Z \geq Z_c] = \int_{Z_c}^{Z_{\max}} Z \frac{f(Z)}{(1 - F(Z_c))} dZ - \delta =$$

$$= \frac{Z_{\max} + 2Z_c}{3} - \delta. \tag{4}$$

Inserting (4) into (2)' and combining the two equilibrium conditions (1) and (2)' yields the following simultaneous equation system.

$$\left.\begin{aligned} Z_c &= \frac{c + (\alpha - e)\pi - (1-\alpha)\delta}{\alpha} \\ \pi &= \frac{Z_{\max} + 2Z_c}{3} - \delta \end{aligned}\right\} \tag{5}$$

Solving the equation system (5) yields explicit equilibrium expressions for the takeover premium and the minimum value improvement. Suppressing all other arguments than δ, we can write

$$\pi^*(\delta) = \frac{\alpha Z_{\max} + 2c - (\alpha + 2)\delta}{\alpha + 2e}, \tag{6}$$

$$Z_c^*(\delta) = \frac{(\alpha - e)Z_{\max} + 3c - 3(1-e)\delta}{\alpha + 2e}. \tag{7}$$

2.4. The ex-ante maximization problem

Shareholders will seek to maximize their takeover gain *ex ante*; in any takeover bid for less than 100% of the firm, shareholders will maximize the sum of the expected gain on the sold shares (the takeover premium) and the expected value improvement on the retained shares. Expectations and probability beliefs are formed conditional on the observation that the bidder has found a value improvement of at least $Z_c^*(\delta)$. This implies that the shareholders' information-updated assessment of the takeover probability is $1-F\left(Z_c^*(\delta)\right)$. Independent of distribution assumptions, the general maximization problem thus becomes

$$\underset{\delta}{Maximize}\left\{1-F\left(Z_c^*(\delta)\right)\right\}\cdot\left\{\alpha\pi^*(\delta)+(1-\alpha)E\left[Z-\delta\middle|Z\geq Z_c^*(\delta)\right]\right\}, \tag{8}$$

where $\alpha\pi^*(\delta)$ is the takeover-contingent profit from the sold shares, and $(1-\alpha)E\left[Z-\delta\middle|Z\geq Z_c^*(\delta)\right]$ is the corresponding value of the retained shares. Because $\pi^*(\delta) = E\left[Z-\delta\middle|Z\geq Z_c^*(\delta)\right]$ in equilibrium, the maximization problem simplifies to

$$\underset{\delta}{Maximize}\left(1-F\left(Z_c^*(\delta)\right)\right)\cdot\pi^*(\delta)\,. \tag{8'}$$

2.5. Optimal dilution

The proposed maximization problem will typically yield an interior optimum; for some sufficiently low level of dilution (including negative amounts), the probability of a takeover turns zero, and for some sufficiently high level of dilution, the asset drain is so severe that the small shareholders' ex post takeover gain turns nonpositive. Define $\underline{\delta}$ as the (lower) turning point for the dilution amount at which the probability of a takeover turns zero. That is, $\underline{\delta}$ is such that, for all $\delta\leq\underline{\delta}$, we have that $1-F\left(Z_c^*(\delta)\right)=0$, and for all $\delta>\underline{\delta}$, $1-F\left(Z_c^*(\delta)\right)>0$. Similarly, define $\overline{\delta}$ as the (upper) pivotal level of dilution at which the small shareholders' ex post takeover gain (the ex post takeover premium) turns nonpositive. Formally, $\bar{\delta}$ is such that, for all $\delta \geq \bar{\delta}$, $\pi^*(\delta) \leq 0$, and for all $\delta < \bar{\delta}$, $\pi^*(\delta) > 0$.

Under the triangular-distribution assumption, an explicit expression for the optimal dilution amount is derived, applying the first and second order conditions to the maximization problem (8)'. The resulting optimal dilution amount, δ* can be written as follows.

Result 1. *Given the linear distribution assumption, the optimal dilution is*

$$\delta^* = \frac{1}{3} \cdot \underline{\delta} + \frac{2}{3} \cdot \bar{\delta}, \tag{9}$$

where, specifically, $\underline{\delta} = \dfrac{c - eZ_{\max}}{1 - e}$*, and* $\bar{\delta} = \dfrac{\alpha Z_{\max} + 2c}{2 + \alpha}$.

A full derivation of the result is presented in Appendix A.[13] Result 1 reflects the fact that the interior optimal solution is a weighted average of the pivotal point where the takeover probability turns zero, $\underline{\delta}$, and the point where the ex post premium turns zero, $\bar{\delta}$. Under the alternative assumption of a uniform distribution, we will receive a similar structure for the optimal-dilution expression, however, where the expression for $\bar{\delta}$ is slightly different and the weights are equal. Alternative expressions are examined in Section 5.2. The following section examines the equilibrium properties of the model with dilution.

3. Equilibrium properties

In examination of their model's equilibrium properties, Shleifer and Vishny present comparative statics for changes in bidder toehold and takeover costs. In this section, I

[13] Section 5 presents two alternative formulas for the optimal dilution, using the uniform and the exponential distributions as benchmark distribution assumptions.

conduct a similar examination of equilibrium properties of the dilution-extended model. In particular, I analyze the situation in which optimal dilution contracts (δ^*) are assumed to be maintained. These *in-optimum* results are then compared to the situation where dilution is assumed to be fixed at an arbitrary level. Notably, the Shleifer-Vishny zero-dilution case falls out as a special case of the fixed-dilution analysis. Hence, a straightforward comparison with the results of Shleifer and Vishny is readily available. The stylized results are followed by numerical examples and graphical illustrations. Formal derivation of the results are presented in Appendix B.

3.1. Effects of changes in the bidder's toehold

Result 2. *The optimal takeover probability is constant with respect to changes in the bidder's toehold*.

According to Result 2, firms that consistently apply optimal dilution contracts, adjust the dilution amount with respect to changes in *e* so as to maintain a constant (optimal) takeover probability. Maintaining the optimal dilution level implies that the bidder's minimum profitable value improvement, Z_c^*, is held at a constant level in relation to the bidder's toehold. As the takeover probability is ultimately a function of Z_c^*, also the optimal takeover probability will remain constant with respect to *e*.

How does the in-optimum result compare to the fixed-dilution case? Similar to the fact that there exists some (possibly negative) dilution amount, $\underline{\delta}$, at which the takeover probability turns zero, there is some upper pivotal point, $\overline{\overline{\delta}}$, where the probability turns 100%. Specifically, we have that $\overline{\overline{\delta}} = \underline{\delta} + \frac{(\alpha + 2e)(Z_{\max} - Z_{\min})}{3(1-e)}$, where $\underline{\delta}$ is defined as before. For arbitrary dilution amounts, we get the result that the takeover probability is *strictly increasing* in *e* for any $\delta \in (\underline{\delta}, \overline{\overline{\delta}})$, while it is *constant* w.r.t. *e* otherwise.[14]

While the takeover probability under fixed dilution (and in particular in no-dilution firms) tends to increase with increases in the bidder's initial stake, the probability of a takeover is unaffected by changes in *e* in dilution-optimizing companies.

Result 3. *The optimal takeover premium increases in the bidder's toehold.*

An increase the bidder's toehold implies an adjustment of the optimal dilution amount resulting in an increase in the ex post takeover premium. This result can be decomposed into a direct and an indirect effect. The direct effect, which is negative, is given by the partial derivative of the expression for the equilibrium takeover premium [equation (6)] w.r.t. *e*., while holding δ constant. This strictly negative, direct effect is the fixed-dilution result. However, when the firm maintains optimal dilution, δ is given by δ^* [equation (9)], which is strictly decreasing in *e*. Because the equilibrium takeover premium is decreasing in δ, the effect of lowering the dilution level is positive. This positive effect dominates over direct effect.

Hence, with optimal dilution, the bidder toehold effect on the takeover premium is opposite to that under fixed dilution in general and under no dilution in particular. Optimal dilution thus implies a qualitative difference to Shleifer-Vishny's lemma 1.

The results of constant optimal takeover probability and an increasing optimal takeover premium with respect to increases in bidder toehold imply the following corollary.

Result 4. *The maximal ex ante takeover gain resulting from optimal dilution will increase with an increase the bidder's toehold.*

[14]For any given level of value improvement, the successful bidder will make a larger capital gain the larger his initial holding. Dually, the shareholders will assess a larger probability that the bidder will find a sufficiently high value improvement the larger his toehold.

When optimal dilution is consistently applied, the total bidder toehold effects on the shareholders' ex ante takeover gain is positive, which is also the case in the Shleifer-Vishny zero-dilution case. However, with optimal dilution, the dynamics are such that this is induced rather by a positive premium effect and a zero probability effect than a positive probability effect overshadowing the negative effect on the premium. This constitutes a distinction towards Shleifer-Vishny's Proposition 1.

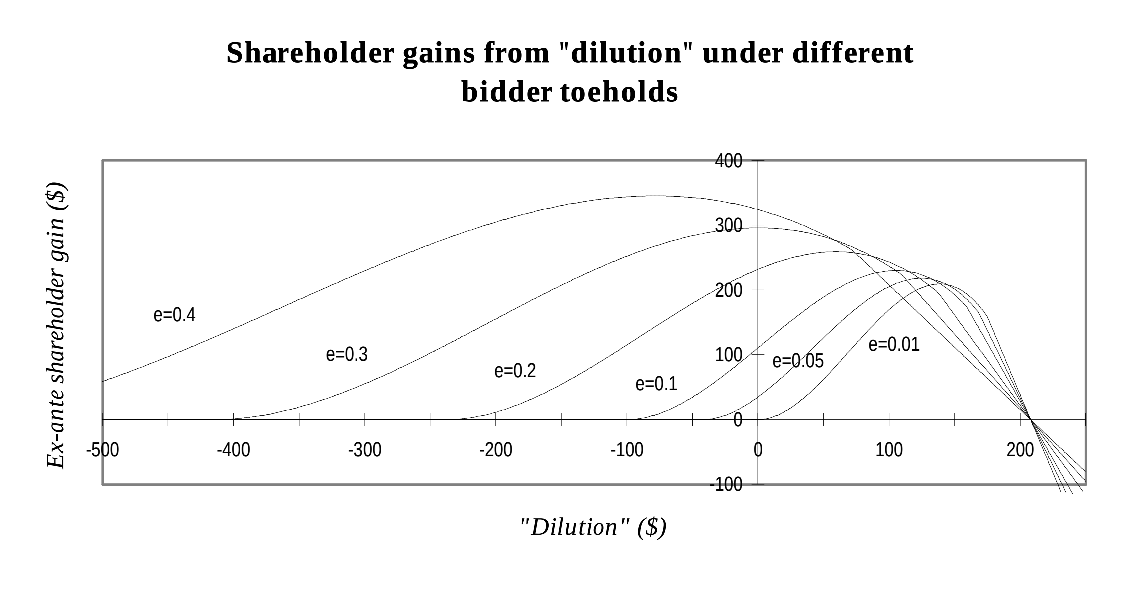
Now consider the general fixed-dilution case. Suppose that the dilution amount is fixed at an arbitrary level. An increase in bidder toehold will cause the shareholders' ex ante takeover gain to *increase* at any dilution amount on the interval $\delta \in (\underline{\delta}, \delta^0)$, where $\delta^0 = \dfrac{Z_{\max}(\alpha - e) + 3c}{\alpha - e + 3}$, *decrease* at any dilution amount on the interval $\delta \in (\delta^0, \bar{\delta})$, and remain *unchanged* at all other dilution amounts. Hence the fixed-dilution analysis also provides a refinement of the Shleifer and Vishny zero-dilution result, in that it displays the existence of a cutoff level, δ^0, above which the total ex ante wealth effect turns negative.

The results of changes in toeholds are illustrated numerically and graphically in Example 1.

Example 1

Figure 1 depicts the shareholders' ex-ante takeover gain, $\left(1 - F\left(Z_c^*(\delta)\right)\right) \cdot \pi^*(\delta)$, as a function of the dilution amount, δ. To capture the impact of changes in bidder toehold, the takeover-gain function is reproduced for a sequence of bidder toeholds: *e* equals 40%, 30%, 20%, 10%, 5%, and 1%, respectively. In the particular example, I assume that $Z_{\max}$ = 1,000, $Z_{\min}$ = 0, α = 50%, and c = 10 as parameters.

Figure 1



First, consider the result when no dilution is present (where the vertical axis and the ex-ante-takeover-gain curves cross). At the 1% toehold (*e* = 0.01), the ex ante takeover gain is zero. When the bidder's toehold is increased to 5%, 10%, 20%, 30% and 40%, respectively, the shareholders' corresponding expected takeover gains increase to $34.70, $110.50, $231.80, $295.50, and $324, respectively. This positive effect in the zero-dilution case reflects the toehold's positive effect on the *a priori* takeover probability, hence effectively illustrating Shleifer-Vishny's Proposition 1.

However, with the right amount of dilution, shareholder wealth can be increased for each level of ownership concentration. In the case with the most disperse ownership concentration, *e* = 0.01, the shareholders will require a takeover premium of $1,000 when no dilution is present. Due to the positive takeover cost, a takeover will never be profitable to the bidder, since $Z_{\max}$ = $1,000. Hence, the probability of a takeover is *zero* without dilution. However, by allowing the bidder a dilution amount of $138.67, the shareholders will be able to extract a maximal ex-ante takeover gain of $209.09 —the top of the bell-shaped curve for *e* = 0.01. At this level of dilution, the takeover probability is increased to 62.7% as the updated conditional expectation of shareholder gain is decreased. This is the optimal takeover probability addressed in Result 2. The equilibrium takeover premium is reduced by two thirds (from $1,000 in the zero dilution case): $\pi^*(\delta = 138.67) = E\left[Z - 138.67 \middle| Z \geq Z_c^*(\delta = 138.67)\right] =$ $333.33.

Now consider the other extreme. At $e = 0.4$, the ex-ante takeover probability is as high as 81% when no dilution is specified. With a conditional equilibrium takeover premium of $400, the scope of a takeover is worth $324 *ex ante* to the shareholders (the intersection of the $e = 0.4$ curve and the vertical axis). This can be improved by specifying a poison pill. By requiring the successful bidder to insert $78 into the company, shareholders may increase their wealth to $345 (the top of the $e = 0.4$ curve). At this level of (negative) dilution, the conditional takeover premium is increased to $550, and the takeover probability is reduced to 62.7% (i.e., the optimal takeover probability). Hence, we have an illustration of Results 2 and 3 —the constant optimal probability and the positive premium effect.

Result 4 is illustrated as the difference in optimal takeover gains under the various levels of toeholds. For each increase in toehold, the height of the bell-shaped curve is increased. The takeover gain at the 40% level minus the gain at a 1% toehold is $345 – $209 = $136. We also observe that the optimal dilution amount decreases in bidder toehold. As a consequence, poison pills will tend to be comparatively more desirable in firms with high ownership concentration as compared to more widely held companies. All in all, the small shareholders will typically want dilution contracts that stipulate smaller dilution amounts (possibly negative amounts) the higher the prebid ownership concentration.

3.2 Changes in takeover costs

The effects of changes in the takeover costs are roughly the opposite to those of changes in bidder toehold.

Result 5. *The optimal takeover premium tends to decrease in the takeover cost.*

Applying optimal dilution implies that the optimal takeover premium will *decrease* in takeover cost. This follows from the fact that the optimal dilution, δ^*, is increasing in c; and the takeover premium is decreasing in δ.

The results under optimal dilution contrast with the fixed-dilution result. Specifically, when dilution is fixed, the takeover premium is increasing in the takeover cost. As a consequence, Shleifer-Vishny's Proposition 2 (stating that an increase in the legal and administrative costs of a takeover will result in a rise in the takeover premium, but a fall in the market value of the firm), is partially modified when optimal dilution is added.

Result 6. *The optimal takeover probability will tend to decrease in the takeover cost.*

The *in-optimum* direction of the cost effect does not differ from the *out-of-optimum* fixed-dilution tendency; both are negative.[15]

Results 5 and 6 together imply a decrease in shareholders' takeover gain:

Result 7. *The maximal level of ex ante takeover gain will decrease with an increase in the takeover costs.*

This *in-optimum* result for the wealth effect corresponds with that stated in Shleifer-Vishny's Proposition 2, with the modification that *both the premium* and *the probability effect decrease instead* of being traded off to create the same qualitative result.

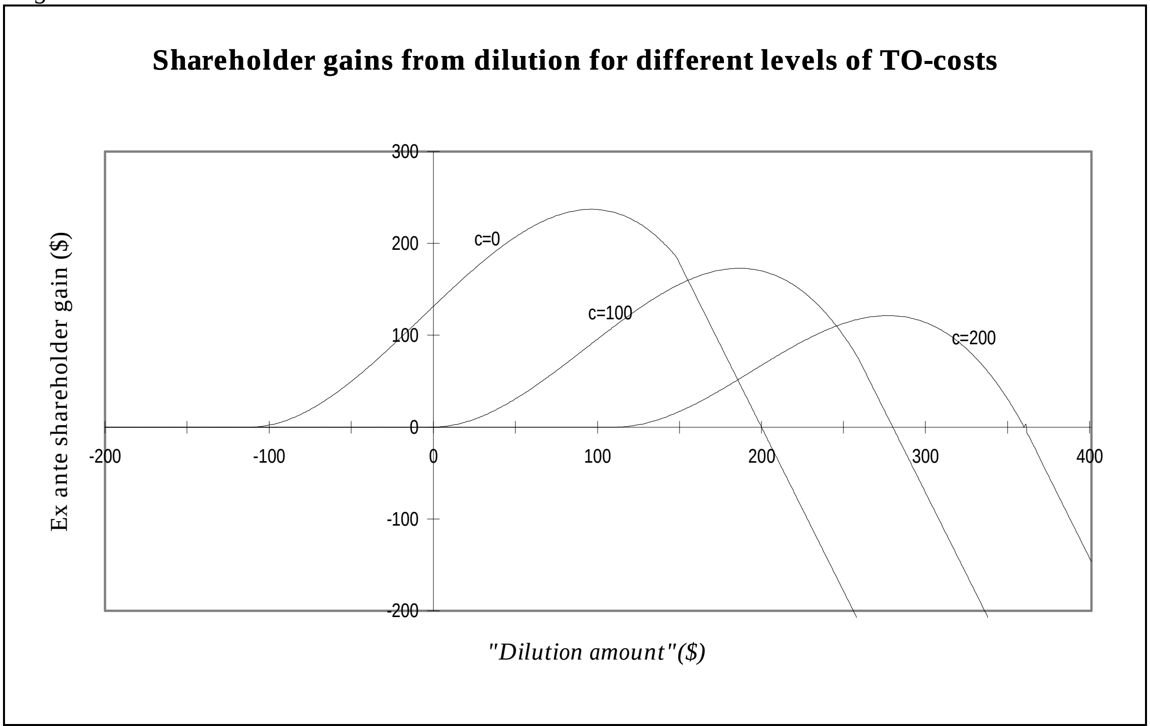
Similarly to the analysis of changes in the bidder's toehold, the general fixed-dilution analysis of cost effects displays the existence of a cutoff point. Let δ^0 be defined as before. Specifically, we have that an increase in the takeover cost will cause the shareholders' ex ante takeover gain to (i) *decrease* for any dilution amount on the interval $\delta \in (\underline{\delta}, \delta^0)$, (ii) *increase* for any dilution amount on the interval $\delta \in (\delta^0, \bar{\delta})$, and (iii) remain *unchanged* for all other dilution amounts. The results are illustrated graphically and numerically below.

Example 2

Figure 2 depicts the impact of variations in takeover costs. The shareholder-gain function is reproduced for three different takeover-cost levels (c = 0, c = 100, and c =

200). The bidder's initial toehold is assumed to 10%. Other parameters are the same as before.

Figure 2



First, consider the zero-dilution case. Only with zero takeover costs will shareholders receive a positive expected takeover gain if no dilution is added; the $c = 0$ curve intersects with the vertical axis at $131.20. Here, the takeover probability is 18.4% and the equilibrium premium is $714.30. At the higher cost levels represented, the probability of a takeover, and consequently ex ante takeover gain, is zero without any dilution.

By specifying a dilution contract, the shareholders will improve their wealth in all three cases. In the zero-cost case, an (optimal) dilution amount of $96.30 will yield an ex ante takeover gain of $237, which is an improvement by $106 (=$237–$131). When c = $100, letting the bidder extract $186.67 will improve small the shareholders' wealth by $172.80. At c = $200, the optimal dilution amount is $277, resulting in a wealth improvement of $121.40.

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[15] Employing a negative exponential distribution assumption (see Section 5.2), yields the result that the takeover premium is constant rather than decreasing in the takeover cost.

The graph illustrates Result 7 by the fact that the height of the bell-shaped curves decrease when takeover costs rise. Simultaneously, the optimal dilution amount increases with *c*. Hence, and not very surprisingly, the effects from increased takeover costs are almost the opposite from the effects of increases in the bidder toehold.

3.3 The optimality of poison pills vs. dilution

The previous examples suggest that positive dilution will tend to be optimal in pools of firms exhibiting relatively low initial ownership concentration and high takeover costs. In such firms, increased dilution will tend to benefit shareholders by enhancing the probability of takeovers. Conversely, poison pill defense will tend to be relatively more desirable in pools of firms exhibiting high initial ownership concentration and low takeover costs. The shareholders are expected to benefit from increased takeover premiums in firms with such characteristics. Suppose that the control threshold is 50%, as defined by the simple-majority rule present in most firms. Given the triangular distribution assumption we get the following result.

Result 8.

(i) Positive dilution will be optimal for bidder toeholds contained on the interval

$e \in \left[0, \frac{13c + 2Z_{\max}}{8c + 7Z_{\max}}\right)$, *and*

(ii) negative dilution (poison pills) will be optimal for bidder toeholds contained on the interval $e \in \left(\frac{13c + 2Z_{\max}}{8c + 7Z_{\max}}, 1/2\right)$.

Result 8 formalizes the intuition that poison pills will tend to increase shareholder wealth under parameter configurations that imply *high a priori takeover probabilities*. This intuition is also confirmed by recent empirical findings. Comment and Schwert (1995) present evidence of this endogenous nature of the decision to adopt poison pills. In particular, from studying a sample of 960 adoptions of original poison pills by exchange-listed firms in the period 1983-90, they find a clear-cut tendency for managers to adopt pills when the likelihood of a takeover is unusually high.[16,17]

4. Changing the threshold for control

The level of ownership that is needed for seizing control can vary between companies. There are several reasons for this. A firm may have adopted a supermajority provision, a multiple-class voting structure, or some other charter amendment affecting the proportion of shares needed for control. Furthermore, control limits may differ between states/countries due to different regulatory frameworks. In subsection 4.1, I analyze the impact of (infinitesimal) changes in the control threshold, α, when this is initially any arbitrary fraction strictly between zero and one. In subsection 4.2, I proceed to examine the wealth effects of the adoption of a particular takeover regulation, the *mandatory bid rule*.

4.1 Changes in α (from arbitrary levels)

Result 9. *The optimal takeover premium is constant with respect to changes in the control threshold*.

In the fixed-dilution case, the takeover premium will increase in the control threshold for all fixed $\delta > \underline{\delta}$. This is triggered by the fact that the minimum value improvement

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[16] See Comment and Schwert (1995), pp. 21-23.
[17] The empirical evidence on actual takeover costs is very limited. However, proxies for the cost of acquiring companies do exist. Administrative acquisition costs will typically depend on the particular legal environment for takeovers. The adoption of the 1968 Williams Act is believed to have increased the legal and administrative costs associated with takeovers. Consistent with this, Jarrell and Bradley (1980) report increasing takeover premia in response to the announcement of the new takeover code. There are many variables that could proxy for the information cost associated with takeovers. Presumably, small firms are associated with higher information costs as larger firms are more closely monitored by the market. A small book-to-market equity ratio may indicate a large proportion of intangibles in the firm's assets, which may render information-production more costly to the acquirer.

needed to make the acquisition profitable will increase as the bidder is required to purchase a larger fraction of the firm in order to get control. However, an increase in the control threshold implies an increase in the optimal dilution amount. This increase in dilution will result in an adjustment of the takeover premium so that the raise implied under fixed dilution is exactly offset. Hence, under optimal dilution, the takeover premium will at the same level independently of the control threshold. Hence, Result 9 implies a refinement in relation to the fixed-dilution case. However, the qualitative result for optimal takeover probability does not differ from the fixed-dilution case:

Result 10. *The optimal takeover probability will decrease with increases in the control threshold.*

Specifically, for fixed dilution amounts, the takeover probability will decrease with increases in the control threshold for all $\delta \in (\underline{\delta}, \bar{\bar{\delta}})$, and remain unchanged otherwise. Combining Results 9 and 10, it is evident that the in-optimum effect on an increase in the control threshold on the total shareholder wealth will be negative:

Result 11. *The maximal ex ante takeover gain (implied by optimal dilution) will decrease with increases in the control threshold.*

Result 11, informs us that shareholders uniformly lose by increasing the threshold for control given that optimal dilution can be maintained. However, similarly to the previous fixed-dilution analyses, there is a cutoff level that determines the sign on the takeover-gain effect when dilution is fixed. Specifically, we have that an increase in α will cause the ex ante takeover gain to *decrease* for any $\delta \in (\underline{\delta}, \delta^0)$, *increase* for any $\delta \in (\delta^0, \bar{\delta})$, and remain *unchanged* at all other dilution levels, and where $\underline{\delta}$, $\bar{\delta}$, and δ^0 are defined as before.

To examine this result a little closer, the effects of imposing a specific regulation like the mandatory bid rule is analyzed in the following subsection.

4.2 The Mandatory Bid Rule

Shleifer and Vishny suggested (Proposition 3) that bids for more than a controlling portion of the firm will not constitute a pure strategy sequential equilibrium supported by credible beliefs. *L* will typically never be better off making a bid for more than 50% of the voting shares. However, in some countries, in particular, in Europe, bidders are required, by law or regulation, to offer to purchase *any or all* of the shares.[18] In this subsection, I examine how increases in the control threshold implied by this type of regulation, often called the mandatory bid rule (henceforth: the MBR), affect shareholder wealth in a dilution context.

When specifically comparing the results of a 50% control threshold (a partial bid) to a the situation in which the bidder is required to extend a nonpartial bid for 100% of the firm (under the MBR), there will be a cutoff level of dilution at which shareholders will be exactly indifferent between a partial and a nonpartial bid. Specifically, with the triangular distribution, we get the following result.

Result 12. *Given the distribution assumption, shareholders will be strictly better off ex ante*

(i) when partial bids are allowed if and only if dilution is constrained to the interval $\delta \in \left(\underline{\delta}, \delta^{MBR}\right)$,

(ii) when partial bids are prohibited if and only if $\delta > \delta^{MBR}$,

where $$\delta^{MBR} = \frac{e\left((32Z_{\max})e^2 - 96ce - 12Z_{\max} - 72c\right) - 3Z_{\max} - 14c}{e\left(32e^2 - 96e - 84\right) - 17}.$$

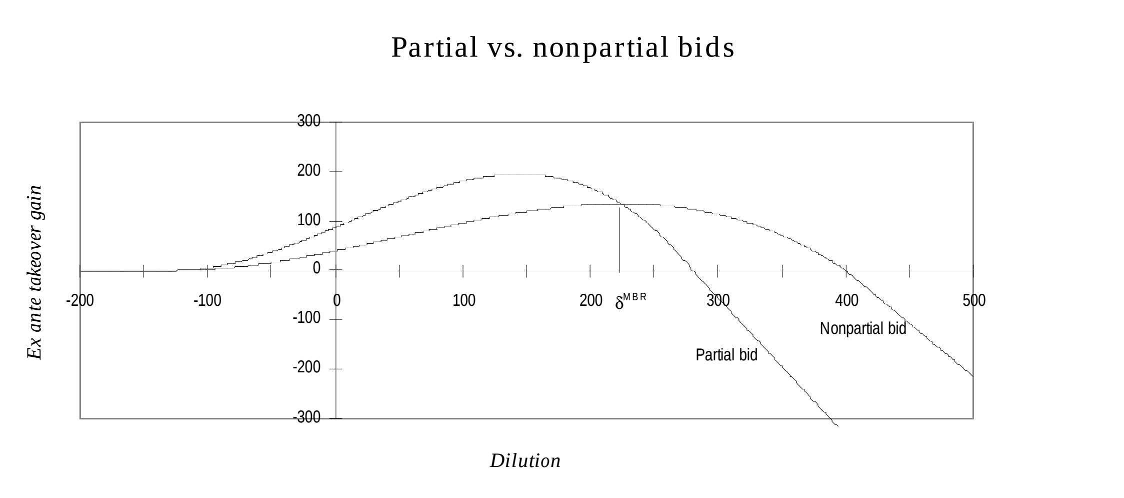
Result 12, with its hideous expression for the cutoff point, is more digestibly illustrated in Figure 3 below.

[18] The Mandatory bid rule is present in the British self-regulatory framework *The City Code on Takeovers and Mergers*. So far, also France, Italy and Norway have adopted the rule.

Example 3

The curve labeled "Partial bid" represents the case when the control threshold is 50%. The curve named "Nonpartial bid" represents the situation in which the mandatory bid rule is operative, that is, when α = 100%. Other parameters are held constant at c = 100, e = 0.2, Z_{max} = 1,000 and Z_{min} = 0.

Figure 3



The two curves intersect at the cutoff point, δ^{MBR} = \$223.84. At this level of dilution, the shareholders will be indifferent between the two bidforms. At δ^{MBR}, a partial bid for 50% of the shares results in a takeover premium of \$156 and a takeover probability of 86.5%. At the same dilution amount, a bid for 100% results in a takeover premium of \$377.49 and a takeover probability of 35.8%. The total ex ante gain at δ^{MBR} is just below \$135 for both bidforms. Shareholders tend to be better off with partial bids if dilution is lower than δ^{MBR}, and benefit from nonpartial bids if dilution is higher.

At zero dilution, the nonpartial bidform yields an ex ante takeover gain of \$39.36 and the partial bidform is results in ex ante takeover gain of \$86.42.

The optimal takeover premium is \$375 for both bidforms (compare Result 9). When the mandatory bid rule is fully adopted, the optimal dilution amount will be given by δ^* = \$225 (i.e., slightly more than δ^{MBR}) yielding a takeover probability of 36% and an ex ante takeover gain of exactly \$135. When partial bids are allowed, shareholder wealth is maximized by a dilution amount of δ^* = \$145. At this dilution level, the probability of a takeover is 51.84% and the resulting ex ante takeover gain is \$194.40 .

5. Sensitivity analysis

In order to generate illustrative results, the analysis has so far been predicated on a specific distribution assumption for future value improvements, the "triangular" distribution. In this section, I discuss the impact of distribution parameters in this particular distribution. I also check the robustness of the results by specifying two alternative distributions.

5.1. Changes in distribution boundaries

First, consider the effects of the distribution boundaries in the applied triangular distribution. The general *in-optimum* result of an increase in $Z_{\max}$ or in $Z_{\min}$ is a corresponding increase in the level of shareholder wealth. Specifically, given the regularity condition $Z_{\min} < c < Z_{\max}$, we have the following in-optimum results.

Result 13. *Given optimal dilution, an increase in Z_{max} will result in an increase in the (i) takeover probability, (ii) the takeover premium, and consequently also in (iii) the shareholders' ex ante takeover gain.*

Result 14. *Given optimal dilution, an increase in Z_{min} will result in (i) an increase in the takeover probability, (ii) no change the takeover premium, and hence (iii) an increase in the shareholders' ex ante takeover gain.*

5.2 Changes in distribution assumptions

In lieu of a general derivation, I check for robustness by making two alternative distribution assumptions, a uniform and an exponential distribution. Under the uniform distribution, the probability density function for Z can be written as

$$f_u(Z) = \frac{1}{Z_{max} - Z_{min}}, \tag{10}$$

for $Z \in [Z_{min}, Z_{max}]$. Under this distribution assumption, all value improvements within the bounds are equally probable. Alternatively, I assume a negative exponential distribution. Under this distribution assumption, the probability density function for Z is

$$f_{exp}(Z) = \frac{1}{m} \cdot \exp\left(-\frac{Z}{m}\right), \tag{11}$$

for $Z \geq 0$, and where m denotes the unconditional expectation of Z. This distribution implies that value improvements are bounded below by zero, and are unbounded above. Moreover, the probability of a value improvement decreases rapidly in its size.

Specifically, the alternative formulae for optimal dilution under the two benchmark distributions can be expressed accordingly.

Result 15.

(i) *Given the uniform distribution* (10), *the optimal dilution amount is*

$$\delta_u^* = \frac{1}{2} \cdot \underline{\delta} + \frac{1}{2} \cdot \overline{\delta}_u. \tag{12}$$

where, specifically, $\underline{\delta} = \frac{c - eZ_{max}}{1-e}$ (i.e., exactly as before), *and* $\overline{\delta}_u = \frac{\alpha Z_{max} + c}{1+\alpha}$.

(ii) *Given the exponential distribution* (11), *the optimal dilution amount is*

$$\delta_{exp}^* = c + m\left(\alpha - \frac{e}{1-e}\right), \tag{13}$$

where m = E[Z] (and e denotes the bidder's toehold and *not* the exponential function).

Figure 4 provides a graphical comparison between the three model specifications. The particular parametric configuration presented is one where $e = 0.2$, $c = 100$, and $Z_{min} = 0$, and E[Z] = 500.

Figure 4

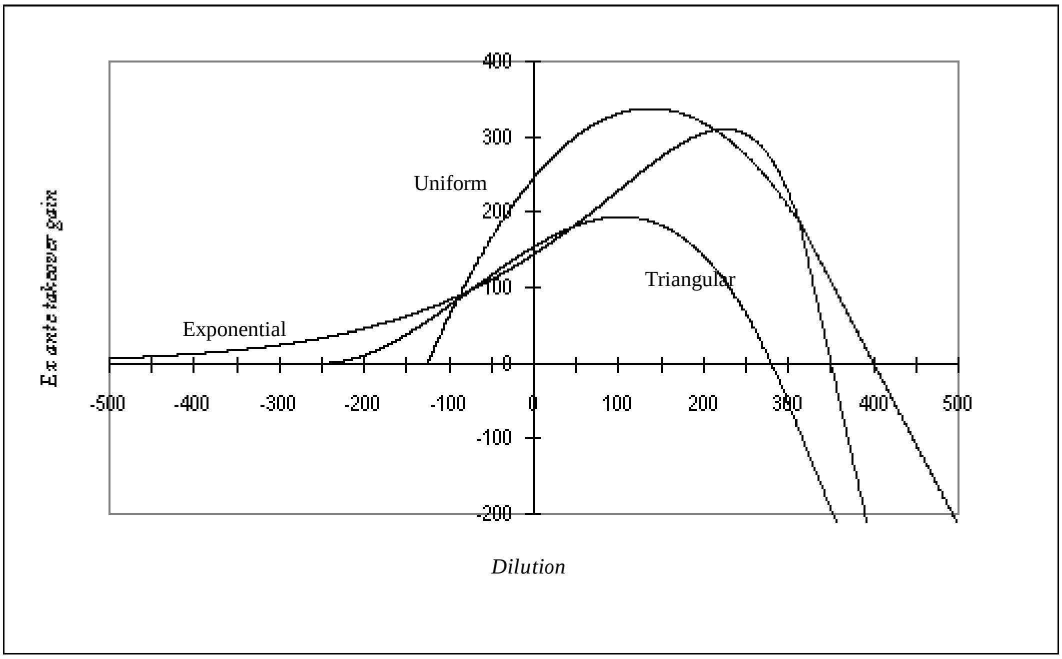


Figure 4 shows that, in relation to the triangular distribution case, the uniform distribution results in a somewhat more symmetric ex-ante-takeover-gain curve, with higher potential takeover gains, while the corresponding curve for the exponential distribution is more skewed to the right. Although the particular formulae for the optimal dilution amount presented in Result 15 differ from the one presented in Result 1, and the shapes of the shareholders' takeover-gain curves are somewhat modified, the basic intuition of the model applies under the alternative distribution assumptions.

In particular, all the stylized properties with respect to changes in the bidder's toehold, the takeover cost, and the level of the control threshold are unchanged, with the minor exception that the optimal ex post takeover premium is constant rather than decreasing in the takeover cost if the exponential distribution is applied. This suggests that the choice of distribution does not have a substantial effect on the qualitative results of the paper. In the following section, I conduct a further discussion of the potential limitations of model and of the empirical evidence.

6. Empirical evidence and discussion

In this section, the consistency between the model's implications and the empirical experience is discussed. In particular, a recent study by Comment and Schwert (1995) presents pertinent empirical results concerning the adoption of poison pills.

The analysis in the present paper implies that poison pills will tend to benefit shareholders the most when the takeover probability is high (as a result of low takeover costs and large bidder toeholds). We should therefore expect more poison pill provisions when such characteristics are present. This prediction receives empirical support by Comment and Schwert (1995).

In a probit analysis for the prediction of takeover probability conducted on a sample of 21,887 firms in the period 1977-1991, Comment and Schwert find that the marginal effect on the takeover frequency of a pill-adoption is positive (2.34%).[19] The literal interpretation of this is that poison pills *increase* the probability of a takeover. However, Comment and Schwert suggest a more plausible interpretation of this result. They hypothesize that poison pills are adopted by managers in anticipation of a takeover attempt. When breaking the poison pill dummy variable into *surprise* and *predictable* components, they obtain a positive marginal effect (2.83%) for the surprise component (when management is likely to have private information of an imminent takeover attempt) and a negative marginal effect (–4.98%) for the predictable one.

Furthermore, Comment and Schwert conduct a probit analysis of predictors for poison pill adoptions (however, this analysis does not include takeover-frequency variables). This analysis generates two economically significant results. First, being incorporated in a state with an antitakeover law appears to increase the likelihood of a

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[19] Comment and Schwert generate some additional evidence on the tendency to adopt pills when takeovers are particularly probable. In a sample of 960 poison pill adoptions in the period 1983-1990, Comment and Schwert examine the cumulative proportion of pill-adopters having received some announcement of takeover interest at specific dates in relation to the announcement date. This is compared to a complementary sample of no-pill firms. (For each of the 960 adoptions, the cumulative proportion of no-pill firms subject to some publicly announced takeover interest is calculated as of the *n*th day relative to the pill adoption announcement.) They find, for pill-adopters, a sharp increase in takeover activity from 2.4% one month before the pill-adoption announcement day to 19.4% one year after. This increase in the cumulative frequency of takeover interest is about double that for no-pill firms. (The corresponding proportion for no-pill firms is 7.8% one month before and 16.2% one year after the pill announcement.) This suggests managers adopt pill defense when the likelihood of a takeover is unusually high.

pill adoption. This suggests that antitakeover laws are complements rather than substitutes to poison pills. Second, firm size is significantly and positively related to the adoption of poison pills. To the extent that the cost of information constitutes a significant part of the takeover costs, this evidence is consistent with the analysis in the paper. Smaller firms presumably incur more costly information production as such firms tend to be less closely monitored by the market than larger firms.

The model presented in this paper also implies that pill adoptions will generate increases in the ex post takeover premium. In a probit analysis attempting to predict the size of takeover premiums, the poison pill dummy receives a significant positive coefficient of 16.27% in a sample of 669 successful takeovers. This suggests that the presence of poison pill coverage is positively related to the size of the takeover premium obtained in an acquisition.

The optimal use of poison pills implied by the model implicitly assumes that management acts in the interest of the shareholders. This is not an unimportant assumption. Anecdotal evidence suggests that managers adopt poison pills to positively deter takeovers in order to protect their private benefits of control. The adequate test of this involves the investigation of whether shareholders gain *ex ante*, and not only ex post. The Comment and Schwert (1995) study shows that not only takeover-conditional takeover premiums increase with the adoption of poison pills, but also that the *unconditional premiums increase*. Specifically, the probit analysis of the full size sample of 21,887 firms (setting the premium equal to zero for the firms in which no takeover activity occurred) yields a significant and positive coefficient of 1.44% for the poison pill indicator variable. This implies that *shareholders benefit ex ante* from the actual use of poison pill defense during the sample period (1975-1991).[20]

Other studies of the impact of antitakeover provisions, exhibit results that partly contrast with those obtained by Comment and Schwert (1995). The typical market reaction to announcements of most types of antitakeover measures appears to be an approximate 1% (or less) *decline* in the stockprice. Ryngaert (1986) examines the

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[20] In addition, Comment and Jarrell (1987) report that two thirds of all takeover attempts – whether hostile or not – involving tender offers for exchange-listed firms between 1981 and 1984 were eventually approved by management. Specifically, in all takeover attempts, 50% of all bidders obtained a merger agreement *before* starting an offer. 22% of all takeover attempts started out as hostile but ended up as successful, negotiated bids, while 12% started as hostile but ended with no shares purchased by any

market reaction to 283 announcements of poison pill adoptions and finds an average two-day abnormal return of –0.34%. In a similar study of 132 poison pills adoptions, Malatesta and Walkling (1988) detect an average two-day abnormal return of –0.92%. These observations of negative average market reactions to adoptions of antitakeover measures appears to contradict the finding that the average ex ante shareholder wealth effect is positive. However, as recognized by Comment and Schwert, these reactions to early poison pill announcements may reflect underestimation of the benefits of increased bargaining power and overestimation of deterrence costs.

When it comes to voluntary dilution, examples of specific methods for this are suggested by Grossman and Hart (1980). These methods include (i) allowing favorable stock issues to the bidder, and (ii) permitting the bidder to sell some of the firm's assets or output. However, whereas observations of poison pill adoptions are abundantly present, the evidence on *explicit contracts* stating a bidder's right to transfer assets from the target is lacking. This may have several reasons. The opportunities to dilute are implicit in the target firm's characteristics and through the very power position held by a controlling owner. There is no lack of examples of post-takeover asset sales and favorable stock issues to acquirers —without any specific dilution contract being drawn up prior to the takeover. In this form, dilution appears to be strongly associated with corporate raiding, and indeed with the general moral hazard problem implied by the separation between ownership and control. Through this "guilt by association", explicit dilution contracts may, hypothetically, be perceived as damaging to the shareholders and would thus not get sufficient approval on stockholders' meetings. However, this explanation is not entirely convincing. To the extent that there exists opportunities for involuntary dilution beyond the optimal amount, these opportunities will not be expanded by the presence of an explicit dilution contract. On the contrary, it seems plausible that a precise contract, by being verifiable in court, will set some limits to potential minority oppression.

A possibly more convincing explanation is that, there may be other ways of increasing the takeover probability that dominate over takeover-triggered dilution. For example, a favorable equity private placement offer allows the firm to approach a specific investor with desirable characteristics to serve as a bidder candidate. Despite

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bidder, and 16% were executed without management's approval. This evidence seems to indicate that the presence of substantial discounts, announcements of equity private placements generate positive market reactions. For example, Wruck (1989) reports an average four-day announcement effect of 4.41%. In a sample of 106 private placement announcement during the period 1980-1987, Hertzel and Smith (1993) obtain an average abnormal return of 1.74%.[21] The average private placement discount in relation to the current market price is 20.1% in the Hertzel and Smith sample. Although there are alternative explanations for this observed behavior, such as the information-revelation hypothesis and the increased-monitoring hypothesis, the empirical evidence is consistent with implications of the present model framework. In particular, it suggests that the use of dilution measures is on average beneficial to shareholders.

Another possible explanation for the lack of explicit dilution contracts is that, in contrast with poison pills, dilution is not naturally aligned with management interest. It seems natural to assume some psychological resistance to suggesting a provision that (i) recognizes that the company possibly can be better run by someone else, and, in consequence of this, (ii) reduces the probability of keeping one's job.

Lastly, because the development of security design is an evolutionary process, a standard dilution contract is possibly yet to be innovated. Notably, after the first introduction of poison pills by the Wall Street law firm Wachtel Lipton in 1983, the coverage of poison pills provisions in U.S. firms has exploded; from trivial levels before 1986, it had reached 35% of all exchange-listed firms by 1991.

Notably, in distinction to the simplified outlining of the presented model, real-life takeover-defensive and takeover-stimulating strategies seem to be adopted in the form of security issues rather than as corporate charter amendments. An evident reason for this preference for security issues is the relatively higher level of flexibility compared to charter amendments. This is important if the uncertainty about future bidder characteristics is substantial. The cost of allowing too much dilution or too effective takeover defense can be very high if, for instance, the management's assessment of the distribution of future value improvements is severely mistaken. In a dynamic environment, a firm's characteristics as a target as well as the properties of potential acquirers are likely to change over time. Therefore, it will be difficult to

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deterrence is not management's principal objective with poison pill provisions.

[21] In a similar study of private placements on the Stockholm Stock Exchange, Molin (1995) encounters a significant positive event-day average abnormal.

rationally assess the distributive characteristics of a future bidder in such a way that the assessment will be valid for very long periods. The corporate charter, being the constitution of the firm, is typically set once and for all and is not easily changed. A security issue, on the other hand, can be triggered when, for example, a takeover attempt is believed to be imminent, and can be aimed at the stimulation or discouragement of takeovers by particular bidders. In this way, management can improve the bargaining power while ensuring that the ex ante reduction in takeover probability is moderate. The empirical evidence suggests that this flexibility is important.

7. Conclusions

This paper explores the wealth effects of voluntary dilution and poison pills. In particular, the model presents a theoretical alternative to the commonly held belief that poison pill defense is detrimental to shareholder wealth. The proposed explanation receives partial support by recent empirical evidence. In particular, the general hypothesis underlying the analysis of optimal dilution, that management, at least on average, acts in the shareholders' interest when adopting poison pills and measures of dilution, receives empirical support by the observation of positive unconditional takeover premiums [Comment and Schwert (1995)] and by the positive market reactions to announcement of equity private placements [See, e.g., Wruck (1989), and Hertzel and Smith (1993)]. However, empirical studies also report negative announcement effects for poison pill adoptions [Ryngaert (1986), Malatesta and Walkling (1988)]. A possible explanation for this is that the market has underestimated the benefits of added bargaining power and overestimated the costs of deterrence.

In terms of specific results, some modifications of the propositions reported in Shleifer and Vishny (1986) are derived. In particular, the takeover probability implied by the use of optimal dilution is shown to be constant with respect to changes in the bidder's toehold, and (ii) the takeover premium implied by the use of optimal dilution is shown to increase in the bidder's toehold and to decrease in the takeover cost.

The empirical implications that optimal use of poison pills do not lower the takeover probability while the takeover premium is increased receive empirical support by Comment and Schwert (1995). Also the model's prediction that poison pills are optimal under parameter configurations that imply a high *a priori* takeover probability is consistent with the empirical evidence.

In an analysis of the effects of changes in the control threshold, it is found that, given that optimal dilution can be maintained, shareholders will *uniformly lose* by an increase the fraction needed to obtain control (as is implied by, e.g., imposing supermajority rules, a mandatory bid rule, etc.). However, in a specific analysis of the mandatory bid rule, it is shown that, if dilution is exogenous, there exists a cutoff point above which nonpartial bids are preferable to partial bids.

References

Ambrose, Brent W. and William L. Megginson, 1992, The role of asset structure, ownership structure, and takeover defenses in determining acquisition likelihood, *Journal of Financial and Quantitative Analysis* 27, 575-589.

Bagnoli, Mark and Barton L. Lipman, 1988, Successful takeovers without exclusion, *Review of Financial Studies* 1, 89-110.

Bebchuk, Lucian A., 1985, Toward undistorted choice and equal treatment in corporate takeovers, *Harvard Law Review* 98.

Bebchuk, Lucian A., 1987, The pressure to tender: An analysis and a proposed remedy, *Delaware Journal of Corporate Law* 12.

Bebchuk, Lucian A., 1988, Takeover bids below the expected value of minority shares, *National Bureau of Economic Research* Working paper No. 2524.

Bergström, Clas and Peter Högfeldt, 1994, An analysis of the Mandatory Bid Rule, *Stockholm School of Economics, Department of Finance* Working Paper No. 61.

Bergström, Clas, Peter Högfeldt, and Johan Molin, 1994, The optimality of the Mandatory Bid Rule, *Stockholm School of Economics, Department of Finance* Working Paper No. 65.

Bradley, Michael, Anand Desai, and E. Han Kim, 1988, Synergistic gains from corporate acquisitions and their division between the stockholders of the target and acquiring firms, *Journal of Financial Economics* 21, 3-40.

Brickley, James A., Jeffrey L. Coles, and Rory L. Terry, 1994, Outside directors and the adoption of poison pills, *Journal of Financial Economics* 35, 371-390.

Burkart, Mike, 1996, Initial shareholdings and overbidding in takeover contests, *The Journal of Finance* 50, 1491-1515.

Coffee, John C., Jr., 1984, Regulating the market for corporate control: A critical assessment of the tender offer's role in corporate governance, *Columbia Law Review* 84.

Comment, Robert and G. William Schwert, 1995, Poison or placebo? Evidence on the deterrence and wealth effects of modern antitakeover measures, *Journal of Financial Economics* 39, 3-43.

Dodd, Peter and Richard Ruback, 1977, Tender offers and stockholder returns: An empirical analysis, *Journal of Financial Economics* 5, 351-373.

Easterbrook, Frank H. and Daniel R. Fischel, 1981, The proper role of target's management in responding to a tender offer, *Harvard Law Review* 94, 1161-1204.

Easterbrook, Frank H. and Daniel R. Fischel, 1982, Corporate control transactions, *Yale Law Journal* 91, 698-737.

Easterbrook, Frank H. and Daniel R. Fischel, 1990, The corporate contract, in ed.: L.A. Bebchuk: *Corporate Law and Economic Analysis*, Cambridge University Press.

Fama, Eugene F. and Michael C. Jensen, 1983, Agency problems and residual claims, *Journal of Law and Economics* 26, 327-349.

Grossman, Sanford J. and Oliver D. Hart, 1980, Takeover bids, the free-rider problem, and the theory of the corporation, *Bell Journal of Economics* 11, 42-64.

Grossman, Sanford J. and Oliver D. Hart, 1988, One share - one vote and the market for corporate control, *Journal of Financial Economics* 20, 175-202.

Grossman, Sanford J. and Motty Perry, 1984a, Perfect sequential equilibrium, *Journal of Economic Theory* 39, 97-119.

Grossman, Sanford J. and Motty Perry, 1984b, Sequential bargaining under asymmetric information, *Journal of Economic Theory* 39, 120-154.

Hertzel, Michael and Richard L. Smith, 1993, Market discounts and shareholder gains for placing equity privately, *The Journal of Finance* 48, 459-485.

Hirshleifer, David and Sheridan Titman, 1990, Share tendering strategies and the success of hostile takeover bids, *Journal of Political Economy* 98, 295-324.

Jarrell, Gregg A. and Michael Bradley, 1980, The economic effects of federal and state regulations of cash tender offers, *Journal of Law and Economics* 19, 371-407.

Jarrell, Gregg A. and Annette B. Poulsen, 1987, Shark repellents and stock prices: The effects of antitakeover amendments since 1980, *Journal of Financial Economics* 19, 127-168.

Jarrell, Gregg A., James A. Brickley, and Jefrey M. Netter, 1988, The market for corporate control: The empirical evidence since 1980, *Journal of Economic Perspectives* 2, 49-68.

Jensen, Michael C. and William H. Meckling, 1976, Theory of the firm: manager behavior, agency costs and ownership structure, *Journal of Financial Economics* 3, 305-360.

Jensen, Michael C. and Richard S. Ruback, 1983, The market for corporate control: The scientific evidence, *Journal of Financial Economics* 11, 5-50.

Malatesta, Paul H. and Ralph A. Walkling, 1988, Poison pill securities: Stockholder wealth, profitability, and ownership structure, *Journal of Financial Economics* 20,347-376.

McWilliam, Victoria B., 1990, Managerial share ownership and the stock price effects of antitakeover amendment proposals, *The Journal of Finance* 45, 1627-1640.

Molin, Johan, 1995, Shareholder gains from equity private placement: Evidence from the Stockholm Stock Exchange, Department of Finance, Stockholm School of Economics, *mimeo*.

Morck, Randall, Andrei Shleifer and Robert W. Vishny, 1988, Characteristics of target of hostile and friendly takeover, in: Alan J. Auerbach, ed., *Corporate takeovers: Causes and consequences*, National Bureau of Economic Research, Chicago, 101-129.

Nathan, Kevin S. and Terrence B. O'Keefe, 1989, The rise in takeover premiums: An exploratory study, *Journal of Financial Economics* 23, 101-119.

Ryngaert, Micahel, 1988, The effect of poison pill securities on shareholder wealth, *Journal of Financial Economics* 20, 377-417.

Shleifer, Andrei and Robert W. Vishny, 1986, Large shareholders and corporate control, *Journal of Political Economy* 94, 461-488.

Stulz, René M., Ralph A. Walkling, and Moon H. Song, 1990, The distribution of target ownership and the division of gains in successful takeovers, *The Journal of Finance* 45, 817-833.

Walkling, Ralph A., 1985, Predicting tender offer success: A logistic analysis, *Journal of Financial and Quantitative Analysis* 20, 461-478.

Walkling, Ralph A. and Robert O. Edminster, 1985, Determinants of tender offer premiums, *Financial Analyst Journal* 41, 27-37.

Weston, J. Fred, Kwang S. Chung, and Susan E. Hoag, 1990, *Mergers, restructuring, and corporate control*, Prentice-Hall, Inc.

Wruck, Karen Hopper, 1989, Equity ownership concentration and firm value: Evidence from private equity financings, *Journal of Financial Economics* 23, 3-28.

Yarrow, G. K., 1985, Shareholder protection, compulsory acquisition and the efficiency of the takeover process, *Journal of Industrial Economics* 34, 3-16.

Appendix A: Derivation of optimal dilution

Generally, the shareholders will maximize the following problem:

$$\underset{\delta}{Maximize}\, W(\cdot) \; = \left(1 - F\left(Z_c^*(\delta)\right)\right) \cdot E\left[Z - \delta \middle| Z \geq Z_c^*(\delta)\right].$$

Under the triangular distribution assumption, the takeover probability is

$$1 - F\left(Z_c^*(\cdot)\right) = \begin{cases} 0 & if \quad \delta \quad \leq \quad \underline{\delta} \\ \left(\dfrac{3(1-e)(\underline{\delta}-\delta)}{(\alpha+2e)(Z_{\max}-Z_{\min})}\right)^2 & if \quad \delta \quad \in \quad (\underline{\delta}, \overline{\overline{\delta}}), \\ 1 & if \quad \delta \quad \geq \quad \overline{\overline{\delta}} \end{cases}$$

where $\underline{\delta} = \dfrac{c - eZ_{\max}}{1-e}$ is such that, $\forall \delta \leq \underline{\delta}$, $1 - F\left(Z_c^*(\delta)\right) = 0$, and $\forall \delta > \underline{\delta}$, $1 - F\left(Z_c^*(\delta)\right) > 0$, while $\overline{\overline{\delta}} = \underline{\delta} + \dfrac{(\alpha + 2e)(Z_{\max} - Z_{\min})}{3(1-e)}$ is such that, $\forall \delta \geq \overline{\overline{\delta}}$, $1 - F\left(Z_c^*(\cdot)\right) = 1$, and $\forall \delta < \overline{\overline{\delta}}$, $1 - F\left(Z_c^*(\cdot)\right) < 1$.

The equilibrium takeover premium is

$$\pi^*(\delta) = E\left[Z - \delta \middle| Z \geq Z_c^*(\cdot)\right] = \frac{(2+\alpha)(\overline{\delta}-\delta)}{(\alpha+2e)},$$

where $\overline{\delta} = \dfrac{\alpha Z_{\max} + 2c}{2+\alpha}$ is such that, $\forall \delta \geq \overline{\delta}$, $\pi^*(\delta) \leq 0$, and $\forall \delta < \overline{\delta}$, $\pi^*(\delta) > 0$.

We can write the maximization problem as

$$\underset{\delta \in \left(\underline{\delta}, \overline{\overline{\delta}}\right)}{Maximize}\, W(\cdot) \; = \frac{9\left(c - eZ_{\max} - (1-e)\delta\right)}{(\alpha+2e)^2\left(Z_{\max} - Z_{\min}\right)^2} \cdot \frac{\left(\alpha Z_{\max} + 2c - (2+\alpha)\delta\right)}{(\alpha+2e)}.$$

Differentiating $W(\cdot)$ w.r.t. δ generates the following partial derivative

$$W' \; = \frac{-9\left(c - eZ_{\max} - (1-e)\delta\right) \cdot \left\{2(1-e)\left(\alpha Z_{\max} + 2c\right) + c(2+\alpha)(c-e) - 3(1-e)(2+\alpha)\delta\right\}}{(\alpha+2e)^3\left(Z_{\max} - Z_{\min}\right)^2}$$

The first order condition for optimum is given by setting $W' = 0$. This is equivalent to

$$\left(c - eZ_{\max} - (1-e)\delta\right) \cdot \left\{2(1-e)\left(\alpha Z_{\max} + 2c\right) + c(2+\alpha)(c-e) - 3(1-e)(2+\alpha)\delta\right\} = 0 \qquad \text{(FOC)}$$

The dilution amount satisfying the first and second order conditions is

$$\delta^* = \frac{1}{3} \cdot \frac{c - eZ_{\max}}{1-e} + \frac{2}{3} \cdot \frac{\alpha Z_{\max} + 2c}{2+\alpha} .$$

The optimal dilution amount can be rewritten as

$$\delta^* = \frac{1}{3} \cdot \underline{\delta} + \frac{2}{3} \cdot \overline{\delta} ,$$

where $\underline{\delta} = \frac{c - eZ_{\max}}{1-e}$ defines the (lower) turning point for the dilution amount at which the probability of takeover becomes zero, while $\overline{\delta} = \frac{\alpha Z_{\max} + 2c}{2+\alpha}$ is interpreted as the (upper) pivotal level of dilution at which the small shareholders' ex post takeover gain will become non-positive.

Appendix B: Derivation of equilibrium properties

B.1 Comparative statics under fixed dilution

B.1.1 Effects on the minimum profitable value improvement

For any fixed level of dilution, the minimum profitable value improvement can be written

$$Z_c^*(\cdot) = \frac{(\alpha - e)Z_{\max} + 3c - 3(1-e)\delta}{\alpha + 2e}. \qquad \text{(B1)}$$

Partial derivatives

$$\frac{\partial}{\partial e} Z_c^*(\cdot) = \frac{-3(\alpha+2)(\bar{\delta} - \delta)}{(\alpha + 2e)^2} < 0 \qquad \forall \delta < \bar{\delta} ,$$

$$\frac{\partial}{\partial c} Z_c^*(\cdot) = \frac{3}{\alpha + 2e} > 0,$$

$$\frac{\partial}{\partial \alpha} Z_c^*(\cdot) = \frac{3(\alpha+2)(\delta - \underline{\delta})}{(\alpha + 2e)^2} > 0 \qquad \forall \delta > \underline{\delta}$$

$$\frac{\partial}{\partial Z_{\max}} Z_c^*(\cdot) = \frac{\alpha - e}{\alpha + 2e} > 0$$

$$\frac{\partial}{\partial Z_{\min}} Z_c^*(\cdot) = 0.$$

B.1.2 Effects on the takeover probability

For any fixed level of dilution on the interval $(\underline{\delta}, \bar{\bar{\delta}})$, the probability of a takeover can be written

$$\left(1 - F\left(Z_c^*(\cdot)\right)\right) = \left(\frac{3(1-e)(\underline{\delta} - \delta)}{(\alpha + 2e)(Z_{\max} - Z_{\min})}\right)^2. \qquad \text{(B2)}$$

The partial derivatives are

$$\frac{\partial}{\partial e}\left(1 - F\left(Z_c^*(\cdot)\right)\right) = \frac{18(1-e)(2+\alpha)(\delta - \underline{\delta})(\bar{\delta} - \delta)}{(\alpha + 2e)^3 (Z_{\max} - Z_{\min})^2} > 0 ,$$

for all $\delta \in (\underline{\delta}, \bar{\bar{\delta}})$, and zero other wise.

$$\frac{\partial}{\partial c}\left(1 - F\left(Z_c^*(\cdot)\right)\right) = \frac{-18(1-e)(\delta - \underline{\delta})}{(\alpha + 2e)^2 (Z_{\max} - Z_{\min})^2} < 0$$

for all $\delta \in (\underline{\delta}, \overline{\overline{\delta}})$, and zero otherwise.

$$\frac{\partial}{\partial \alpha}\left(1 - F\left(Z_c^*(\cdot)\right)\right) = \frac{-18\left((1-e)(\delta - \underline{\delta})\right)^2}{(\alpha + 2e)^3 (Z_{\max} - Z_{\min})^2} < 0$$

for all $\delta \in (\underline{\delta}, \overline{\overline{\delta}})$, and zero otherwise.

$\frac{\partial}{\partial Z_{\max}}\left(1 - F\left(Z_c^*(\cdot)\right)\right) = \frac{-18(1-e)^2 (\underline{\delta} - \delta)(\delta^Z - \delta)}{(\alpha + 2e)^2 (Z_{\max} - Z_{\min})^3}$ >0 for all $\delta \in (\underline{\delta}, \delta^Z)$, and < 0 for all $\delta \in (\delta^Z, \overline{\overline{\delta}})$, and zero otherwise, where $\delta^Z = \frac{c - eZ_{\min}}{1-e}$.

$\frac{\partial}{\partial Z_{\min}}\left(1 - F\left(Z_c^*(\cdot)\right)\right) = \frac{18\left((1-e)(\delta - \underline{\delta})\right)^2}{(\alpha + 2e)^2 (Z_{\max} - Z_{\min})^3} < 0$ for all $\delta \in (\underline{\delta}, \overline{\overline{\delta}})$, and zero otherwise.

B.1.3 Effects on the takeover premium

For any fixed level of dilution, the takeover premium can be written

$$E\left[Z - \delta \middle| Z \geq Z_c^*(\cdot)\right] = \frac{(2+\alpha)(\overline{\delta} - \delta)}{(\alpha + 2e)}. \qquad \text{(B3)}$$

We obtain the following partial derivatives:

$\frac{\partial}{\partial e} E\left[Z - \delta \middle| Z \geq Z_c^*(\cdot)\right] = \frac{-2(\alpha + 2)(\overline{\delta} - \delta)}{(\alpha + 2e)^2} < 0$ for all $\delta < \overline{\delta}$,

$\frac{\partial}{\partial c} E\left[Z - \delta \middle| Z \geq Z_c^*(\cdot)\right] = \frac{2}{\alpha + 2e} > 0$,

$\frac{\partial}{\partial \alpha} E\left[Z - \delta \middle| Z \geq Z_c^*(\cdot)\right] = \frac{2(1-e)(\delta - \underline{\delta})}{(\alpha + 2e)^2} > 0$ for all $\delta > \underline{\delta}$,

$\frac{\partial}{\partial Z_{\max}} E\left[Z - \delta \middle| Z \geq Z_c^*(\cdot)\right] = \frac{\alpha}{\alpha + 2e} > 0$,

$\frac{\partial}{\partial Z_{\min}} E\left[Z - \delta \middle| Z \geq Z_c^*(\cdot)\right] = 0$.

B.1.4 Effects on the shareholders' ex ante takeover gain

For any fixed level of dilution, the ex ante takeover gain can be written

$$\left(1-F\left(Z_c^*(\cdot)\right)\right)\cdot E\left[Z-\delta \middle| Z \geq Z_c^*(\cdot)\right] = \left(\frac{3(1-e)(\underline{\delta}-\delta)}{(\alpha+2e)(Z_{\max}-Z_{\min})}\right)^2 \cdot \frac{(2+\alpha)(\bar{\delta}-\delta)}{(\alpha+2e)}. \qquad \text{(B4)}$$

The partial derivatives, holding δ fixed, are the following:

$$\frac{\partial}{\partial e}\left(1-F\left(Z_c^*(\cdot)\right)\right)\cdot E\left[Z-\delta \middle| Z \geq Z_c^*(\cdot)\right] =$$

$$= \frac{18(1-e)(2+\alpha)(\delta-\underline{\delta})(\bar{\delta}-\delta)\left((\alpha-e)(Z_{\max}-\delta)-3(\delta-c)\right)}{(\alpha+2e)^4(Z_{\max}-Z_{\min})^2} > 0 \text{ for all } \delta$$

$\in (\underline{\delta},\delta^0)$, and < 0 for all $\delta \in (\delta^0,\bar{\delta})$, and zero otherwise, where

$$\delta^0 = \frac{Z_{\max}(\alpha-e)+3c}{\alpha-e+3}.$$

$$\frac{\partial}{\partial c}\left(1-F\left(Z_c^*(\cdot)\right)\right)\cdot E\left[Z-\delta \middle| Z \geq Z_c^*(\cdot)\right] =$$

$$= \frac{-18(1-e)(\delta-\underline{\delta})\left((\alpha-e)(Z_{\max}-\delta)-3(\delta-c)\right)}{(\alpha+2e)^3(Z_{\max}-Z_{\min})^2} < 0 \text{ for all } \delta \in (\underline{\delta},\delta^0), \text{ and } > 0 \text{ for all } \delta$$

$\in (\delta^0,\bar{\delta})$, and zero otherwise.

$$\frac{\partial}{\partial \alpha}\left(1-F\left(Z_c^*(\cdot)\right)\right)\cdot E\left[Z-\delta \middle| Z \geq Z_c^*(\cdot)\right] =$$

$$= \frac{-18\left((1-e)(\delta-\underline{\delta})\right)^2\left((\alpha-e)(Z_{\max}-\delta)+3(\delta-c)\right)}{(\alpha+2e)^4(Z_{\max}-Z_{\min})^2} < 0 \text{ for all } \delta \in (\underline{\delta},\delta^0), \text{ and } > 0 \text{ for}$$

all $\delta \in (\delta^0,\bar{\delta})$.

B.2 Comparative statics under optimal dilution

B.2.1 Effects on the optimal dilution amount

The optimal dilution is written

$$\delta^* = \frac{1}{3}\cdot\underline{\delta} + \frac{2}{3}\cdot\overline{\delta} = \frac{1}{3}\cdot\frac{c - eZ_{\max}}{1-e} + \frac{2}{3}\cdot\frac{\alpha Z_{\max} + 2c}{2+\alpha}. \qquad \text{(B5)}$$

The partial derivatives are

$$\frac{\partial}{\partial e}\,\delta^* = -\frac{1}{3}\cdot\frac{Z_{\max} - c}{(1-e)^2} < 0$$

$$\frac{\partial}{\partial c}\,\delta^* = \frac{1}{3}\cdot\left(\frac{1}{(1-e)} + \frac{4}{2+\alpha}\right) > 0$$

$$\frac{\partial}{\partial \alpha}\,\delta^* = \frac{4}{3}\cdot\frac{Z_{\max} - c}{(2+\alpha)^2} > 0$$

$$\frac{\partial}{\partial Z_{\max}}\,\delta^* = \frac{1}{3}\cdot\left(\frac{2\alpha}{2+\alpha} - \frac{e}{1-e}\right) > 0 \text{ for } e < \frac{2\alpha}{2+3\alpha} \text{ (and nonpositive otherwise)}^{22}$$

$$\frac{\partial}{\partial Z_{\min}}\,\delta^* = 0.$$

B.2.2 Effects on the optimal minimum profitable value improvement

Inserting (B5) into (B1) yields an expression for the optimal minimum profitable value improvement:

$$Z_c^*(\delta^*,\cdot) = \frac{\alpha Z_{\max} + 2c}{2+\alpha} \;(=\overline{\delta}). \qquad \text{(B6)}$$

We obtain the following *in-optimum* partial derivatives

$$\frac{\partial}{\partial e}\,Z_c^*(\delta^*,\cdot) = 0,$$

$$\frac{\partial}{\partial c}\,Z_c^*(\delta^*,\cdot) = \frac{2}{2+\alpha} > 0,$$

[22] In particular, for $\alpha = 0.5$, the optimal dilution will increase in $Z_{\max}$ if and only if e < $^2/_7$.

$$\frac{\partial}{\partial \alpha} Z_c^*(\delta^*,\cdot) = \frac{2(Z_{\max} - c)}{(2+\alpha)^2} > 0,$$
$$\frac{\partial}{\partial Z_{\max}} Z_c^*(\delta^*,\cdot) = \frac{\alpha}{2+\alpha} > 0,$$
$$\frac{\partial}{\partial Z_{\min}} Z_c^*(\delta^*,\cdot) = 0.$$

B.2.3 Effects on the optimal takeover probability

Substituting (B5) into (B2) produces an expression for the optimal takeover probability:

$$1 - F\left(Z_c^*(\delta^*,\cdot)\right) = \left(\frac{2(Z_{\max} - c)}{(2+\alpha)(Z_{\max} - Z_{\min})}\right)^2. \tag{B7}$$

Partial derivatives are

$$\frac{\partial}{\partial e}\left(1 - F\left(Z_c^*(\delta^*,\cdot)\right)\right) = 0$$
$$\frac{\partial}{\partial c}\left(1 - F\left(Z_c^*(\delta^*,\cdot)\right)\right) = \frac{-8(Z_{\max} - c)}{(2+\alpha)^2(Z_{\max} - Z_{\min})^2} < 0$$
$$\frac{\partial}{\partial \alpha}\left(1 - F\left(Z_c^*(\delta^*,\cdot)\right)\right) = \frac{-8(Z_{\max} - c)^2}{(2+\alpha)^3(Z_{\max} - Z_{\min})^2} < 0$$
$$\frac{\partial}{\partial Z_{\max}}\left(1 - F\left(Z_c^*(\delta^*,\cdot)\right)\right) = \frac{8(Z_{\max} - c)(c - Z_{\min})}{(2+\alpha)^2(Z_{\max} - Z_{\min})^3} > 0 \qquad \text{for } Z_{\min} < c < Z_{\max}$$
$$\frac{\partial}{\partial Z_{\min}}\left(1 - F\left(Z_c^*(\delta^*,\cdot)\right)\right) = \frac{8(Z_{\max} - c)^2}{(2+\alpha)^2(Z_{\max} - Z_{\min})^3} > 0$$

B.2.4 Effects on the optimal takeover premium

Substituting (B5) into (B3) results in the following expression for the optimal takeover premium:

$$E\left[Z - \delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = \frac{1}{3}\cdot\frac{Z_{\max} - c}{1-e}. \tag{B8}$$

Partial derivatives are

$$\frac{\partial}{\partial e}\ E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = \frac{1}{3}\cdot\frac{Z_{\max}-c}{(1-e)^2} > 0$$

$$\frac{\partial}{\partial c}\ E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = -\frac{1}{3}\cdot\frac{1}{(1-e)} < 0$$

$$\frac{\partial}{\partial \alpha}\ E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = 0$$

$$\frac{\partial}{\partial Z_{\max}}\ E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = \frac{1}{3}\cdot\frac{1}{(1-e)} > 0$$

$$\frac{\partial}{\partial Z_{\min}}\ E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = 0$$

B.2.5 Effects on the optimal ex-ante takeover gain

Inserting (B5) into (B4), or equivalently, multiplying the respective RHSs of (B7) and (B8) produces the following expression for the maximal ex ante takeover gain:

$$\left(1-F\left(Z_c^*(\delta^*,\cdot)\right)\right) E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = \frac{4}{3}\cdot\frac{(Z_{\max}-c)^3}{(1-e)(2+\alpha)^2(Z_{\max}-Z_{\min})^2}. \qquad \text{(B9)}$$

The in-optimum partial derivatives are

$$\frac{\partial}{\partial e}\left(1-F\left(Z_c^*(\delta^*,\cdot)\right)\right) E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = \frac{4}{3}\cdot\frac{(Z_{\max}-c)^3}{(1-e)^2(2+\alpha)^2(Z_{\max}-Z_{\min})^2} > 0$$

$$\frac{\partial}{\partial c}\left(1-F\left(Z_c^*(\delta^*,\cdot)\right)\right) E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = -\frac{4}{3}\cdot\frac{(Z_{\max}-c)^2}{(1-e)(2+\alpha)^2(Z_{\max}-Z_{\min})^2} < 0$$

$$\frac{\partial}{\partial \alpha}\left(1-F\left(Z_c^*(\delta^*,\cdot)\right)\right) E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = -\frac{8}{3}\cdot\frac{(Z_{\max}-c)^3}{(1-e)(2+\alpha)^2(Z_{\max}-Z_{\min})^3} < 0$$

$$\frac{\partial}{\partial Z_{\max}}\left(1-F\left(Z_c^*(\delta^*,\cdot)\right)\right) E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] =$$

$$= \frac{4}{3}\cdot\frac{(Z_{\max}-c)^2\left((Z_{\max}-Z_{\min})-2(Z_{\min}-c)\right)}{(1-e)^2(2+\alpha)^2(Z_{\max}-Z_{\min})^2} > 0$$

$$\frac{\partial}{\partial Z_{\min}}\left(1-F\left(Z_c^*(\delta^*,\cdot)\right)\right) E\left[Z-\delta \middle| Z \geq Z_c^*(\delta^*,\cdot)\right] = \frac{8}{3}\cdot\frac{(Z_{\max}-c)^3}{(1-e)(2+\alpha)^2(Z_{\max}-Z_{\min})^3} > 0$$