Multimarket Contact, Concavity, and Collusion: on Extremal Equilibria of Interdependent Supergames*

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SSE/EFI Working Paper in Ec. and Fin. No.104 (Febr. 1996)
First draft, September 1995. This version: November 30, 1998.

Abstract

Following Bernheim and Whinston (1990), this paper addresses the effects of multimarket contact on firms' ability to collude in repeated oligopolies. Managerial incentives, taxation, and financial market imperfections tend to make firms' objective function strictly concave in profits and market games "interdependent"; firms' payoffs in each market depend on how they are doing in others. In this case multimarket contact always facilitates collusion, and may make it sustainable in all markets even when otherwise it would not be sustainable in any. The effects of conglomeration and horizontal mergers are discussed. The results extend to non-oligopolistic supergames with objective functions submodular in material payoffs.

JEL CLASSIFICATION: C72, D43, L13, L21.

KEYWORDS: Repeated games, oligopoly, collusion, cooperation, conglomeration, mergers.

*Many thanks to Gaetano Bloise, Paolo Buccirossi, Peter Högfeldt, John Scott, Klaus Wallner, and particularly Douglas Bernheim and Jörgen W. Weibull for useful comments on earlier versions of the paper. Thanks also to Marcus Asplund, David Brown, Tore Ellingsen, Yeongjae Kang, Karl Wärneryd, and participants at economic theory workshops at the Stockholm School of Economics, at seminars at the University of Rome "La Sapienza," and at ESEM97 (Toulouse) for comments or discussions. Of course, I am responsible alone for all remaining errors.

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1 Introduction

The traditional view about multimarket contact is that it always enhances firms' ability to sustain tacit collusion by allowing for "mutual forbearance" in the different markets. Sticking to an established convention in recent studies on the subject, we report Corwin Edwards' words (1955), probably the first clear statement on the effects of multimarket contact:

> "When one large conglomerate enterprise competes with another, the two are likely to encounter each other in a considerable number of markets. The multiplicity of their contact may blunt the edge of their competition. A prospect of advantage from vigorous competition in one market may be weighted against the danger of retaliatory forays by the competitor in other markets. Each conglomerate competitor may adopt a live-and-let-live policy designed to stabilize the whole structure of the competitive relationship." [As quoted by Scherer (1980, p. 340).]

The view that multimarket contact facilitates collusion "in general" has not been supported by Douglas Bernheim and Michael Whinston's (1990) rigorous supergame-theoretic analysis. In fact, although these authors do conclude that in a wide range of circumstances multimarket contact does facilitate collusive behavior, they also begin stating an irrelevance result: when firms and markets are identical and there are constant returns to scale, multimarket contact does not strengthen firms' ability to collude. Bernheim and Whinston then proceed by relaxing the assumptions behind this result to understand under which circumstances multimarket contact helps to sustain collusion. Among the many conditions they identify are that firms' production costs, the number of competitors, or the demand growth rates differ across markets, or that a single firm maintains an absolute cost advantage. Most of these conditions imply asymmetries between strategic interactions, due to differences between firms or markets, so that multimarket contact facilitates collusion by allowing the transfer of the slack of enforcing power (of net expected gains from collusion) which may be present in some markets to other markets in which it lacks such power.

This paper identifies a further condition under which multimarket contact facilitates collusion, one that is independent of asymmetries and that brings grist to the mill of the traditional view. We show that when firms' objective function is strictly concave the irrelevance result disappears and multimarket contact *always* facilitates collusion. When ownership is separated from control, so that managerial objectives become relevant, or when corporate taxes are non-linear or financial markets are imperfect, the firms' objective function tends to display decreasing marginal utility for profits within each time period. A strictly concave static objective function makes the repeated strategic interactions *interdependent*: firms' evaluation of profits from one market depends on profits realized in other markets. Then, expected losses from simultaneous retaliation in more markets - a threat available only with multimarket contact - are always larger than (the sum of) those from independent retaliations. Further, short-run profits from a simultaneous deviation from collusion in more markets are always less valuable than (the sum of) short-run profits from independent deviations. These two effects both facilitate collusion, whatever the type of repeated oligopoly considered. They can be reinforced by Bernheim and Whinston's conditions, but they will be present even with identical firms, identical markets, and constant returns to scale.

The wealth effect induced by a concave static objective function is also shown to generate "scale economies" in collusion (or cooperation); with multimarket contact collusion can be viable in a set of markets even when, in the absence of multimarket contact, it could not have been supported in *any* of these markets.

A complementary interpretation of these results is that *conglomeration* has negative effects on firms' ability to collude, as it creates independent sub-markets which "insure" firms against too low levels of profits during punishments. Multimarket contact then facilitates collusion by restoring the situation preceding conglomeration.

The effects of horizontal mergers without multimarket contact on the minimum discount factor at which collusion is supportable are ambiguous, as the wealth effect they generate may have different effects at different levels of profitability. On the other hand, we find that mergers always facilitate collusion when the discount factor is relatively low, and conversely hinder collusion when the discount factor is relatively high.

The mechanism behind the results is quite general. We show that "multi-game" contact facilitates cooperation in repeated strategic interactions that differ from oligopolistic ones as long as agents' static objective function is strictly *submodular* in stage-games' material payoffs.

Section 2 discusses firms' objective function; Section 3 considers Bernheim and Whinston's model; the general result is presented in Section 4; Section 5 discusses extensions; and Section 6 concludes. An appendix contains all proofs.

2 On the concavity of firms' objective function

When (a) firms are led by owners and (b) financial markets are perfect the only interesting and relevant modeling assumption is the standard one of profit-maximizing firms. This is, of course, because owners can freely reallocate wealth in time through the financial market to satisfy their intertemporal preferences, so that they will only care about the discounted value of firms' profits. In this case there is little more to say on multimarket contact and collusion apart from what has already been stated by Bernheim and Whinston.

However, in the real world it often happens that at least one of the two conditions above is violated. For example, when ownership is separated from control firms tend to pursue objectives different from profit-maximization (see, e.g., Herbert Simon, 1957; William Baumol, 1958; Oliver Williamson, 1964; or Michael Jensen and William Meckling, 1976).

In what follows we briefly explain why we think that the most interesting non-standard assumption to study is that of a strictly concave utility function.[1]

2.1 Empirical evidence

Income smoothing. The terms "income smoothing" and "earnings management" in the accounting and management literature refer to the apparently common practice of manipulating accounts and tuning production decisions in order to reduce the variability of firms' book profits. There is a long series of robust empirical results on this phenomenon revealing that real world top managers are strongly averse to intertemporal substitution in firm profits, that is, they have a strictly concave static objective function.[2]

Hedging. Companies invest large amounts of resources in order to hedge risks through various kinds of derivatives (e.g. Christopher Géczy *et al.*, 1997). They even hire specialized staff and create offices for "risk management." The amount of resources spent on hedging risks is tangible evidence that real-world firms are usually risk-averse.

2.2 Theoretical explanations: managerial objectives

Managerial incentive contracts. Healy (1985) explains income smoothing by the fact that managers' monetary bonuses are usually bounded above. This gives managers incentives to transfer income from periods in which it is above the upper bound of the incentive scheme to periods in which it is below it. Healy provides some empirical support for his view. Joskow and Rose (1994) find further evidence that boards discount extreme performance realizations when dealing with managers' compensation, i.e. that managers' bonuses tend to be capped. Capped incentives make managers averse to intertemporal substitution in firms' profits.

Managerial rents, asymmetric information, and career concerns. Fudenberg and Tirole (1995) propose an alternative explanation of income smoothing. They build an optimal contracting model in which incumbent managers earn rents, owners cannot

[1] Financial economists - who are closer to the real world than others - often consider risk-averse firms/agents to be the "standard" assumption.

[2] See, for example, Mark De Fond and Chul Park (1997); Francoise Degeorge *et al.* (1997); Eero Kasanen *et. al.* (1996); Robert Holthausen *et al.* (1995); Jennifer Gaver *et al.* (1995); Kenneth Merchant (1989); Mary Greenawalt and Joseph Sinkey (1988); or Paul Healy (1985).

commit to long-term contracts, and performance measures are subject to "information decay" (i.e. new performance measurements are better signals than old ones). The result is that in equilibrium managers are willing to incur positive costs in order to smooth reported profits and dividends. The point is that with information asymmetries and information decay after some periods of low profits, shareholders may find it optimal to replace the manager even if low profit periods follow high profit ones. Because they enjoy rents managers incur costs if they are fired. This generates a managerial "aversion to low profits" which here translates into a strictly concave objective function.

Managerial risk-aversion. The explanation that financially constrained risk-averse managers smooth their own income in time is one of the first offered for income smoothing (e.g. Richard Lambert, 1984; Ronald Dye, 1988) and for hedging (e.g. Clifford Smith and Rene Stulz, 1985). In the analysis of the shareholders/managers relation as a principal-agent problem, the typical trade-off between incentives and risk-sharing is obtained exactly because managers are assumed to be more averse to risk than owners.

Managerial discretion. Managers are usually thought to be interested in power, in the firm's growth, in "pet projects," etc. They may want to invest even when the expected returns from the investment project are negative. For these investments managers are financially constrained. Because within each time period such projects tend to have diminishing marginal value, managers prefer to have some free cash flow in each period to invest discretionally (e.g. Jensen, 1986). This makes them averse to intertemporal substitution with respect to the firm's profits.

Debt and managerial bankruptcy-aversion. Debt financing may lead firms to behave in an "as if" risk-averse manner. Debt implies the risk of bankruptcy, an event to which managers are strongly averse in that it ruins their future earning opportunities (Stuart Gilson, 1989; Gilson and Michael Vetsuypens, 1993). Bondholders will also want to reduce the variability of earnings to minimize the probability of financial distress and associated bankruptcy costs (e.g. Smith and Stulz, 1985). In the presence of stochastic shocks, this keeps managers from maximizing expected profits and leads them to avoid bankruptcy, i.e. to behave "as if" they had a strictly concave objective function (e.g. Bruce Greenwald and Joseph Stiglitz, 1990, 1993).

2.3 Theoretical explanations: external factors

Financial market's imperfections. Work on both capital structure (e.g. Stuart Myers and Nicholas Majluf, 1984) and on "the credit channel of monetary policy" (e.g. Steven Fazzari *et al.,* 1988; Robert Hubbard *et al.,* 1993; Ben Bernanke and Mark Gertler, 1995) indicates that because of information asymmetries in capital markets, firms' cost of external finance is strictly convex. In this case firms will prefer smooth earnings paths so that some internal funds are always available and suboptimal investment policies can be avoided. This leads them to maximize a strictly concave objective function. In fact, the convex cost of external finance has been proposed as the first reason why firms *should* smooth profits by hedging.[3]

Taxation. Other arguments made for hedging all imply that firms have a strictly concave objective function. For example, it has been argued that firms should hedge to reduce their tax bill.[4] This is because corporate taxes are not perfectly linear. For example, items such as tax credits generate convexity in firms' tax liability (concavity in firms' objective function) because the present value of unused credits diminishes during carry-forward to future periods (e.g. DeAngelo and Masulis, 1980).

Investors' preferences. Many investors seem to value more assets with smooth returns (e.g. Allen and Michaely, 1995). Some institutional investors have constraints that make them prefer assets which pay out stable dividends (e.g., the allowance to spend income but not capital gains). Small investors may face transaction costs when selling their assets and may want smooth returns for consumption reasons. Equityholders may gain from a reduced variance in earnings through improvements in portfolio optimization decisions (e.g. Peter DeMarzo and Durrell Duffie, 1991). In fact, one of the first explanations proposed for income smoothing is that – by reducing the perceived volatility of cash flow – they increase the market valuation of firms by risk-averse investors (e.g. Brett Trueman and Titman, 1988; Ronen and Sadan, 1981).

3 Repeated Bertrand competition

3.1 Bernheim and Whinston's irrelevance result

Consider first Bernheim and Whinston's model of repeated Bertrand competition. Time is discrete and in each market k trade occurs simultaneously in each period t, $t = 1, 2, ...$ In each market and in each period demand is a decreasing and continuous function $Q_k(p_k)$ of price p_k. Entry barriers limit the number of firms in each market to two. Each firm i is active in two markets and at every point in time it announces its current prices. When in a market the two firms announce identical prices, half of the consumers buy from each firm. When prices differ, all the consumers buy from the firm which quoted the lowest price. Firms must meet all the demand at the announced price. Let c_{ik} denote the constant marginal cost of production for firm i in market k. An equilibrium in market k will be a path of prices and associated profits, $\{p_{zk}, \pi_{zk}\}_{t=0}^{\infty}$, where $z = \{i \in I \,|i \text{ is active in market } k\}$. When firms are identical and markets are identical we write $Q_k = Q$ and $c_{ik} = c$.

[3] For example, Alan Shapiro and Sheridan Titman (1986); Donal Lessard (1990); Rene Stulz (1990); Kenneth Froot *et al.* (1993); Géczy *et al.* (1997).

[4] See, e.g., Stulz (1984); Smith and Stulz (1985); Froot *et al.* (1993); and The Economist (1996).

Under the assumptions that industry profits are concave in price and that firms use trigger strategies to sustain symmetric stationary collusive agreements, Bernheim and Whinston derive the irrelevance result: *"When identical firms with identical constant returns to scale technologies meet in identical markets, multimarket contact does not aid in sustaining collusive outcomes"* (1990, p.5). The proof is straightforward: consider any pair of identical markets, say A and B, and call p^m the monopoly price common to these markets. When different firms interact in the two markets, a stationary collusive price $p \in [c, p^m]$ can be sustained in subgame-perfect equilibrium if

$$\frac{1}{1-\delta}(p-c)\frac{Q(p)}{2} - (p-c)Q(p) \geq 0,$$

which implies $\delta \geq \frac{1}{2}$. On the other hand with multimarket contact between firms, say 1 and 2, prices (p_A, p_B) and market shares $\lambda_{ik}, \lambda_{jk}{}_{k=A,B}$ are sustainable as a symmetric-payoffs stationary equilibrium if, for i=1, 2

$$\sum_{k\in(A,B)} \left[\frac{1}{1-\delta}\lambda_{ik}(p_k - c)Q(p_k) - (p_k - c)Q(p_k)\right] \geq 0,$$

with $p_k \in [c, p^m]$. Summing over i they obtain

$$\sum_{k=(A,B)} (p_k - c)Q(p_k)(\delta - \frac{1}{2}) \geq 0,$$

which also requires $\delta \geq \frac{1}{2}$.

3.2 Concavity and collusion

Keeping all the other assumptions, let now the static objective function of a firm i active in markets A and B be $U_i = \ln(1 + \pi_{iA} + \pi_{iB})$. If firms i and j are active on the same two identical markets, i.e. with multimarket contact, firms are able to sustain a constant sequence of monopoly prices in subgame-perfect equilibrium in both markets if

$$\frac{1}{1-\delta}\ln\left(1 + 2[(p^m - c)\frac{Q(p^m)}{2}]\right) - \ln\left(1 + 2(p^m - c)Q(p^m)\right) \geq 0,$$

or, equivalently, if

$$\delta \geq \delta^* = \frac{\ln\left(1 + 2(p^m - c)Q(p^m)\right) - \ln\left(1 + (p^m - c)Q(p^m)\right)}{\ln\left(1 + 2(p^m - c)Q(p^m)\right)}. \quad (1)$$

In the absence of multimarket contact, in each market only the threat of punishment in that same market can be used; firms' profits from the other market must be taken as a given. Therefore, in this case a constant sequence of joint monopoly prices will be supportable in subgame-perfect equilibrium in each market if

$$\frac{1}{1-\delta}\ln\left(1+2[(p^m-c)\frac{Q(p^m)}{2}]\right)-\ln\left(1+\frac{3}{2}(p^m-c)Q(p^m)\right)+$$

$$-\frac{\delta}{1-\delta}\ln\left(1+\frac{1}{2}(p^m-c)Q(p^m)\right)\geq 0,$$

or, equivalently, if

$$\delta \geq \delta^{**}=\frac{\ln\left(1+\frac{3}{2}(p^m-c)Q(p^m)\right)-\ln\left(1+(p^m-c)Q(p^m)\right)}{\ln\left(1+\frac{3}{2}(p^m-c)Q(p^m)\right)-\ln\left(1+\frac{1}{2}(p^m-c)Q(p^m)\right)}. \tag{2}$$

As expected, both δ^* and δ^{**} are positive and less than one when $(p^m-c)Q(p^m)>0$, but it turns out that $\delta^{**}>\delta^*$, for any p^m, c, and $Q()$. Therefore, when δ is $\delta^{**}>\delta\geq\delta^*$ collusion will be supportable only when there is multimarket contact. In other words, with a logarithmic objective function multimarket contact facilitates collusion even with identical firms, identical markets, and constant returns to scale.

4 A more general result

Consider a finite set of oligopolistic markets $\Omega=\{A,B,C,...\}$ and a finite set of firms I, $I=\{1,2,3,...,N\}$ interacting repeatedly in several of these markets and having a common intertemporal discount factor $\delta<1$. Let S_{ik} denote the pure strategy set of a firm i in the static (one-shot) strategic interaction in market k, let $s_{ik}\in S_{ik}$ denote one particular pure strategy and, abusing notation, let $\hat{s}_{ik}(s_{-ik})$ indicate firm i's static best response to its opponents' strategy profile $s_{-ik}\in S_{-ik}$, where $S_{-ik}=\prod_{j\neq i}S_{jk}$. Let $\pi_{ik}(.)$ denote firm i's profit function in market k, so that $\pi^*_{ik}=\pi_{ik}(s^*_{ik},s^*_{-ik})$ indicates firm i's profits from the strategy profile $s^*_k=(s^*_{ik},s^*_{-ik})$ and $\hat{\pi}^*_{ik}=\pi_{ik}(\hat{s}_{ik}(s^*_{-ik}),s^*_{-ik})$ its profits from the static best response strategy $\hat{s}_{ik}(s^*_{-ik})$ in market k. The standard approach is to assume that each firm i's static payoff function is simply $\sum_{k\in\Omega}\pi_{ik}(s_{ik},s_{-ik})$, so that firms' evaluation of payoffs from each market is independent of conditions in other markets. Instead, we let $U=U(\sum_{k\in\Omega}\pi_{ik}(s_{ik},s_{-ik}))$ denote firms' static objective function, and we assume U to be continuous, monotonically increasing and strictly concave in total profits. Now firms' evaluation of per-period profits from one market depends on profits realized in other markets. We assume, as Bernheim and Whinston implicitly do, that when a firm faces different opponents in the markets in which it is active, these opponents are not able or willing to coordinate their strategies. Also, we follow Bernheim and Whinston in focusing on stationary equilibrium paths sustained by trigger strategies (e.g. Friedman, 1971).[5] Let $\underline{\pi}_{ik}$ denote firm i's monetary payoff in one period of the punishment phase. Then one can state what follows.

Proposition 1 *Suppose firms' static objective function is strictly concave in profits. Then multimarket contact (always) relaxes the necessary and sufficient conditions for any set of profit streams to be supportable in subgame-perfect equilibrium by stationary punishment strategies in any set of infinitely repeated oligopoly games.*

The necessary and sufficient conditions for firms to be willing to stick to a collusive agreement are functions of the discount factor, of the collusive agreement chosen, and of market structure ($\hat{\pi}^*_{ik}$ and $\underline{\pi}_{ik}$). It follows that rearranging the same proof we could alternatively state:

i) "Suppose (...). Then multimarket contact reduces the minimum level of the discount factor at which any given set of profit streams can be supported in subgame-perfect equilibrium in any given set of oligopolistic supergames."

ii) "Suppose (...). Then, given the discount factor, the set of stationary profit streams supportable in subgame-perfect equilibrium with multimarket contact in any given set of repeated oligopolies is no smaller (in the sense of inclusion) than the set supportable without multimarket contact. Further, there exist $\underline{\delta}$ and $\overline{\delta}$, $0 < \underline{\delta} < \overline{\delta} < 1$, such that for $\underline{\delta} < \overline{\delta}$ multimarket contact strictly enlarges the set of supportable stationary profit streams."

iii) "Suppose (...). Then, given the discount factor, multimarket contact strictly enlarges (in the sense of inclusion) the set of oligopolistic supergames in which any (set of) collusive profit stream(s) is supportable in subgame-perfect equilibrium."

The intuition behind this result is straightforward. When a firm faces different opponents in the two markets and these opponents play their market games independently, the threat used to enforce the first firm's respect of a tacit collusive agreement in each of the markets is that of reverting to the static Nash equilibrium in that market only, taking for granted what is happening in other markets. Multimarket contact allows firms to use the threat of a simultaneous punishment in more markets to enforce collusive agreements. A simultaneous punishment is "heavier" because when a firm is already being punished in one market it has a higher marginal valuation of profits, so that it has a relatively greater fear of the loss of gains from cooperation caused by punishments in other markets. Furthermore, with multimarket contact, a firm which decides to "cheat" on a collusive agreement will find it convenient to deviate in all markets simultaneously. Because the marginal utility of profits is decreasing

[5] It is straightforward to check that all results apply to the case when the length of the punishment phase is bounded by finite renegotiation costs, as in McCutcheon (1997) (see also Blume, 1994). In section 5.4 we extend the results to "repentance" punishment strategies, such as those introduced by van Damme (1989) for the Prisoner's Dilemma, which are renegotiation-proof in the sense of Farrel and Maskin (1989).

within each period, the simultaneity of the deviation makes the short-run monetary gains from deviating in each market less valuable relative to the case when the firm is cheating in one market only. These two effects both facilitate collusion.

Note that the proposition was proved without reference to any specific market structure, so that the result applies to any type of repeated oligopoly (symmetric, asymmetric, Cournot, differentiated Bertrand, with or without capacity constraints, etc.).

5 Extensions

5.1 "Increasing returns" in collusion

Because of the wealth effects induced by concave objective functions, multimarket contact may allow firms to sustain collusive outcomes in all markets even when without multimarket contact collusion could not be sustained in any of them. To see this, consider the modified Bernheim and Whinston model of Section 3.2. In that model collusion in only one of the existing markets is sustainable if

$$\frac{1}{1-\delta}\ln\left(1+(p^m-c)\frac{Q(p^m)}{2}\right)-\ln\left(1+(p^m-c)Q(p^m)\right)\geq 0,$$

or, equivalently, if

$$\delta \geq \delta' = \frac{\ln\left(1+(p^m-c)Q(p^m)\right)-\ln\left(1+\frac{1}{2}(p^m-c)Q(p^m)\right)}{\ln\left(1+(p^m-c)Q(p^m)\right)}. \tag{3}$$

It easy to check that $\delta^{**} > \delta' > \delta^*$ for any p^m, c, and $Q()$. When $\delta^{**} > \delta \geq \delta'$, with multimarket contact collusion can be sustained in both markets, while in the absence of multimarket contact collusion can be sustained only in one of the markets. However, when $\delta' > \delta \geq \delta^*$, with multimarket contact the collusive price can still be sustained in both markets, while without multimarket contact it cannot be sustained in *either* of them.

This effect depends both on the shape of the objective function and on the structure of monetary payoffs in the different markets; therefore it is difficult to generalize. However, it is easy to check that for sufficiently similar games (markets) the scale effect is present with all most commonly used utility functions (quadratic, logarithmic, and other hyperbolic CRRA functions with elasticity of substitution lower than one).

5.2 Conglomeration, mergers, and collusion

As mentioned in the introduction, the mechanism behind Proposition 1 is open to a complementary interpretation. One can argue that it is the process of *conglomeration* that, by leading firms to operate in several segregated markets, insures them against too harsh punishments and reduces their ability to sustain collusive agreements. Multimarket contact then facilitates collusion by restoring such an ability at the pre-conglomeration level.[6]

This interpretation is valid as long as "conglomeration" denotes the process by which a firm becomes active in more than one market while maintaining the same level of overall activity. For example, consider a situation in which two firms of a given size are first active in one market (no conglomeration). Suppose that in a second period firms reduce their operations in the original market to increase them in other markets where they face different competitors (conglomeration without multimarket contact). The effect of this process is clearly the inverse of that behind Proposition 1, so collusion will be harder to support than before conglomeration.

When conglomeration implies an increase in the size of the firm, the effect on the firm's ability to collude is less clear. To see this, consider how *horizontal mergers* affect firms' ability to collude. Suppose a firm initially active in one market acquires another firm active in a different market, and that the acquired firm is colluding in its own market. The acquisition guarantees the acquiring firm an independent stream of profits which makes it less afraid of punishments, but also less interested in short-run gains from deviating in its original market. Such a wealth effect may enhance or worsen the acquiring firm's ability to sustain collusion in its original market, depending on the exact shape of its objective function. Let $\alpha = \frac{U(\pi^*_{iA})-U(\underline{\pi}_{iA})}{U(\hat{\pi}^*_{iA})-U(\underline{\pi}_{iA})}$. For the marginal merger one can state the following result.

Corollary 1 *Suppose firms' static objective function is strictly concave in profits. Then a horizontal merger with a marginally profitable firm – in the absence of multimarket contact – reduces (increases) the minimum discount factor at which firm* i *can sustain a collusive agreement in its original market A when*

$$\alpha U'(\hat{\pi}^*_{iA}) + (1-\alpha)U'(\underline{\pi}_{iA}) < (>) U'(\pi^*_{iA}). \tag{4}$$

Condition (5.2) is more easily satisfied when firms' marginal valuation of profits is particularly high at low levels of profits. Then the effect of the independent profit stream on firms' evaluation of losses from punishments in its market tends to dominate.

Because a merger affects firms' evaluation of profits for many periods, the discount factor plays an important role. In fact, for a given discount factor we can state a stronger result for the relation between mergers and collusion.[7]

Corollary 2 *Suppose firms' static objective function is strictly concave in profits. Then, in the absence of multimarket contact, any horizontal merger increases (diminishes) the ability of a firm* i *to sustain collusive agreements in its original market*

[6] I thank Douglas Bernheim who let me note this alternative interpretation.

[7] I am grateful to an anonymous referee whose comments persuaded me to formalize this implication.

A when the discount factor is lower (higher) than a well defined intermediate level $0 < \widetilde{\delta} < 1$.

The intuition is, of course, that at low enough discount factors the negative effect of a merger on the firm's evaluation of present short-run gains from deviations dominates the negative effect on future losses from the punishment phase.

5.3 Interdependent supergames

The mechanism behind Proposition 1 applies to any set of repeated strategic interactions other than oligopolistic games (Prisoner's Dilemmas, implicit contracts, reciprocal exchanges, etc.). When players face simultaneously several repeated games, payoffs from some of these may affect agents' evaluation of payoffs from others even though payoffs are of a different nature. Let $\mu_i = (\mu_{i1}, ..., \mu_{in})$ represent the vector of material payoffs from the n stage-games that an agent i plays simultaneously in each time period.

Definition 1 *An agent's static objective function U is strictly supermodular (submodular) in n stage-games' material payoffs if, for any two possible material payoff vectors $\mu_i^{'} = (\mu_{i1}^{'}, ..., \mu_{in}^{'})$ and $\mu_i^{''} = (\mu_{i1}^{''}, ..., \mu_{in}^{''})$ such that $\mu_i^{'}$ and $\mu_i^{''}$ are not comparable with respect to $\geq$,*

$$U(\mu_i^{'}) + U(\mu_i^{''}) < (>) U\left[\min(\mu_{i1}^{'}, \mu_{i1}^{''}), ..., \min(\mu_{in}^{'}, \mu_{in}^{''})\right] + \\ + U\left[\min(\mu_{i1}^{'}, \mu_{i1}^{''}), ..., \min(\mu_{in}^{'}, \mu_{in}^{''})\right].$$

Supermodularity (submodularity) for a function is a generalization of the concept of complementarity (substitutability) of its arguments. An alternative – but in this framework equivalent – generalization of complementarity (substitutability) is the concept of increasing (decreasing) differences:

Definition 2 *An agent i's static objective function U has strictly increasing (decreasing) differences in (μ_{ik}, μ_{ih}) if for all $(\mu_{ik}^{'}, \mu_{ik}^{''})$ and $(\mu_{ih}^{'}, \mu_{ih}^{''})$ such that $\mu_{ik}^{'} > \mu_{ik}^{''}$ and $\mu_{ih}^{'} > \mu_{ih}^{''}$,*

$$U(\mu_{ik}^{'}, \mu_{ih}^{'}, \mu_{i-k-h}) - U(\mu_{ik}^{''}, \mu_{ih}^{'}, \mu_{i-k-h}) > (<) U(\mu_{ik}^{'}, \mu_{ih}^{''}, \mu_{i-k-h}) - U(\mu_{ik}^{''}, \mu_{ih}^{''}, \mu_{i-k-h}).$$

Then one can obtain results analogous to those in previous sections. Here we prove only the result corresponding to Proposition 1.

Proposition 2 *Suppose agents' static objective function is strictly submodular (or has strictly decreasing differences) in stage games' material payoffs. Then "multi-game contact" relaxes the necessary conditions for any stationary cooperative outcome to be supportable in subgame-perfect equilibrium by stationary punishment strategies. (The converse does not hold for supermodular objective functions or functions with increasing differences.)*

Strict submodularity of (or strictly decreasing differences in) the objective function implies that the payoffs from different games are a kind of substitutes. When this is the case, agents who are doing well in one strategic interaction value material payoffs from other interactions less, and vice versa. This is enough to replicate the effects of concavity on agents' evaluation of gains from deviations and of losses from punishments behind Proposition 1.

The converse does not hold for supermodular functions because then short-run gains from simultaneous deviations are more valuable and simultaneous punishments less harsh. Agents can choose to deviate (and be punished) simultaneously in several strategic interactions whether or not there is "multi-game" contact; therefore "multi-game" contact cannot make agents' incentive constraints more stringent.

5.4 Renegotiation-proof strategies

Simple threats based on Nash reversion are widely used in the literature because they are subgame-perfect and easy to handle, both for researchers in models and for firms in markets (agreements and communication between oligopolistic firms are forbidden, so simple threats may greatly reduce coordination problems). However, these threats may be subject to ex-post renegotiation which may undermine their credibility (e.g. Joseph Farrell and Eric Maskin, 1989; Douglas Bernheim and Debray Ray, 1989).

Suppose simple renegotiation-proof strategies are used to support cooperation. Consider a standard repeated symmetric Cournot duopoly with profit functions

$$\pi_{ik}(q_{ik}, q_{jk}) = P(q_{ik} + q_{jk})q_{ik} - c(q_{ik}),$$

where q_{ik} denotes firm i's output, $P(.)$ is the inverse demand function and $c(.)$ is firms' cost function. We adopt the standard assumptions that the inverse demand function satisfies $P' < 0$, $P'' \geq 0$, that profits are concave in output, and that marginal profits are decreasing in rivals' output so that one-shot reaction functions are continuous and downward sloping. Assume firms support collusion by simple two-phase renegotiation-proof "repentance" strategies of the kind proposed by van Damme (1989):

> *Phase 1*: stick to the collusive output q^*_{ik} as long as the other firm did the same in the past; if the other firm deviates, start Phase 2;
>
> *Phase 2:* produce the full monopoly output q^M_{ik} as long as the other firm's output is positive (or larger than some low "repentance" level $\underline{q}_i$); if for one period the other firm's output is zero (or below $\underline{q}_i$), restart Phase 1.

Then one can to state the following.

Corollary 3 *Suppose firms' static objective function is strictly concave in profits and firms use the two-phase renegotiation-proof strategies defined above. Then multimarket contact always facilitates collusion by relaxing the necessary and sufficient conditions for any profit stream to be supportable in subgame-perfect equilibrium.*

6 Concluding Remarks

We have studied the effects of multimarket contact on firms' ability to sustain tacit collusive agreements in an infinitely repeated oligopoly framework. Managerial incentives, taxation, financial market imperfections, and other features of reality tend to make firms' static objective function strictly concave in profits. In this case multimarket contact always facilitates collusive behavior and may even generate "increasing returns" in collusion. The same result applies to infinitely repeated games with non-monetary material payoffs whenever agents' objective function is strictly submodular in the material payoffs vector.

Empirical studies should help to understand how relevant the above arguments are to the real world. Also, it should be relatively easy test these results through experimental work.

We conclude with two direct implications of our results. First, improvements in the incentive power of top managers' compensation and in the efficiency of financial markets should reduce the pro-collusive effects of multimarket contact, as they should reduce firms' aversion to intertemporal substitution in profits. Second, shareholders of conglomerates involved in multimarket contact might find it convenient to delegate control to managers strongly averse to intertemporal substitution (or to create incentives in such a direction, such as capped bonuses, rents, etc.) in order to facilitate collusion and increase profits.[8]

[8] In a repeated version of the strategic delegation game introduced by John Vickers (1985) and Chaim Fershtman and Kenneth Judd (1987), Giancarlo Spagnolo (1996b) finds that even in single repeated interactions, that is, independent of multimarket contact, delegation to managers with strictly concave objective functions is a powerful collusive device.

7 Appendix

7.1 Proof of Proposition 1

We prove the proposition for the case of two markets, A and B. The generalization to N markets is straightforward. This simple lemma will be useful:

Lemma 1 *Let U:R→R be a strictly concave function. Then for every* x, y *in* R_{++} *and* z *in* R_{+}, $U(z) + U(x + y + z) < U(x + z) + U(y + z)$.

Proof: Define $s = (x + y)$, $\mu_x = \frac{x}{s}$ and $\mu_y = \frac{y}{s}$, so that $0 < \mu_i < 1$, $i = x, y$ and $g = (x + y + z)$. By the definition of strict concavity $U[\mu_i g + (1 - \mu_i)z] > \mu_i U(g) + (1 - \mu_i)U(z)$. Solving inside the squared brackets and summing over i we obtain the expression above. **Q.E.D.**

Without multimarket contact, a firm i which is active in markets A and B will not deviate from a collusive agreement in market A which leads to a stationary sequence of monetary payoffs $\{\pi_{iA}^*\}_t^\infty$ if

$$\frac{1}{1-\delta}U(\pi_{iA}^* + \pi_{iB}) - U(\hat{\pi}_{iA}^* + \pi_{iB}) - \frac{\delta}{1-\delta}U(\underline{\pi}_{iA} + \pi_{iB}) \geq 0.$$

Analogously, firm 1 will not deviate from a collusive agreement in market B which leads to a stationary sequence of monetary payoffs $\{\pi_{iB}^*\}_t^\infty$ if

$$\frac{1}{1-\delta}U(\pi_{iA} + \pi_{iB}^*) - U(\pi_{iA} + \hat{\pi}_{iB}^*) - \frac{\delta}{1-\delta}U(\pi_{iA} + \underline{\pi}_{iB}) \geq 0.$$

Because firms are identical, these two conditions are necessary and sufficient for a collusive agreement to be supportable in each market. This implies that their sum, the pooled incentive constraint across the two markets, will be a *necessary* condition for a collusive agreement leading to a stationary sequence of monetary payoffs $\{\pi_{iA}^*, \pi_{iB}^*\}_t^\infty$ to be simultaneously supportable in both markets without multimarket contact. The following condition must be satisfied

$$\frac{2}{1-\delta}U(\pi_{iA}^* + \pi_{iB}^*) - U(\hat{\pi}_{iA}^* + \pi_{iB}^*) +$$

$$-U(\pi_{iA}^* + \hat{\pi}_{iB}^*) - \frac{\delta}{1-\delta}[U(\underline{\pi}_{iA} + \pi_{iB}^*) + U(\pi_{iA}^* + \underline{\pi}_{iB})] \geq 0. \quad \text{(A1)}$$

With multimarket contact, instead, a collusive outcome generating the stationary sequence of payoffs $\{\pi_{iA}^*, \pi_{iB}^*\}_t^\infty$ will be supportable if

$$\frac{1}{1-\delta}U(\pi_{iA}^* + \pi_{iB}^*) - U(\hat{\pi}_{iA}^* + \hat{\pi}_{iB}^*) - \frac{\delta}{1-\delta}U(\underline{\pi}_{iA} + \underline{\pi}_{iB}) \geq 0. \quad \text{(A2)}$$

If we can show that (A2) is always satisfied when (A1) is, and that for some games (A2) is satisfied but (A1) is not, we will have proved the proposition. To do this we can subtract the LHS of condition (A2) from the LHS of condition (A1) and check if such a difference is always positive. Subtracting and simplifying, we obtain

$$-\frac{1}{1-\delta}U(\pi_{iA}^*+\pi_{iB}^*)-U(\hat{\pi}_{iA}^*+\hat{\pi}_{iB}^*)+U(\hat{\pi}_{iA}^*+\pi_{iB}^*)+U(\pi_{iA}^*+\hat{\pi}_{iB}^*)+$$

$$+\frac{\delta}{1-\delta}[U(\underline{\pi}_{iA}+\pi_{iB}^*)+U(\pi_{iA}^*+\underline{\pi}_{iB})-U(\underline{\pi}_{iA}+\underline{\pi}_{iB})]>0,$$

and then

$$U(\hat{\pi}_{iA}^*+\hat{\pi}_{iB}^*)-U(\underline{\pi}_{iA}+\underline{\pi}_{iB})+U(\underline{\pi}_{iA}+\pi_{iB}^*)+U(\pi_{iA}^*+\underline{\pi}_{iB})-U(\hat{\pi}_{iA}^*+\pi_{iB}^*)-U(\pi_{iA}^*+\hat{\pi}_{iB}^*)>$$

$$>\frac{1}{\delta}\left[U(\pi_{iA}^*+\pi_{iB}^*)+U(\hat{\pi}_{iA}^*+\hat{\pi}_{iB}^*)-U(\hat{\pi}_{iA}^*+\pi_{iB}^*)-U(\pi_{iA}^*+\hat{\pi}_{iB}^*)\right].$$

By Lemma 1 the RHS of this inequality is negative, while the LHS may be either positive or negative. If it is positive the inequality is satisfied. If it is negative, we can multiply everything by -1 and rearrange as

$$\delta<\frac{-U(\pi_{iA}^*+\pi_{iB}^*)-K}{-U(\underline{\pi}_{iA}+\pi_{iB}^*)-U(\pi_{iA}^*+\underline{\pi}_{iB})+U(\underline{\pi}_{iA}+\underline{\pi}_{iB})-K},$$

where $K=\left[U(\hat{\pi}_{iA}^*+\hat{\pi}_{iB}^*)-U(\hat{\pi}_{iA}^*+\pi_{iB}^*)-U(\pi_{iA}^*+\hat{\pi}_{iB}^*)\right]$. By Lemma 1 it is $U(\pi_{iA}^*+\pi_{iB}^*)<U(\underline{\pi}_{iA}+\pi_{iB}^*)+U(\pi_{iA}^*+\underline{\pi}_{iB})-U(\underline{\pi}_{iA}+\underline{\pi}_{iB})$, therefore in the RHS of the last inequality the numerator is strictly larger than the denominator, and because $\delta\leq 1$ the condition is always satisfied. **Q.E.D.**

7.2 Proof of Corollary 1

Before acquisition the incentive constraint for firm i to sustain a collusive agreement leading to a stationary sequence of monetary payoffs $\{\pi_{iA}^*\}_t^\infty$ is

$$\frac{1}{1-\delta}U(\pi_{iA}^*)-U(\hat{\pi}_{iA}^*)-\frac{\delta}{1-\delta}U(\underline{\pi}_{iA})\geq 0,$$

or, equivalently,

$$\delta\geq\underline{\delta}(0)=\frac{U(\hat{\pi}_{iA}^*)-U(\pi_{iA}^*)}{U(\hat{\pi}_{iA}^*)-U(\underline{\pi}_{iA})}.$$

After the acquisition(s), in each period firm i will obtain some positive profits, say π_{iB}, from acquired firms so that the incentive constraint becomes

$$\frac{1}{1-\delta}U(\pi_{iA}^*+\pi_{iB})-U(\hat{\pi}_{iA}^*+\pi_{iB})-\frac{\delta}{1-\delta}U(\underline{\pi}_{iA}+\pi_{iB})\geq 0,$$

or, equivalently,

$$\delta \geq \underline{\delta}(\pi_{iB}) = \frac{U(\hat{\pi}^*_{iA} + \pi_{iB}) - U(\pi^*_{iA} + \pi_{iB})}{U(\hat{\pi}^*_{iA} + \pi_{iB}) - U(\underline{\pi}_{iA} + \pi_{iB})}.$$

The acquisition worsens firm's ability to collude when $\underline{\delta}(0) < \underline{\delta}(\pi_{iB})$. Therefore the marginal acquisition facilitates collusion when:

$$Sign \left\{ \left. \frac{\partial \underline{\delta}(\pi_{iB})}{\partial \pi} \right|_{\pi_{iB}=0} \right\} < 0,$$

which leads to

$$[U'(\hat{\pi}^*_{iA}) - U'(\pi^*_{iA})] \, [U(\hat{\pi}^*_{iA}) - U(\underline{\pi}_{iA})] - [U'(\hat{\pi}^*_{iA}) - U'(\underline{\pi}_{iA})] \, [U(\hat{\pi}^*_{iA}) - U(\pi^*_{iA})] < 0,$$

and then to

$$\frac{U(\pi^*_{iA}) - U(\underline{\pi}_{iA})}{U(\hat{\pi}^*_{iA}) - U(\underline{\pi}_{iA})} U'(\hat{\pi}^*_{iA}) + \frac{U(\hat{\pi}^*_{iA}) - U(\pi^*_{iA})}{U(\hat{\pi}^*_{iA}) - U(\underline{\pi}_{iA})} U'(\underline{\pi}_{iA}) < U'(\pi^*_{iA}).$$

Q.E.D.

7.3 Proof of Corollary 2

A merger with a firm active in market B facilitates collusion in market A as long as the firm's incentive compatibility constraint after the merger is less stringent than that before the merger, that is if

$$\frac{1}{1-\delta} U(\pi^*_{iA}) - U(\hat{\pi}^*_{iA}) - \frac{\delta}{1-\delta} U(\underline{\pi}_{iA}) <$$

$$\frac{1}{1-\delta} U(\pi^*_{iA} + \pi_{iB}) - U(\hat{\pi}^*_{iA} + \pi_{iB}) - \frac{\delta}{1-\delta} U(\underline{\pi}_{iA} + \pi_{iB}).$$

With few algebraic manipulations the inequality reduces to

$$\delta < \widetilde{\delta} = \frac{[U(\pi^*_{iA} + \pi_{iB}) - U(\pi^*_{iA})] - [U(\hat{\pi}^*_{iA} + \pi_{iB}) - U(\hat{\pi}^*_{iA})]}{[U(\underline{\pi}_{iA} + \pi_{iB}) - U(\underline{\pi}_{iA})] - [U(\hat{\pi}^*_{iA} + \pi_{iB}) - U(\hat{\pi}^*_{iA})]},$$

and because $U' > 0$ and $U'' < 0$ imply

$$U(\underline{\pi}_{iA} + \pi_{iB}) - U(\underline{\pi}_{iA}) > U(\pi^*_{iA} + \pi_{iB}) - U(\pi^*_{iA}) > U(\hat{\pi}^*_{iA} + \pi_{iB}) - U(\hat{\pi}^*_{iA}),$$

it is always $0 < \widetilde{\delta} < 1$. **Q.E.D.**

7.4 Proof of Proposition 2

Again, we prove the corollary for the case of agents active in two repeated games, A and B (the extension to more than two repeated games is straightforward, although cumbersome). Let μ^*_{ik} denote player i's static material payoff from supergame k when the stationary cooperative agreement at stake is being respected, $\widehat{\mu}^*_{ik}$ denote player i's static material payoff from deviating by choosing a static best response strategy to such agreement, and $\underline{\mu}_{ik}$ denote his material payoff in a period of the punishment phase which follows the deviation.

We can follow the same steps as in the proof of Proposition 1. Without multimarket contact, agent i playing supergames A and B will not deviate from a cooperative agreement in A leading to the sequence of material payoffs $\{\mu^*_{iA}\}^\infty_t$ if

$$\frac{1}{1-\delta}U(\mu^*_{iA},\mu_{iB}) - U(\widehat{\mu}^*_{iA},\mu_{iB}) - \frac{\delta}{1-\delta}U(\underline{\mu}_{iA},\mu_{iB}) \geq 0,$$

and will not deviate from a cooperative agreement in B leading to the sequence of material payoffs $\{\mu^*_{iB}\}^\infty_t$ if

$$\frac{1}{1-\delta}U(\mu_{iA},\mu^*_{iB}) - U(\mu_{iA},\widehat{\mu}^*_{iB}) - \frac{\delta}{1-\delta}U(\mu_{iA},\underline{\mu}_{iB}) \geq 0.$$

The pooled incentive constraint across the two supergames will be a *necessary* condition for the stationary sequence of material payoffs $\{\mu^*_{iA},\mu^*_{iB}\}^\infty_t$ to be simultaneously supportable without "multi-game" contact, while with "multi-game" contact the sequence $\{\mu^*_{iA},\mu^*_{iB}\}^\infty_t$ is supportable if

$$\frac{1}{1-\delta}U(\mu^*_{iA},\mu^*_{iB}) - U(\widehat{\mu}^*_{iA},\widehat{\mu}^*_{iB}) - \frac{\delta}{1-\delta}U(\underline{\mu}_{iA},\underline{\mu}_{iB}) \geq 0.$$

Subtracting the LHS of the pooled incentive constraint from this condition we obtain

$$U(\widehat{\mu}^*_{iA},\widehat{\mu}^*_{iB}) - U(\underline{\mu}_{iA},\underline{\mu}_{iB}) + U(\underline{\mu}_{iA},\mu^*_{iB}) + U(\mu^*_{iA},\underline{\mu}_{iB}) - U(\widehat{\mu}^*_{iA},\mu^*_{iB}) - U(\mu^*_{iA},\widehat{\mu}^*_{iB}) >$$

$$> \frac{1}{\delta}\left[U(\mu^*_{iA},\mu^*_{iB}) + U(\widehat{\mu}^*_{iA},\widehat{\mu}^*_{iB}) - U(\widehat{\mu}^*_{iA},\mu^*_{iB}) - U(\mu^*_{iA},\widehat{\mu}^*_{iB})\right].$$

By Definition 1 (or 2) the RHS of this inequality is always negative, while the LHS may either be positive or negative. If it is positive the inequality is satisfied. If it is negative, we can multiply everything by -1 and rearrange as

$$\delta < \frac{-U(\mu^*_{iA},\mu^*_{iB}) - K'}{-U(\underline{\mu}_{iA},\mu^*_{iB}) - U(\mu^*_{iA},\underline{\mu}_{iB}) + U(\underline{\mu}_{iA},\underline{\mu}_{iB}) - K'}$$

where $K' = \left[U(\widehat{\mu}^*_{iA},\widehat{\mu}^*_{iB}) - U(\widehat{\mu}^*_{iA},\mu^*_{iB}) - U(\mu^*_{iA},\widehat{\mu}^*_{iB})\right]$. By Definition 1 (or 2) $U(\mu^*_{iA},\mu^*_{iB}) < U(\underline{\mu}_{iA},\mu^*_{iB}) + U(\mu^*_{iA},\underline{\mu}_{iB}) - U(\underline{\mu}_{iA},\underline{\mu}_{iB})$, therefore the numerator is strictly larger than the denominator, and given that $\delta \leq 1$ the inequality is always satisfied. **Q.E.D.**

7.5 Proof of Corollary 3

As usual, define $\pi_{ik}^* = \pi_i(q_i^*, q_j^*)$, $\widehat{\pi}_{ik}^* = \pi_{ik}(\widehat{q}_{ik}(q_{jk}^*), q_{jk}^*)$, $\pi_{ik}^M = \pi_{ik}(q_{ik}^M, 0)$, $\widehat{\pi}_{ik}^M = \pi_{ik}(\widehat{q}_{ik}(q_{jk}^M), q_{jk}^M)$, and $\underline{\pi}_{ik} = \pi_{ik}(\underline{q}_{ik} = 0)$. Suppose

$$U(\underline{\pi}_{iA} + \underline{\pi}_{iB}) + \frac{\delta U(\pi_{iA}^* + \pi_{iB}^*)}{1-\delta} \geq \frac{U(\widehat{\pi}_{iA}^M + \widehat{\pi}_{iB}^M)}{1-\delta},$$

$$U(\underline{\pi}_{iA} + \pi_{iB}^*) + \frac{\delta U(\pi_{iA}^* + \pi_{iB}^*)}{1-\delta} \geq \frac{U(\widehat{\pi}_{iA}^M + \pi_{iB}^*)}{1-\delta},$$

$$U(\pi_{iA}^* + \underline{\pi}_{iB}) + \frac{\delta U(\pi_{iA}^* + \pi_{iB}^*)}{1-\delta} \geq \frac{U(\pi_{iB}^* + \widehat{\pi}_{iB}^M)}{1-\delta},$$

so that the strategies are subgame perfect. Then the condition for collusion to be supportable in both markets with no multimarket contact becomes

$$\frac{(1-\delta^2)2U(\pi_{iA}^* + \pi_{iB}^*)}{1-\delta} - U(\widehat{\pi}_{iA}^* + \pi_{iB}^*) - U(\pi_{iA}^* + \widehat{\pi}_{iB}^*) - \delta\left[U(\underline{\pi}_{iA} + \pi_{iB}^*) + U(\pi_{iA}^* + \underline{\pi}_{iB})\right] \geq 0.$$

With multimarket contact the corresponding condition is

$$\frac{(1-\delta^2)U(\pi_{iA}^* + \pi_{iB}^*)}{1-\delta} - U(\widehat{\pi}_{iA}^* + \widehat{\pi}_{iB}^*) - \delta U(\underline{\pi}_{iA} + \underline{\pi}_{iB}) \geq 0.$$

If when subtracting the LHS of the first inequality from that of the second we obtain a positive expression the statement will be proved. Subtracting, we obtain

$$(1-\delta)\left[U(\widehat{\pi}_{iA}^* + \pi_{iB}^*) + U(\pi_{iA}^* + \widehat{\pi}_{iB}^*) - U(\widehat{\pi}_{iA}^* + \widehat{\pi}_{iB}^*)\right] +$$

$$\delta(1-\delta)\left[U(\underline{\pi}_{iA} + \pi_{iB}^*) + U(\pi_{iA}^* + \underline{\pi}_{iB}) - U(\underline{\pi}_{iA} + \underline{\pi}_{iB})\right] - (1-\delta^2)U(\pi_{iA}^* + \pi_{iB}^*).$$

Let us define

$$A = U(\widehat{\pi}_{iA}^* + \pi_{iB}^*) + U(\pi_{iA}^* + \widehat{\pi}_{iB}^*) - U(\widehat{\pi}_{iA}^* + \widehat{\pi}_{iB}^*),$$

$$B = U(\underline{\pi}_{iA} + \pi_{iB}^*) + U(\pi_{iA}^* + \underline{\pi}_{iB}) - U(\underline{\pi}_{iA} + \underline{\pi}_{iB}),$$

$$C = U(\pi_{iA}^* + \pi_{iB}^*),$$

so that the difference can be rewritten as

$$(1-\delta)A + \delta(1-\delta)B - (1-\delta^2)C$$

or, equivalently, as

$$(A-C) + \delta(B-A) + \delta^2(C-B).$$

Using Lemma 1 we have (i) $C < A$ (let $z = \pi_{iA}^* + \pi_{iB}^*$, $x = \widehat{\pi}_{iA}^* - \pi_{iA}^*$, $y = \widehat{\pi}_{iB}^* - \pi_{iB}^*$,), and (ii) $C < B$ (let $z = \underline{\pi}_{iA} + \underline{\pi}_{iB}$, $x = \pi_{iA}^* - \underline{\pi}_{iA}$, $y = \pi_{iB}^* - \underline{\pi}_{iB}$). Suppose first $A = B$. Then the inequality becomes

$$(A-C) + \delta^2(C-A) = (1-\delta^2)(A-C) > 0$$

as desired. Suppose now $B > A$. We can rearrange and obtain

$$(A - C) + \delta(B - A) + \delta^2(C - A + A - B) = (1 - \delta^2)(A - C) + \delta(1 - \delta)(B - A) > 0$$

as desired. Finally, suppose $A > B$. We can rearrange obtaining

$$(A - C) + \delta(B - C + C - A) + \delta^2(C - B) = (1 - \delta)(A - C) + (\delta - \delta^2)(B - C) > 0.$$

Q.E.D.

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