Modeling Nordic Stock Returns with Asymmetric GARCH Models*

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Abstract

This paper investigates the presence of asymmetric GARCH effects in a number of equity return series, and compare the modeling performance of seven different conditional variance models, within the parametric GARCH class of models. The data consists of daily returns for 45 Nordic stocks, during the period July 1991 to July 1996. The models investigated are: EGARCH, GJR, TGARCH, A-PARCH, GQARCH, VS-ARCH, and LSTGARCH. In all these models the conditional variance is a function of the sign of lagged residuals. Thus, the models can capture the often reported negative correlation between lagged returns and conditional variance. In the paper I also introduce three new procedures for asymmetry testing. The proposed LM tests, which are based on the results of Wooldridge [1991], allow for heterokurtosis under the null. Asymmetries are detected for only 12 of the 45 series. The specifications GJR, TGARCH, and GQARCH appear to be superior for modeling the dynamics of the conditional variance. Furthermore, I show that the use of robust test statistics is advisable.

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1 Introduction

During the last 15 years, an enormous amount of effort has been devoted to the modeling of conditional volatility in financial markets data. The seminal work in this area of research is by Engle [1982], who introduces the standard autoregressive conditional heteroskedasticity model, thereby initiating the development of the ARCH class of models. Today the ARCH literature has grown to spectacular proportions. An excellent survey of the literature is Bollerslev, Engle, and Nelson [1994].

Many of the proposed ARCH models include a term that can capture correlation between returns and conditional variance. Models with this feature are often termed "asymmetric" or "leverage" volatility models. The term *leverage* stems from the empirical observation that the conditional variance of equity returns often increases when returns are negative, i.e. when the financial leverage of the firm increases. One of the earliest asymmetric ARCH models is the EGARCH (Exponential Generalized ARCH) model of Nelson [1991]. Another popular specification is the model of Glosten, Runkle, and Jagannathan [1993], denoted GJR.

This paper investigates to what extent asymmetric GARCH models might have been the data generating process for a number of time series. Data investigated is daily observations from 45 Nordic stocks. The investigated period is July 1991 to July 1996. Furthermore, this paper is concerned with the relative in sample performance of seven different parametric asymmetric ARCH models. The models studied are: EGARCH, GJR, TGARCH of Zakoïan [1994], A-PARCH of Ding, Granger, and Engle [1993], GQARCH of Sentana [1995], VS-ARCH of Fornari and Mele [1996], and LSTGARCH of Hagerud [1996], and González-Rivera [1996].

To estimate the unknown parameters of the models, iterative numerical methods are required. These procedures are often time consuming, and if the model in question explains the data badly the estimation might not converge. Specification tests are therefore used to investigate whether a certain model might have been the data generating process of a time series. The test of no ARCH, developed by Wooldridge [1990] is used to test for general ARCH effects. This test is a robust version of Engle's [1982] test of no ARCH. To test for asymmetric effects, robust versions of two tests proposed by Hagerud [1997] are used. Hagerud's test are derived under conditional normality, and since the data shows a high level of conditional heterokurtosis, robustification is necessary.

It is found that 32 of the 45 series show signs of heteroskedasticity. Of these 32 securities, twelve could have been generated by an asymmetric GARCH model. Furthermore, for the subsample of twelve series, it is concluded that the models GJR, TGARCH, and GQARCH are superior in modeling the asymmetric dynamics of the conditional variance.

This article is organized as follows. Section 2 describes the conditional variance models and the estimation procedures used. In Section 3, the tests used for the specification of the conditional mean and variance are formulated. The data is described in Section 4. Results are presented in Section 5, and the conclusions in Section 6.

2 Models and Estimation

Let the price of a stock at time t be denoted by P_t. Returns, measured as $ln(P_t/P_{t-1})$, are assumed to follow the AR(p)-process

$$r_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \varepsilon_t, \tag{1}$$

where ε_t denotes a discrete-time stochastic process with the form

$$\varepsilon_t = z_t h_t^{1/2}, \tag{2}$$

where $z_t \sim iid(0,1)$, and h_t is the conditional variance of return at time t, whose dynamics GARCH specifications wish to model.

The seven asymmetric GARCH models that will be investigated, are listed below. But first is a specification of the symmetric GARCH(1,1) model that is used as the null hypothesis when the presence of asymmetry is tested for. The models will only be studied in their most simple structure, when the lag lengths are equal to one. In many empirical investigations, these parsimonious models have proven to perform well. The description below is very brief. A more detailed presentation of the asymmetric models can be found in Hagerud [1997].

Bollerslev's [1986] GARCH(1,1) model assumes that the conditional variance is generated by

$$h_t = \gamma + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{3}$$

where γ, α and β are non-negative constants. For the GARCH process to be defined, it is required that $\alpha > 0$.

The EGARCH model of Nelson [1991] is

$$\ln h_t = \gamma + \beta \ln h_{t-1} + \lambda z_{t-1} + \varphi \left[|z_{t-1}| - \sqrt{2/\pi} \right], \tag{4}$$

where γ, β, λ, and φ are constant parameters, and z_t is defined as in (2).

The GJR model of Glosten, Jagannathan, and Runkle [1993] is

$$h_t = \gamma + \alpha\varepsilon_{t-1}^2 + \omega S_{t-1}^- \varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{5}$$

where γ, α, β, and ω are constant parameters, and S_{t-1}^- is an indicator function that takes the value one when $\varepsilon_{t-1} < 0$ and zero otherwise.

The Threshold GARCH model is introduced in Zakoïan [1994]. The TGARCH(1,1) model is

$$h_t^{1/2} = \gamma + \alpha^+ \varepsilon_{t-1}^+ - \alpha^- \varepsilon_{t-1}^- + \beta h_{t-1}^{1/2}, \tag{6}$$

where $\varepsilon_t^+ = \max(\varepsilon_t, 0)$, and $\varepsilon_t^- = \min(\varepsilon_t, 0)$. Note that (6) can be reparameterized as

$$h_t^{1/2} = \gamma + \alpha |\varepsilon_{t-1}| + \omega S_{t-1}^- \varepsilon_{t-1} + \beta h_{t-1}^{1/2}.$$

Thus, in the TGARCH(1,1) model, the conditional standard deviation has the same functional form as the conditional variance has in the GJR model (5).

Ding, Granger, and Engle [1993] propose the Asymmetric Power ARCH model. The A-PARCH(1,1) model is

$$h_t^{\delta/2} = \gamma + \alpha \left(|\varepsilon_{t-1}| - \eta\varepsilon_{t-1}\right)^{\delta} + \beta h_{t-1}^{\delta/2}, \tag{7}$$

where γ, α, β, η and $\delta \geq 0$ are constant parameters.

Sentana [1995] introduces the Quadratic ARCH model. The term *quadratic* is used since the QARCH model can be interpreted as a second-order Taylor approximation to the unknown conditional variance function. The Generalized QARCH(1,1) model is

$$h_t = \gamma + \zeta\varepsilon_{t-1} + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{8}$$

where γ, ζ, α, and β are constant parameters.

The VS-ARCH (Volatility Switching) model of Fornari and Mele [1996] is

$$h_t = \gamma + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} + \xi S_{t-1}\upsilon_{t-1}^2, \tag{9}$$

where

$$\begin{aligned} S_t &= 1 \ \text{ if } \varepsilon_t > 0 \\ S_t &= 0 \ \text{ if } \varepsilon_t = 0 \\ S_t &= -1 \ \text{ if } \varepsilon_t < 0, \end{aligned}$$

and υ_t^2 is defined as ε_t^2/h_t. The parameters of the model are γ, α, β, and ξ.

The logistic smooth transition GARCH$(1,1)$ model of Hagerud [1996], and González-Rivera [1996] is given by

$$h_t = \gamma + \left[\alpha_1 + \alpha_2 F\left(\varepsilon_{t-1}\right)\right]\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{10}$$

where $F(.)$ is a transition function with the form

$$F\left(\varepsilon_{t-1}\right) = \left(1 + \exp\left[-\theta\varepsilon_{t-1}\right]\right)^{-1} - \frac{1}{2}, \ \theta > 0.$$

The parameters of the model are therefore γ, α_1, α_2, β, and θ. Note that GJR will result as a limiting case of LSTGARCH, when $\theta \to \infty$.

To estimate the parameters of the models, a quasi-maximum likelihood approach is used. The innovations z_t are assumed to be distributed independently normal, and a normal log-likelihood function is maximized, using standard numerical methods. Bollerslev and Wooldridge [1992] show that when the normality is violated, the quasi-maximum likelihood estimators (QMLE) are generally consistent and have a limiting normal distribution. In their article, Bollerslev and Wooldridge also present asymptotic standard errors of the estimators valid under non-normality.

The parameters of the conditional mean model are estimated simultaneously with the conditional variance model. Engle [1982] and Bollerslev [1986] show that when the ARCH model is symmetric with respect to lagged returns, the two models can be estimated separately. This result is used when the standard GARCH(1,1) model is estimated, in the testing procedure. Unfortunately, this simplification cannot be used for the other models.

3 Specification Tests

Following the recommendations of Wooldridge [1991], a bottom-up strategy is used when performing specification tests. Thus, first the conditional first movement is specified. Once the conditional mean is formulated and estimated satisfactorily, tests for the conditional variance specification are performed.

When attempting to specify the conditional mean, only possible autocorrelation in the returns is tested for. Thus, any possible non-linearity in the conditional mean is disregarded. Furthermore, the possibility of the conditional variance to be an explanatory variable of return is not considered. To test for autocorrelation, a test developed by Richardson and Smith [1994] is used. The test, in the form use here, is a robust version of a standard Box and Pierce [1970] procedure. Letting $\widehat{\rho}_i$ be the estimated autocorrelation between the returns at time t and $t-i$, the test is formulated as

$$RS(k) = T \sum_{i=1}^{k} \frac{\widehat{\rho}_i^2}{1+c_i}. \tag{11}$$

The terms c_i is an adjustment factor for heteroskedasticity, and it is calculated as

$$c_i = \frac{cov[\overline{r}_t^2, \overline{r}_{t-i}^2]}{var[r_t]^2}, \tag{12}$$

where $\overline{r}_t$ is the demeaned return at time t. Under the null of no autocorrelation, the statistic is distributed asymptotically χ^2 with k degrees of freedom. Since the sample sizes are relatively large, any adjustments of the statistic, in the spirit of Ljung and Box [1978], are unnecessary.

If the null of no autocorrelation cannot be reject, it is concluded that returns are equal to a constant plus a residual ε_t, i.e. the conditional mean specification is model (1), with p equal to zero. If the null is rejected, an AR(1) model is estimated on the series. To ensure that this model captures the detected autocorrelation, test (11) was once more applied. In this case, the test is run on the estimated residuals from the AR(1) model. Following the recommendations of Box and Pierce [1970] and Ljung and Box [1978], the value of the statistic in this case is compared to a χ^2 distribution with $k-1$ degrees of freedom. If the null cannot be rejected, it is concluded that returns are generated by an AR(1) model. If the null is rejected, the procedure is continued with higher order AR models, until test (11) is not rejected.

After the conditional mean model is deemed satisfactory, tests for possible heteroskedasticity are performed. First, the null of homoskedasticity against the alternative of heteroskedasticity is tested. The most commonly used test for this hypothesis is the LM test of no ARCH of Engle [1982]. However, Engle's test requires that the fourth conditional moment of ε_t is constant and finite, as shown by Koenker [1981]. To overcome that complication, and to ensure that the test has a satisfactory asymptotic size, a robust test of no ARCH developed by Wooldridge [1990] is used. This test is calculated using the procedure:

1. Estimate the sample variance under the null of homoskedasticity

$$\widehat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \widehat{\varepsilon}_t^2,$$

   where $\widehat{\varepsilon}_t$ is the consistently estimated residuals from the model (1).

2. Regress 1 on

$$\left(\widehat{\varepsilon}_t^2-\widehat{\sigma}^2\right)\left(\widehat{\varepsilon}_{t-1}^2-\widehat{\sigma}^2\right),...,\left(\widehat{\varepsilon}_t^2-\widehat{\sigma}^2\right)\left(\widehat{\varepsilon}_{t-q}^2-\widehat{\sigma}^2\right) \tag{13}$$

The statistic is equal to $LM_0 = T \cdot R_u^2 = T - RSS$, where R_u^2 is the uncentered coefficient of determination from the regression (13), and RSS is the residual sum of squares. The statistic converges in distribution to a χ^2 variable with q degrees of freedom.

If the null of no ARCH(q) cannot be rejected, the investigation continues with tests for asymmetric GARCH. This is done with two new test procedures, based on a pair of LM statistics proposed by Hagerud [1997]. In both tests, the conditional variance follows a GARCH(1,1) process under the null. In test number one, the alternative hypothesis is the GQARCH model (8), and in test number two, the alternative hypothesis is the LSTGARCH model (10). Hagerud [1997] shows that these two tests have superior power properties, compared to the standard asymmetric ARCH tests developed by Engle and Ng [1993]. This superiority remains valid even when the true data generating process is not the GQARCH or the LSTGARCH models, but any other of the asymmetric models presented in Section 2.

Hagerud's [1997] tests are derived under the assumption that ε_t is distributed conditionally normal. This assumption is unlikely to be fulfilled in the data set considered here. Non-normality might give the statistics the wrong asymptotic size. The statistics are therefore robustified using a method presented by Wooldridge [1991]. For these statistics to be of the correct asymptotic size, only general distributional assumptions have to be made (see Wooldridge [1990]). Furthermore, the information matrix between the conditional mean and the conditional variance parameters does not have to be block-diagonal. Derivations of statistics LM_1 and LM_2, presented below, can be found in the appendix.

The hypothesis for the test of no GQARCH can be formulated

$$\begin{aligned} H_0 &: \zeta = 0, \\ H_1 &: \zeta \neq 0, \end{aligned}$$

and the proposed test procedure is:

1. Estimate a GARCH(1,1) model on the series of estimated residuals $\{\widehat{\varepsilon}_t\}_{t=1}^T$, and form the series of conditional variance under the null $\{h_{0t}\}_{t=1}^T$. Let $\widehat{\beta}$ be the estimated parameters β in the GARCH(1,1) model.

2. Regress

$$\frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}}{h_{0t}},$$

on

$$y_t = \left\{\frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{0t-i}}{h_{0t}}\right\},$$

and let $\widehat{a}$ be the vector of estimated parameters. Form the series of residuals

$$x_t = \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}}{h_{0t}} - \widehat{a}y_t'.$$

3. Regress 1 on
$$\left(\frac{\widehat{\varepsilon}_t^2}{h_{0t}} - 1\right) \cdot x_t \tag{14}$$
The statistic is equal to $LM_1 = T \cdot R_u^2 = T - RSS$, from the regression (14).

The statistic LM_1 converges in distribution to a χ^2 variable with one degree of freedom, corresponding to the number of restrictions.

The test of no LSTGARCH can be formulated[1]

$$\begin{aligned} H_0 &: \alpha_2 = 0 \\ H_1 &: \alpha_2 \neq 0, \end{aligned}$$

and the proposed test procedure is:

1. Estimate a GARCH(1,1) model on the series $\{\widehat{\varepsilon}_t\}_{t=1}^T$, and form $\{h_{0t}\}_{t=1}^T$.

2. Regress
$$\frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}^3}{h_{0t}},$$
on y_t, defined above, and let $\widehat{b}$ be the vector of estimated parameters. Form the series of residuals
$$s_t = \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}^3}{h_{0t}} - \widehat{b} y_t'.$$

3. Regress 1 on
$$\left(\frac{\widehat{\varepsilon}_t^2}{h_{0t}} - 1\right) \cdot s_t \tag{15}$$
The statistic is equal to $LM_2 = T \cdot R_u^2 = T - RSS$.

The statistic LM_2 converges in distribution to a χ^2 variable with one degree of freedom.

While performing test LM_1 or LM_2, and given that the series of conditional variance is estimated with quasi-maximum likelihood, the normalized residual, $\upsilon_t = \varepsilon_t / h_{0t}^{1/2}$, should be orthogonal to the vector

$$y_t = \left\{ \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{0t-i}}{h_{0t}} \right\}.$$

This should be true independently of whether the null is true or not. However, in practice, exact orthogonality cannot always be guaranteed. This is noted by Engle and Ng [1993], and from a Monte Carlo study, the authors concluded that it is advisable to adjust υ_t, such that orthogonality is guaranteed. However, since the small sample properties of LM_1 or LM_2 are unknown, no such adjustment will be performed.

If at least one of the tests LM_1 and LM_2 are rejected, the models (4) through (10) will be estimated on the series. To decide whether a certain model is able to capture the heteroskedasticity adequately,

[1] To test for $\alpha_2 = 0$ in the LSTGARCH(1,1) model will cause an identification problem. Hagerud [1997] shows how this problem can be solved by making a Taylor approximation of the model.

a number specification tests are once more performed. First, the skewness and kurtosis of the series of normalized residuals, $\widehat{\varepsilon}_t/h_t^{1/2}$, are calculated, where h_t is the estimated conditional variance. Second, a new test procedure to test for further asymmetric ARCH effects is used. This test procedure is a robust version of the *Sign bias test* of Engle and Ng's [1993]. The sign bias test is a test for general asymmetry. The robustification is, as above, done using the method of Wooldridge [1991]. The null is now the asymmetric model that was used to estimate h_t, and the alternative is the same asymmetric model augmented by a term $\tau \cdot S_{t-1}^{-}$. Thus, the alternative model considered is

$$g_t = h_t + \tau \cdot S_{t-1}^{-},$$

where h_t is any of the asymmetric models (4) to (10). The test is formulated as follows

$$\begin{aligned} H_0 &: \quad \tau = 0, \\ H_1 &: \quad \tau \neq 0, \end{aligned}$$

and the robust test procedure is:

1. Estimate the asymmetric GARCH model with quasi-maximum likelihood, and form $\{h_t\}_{t=1}^{T}$.

2. Regress
$$\frac{1}{h_t}\frac{\partial g_t}{\partial \tau},$$
on
$$w_t = \frac{1}{h_t}\frac{\partial h_t}{\partial \boldsymbol{\beta}'}$$
where $\boldsymbol{\beta}$ is the vector of parameters in the asymmetric model. Let $\widehat{c}$ be the vector of estimated parameters. Form the series of residuals
$$r_t = \frac{1}{h_t}\frac{\partial g_t}{\partial \tau} - \widehat{c}w_t'.$$

3. Regress 1 on
$$\left(\frac{\widehat{\varepsilon}_t^2}{h_t} - 1\right) \cdot r_t. \tag{16}$$
The statistic is equal to $LM_3 = T \cdot R_u^2 = T - RSS$.

The statistic LM_3 converges in distribution to a χ^2 variable with one degree of freedom. Note that to perform the test, it is necessary to calculate the partial derivatives $\partial g_t/\partial \tau$, and $\partial h_t/\partial \boldsymbol{\beta}$. These will differ depending on the model considered under the null. These derivatives for all the seven models are listed in the appendix.

4 Data

Original stock price observations are daily close prices from the exchanges in Helsinki, Stockholm, Oslo, and Copenhagen. The stocks are the most actively traded securities in each of the markets. The period investigated is July 1, 1991 to July 1, 1996. This gives approximately 1,260 price observations per security. The price observations are not adjusted for dividends, but since dividends are paid on a yearly basis, only five return observations per series are affected by dividends. The data is collected from Datasteam.

The investigated securities are listed in Table 1, and some summary statistics are given. 17 stocks are from Stockholm, 14 from Copenhagen, 8 from Oslo, and 6 from Helsinki. The prices of most of the stock have increased over the investigated period of five years. The highest return is observed for the Finnish telecommunication company Nokia, with 63 percent average yearly yield. The worst investment during the period is the Norwegian shipping company I.m. Skaugen. The average yearly yield for the 45 companies is 8.9 percent.

From the last column in Table 1, it can be seen that the volatility, measured as estimated constant standard deviation p.a., varies considerably in the sample. The lowest volatility is reported for the Danish pharmaceutical company NovoNordisk, with 15 percent, and the highest is reported for I.m. Skaugen, with 70 percent. However, a majority of the stocks have a volatility in the interval 20 to 30 percent.

5 Results

Table 2 reports the results from the tests performed to specify the conditional mean. Richardson and Smith's [1994] test (11), calculated on eight autocorrelations, indicate that twelve companies show signs of autocorrelation, on five percent significance level. For these companies, AR(1) models were estimated. Column three in the table reports p-values for Richardson and Smith's [1994] test calculated on the estimated residuals from these AR(1) models. No autocorrelation can be detected in these series of residuals. It is therefore concluded that for 33 securities, a suitable mean equation is (1) with $p = 0$, and for the remaining twelve securities an AR(1) model is appropriate.

Column two in Table 3 reports the results from Wooldridge's [1990] tests of no ARCH. The test is calculated with q equal to eight.[2] Thirteen companies show no signs of heteroskedasticity, on five percent significance level. These companies will not be considered further. Column three contain p-values for Engle's [1982] test of no ARCH, calculated on eight lagged squared residuals. This is the non-robust alternative to Wooldridge's test. Note that if the non-robust test had been used, quite different conclusions about the presence of heteroskedasticity in the sample would have been drawn.

Column four in Table 3 shows excess kurtosis for the series of estimated residuals, and column five reports the first autocorrelation in squared residuals. Teräsvirta [1996] shows that a GARCH(1,1) model with normal errors cannot generate data with high excess kurtosis and low first-order autocorrelation in

[2] Wooldridge's test of no ARCH was also calculated on ten and fifteen lagged residuals. The results from those tests were only slightly different from the ones shown in Table 3. Thus, the general level of detected heteroskedasticity was not altered when the number of lags in the statistic was increased.

squared residuals. Many of the series in this sample show such a pattern. However, since the primary interest is finding an estimate of the series of conditional variance, $\{h_t\}_{t=1}^{T}$, this should not affect the results. This follows from the fact that the series of conditional variance only depends on the consistently estimated parameters α, β, and γ, and not on the distributional assumption made. This is obviously only true if the assumptions of Bollerslev and Wooldridge [1992] are fulfilled, so that the QMLEs are consistent.

Table 4 reports the results from the two tests of no asymmetric GARCH. According to column two, the null of GARCH(1,1) can be rejected against the alternative of GQARCH(1,1), for eleven securities on the five percent significance level. The null of GARCH(1,1) against LSTGARCH(1,1), can be rejected for three securities. In total, asymmetries have been detected for twelve securities, out of the 32 securities that showed signs of heteroskedasticity. For this subsample of twelve securities, the investigation continues by fitting the seven asymmetric models (4) to (10) to the return observations.

In Table 4, are also the p-values for the corresponding non-robust test procedures of Hagerud [1997]. The results for the test of GARCH(1,1) against GQARCH(1,1), are given in column three, and column five reports the results for the test of GARCH(1,1) against LSTGARCH(1,1). In number cases different conclusions would have been drawn if the non-robust tests had been used. Thus, it is advisable to use the proposed robust test statistics.

As pointed out in Section 2, a quasi-maximum likelihood approach is used to estimate the seven models on the twelve return series. For most securities, all seven models could be estimated without difficulty. Out of a total of 84 estimations, convergence was not reached in eight cases. These cases are indicated as failures in Table 7. It is primarily the A-PARCH model that is hard to estimate. Failures often occur because the series of conditional variance is given a negative value, or because stationarity conditions on the estimated parameters could not be met. The two samples that proved to be most troublesome to estimate are the price series for SCA, a Swedish forestry company, and the series for Unibank, a Danish bank.

In all models but one, the asymmetries were estimated to be in agreement with the folkloristic view that the conditional volatility increases in bear markets. Thus, the parameters of the models were estimated to such values that if a certain model had been the true data generating process, a negative correlation between lagged residuals and conditional variance would result. The exception is the VS-ARCH model estimated for I.m. Skaugen.

One of the objectives of this study was to investigate which of the asymmetric ARCH specifications (4) to (10) models the conditional variance "best." Two simplistic selection criteria for finding the best model are used: the value of the likelihood function, and the BIC information criteria of Schwarz [1978]. In addition, it is required that the best model, should not perform worse than the other models, with regard to the specification tests presented in Section 3.

Table 5 reports, for each security, the top three ranked models maximizing the likelihood function. Table 6 gives the analogous results for BIC. Not surprisingly, the models with most parameters, A-PARCH and LSTGARCH, often maximize the likelihood function. However, in some cases, the more parsimonious models GJR, TGARCH, and GQARCH perform better. When the number of parameters is given consideration, as in the BIC, the three models GJR, TGARCH, and GQARCH, seems to be superior. Both GJR and TGARCH minimize BIC in five cases, and GQARCH is ranked first in two cases.

P-values for the robust sign bias test, LM_3, performed on the 76 models that were successfully estimated, are reported in Table 7. Only in two cases, GQARCH for SCA B, and EGARCH for Handelsbanken A, can the null of no additional asymmetry be rejected, on five percent significance level. In all other cases, it is hard to find any evidence which favors one model over the other. It is therefore concluded that the models seem to capture the same dynamics of heteroskedasticity, but do so more or less well. However, this conclusion should be taken with a pinch of salt, since there might be asymmetries in the data that cannot be detected with the rather general hypothesis of the sign bias test.

The investigation now focuses on the series of estimated normalized residuals, $\left\{\widehat{\varepsilon}_t / h_t^{1/2}\right\}_{t=1}^{T}$. Table 8 reports estimated standardized third moment, *skewness*, of the series. An assumption that the distribution of z_t in (2) is symmetric appears to be appropriate for seven of the series, on five percent significance level. For EAC, Modo B, Orkla A, I.m. Skaugen, and Volvo B, the estimated skewness indicates that the distribution is skewed. Alternatively, this might be an indication that a different specification of the conditional variance should be used. Analogously to the results of the sign bias test, no model is better than the rest in reaching a zero skewness.

The estimated excess kurtosises of $\left\{\widehat{\varepsilon}_t / h_t^{1/2}\right\}_{t=1}^{T}$, for the 76 models, are reported in Table 9. The hypothesis that the coefficient of excess kurtosis is equal to zero is rejected for all models. Thus, it can be concluded that an assumption that z_t is normally distributed is inappropriate. The use of a distribution that can generate large innovations more often is recommended. A possible candidate is the Student-t distribution. It is interesting to note once again that all models seem to capture a similar structure for the conditional variance series. The excess kurtosis for a particular security is almost the same across the seven models.

Based on the selection criteria, and on the results of the specification tests presented in Table 7 to 9, the GJR specification appears to have many advantageous properties. Only for EAC and I.m. Skaugen does the model not rank among the three best based on BIC. The model is also relatively easy to estimate. Convergence was reached for all twelve securities. For the Orkla A, SCA B, Skandia, and Unidanmark A, the parameter α in the GJR model was estimated to a negative value. A very large positive residual might then give a negative conditional variance. However, in the four series, this does not occur, indicating that it might only be a theoretical problem. In none of the 76 models that were successfully estimated did any negative variance occur.

The LSTGARCH model is, as noted in Section 2, a generalization of the GJR model. In six cases, the estimate of the parameter θ is so high that the two models coincide. As was the case with GJR, the estimated parameters for Orkla A, SCA B, and Skandia might in extreme cases give rise to negative conditional variance. The LSTGARCH is somewhat harder to estimate, and for Unidanmark A no convergence is reached. Based on these results, it is concluded that the GJR specification is a good approximation of the LSTGARCH model.

The TGARCH model appears to be a good complement to the GJR model. For six securities, the model ranks among the best three, both according to the values on the likelihood function and BIC. The model's simple structure makes estimation easy. However, for SCA B no convergence is reached. For Orkla A, and Unidanmark A, the parameter α^{+} is estimated to a negative value. This allows negative conditional variances to appear for large positive return realizations, but this does not occur for the two series.

The results for the EGARCH model are slightly disappointing. In five cases, both according to the value of the likelihood function and BIC, the model ranks among the top three models, but is always outperformed by TGARCH. The model is fairly hard to estimate. For SCA A, and Unidanmark A, no convergence was reached. The persistence parameter β is often estimated to a value just below unity. Only in two cases is β estimated to values below 0.99.

Even though the GQARCH(1,1) model is extremely simple in its structure, it performs well. For ABB A and Volvo B, the model maximizes the likelihood function. It is among the top three models, according to BIC, for four securities. The model is easily estimated for all twelve securities.

The A-PARCH model maximizes the likelihood function for five securities, and ranks among the top three models in four cases, according to BIC. The model's very general structure therefore seems to capture the dynamics of the conditional variance well. However, one of the model's disadvantages is that it is hard to estimate. For successful estimation, the choice of start values is important. For four securities convergence could not be reached.

In three cases the VS-ARCH model ranks among the top three models, both according to the value of the likelihood function and BIC. The model is easily estimated and convergence is reached for all securities. Unfortunately, the rather different asymmetry structure of the model does not appear to be successful in the sample investigated.

6 Summary and Conclusion

This paper has presented results from an empirical investigation of 45 equity return series, from the Nordic stock exchanges in Helsinki, Stockholm, Oslo, and Copenhagen. The study investigated whether some asymmetric GARCH model might have been the data generating process of the series. For this, two novel test procedures that are robust to non-normality were used. Evidence of asymmetry was found for twelve securities. It was also shown that the results of the proposed tests differ from those of non-robust tests. Thus, for this sample, using robust tests seems to be advisable.

For the twelve series that indicated asymmetries, the seven GARCH models, EGARCH, GJR, TGARCH, A-PARCH, GQARCH, VS-ARCH, and LSTGARCH, were estimated . These models all allow for correlation between the conditional variance and lagged returns. In 76 cases of 84 the models were successfully estimated. Using the value on the likelihood function, and the information criteria of Schwarz [1978], BIC, an attempt was made to identify which specifications modeled the conditional variance best. The models GJR, TGARCH, and GQARCH were found to be superior.

To investigate whether the seven models were able to capture the asymmetry present in the data, a robust version of the sign bias test was performed . Only in two cases of 76 could the hypothesis of no additional asymmetry be rejected. Finally, the skewness and kurtosis of the series of normalized residuals were calculated. These coefficients were very stable across the models. For seven securities the hypothesis of zero skewness could not be rejected. The hypothesis of no excess kurtosis was rejected for all 76 models. It is therefore concluded that an assumption that the distribution of z_t is the standard normal is most likely incorrect.

This study has focused on in sample properties for a number of different parametric GARCH models. This naturally give rise ti the question of how these results can be used in a practical modeling situation. This will depend on the purpose of the exercise. GARCH models are commonly used by professionals in the option markets to forecast volatility for securities. If it is believed that the in sample properties of a model reflect the forecasting ability of the model, the results presented here are of major importance. For the practitioner, it should be comforting to note that the modeling performance of the relatively simple models, GJR, TGARCH, and GQARCH are at least as good as that of the more complicated models. These models are easier to estimate, and much easier to use for forecasting. However, the reader must be warned not to equate in sample properties with forecasting ability. This subject calls for further research.

In many other situations where GARCH models are used, the mean specification is more complex than the one used here. For econometricians working with such models, it is hoped that the methodological part of this paper is of interest. The use of the proposed tests for asymmetry is not limited to the simple model structure considered here. However, in this respect, further research into the small sample properties of the statistics is needed.

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Appendix

1. Test of H_0: GARCH(1,1), H_1: GQARCH(1,1)

The test considered is

$$\begin{aligned} H_0 &: \zeta = 0, \\ H_1 &: \zeta \neq 0. \end{aligned}$$

Hagerud [1997] shows that given that the residuals are distributed conditionally normal, a Lagrange multiplier test statistics for the hypothesis is

$$\frac{1}{2}\left\{\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\widehat{\varepsilon}_t^2}{h_{0t}}-1\right]\frac{\partial h_t}{\partial \boldsymbol{\beta}}\right\}'\left\{\sum_{t=1}^{T}\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial \boldsymbol{\beta}}\right]\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial \boldsymbol{\beta}}\right]'\right\}^{-1}$$
$$\left\{\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\widehat{\varepsilon}_t^2}{h_{0t}}-1\right]\frac{\partial h_t}{\partial \boldsymbol{\beta}}\right\}, \tag{17}$$

where

$$\frac{\partial h_t}{\partial \boldsymbol{\beta}'} = \left[\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}^2, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{0t-i}, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}\right],$$

h_{0t} is the conditional variance under the null of GARCH(1,1), $\widehat{\varepsilon}_t$ is the consistently estimated residual, $\boldsymbol{\beta}'$ is the vector of conditional variance parameters $(\gamma, \alpha, \beta, \zeta)$, and $\widehat{\beta}$ is the estimated parameter β in the GARCH(1,1) model. Based on test (17) it is straightforward to derive the asymptotically equivalent test TR_u^2 from the regression

$$\left(\frac{\widehat{\varepsilon}_t^2}{h_{0t}}-1\right)$$

on

$$\left\{\frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{0t-i}}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}}{h_{0t}}\right\}. \tag{18}$$

If conditional normality fails to hold, (18) will have the wrong asymptotic size. In *Procedure 4.1* in Wooldridge[1991], the author shows how a regression-based specification test of this type can be robustified. The resulting test statistic will be valid under the rather general regularity conditions of Wooldridge [1990]. Using the method proposed by Wooldridge [1991], the robust version of test (18) is:

1. Regress

   $$\frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}}{h_{0t}},$$

   on

   $$y_t = \left\{\frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{0t-i}}{h_{0t}}\right\}.$$

   Let $\widehat{a}$ be the vector of estimated parameters. Form the series of residuals

   $$x_t = \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}}{h_{0t}} - \widehat{a}y_t'.$$

2. Regress 1 on

$$\left(\frac{\widehat{\varepsilon}_t^2}{h_{0t}}-1\right)\cdot x_t$$

The statistic is equal to $T\cdot R_u^2=T-RSS$, from this last regression, which is equivalent to the statistic LM_1 presented in Section 3. The statistic converges in distribution to a χ^2 variable with one degree of freedom.

2. Test of $\mathbf{H}_0$: GARCH(1,1), $\mathbf{H}_1$: LSTGARCH(1,1)

The test considered is

$$\begin{aligned} H_0 &: \alpha_2=0, \\ H_1 &: \alpha_2\neq 0. \end{aligned}$$

Hagerud [1997] shows that given that the residuals are distributed conditionally normal, a Lagrange multiplier test statistics for the hypothesis is

$$\frac{1}{2}\left\{\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\widehat{\varepsilon}_t^2}{h_{0t}}-1\right]\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right\}'\left\{\sum_{t=1}^{T}\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]'\right\}^{-1}$$
$$\left\{\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\widehat{\varepsilon}_t^2}{h_{0t}}-1\right]\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right\},$$

where

$$\frac{\partial h_t}{\partial\boldsymbol{\beta}'}=\left[\sum_{i=1}^{t-1}\widehat{\beta}^{i-1},\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}^2,\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{0t-i},\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}^3\right].$$

To derive the robust test LM_2 presented in Section 3, the same method employed in the GQARCH case above was used.

3. $\partial g_t/\partial\tau$, and $\partial h_t/\partial\boldsymbol{\beta}$ for the models (4) to (10)

When the model under the null is the EGARCH model (4), and $\boldsymbol{\beta}'$ is $(\gamma,\beta,\lambda,\varphi)$, the partial derivatives are

$$\begin{aligned} \frac{\partial g_t}{\partial\tau} &= S_{t-1}^-+h_t\sum_{i=2}^{t-1}\widehat{\beta}^{i-1}h_{t-i}^{-1}S_{t-i}^-, \\ \frac{\partial h_t}{\partial\boldsymbol{\beta}'} &= \Bigg[h_t\sum_{i=1}^{t-1}\widehat{\beta}^{i-1},h_t\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\ln h_{t-i},h_t\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}z_{t-i}, \\ &\qquad h_t\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\left(|z_{t-i}|-\sqrt{2/\pi}\right)\Bigg]. \end{aligned}$$

When the model under the null is the GJR model (5), and $\boldsymbol{\beta}'$ is $(\gamma,\alpha,\omega,\beta)$, the partial derivatives are

$$\begin{aligned} \frac{\partial g_t}{\partial\tau} &= \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}S_{t-i}^-, \\ \frac{\partial h_t}{\partial\boldsymbol{\beta}'} &= \left[\sum_{i=1}^{t-1}\widehat{\beta}^{i-1},\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\widehat{\varepsilon}_{t-i}^2,\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}S_{t-i}^-\widehat{\varepsilon}_{t-i}^2,\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{t-i}\right]. \end{aligned}$$

When the model under the null is the TGARCH model (6), and $\boldsymbol{\beta}'$ is $(\gamma, \alpha^+, \alpha^+, \beta)$, the partial derivatives are

$$\begin{aligned}\frac{\partial g_t}{\partial \tau} &= S_{t-1}^- + h_t^{1/2} \sum_{i=2}^{t-1} \widehat{\beta}^{i-1} h_{t+1-i}^{-1/2} S_{t-i}^-, \\ \frac{\partial h_t}{\partial \boldsymbol{\beta}'} &= \Bigg[2h_t^{1/2} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1}, 2h_t^{1/2} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}^+, 2h_t^{1/2} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}^-, \\ &\quad 2h_t^{1/2} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i}^{1/2} \Bigg].\end{aligned}$$

When the model under the null is the A-PARCH model (7), and $\boldsymbol{\beta}'$ is $(\gamma, \alpha, \eta, \beta, \delta)$, the partial derivatives are

$$\frac{\partial g_t}{\partial \tau} = S_{t-1}^- + h_t^{1-\delta/2} \sum_{i=2}^{t-1} \widehat{\beta}^{i-1} h_{t+1-i}^{\delta/2-1} S_{t-i}^-,$$

$$\frac{\partial h_t}{\partial \boldsymbol{\beta}'} =$$

$$\Bigg[\frac{2h_t^{1-\delta/2}}{\delta} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1}, \frac{2h_t^{1-\delta/2}}{\delta} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} (|\widehat{\varepsilon}_{t-i}| - \eta \widehat{\varepsilon}_{t-i})^{\delta},$$

$$2\alpha h_t^{1-\delta/2} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \left\{ (|\widehat{\varepsilon}_{t-i}| - \eta \widehat{\varepsilon}_{t-i})^{\delta} \widehat{\varepsilon}_{t-i} \right\}, \frac{2h_t^{1-\delta/2}}{\delta} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i}^{\delta/2},$$

$$\frac{2h_t^{1-\delta/2}}{\delta} \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \Big(-h_{t+1-i}^{\delta/2} \ln h_{t+1-i}^{\delta/2} + 2\alpha (|\widehat{\varepsilon}_{t-i}| - \eta \widehat{\varepsilon}_{t-i})^{\delta}$$

$$\ln(|\widehat{\varepsilon}_{t-i}| - \eta \widehat{\varepsilon}_{t-i}) + \beta h_{t-i}^{\delta/2} \ln h_{t-i}^{\delta/2} \Big) \Bigg]$$

When the model under the null is the GQARCH model (8), and $\boldsymbol{\beta}'$ is $(\gamma, \zeta, \alpha, \beta)$, the partial derivatives are

$$\begin{aligned}\frac{\partial g_t}{\partial \tau} &= \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} S_{t-i}^-, \\ \frac{\partial h_t}{\partial \boldsymbol{\beta}'} &= \left[\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}^2, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i} \right].\end{aligned}$$

When the model under the null is the VS-ARCH model (9), and $\boldsymbol{\beta}'$ is $(\gamma, \alpha, \beta, \xi)$, the partial derivatives are

$$\begin{aligned}\frac{\partial g_t}{\partial \tau} &= \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} S_{t-i}^-, \\ \frac{\partial h_t}{\partial \boldsymbol{\beta}'} &= \left[\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}^2, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i} \varepsilon_{t-i}, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} S_{t-i} v_{t-i}^2 \right].\end{aligned}$$

When the model under the null is the LSTGARCH model (10), and $\boldsymbol{\beta}'$ is $(\gamma, \alpha_{1,}, \alpha_2, \beta, \theta)$, the partial derivatives are

$$
\begin{aligned}
\frac{\partial g_t}{\partial \tau} &= \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} S_{t-i}^{-}, \\
\frac{\partial h_t}{\partial \boldsymbol{\beta}'} &= \Bigg[\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{\varepsilon}_{t-i}^2, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \widehat{F}(\widehat{\varepsilon}_{t-i}|\widehat{\theta}) \widehat{\varepsilon}_{t-i}^2, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i}, \\
&\quad \alpha_2 \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \frac{e^{-\widehat{\theta \varepsilon}_{t-i}}}{\left(1 + e^{-\widehat{\theta \varepsilon}_{t-i}}\right)^2} \widehat{\varepsilon}_{t-i}^3 \Bigg],
\end{aligned}
$$

where $\widehat{F}(\widehat{\varepsilon}_t|\widehat{\theta})$ is the estimated value of the transition function at time t, under the null. When θ is estimated to a high value, such that the LSTGARCH model coincides with GJR model, the columns of the matrix $\partial \mathbf{h}/\partial \boldsymbol{\beta}' = \partial(h_1, ..., h_T)'/\partial \boldsymbol{\beta}'$ might not be independent. To overcome this complication the partial derivatives of the GJR model should be used instead of those of the LSTGARCH model. This is the practice followed in this paper.

Table 1. Summary data for the investigated series

This table lists the 45 Nordic stocks investigated. The column labeled "$\overline{\mu}$ in %", reports yearly yield for the security during the investigated period of five years. The column labeled "σ in %", reports the estimated constant standard deviation in return, on a yearly basis. The period investigated is July 1, 1991 to July 1, 1996.

Security	Exchange	Industry	$\overline{\mu}$ in %	σ in %
ABB A	SE	Engineering	12.9	19.8
Aker B	NO	Engineering	5.0	50.5
Astra A	SE	Pharmaceuticals	24.8	24.5
Atlas Copco A	SE	Engineering	19.2	25.6
Bergesen d.y. B	NO	Shipping	−3.8	30.5
Carlsberg B	DK	Food and beverage	1.3	20.2
Cultor 2	FI	Food and beverage	40.2	44.1
Danisco	DK	Food and beverage	8.7	19.5
Den Danske Bank	DK	Financial	2.5	21.3
D/S 1912 B	DK	Shipping	5.2	21.1
D/S Svendborg B	DK	Shipping	5.6	20.2
EAC	DK	Shipping	−7.8	35.4
Electrolux B	SE	Household durables	4.2	28.8
Enso R	FI	Forestry	21.0	33.3
FLS Industries B	DK	Engineering	−2.0	25.1
Norsk Hydro	NO	Oil and chemical	11.3	25.2
Investor B	SE	Investment company	10.3	33.6
ISS B	DK	Service	0.3	29.3
Jyske Bank	DK	Financial	0.7	18.3
Kvaerner B	NO	Engineering	3.4	29.9
LM Ericsson B	SE	Telecommunication	23.9	36.4
Modo B	SE	Forestry	5.6	43.9
Nokia	FI	Telecommunication	63.2	40.0
Novo Nordisk B	DK	Pharmaceuticals	12.0	15.0
Norske Skog B	NO	Forestry	3.6	43.4
Orkla A	NO	Food and beverage	14.0	29.7
Outokompu A	FI	Metal and mining	31.1	36.7
Pohjola B	FI	Insurance	8.6	47.9
Saga B	NO	Oil and chemical	−5.5	33.5
Sandvik B	SE	Engineering	16.6	24.7
SAS Danmark	DK	Transportation	17.6	37.9
SCA B	SE	Forestry	3.6	30.3
Skandia	SE	Insurance	0.9	36.7
S-E Banken A	SE	Financial	1.7	35.0
Handelsbanken A	SE	Financial	8.4	47.1
Skanska B	SE	Construction	4.3	38.5
I.m. Skaugen	NO	Shipping	−17.9	69.7
SKF B	SE	Engineering	7.4	30.4
Sophus Berendsen B	DK	Service	21.7	16.5
Stora A	SE	Forestry	2.1	31.3
Topdanmark	DK	Insurance	−9.5	24.2
Trelleborg B	SE	Metal and mining	−6.7	44.2
Unidanmark A	DK	Financial	−1.1	27.1
UPM Kymmene	FI	Forestry	15.5	35.1
Volvo B	SE	Automotive	18.3	30.3

Table 2. Results from tests of autocorrelation

This table reports results from tests performed to specify the conditional mean equation. Column two gives p-values for Richardson and Smith's [1994] test for autocorrelation, (11), calculated on demeaned returns. Column three reports p-values for the same statistic, but calculated on estimated residuals from an AR(1) model. Both statistics are calculated on eight autocorrelations, i.e. in formula (11) k is equal to eight.

Security	$RS(8)$ on $\overline{r}_t$ (p-value)	$RS(8)$ on $\widehat{\varepsilon}_t$ from AR(1) (p-value)
ABB A	0.107	-
Aker B	0.305	-
Astra A	0.501	-
Atlas Copco A	0.157	-
Bergesen d.y. B	0.462	-
Carlsberg B	0.137	-
Cultor 2	0.610	-
Danisco	0.005	0.320
Den Danske Bank	0.012	0.419
D/S 1912 B	0.417	-
D/S Svendborg B	0.752	-
EAC	0.075	-
Electrolux B	0.003	0.642
Enso R	0.807	-
FLS Industries B	0.031	0.797
Norsk Hydro	0.354	-
Investor B	0.442	-
ISS B	0.039	0.803
Jyske Bank	0.018	0.739
Kvaerner B	0.250	-
LM Ericsson B	0.609	-
Modo B	0.862	-
Nokia	0.333	-
Novo Nordisk B	0.005	0.674
Norske Skog B	0.870	-
Orkla A	0.455	-
Outokompu A	0.981	-
Pohjola B	0.345	-
Saga B	0.677	-
Sandvik B	0.146	-
SAS Danmark	0.062	-
SCA B	0.519	-
Skandia	0.214	-
S-E Banken A	0.827	-
Handelsbanken A	0.361	-
Skanska B	0.434	-
I.m. Skaugen	0.041	0.365
SKF B	0.031	0.707
Sophus Berendsen B	0.047	0.826
Stora A	0.047	0.941
Topdanmark	0.431	-
Trelleborg B	0.457	-
Unidanmark A	0.001	0.607
UPM Kymmene	0.688	-
Volvo B	0.206	-

Table 3. Results from tests of ARCH(q)

This table reports results from tests performed to specify the conditional variance equation. Column two reports p-values for Wooldridge's [1990] test for no ARCH(8), (13). Column three gives p-values for Engle's [1982] test of no ARCH, calculated on eight squared residuals. Column four reports the coefficient of excess kurtosis calculated on estimated residuals. Column five reports first-order autocorrelation in squared estimated residuals.

Security	no $ARCH(8)$ (p-value)	non-robust	$\kappa(\widehat{\varepsilon})$	$\widehat{\rho}(\widehat{\varepsilon}_t^2, \widehat{\varepsilon}_{t-1}^2)$
ABB A	0.001	0.000	5.2	0.089
Aker B	0.057	0.000	6.9	0.136
Astra A	0.049	0.001	1.8	0.122
Atlas Copco A	0.452	0.000	4.7	0.209
Bergesen d.y. B	0.001	0.000	3.7	0.101
Carlsberg B	0.237	0.000	3.2	0.150
Cultor 2	0.502	0.059	33.7	0.001
Danisco	0.002	0.000	4.7	0.162
Den Danske Bank	0.000	0.000	3.5	0.108
D/S 1912 B	0.012	0.000	3.0	0.158
D/S Svendborg B	0.160	0.000	4.4	0.125
EAC	0.000	0.000	7.6	0.278
Electrolux B	0.000	0.000	2.8	0.202
Enso R	0.306	0.070	4.4	0.048
FLS Industries B	0.011	0.054	7.2	0.087
Norsk Hydro	0.002	0.000	6.6	0.200
Investor B	0.024	0.000	14.7	0.188
ISS B	0.202	0.000	51.8	0.028
Jyske Bank	0.488	0.090	50.1	0.102
Kvaerner B	0.031	0.000	7.2	0.116
LM Ericsson B	0.014	0.000	3.7	0.149
Modo B	0.040	0.000	13.4	0.281
Nokia	0.025	0.000	7.5	0.100
Novo Nordisk B	0.026	0.001	3.7	0.110
Norske Skog B	0.060	0.000	10.3	0.124
Orkla A	0.039	0.000	8.5	0.115
Outokompu A	0.455	0.000	38.9	0.150
Pohjola B	0.002	0.000	7.1	0.156
Saga B	0.073	0.000	9.4	0.153
Sandvik B	0.003	0.000	3.0	0.190
SAS Danmark	0.034	0.000	8.0	0.172
SCA B	0.034	0.000	7.8	0.182
Skandia	0.031	0.000	5.7	0.353
S-E Banken A	0.004	0.000	18.9	0.347
Handelsbanken A	0.000	0.000	12.4	0.365
Skanska B	0.006	0.000	14.1	0.081
I.m. Skaugen	0.000	0.000	8.4	0.186
SKF B	0.002	0.000	2.5	0.166
Sophus Berendsen B	0.072	0.094	4.1	0.088
Stora A	0.010	0.000	6.0	0.193
Topdanmark	0.156	0.000	9.2	0.294
Trelleborg B	0.010	0.000	5.1	0.185
Unidanmark A	0.000	0.000	2.9	0.132
UPM Kymmene	0.000	0.000	2.3	0.143
Volvo B	0.000	0.000	5.4	0.202

Table 4. Results from tests of no asymmetric GARCH

This table reports results from tests performed to specify the conditional variance equation. In column two, p-values for the robust test of GARCH(1,1) against GQARCH(1,1), are reported. Column three gives the p-values for the non-robust version of the same test, as formulated in Hagerud [1997]. In column four, p-values for the robust test of GARCH(1,1) against LSTGARCH(1,1), are presented. Column five reports the p-values for the non-robust version of the same test, as formulated in Hagerud [1997].

Security	LM_1	non-robust	LM_2	non-robust
ABB A	0.001	0.009	0.199	0.129
Astra A	0.068	0.053	0.061	0.170
Bergesen d.y. B	0.144	0.419	0.558	0.397
Danisco	0.235	0.018	0.875	0.385
Den Danske Bank	0.447	0.373	0.505	0.933
D/S 1912 B	0.101	0.120	0.109	0.274
EAC	0.043	0.280	0.029	0.313
Electrolux B	0.202	0.232	0.123	0.197
FLS Industries B	0.638	0.757	0.863	0.882
Norsk Hydro	0.215	0.414	0.412	0.241
Investor B	0.063	0.740	0.188	0.753
Kvaerner B	0.572	0.785	0.780	0.631
LM Ericsson B	0.069	0.004	0.748	0.377
Modo B	0.014	0.066	0.092	0.005
Nokia	0.986	0.495	0.307	0.275
Novo Nordisk B	0.960	0.937	0.608	0.795
Orkla A	0.000	0.010	0.784	0.181
Pohjola B	0.450	0.100	0.604	0.176
Sandvik B	0.545	0.887	0.251	0.207
SAS Danmark	0.907	0.904	0.962	0.955
SCA B	0.343	0.131	0.044	0.046
Skandia	0.001	0.000	0.017	0.000
S-E Banken A	0.884	0.323	0.779	0.602
Handelsbanken A	0.032	0.002	0.140	0.033
Skanska B	0.054	0.109	0.213	0.051
I.m. Skaugen	0.046	0.495	0.779	0.552
SKF B	0.040	0.163	0.706	0.400
Stora A	0.475	0.436	0.058	0.115
Trelleborg B	0.001	0.000	0.074	0.005
Unidanmark A	0.030	0.029	0.087	0.024
UPM Kymmene	0.852	0.835	0.425	0.681
Volvo B	0.042	0.022	0.351	0.425

Table 5. Asymmetric models ranked according to value on the likelihood function

Results from quasi-maximum likelihood estimation of the EGARCH, GJR, TGARCH, A-PARCH, GQARCH, VS-ARCH, and LSTGARCH models. The table reports the three models that gave the highest value on the likelihood function, for each security. The column labelled '# 1' reports the highest value, the following column the second highest, and the last column the third highest value on the likelihood function. In those cases where the GJR model gave the same value as the LSTGARCH model, the GJR model is ranked before the LSTGARCH model.

Security	# 1	# 2	# 3
ABB A	GQARCH	LSTGARCH	A-PARCH
EAC	A-PARCH	TGARCH	EGARCH
Modo B	LSTGARCH	A-PARCH	GJR
Orkla A	TGARCH	EGARCH	GJR
SCA B	LSTGARCH	GJR	VS-ARCH
Skandia	GJR	LSTGARCH	TGARCH
Handelsbanken A	A-PARCH	GJR	LSTGARCH
I.m. Skaugen	A-PARCH	TGARCH	EGARCH
SKF B	A-PARCH	TGARCH	EGARCH
Trelleborg B	A-PARCH	TGARCH	EGARCH
Unidanmark A	GJR	VS-ARCH	TGARCH
Volvo B	GQARCH	A-PARCH	VS-ARCH

Table 6. Asymmetric models ranked according to value on BIC

Results from quasi-maximum likelihood estimation of the EGARCH, GJR, TGARCH, A-PARCH, GQARCH, VS-ARCH, and LSTGARCH models. The table reports the three models that gave the lowest value on Schwarz [1978] information criteria, BIC, for each security. The column labelled '# 1' reports the lowest value, the following column the second lowest, and the last column the third lowest value on BIC.

Security	# 1	# 2	# 3
ABB A	GQARCH	GJR	LSTGARCH
EAC	TGARCH	EGARCH	A-PARCH
Modo B	GJR	LSTGARCH	A-PARCH
Orkla A	TGARCH	EGARCH	GJR
SCA B	GJR	LSTGARCH	VS-ARCH
Skandia	GJR	TGARCH	GQARCH
Handelsbanken A	GJR	GQARCH	A-PARCH
I.m. Skaugen	TGARCH	EGARCH	A-PARCH
SKF B	TGARCH	EGARCH	GJR
Trelleborg B	TGARCH	EGARCH	GJR
Unidanmark A	GJR	VS-ARCH	TGARCH
Volvo B	GQARCH	VS-ARCH	GJR

Table 7. Test for higher order asymmetric effects

Results from the robust sign bias test LM_3. The null hypothesis is the estimated asymmetric model, and the alternative is the asymmetric model augmented by the term $\tau \cdot S_{t-1}^{-}$. The table reports p-values.

	Model						
Security	EGARCH	GJR	TGARCH	A-PARCH	GQARCH	VS-ARCH	LSTGARCH
ABB A	0.0513	0.2988	0.1291	0.2943	0.7503	0.3796	0.2942
EAC	0.8562	0.6597	0.5102	0.3837	0.5978	0.7042	0.4380
Modo B	0.7269	0.0935	0.1814	0.4840	0.3291	0.0789	0.4255
Orkla A	0.3378	0.0658	0.3928	Failure	0.3872	0.4671	0.0658
SCA B	Failure	0.9657	Failure	Failure	0.0097	0.7256	0.9570
Skandia	0.0666	0.4562	0.2707	Failure	0.9582	0.5841	0.4560
Hand. A	0.0497	0.8977	0.2342	0.9787	0.1276	0.4146	0.8778
I.m. Ska.	0.6711	0.9206	0.6615	0.6001	0.9523	0.9090	0.9510
SKF B	0.9324	0.9389	0.8305	0.8095	0.5781	0.4076	0.9897
Trell. B	0.6900	0.8063	0.5120	0.6967	0.3664	0.8149	0.7987
Unid. A	Failure	0.2440	0.6663	Failure	0.2884	0.2556	Failure
Volvo B	0.3470	0.2796	0.3299	0.2968	0.9784	0.5343	0.2793

Table 8. Skewness for the series of normalized residuals

The table reports the estimated standardized third moment (skewness) for the series of normalized residuals, $\widehat{\varepsilon}_t / h_t$. The critical value on five percent significance level, for the test of zero skewness against non-zero skewness is approximately, ± 0.14. For underlined figures, the null of zero skewness is rejected on five percent significance level.

	Model						
Security	EGARCH	GJR	TGARCH	A-PARCH	GQARCH	VS-ARCH	LSTGARCH
ABB A	-0.1233	-0.0698	-0.1202	-0.0699	-0.0649	-0.0914	-0.0698
EAC	-0.6039	-0.7254	-0.5921	-0.6013	-0.7070	-0.7393	-0.7497
Modo B	0.1531	0.2104	0.1482	0.2142	0.1669	0.1670	0.2206
Orkla A	-0.3914	-0.4040	-0.3807	Failure	-0.4184	-0.4806	-0.4040
SCA B	Failure	0.0688	Failure	Failure	0.0420	0.1825	0.0685
Skandia	0.0770	0.0462	0.0776	Failure	0.0827	0.0625	0.0462
Hand. A	0.0195	0.0794	0.0109	0.0875	0.0949	0.0605	0.0794
I.m. Ska.	-0.8084	-0.7348	-0.8025	-0.7981	-0.7495	-0.5892	-0.7348
SKF B	0.0377	0.0285	0.0293	0.0251	0.0364	0.0370	0.0376
Trell. B	-0.0077	-0.0425	0.0014	0.0023	-0.0408	-0.0613	-0.0424
Unid. A	Failure	-0.0291	-0.0660	Failure	-0.1297	0.0576	Failure
Volvo B	0.2656	0.2819	0.2656	0.2834	0.2702	0.2693	0.2596

Table 9. Excess kurtosis for the series of normalized residuals

The table reports the estimated standardized fourth moment minus three (coefficient of excess kurtosis), for the series of normalized residuals, $\widehat{\varepsilon}_t/h_t$. The critical value on five percent significance level, for the test of zero excess kurtosis against non-zero excess kurtosis is approximately, ± 0.27. The null can therefore be rejected for all series, and for all models.

	Model						
Security	EGARCH	GJR	TGARCH	A-PARCH	GQARCH	VS-ARCH	LSTGARCH
ABB A	2.5803	2.3695	2.6115	2.3692	2.3737	2.5291	2.3693
EAC	7.3740	9.4624	7.0605	7.2255	9.3352	9.7064	9.7532
Modo B	2.3141	2.0073	2.3467	1.9888	2.4548	2.4052	1.7063
Orkla A	5.2144	5.4453	5.2131	Failure	5.6993	6.2284	5.4453
SCA B	Failure	1.7293	Failure	Failure	2.0850	2.2559	1.7287
Skandia	1.5634	1.6442	1.5368	Failure	1.4757	1.7601	1.6442
Hand. A	3.0726	3.1501	3.0572	3.1512	3.0998	3.3614	3.1501
I.m. Ska.	7.8845	7.3694	7.8293	7.7823	7.4495	6.4197	7.3693
SKF B	0.7471	0.7681	0.7761	0.7909	0.7752	0.7527	0.7460
Trell. B	1.4918	1.6209	1.4497	1.4483	1.6812	1.8034	1.6202
Unid. A	Failure	1.6002	1.9712	Failure	2.4035	1.6721	Failure
Volvo B	1.6459	1.5086	1.6408	1.5365	1.5083	1.4707	1.5087