The Stock Market as a Screening Device and the Decision to Go Public

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Abstract

We argue that many firms become publicly traded on a stock exchange as the first stage of a longer term divestment plan. Making a direct sale of unlisted stock may be associated with great adverse selection costs. The publicly listed stock price reduces adverse selection by aggregating the information of several investors, and this market valuation, rather than the cash infusion, could be the main benefit of an initial public offering.

This theory provides a unified treatment of a whole range of empirical observations, in particular why initial owners frequently exit completely subsequent to an initial public offering (IPO) and why the number of stock market introductions increases with the stock price level. The model also reformulates the “sweet taste” explanation of IPO underpricing in a way which is consistent with recent evidence. Finally, we argue that the number of firms which go public is inefficiently large.

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1 Introduction

Why do firms choose to enter the stock market? Stock market entrants themselves tend to emphasize four different reasons: (i) to obtain new finance; (ii) to enhance a company's image and increase its publicity; (iii) to motivate managers and other employees; (iv) to "cash in" by selling off shares. The deeper question is why firms *go public* to achieve these goals. For example, direct sales of stock and bank loans are alternative sources of funds which could potentially finance new projects or allow the original owners to cash in. Moreover, funds raised through a stock market introduction are often very expensive. Even in the relatively efficient U.S. equity market, a cost of twenty cents per dollar is a reasonable measure of the average, and thirty cents per dollar is common for small firms [see Ritter (1987) and Barry, Muscarella and Vertsuypens (1991)]. The only reasonable explanation for the initial offer is that there are some further future benefits associated with being publicly traded. Indeed, in a recent survey article, Röell (1996) concludes that the real reasons why firms go public are "an informative stock price, a more liquid stock, and increased competition among providers of finance."

In this paper, we shall discuss in some detail why an informative stock price is important to owners who want to cash in on their stock, and how this insight can explain several common features of public offerings. To understand our argument, it may be helpful to begin by thinking about why some owners want to cash in rather than keeping their shares. One story goes as follows: For a variety of reasons, many start-up firms are confined to use internal financing. As a result, the founders are forced to invest a larger portion of their wealth in the firm than would otherwise be optimal. They consume too little and bear too much idiosynchratic risk. The distortion increases as the firm grows and a larger portion of the wealth is tied up in the owned stock. However, when there is inside information, founders may nevertheless prefer the locked-in investment portfolio rather than divesting to outside investors at a low price. This is a classic adverse selection problem.

We demonstrate that an initial public offering (IPO) may be a good way to counteract adverse selection. The idea is that when some of the firms' shares are traded on a stock exchange, the firm gets priced by the market, reflecting the opinions of a large number of traders. Thus, by making an IPO, the founders are not only able to raise some cash through the offer itself; they may also be able to sell further shares under more favorable information conditions. As we shall show, behavior may vary according to the private information that owners have. Owners who have negative information about the firm's prospect are generally more reluctant to go public. These may choose a a cheaper direct sale instead. However, in general it will not be possible perfectly to infer the private information of a stock market entrant.

Our model produces a number of predictions which are consistent with established empirical regularities. First, many old and established firms go public when stock prices in general are high [Loughran, Ritter, and Rydqvist (1994)]. Second, IPOs are frequently followed by substantial seasoned offers in which initial owners cash in [Brennan and Franks (1995), Rydqvist and Högholm (1995)].[1] Third, IPOs are often substantially underpriced [e.g. Ritter (1987)]. Fourth, the probability of seasoned offers is strongly related to the stock price increase in the weeks following the IPO, but is only weakly related to the amount of IPO underpricing (measured as the relative difference between the offer price and the price at the first day of trading) [Jegadeesh, Weinstein and Welch (1993)]. Fifth, on average the stock price falls following a seasoned offer announcement, and this fall is greater the poorer is the firm's performance prior to the announcement [e.g. Asquith and Mullins (1986)]. Given that it is often complained that theorists build special purpose models to explain each individual phenomenon, we would like to emphasize this multiplicity of implications.

Our analysis also suggests some new directions for research. For example, there is a tendency that, in equilibrium, relatively good firms enter the stock market. Thus the P/E–ratio might be higher in IPOs than in direct sales of unlisted stock.

What is the social value of the stock exchange? Apart from the benefit associated with overcoming the adverse selection problem and thereby improving the allocation of ownership rights, in our model the existence of stock exchanges also has a potentially harmful effect in diverting trade through direct sales towards the costlier stock exchange. Thus, even if the listed firms themselves pay the full cost of entering the stock exchange, the number of listed firms is never too small and may well be too large. In the model it is even possible that an abolition of the stock exchange could increase overall welfare.

Let us briefly relate our work to the contributions of others. It is an old insight that the stock market performs an important service in aggregating information which can be used to evaluate the performance and the prospects of firms. Beside direct evidence from prospectuses [Rydqvist and Högholm (1995)], there is a body of empirical research which indirectly supports our contention

[1] For example, Brennan and Franks (1995) find that within seven years more than two thirds of shares of UK main market entrants are sold to outside shareholders. In 36% of all firms which went public on the Swedish stock exchange in the 1980s, the original owners had sold out practically all their shares in the course of five years following the IPO, and also in the remaining firms the original owners tended to reduce their stake substantially the years following the initial offer [Rydqvist and Högholm (1995)].

that firms are taken public for the sake of the market valuation. Grammatikos and Papaioannou (1986) show that stocks with large bid–ask spreads on the OTC market experience significant positive price reaction around the listing announcement time as opposed to stocks with low spreads which experience an insignificant price reaction. Together with evidence that adverse selection constitutes a significant fraction of the bid–ask spread [e.g. Stoll (1989)] and that higher trading volume reduces the part of bid–ask spreads which is due to adverse selection [Easley et al. (1996)], the listing announcement data would seem to suggest that firms are taken public, or listed at more public exchanges, in order to reduce adverse selection costs.

Authors who have explored the usefulness of having stock prices aggregate the information of individual traders, have mainly focussed on the role of stock prices in guiding real investment decisions. An exception is Holmström and Tirole (1993), who argue that an objective evaluation of firm performance, such as a stock market price, is useful for designing managerial compensation schemes. We have not, however, found any formal analysis which directly addresses the role of public listing as a way to reduce adverse selection in the trade of ownership rights.

Recently, a number of authors have been interested in the dynamics of stock market introductions [see e.g. Chemmanur and Fulghieri (1996), Pagano and Röell (1996), Mello and Parsons (1995), and Zingales (1995)], but these typically neglect adverse selection problems altogether. In the literature on seasoned offers on the other hand, adverse selection figures prominently, and like Lucas and McDonald (1990) we show how stock price run–ups prior to seasoned offers and the negative stock price reaction to the announcement of seasoned offers both may be due to adverse selection. However, unlike Lucas and McDonald we focus on the ramifications for the decision to go public in the first place, when adverse selection is likely to be most severe. In emphasizing the seller's private information at the (potential) introduction date, our theory is perhaps most closely related to the signalling models of IPO underpricing [Allen and Faulhaber (1989), Chemmanur (1993), Grinblatt and Hwang (1989), and Welch (1989)]. Their basic adverse selection problem is the same and, as in our model, underpricing benefits high value issuers. However, the purpose of these papers is to explain why IPOs are underpriced, assuming that the firm goes public and may return to the market with a seasoned offer, while our paper endogenizes whether a closely-held firm stays private, is divested through direct sale, or goes public.[2] Unlike the previous work on

[2]Chemmanur (1993) argues that underpricing may improve the precision of the market price. The three other models do not explicitly consider the information embedded in the market price, but they assume that, somehow, new information arrives between the initial and seasoned offer dates.

underpricing by informed incumbents, our story is fully consistent with the evidence of Jegadeesh, Weinstein and Welch (1993) and by Michaely and Shaw (1994), who find that the amount of underpricing is a weak predictor of seasoned offerings.

The rest of the paper is organized as follows: Section 2 describes the model and Section 3 derives the central results. Section 4 concludes.

2 Model

This section first introduces the adverse selection problem associated with the founder's private information. We then describe the founder's choice between direct sale and public offer for reducing the investment in the own firm.

We consider an all-equity financed firm which at the outset is closely-held by its founder. The founder's valuation of his firm is v. Well diversified and liquid investors would, if they were perfectly informed, value the firm at kv, where $k > 1$. This is perhaps the simplest way to introduce gains from trade into the model. However, our results do not hinge on this particular specification.

Clearly, in a world of complete information the founder would divest immediately given these assumptions. But we assume that founder's valuation is private information, reflecting his superior knowledge about the prospects of the firm. Outside investors share an imperfect estimate of v, but they know that v is either high or low: $v \in \{\underline{v}, \overline{v}\}$ where $\underline{v} < \overline{v}$. The prior probability that a firm is of high value is denoted h.

In principle, there are many ways in which the founder can reduce investment in the own firm. For example, he could borrow against the firm's assets, he could pay himself dividends, or pay himself higher wages as the manager of the firm. However, we shall assume that selling equity is the optimal divestiture strategy, because debt financing is constrained by the agency problems of debt, and wage and dividend payments are limited by the firm's liquidity needs. Taxation of wages and dividends may also favor equity sales.

Before we discuss in detail the strategies available to the various agents, let us make some assumptions about the solution concept. Since the founder has private information, and the game has several stages, it is natural to confine attention to perfect Bayesian equilibria. In short, a perfect Bayesian equilibrium (PBE) is a set of strategies and beliefs such that each agent applies a best response to the others' strategies, given his beliefs. Moreover these beliefs should be consistent with Bayes' rule along the equilibrium path. When there are several PBEa, we disregard any equilibrium which is Pareto–dominated by some other PBE. (Other refinements have less cutting power, but yield qualitatively similar results.) While this assumption is debatable, we think it has been given a quite convincing justification in the context of signalling games [see Mailath et al. (1993)], and although our game is a bit more complicated than that, similar arguments appear to be valid.

To simplify the analysis, we shall assume that the founder only sells shares if he receives a price strictly exceeding his reservation value v. Finally, and perhaps most controversially, we assume that whenever the potential buyers of the firm's shares are indifferent between purchasing shares or not, they choose to purchase.[3]

2.1 Direct sale

The founder may divest through direct sale to another, wealthier individual or to an institution which is owned by well-diversified and liquid investors. Note that the parameter k is the same in the direct sale as in the public offer (below), which means that the founder's overinvestment problem is not passed to the new owner in the direct sale, i.e., we assume that the new owner is well diversified and liquid. We also assume that no information is disseminated after a direct sale, so there will be no reason to make such a sale in several stages. Let p_d be the direct sale offer price.

Consider now a game where the founder chooses between keeping the firm and offering it for sale at a price p_d. There is one or several identical buyers, and their strategy is simply to accept or reject the offer.

It is straightforward to see that, under our assumptions, adverse selection prevents equilibria with direct sales of high value firms unless $h > h^*$, where

$$h^* := \frac{\overline{v} - k\underline{v}}{k(\overline{v} - \underline{v})}.$$

More precisely:

Lemma 1 *There is a PBE in which both types of firms are traded exclusively through direct sale only if $h > h^*$.*

Proof: The highest price acceptable to the buyers, given rational beliefs, would be $\overline{p} = k[h\overline{v} + (1-$

[3]This assumption rules out the possibility that high value founders could choose high offer prices to signal high value, trading off the higher price against a larger probability of failure. For a general analysis of mixed buyer strategies in models with perfectly inelastic demand, see Ellingsen (1997). Allowing mixed strategies by the buyer would allow several direct sale equilibrium outcomes, but would not essentially affect our main conclusions.

$h)\underline{v}]$ and the high value seller only accepts prices above $\overline{v}$. Thus, a necessary and sufficient condition for a successful direct sale involving type $\overline{v}$ firms is that $\overline{p} > \overline{v}$, or equivalently that $h > h^*$. ■

Since the seller sets the highest price the buyers are willing to accept, we furthermore see that in any PBE the direct sale price is

$$p_d = \begin{cases} k[h'\overline{v} + (1-h')\underline{v}] & \text{, if } h' > h^*; \\ k\underline{v} & \text{, if } h' \leq h^*, \end{cases} \tag{1}$$

where h' is the proportion of direct sales made by type $\overline{v}$ firms.

2.2 Public offer

The founder may alternatively divest a fraction f of his shares through a public offer.[4] The difference between a public offer and a direct sale is that after a firm has gone public, the stock price might disseminate some information about the value of the firm.

After the introduction, the stock market evaluates the firm, the evaluation being embodied in the stock price. The price formation process is not modelled here, but there are good reasons to believe that investors can learn from observing the stock price [see e.g., Hellwig (1980) and Kyle (1985)].[5] For simplicity, we suppose that there is a fixed time interval beyond which additional information is not reflected in the stock price. Let the information elicited from the stock market trading be represented by the noisy signal $S \in \{G, B\}$ where G denotes good and B denotes bad. The information content of the signal is given by the probability $Pr(S|v)$, and is assumed to satisfy the inequalities

$$0 < Pr(B|\overline{v}) < Pr(G|\overline{v}) < 1. \tag{2}$$

Thus, no signal is fully revealing, and a founder of type $\underline{v}$ ($\overline{v}$) is more likely to obtain a signal B (G). We may think of this signal as representing the direction of movement of the stock price during the first weeks or months of trading, with a signal G indicating that the price has increased

[4]In a real world IPO, the founder has a choice between offering a fraction of his own shares and issuing new shares. If he issues new shares exclusively, the founder delays any divestiture, and the value of the firm's stock after the IPO (to well diversified investors) is $kv + c$, where c denotes the cash raised through the offer. To complicate matters further, the founder himself may take part in the new issue. We return to these considerations in the final section.

[5]One may still wonder if continuous trading in the stock market generates more information than a one-time auction which can be arranged for the direct sale. Is continuous stock market trading better at aggregating existing information, or does it provide stronger incentives to produce information? Whatever the answers to these questions, stock market listing has another advantage over a one–time auction; there are low fixed costs associated with further trade, an advantage that buyers may be willing to pay for (although this effect is not modelled explicitly here).

relative to, say, the close price at the first day of trading.[6]

A disadvantage of public offers is the flotation cost, denoted $a > 0$. (In practice, the flotation cost materializes as fees to the investment bankers and auditors, who produce the offer documents and certify their content, as well as listing fees to the stock exchange and authorities.) For simplicity, we shall assume that both the direct sale and the seasoned offer are costless transactions.[7]

We want to determine the probability that each type of firm goes public as well as their initial offer prices. Let $m \in [0,1]$ be the probability that a high value firm goes public, and let $l \in [0,1]$ be the corresponding probability that a low value firm goes public.

The initial offer price is denoted p_o. Of course, the price of one share depends on the number of shares in the firm. To avoid confusion, whenever we talk about offer prices we shall refer to the price of one share multiplied by the total number of shares in the firm. In principle, the initial offer price could be a random variable (if the founder uses a mixed pricing strategy), and this random variable may depend on the founder's type. Thus, let the functions $g_l(p_o)$ and $g_m(p_o)$ denote the founder's pricing strategy, i.e. the densities of initial offer prices posted by the low respectively high value founder. Thus, in a perfect Bayesian equilibrium

$$l_u(p_o) := \frac{g_l(p_o)l}{g_l(p_o)l + g_m(p_o)m} \quad \text{and} \quad m_u := \frac{g_m(p_o)m}{g_l(p_o)l + g_m(p_o)m}$$

are the posterior probabilities that the firm is of low value respectively high value given only the observation of the initial public offer price.

Let $\lambda(p_o) := l_u(p_o)/m_u(p_o)$. The highest offer price acceptable to an uninformed buyer is then given implicitly by the equation

$$\hat{p}_o(\lambda) := k\left(\frac{h\overline{v}}{h + (1-h)\lambda(\hat{p}_o)} + \frac{(1-h)\lambda(\hat{p}_o)\underline{v}}{h + (1-h)\lambda(\hat{p}_o)}\right). \tag{3}$$

The price $\hat{p}_o$ is also a reasonable estimate of the market price at the moment at which trade starts. Only after some time will the stock price become more informative, as informed traders need to hide behind the noise created by liquidity trades [Kyle (1985)]. If $p_o < \hat{p}_o$, we say that the IPO is *underpriced.*

The posterior probability that the offer is made by a high value founder given the offer price

[6]Whereas in reality the space of signals (possible stock prices) is much larger, the assumption that there are only two signals considerably simplifies the analysis without, we think, generating excessively artificial results.

[7]In reality, issuance costs are significant for any offer, but smaller for seasoned offers than for IPOs [Smith (1986)].

and the stock market signal is

$$h_s(S, \lambda(p_o)) = \frac{Pr(S|\overline{v})h}{Pr(S|\overline{v})h + Pr(S|\underline{v})(1-h)\lambda(p_o)}. \tag{4}$$

In the ensuing analysis, $h_s(S)$ should be understood as $h_s(S, \lambda(p_o))$.

The stock market's valuation of the stock after incorporating the information embodied in the signal is the stock price

$$p_m(S) = k[h_s(S)\overline{v} + (1 - h_s(S))\underline{v}]. \tag{5}$$

Next, the founder must decide whether to make a seasoned offer and if so at what price. Notice that there is no reason to underprice the seasoned offer (since issuers do not return to the market). Thus, if both types of founder make the seasoned offer, the seasoned offer price, $p_s(S)$, is simply equal to $p_m(S)$. If only the low type makes the offer, the price is $k\underline{v}$. High value issuers make the seasoned offer at the long-run market price if and only if $p_s(S) > \overline{v}$, or equivalently if $h_s(S) > h^*$. Low value founders always make a seasoned offer. To summarize, the seasoned offer price can be written

$$p_s(S) = \begin{cases} p_m(S) & \text{, if } h_s(S) > h^*; \\ k\underline{v} & \text{, if } h_s(S) \leq h^*. \end{cases} \tag{6}$$

The expected utility associated with making an IPO is hence

$$\Pi(v) := fp_o + (1-f)\sum_S Pr(S|v)\max\{p_s(S), v\} - a. \tag{7}$$

Let the utility difference between going public and the best alternative be denoted

$$\pi(v) := \Pi(v) - \max\{p_d, v\}. \tag{8}$$

Thus, given our assumptions, a founder of type v goes public if and only if $\pi(v) > 0$. For future reference, let us also define

$$E(\pi) := h\pi(\overline{v}) + (1-h)\pi(\underline{v}).$$

This is the expected gain from going public for a randomly chosen founder.

3 Analysis

We already know what the founder does if he does not go public. Also, we have described the outcome of the seasoned offers. It remains to characterize the probability that a firm is taken public conditional on the founder's type, i.e. l and m, and the initial offer price strategies of each type of founder, g_l and g_m.

The complete analysis of our problem is quite complex, and technical details are therefore relegated to the appendix. Here, we give a more intuitive account of the analysis.

First, we show that there cannot be equilibria in which only one type of firm is taken public. While it is perhaps obvious that there cannot be a PBE in which only low value firms go public (rational buyers would never pay more than $k\underline{v}$ and the founder therefore loses a), there is a slightly more subtle reason why we cannot have exclusively high value firms doing so. The idea here is that in such a fully separating equilibrium, the stock market signal would be uninformative, as the buyers' prior does not allow the possibility that low value firms are taken public. But if it pays for a high value firm to go public, it must then pay for a low value firm as well (as both the IPO and the seasoned offer will be priced according to beliefs of high value), which is a contradiction.

Second, we show that if there is a positive probability of a public offer, then we can restrict attention to equilibria in which $m = 1$; i.e. the high value firm is taken public with certainty. Roughly, the argument is that equilibria with $m < 1$ can either be replicated by, or are payoff dominated by, equilibria in which $m = 1$.

Next, we find that the offer price strategy does not depend on the firm's value, and that the founder uses a pure offer price strategy. (In other words, g_l and g_m are identical degenerate distributions.) The reason why the offer price is identical for both types of founder is that it pays for the low value founder to disguise as well as he can, and not to reveal low value already through the offer price. Given that the offer price reveals no information, it is also quite clear that — given l and m — there is a unique best offer price, and hence that the founder does not use a mixed pricing strategy.

In a pooling equilibrium, with $l = m = 1$, it is easily seen that the best offer price for the founder is equal to the uninformed buyers' valuation, i.e. $p_o = k[h\overline{v} + (1-h)\underline{v}]$. Higher prices are not accepted, and lower prices simply hand rents to the buyers. Hence, it remains to characterize the set of Pareto–optimal semi–separating equilibria, in which $0 < l < m = 1$. The crucial feature of these equilibria is that the low value founder is indifferent whether or not to go public, i.e. $\pi(\underline{v}) = 0$.

Looking at this expression, we see that lower initial offer prices must be balanced by higher seasoned offer prices. Thus, the zero profit condition defines an increasing function $l_e(p_o)$, which maps out a range of possible semi–separating equilibria. As it turns out, one of these equilibria Pareto dominates the others. This equilibrium is given by the offer price p_o^* which solves the equation

$$l_e(p_o^*) = l^* := \frac{h(1-h^*)Pr(B|\overline{v})}{(1-h)h^*Pr(B|\underline{v})}.$$

We call this the *best* semi–separating equilibrium.

Generically, the offer price p_o^* has the property that it is lower than $\hat{p}_o(l^*)$ and hence represents underpricing. The reason why it pays to underprice the initial offer is that it is a credible way to convey that the firm is likely to be of high value. However, as argued above, it never pays to completely eliminate the possibility that the firm is of low value. Indeed, that probability will not be below $l^*/(1+l^*)$. The crucial property of l^* is that it is the lowest l which is consistent with $h_s(B) < h^*$. In other words, if l is lowered further, it would be profitable even for the high value firm to make a seasoned offer after a negative stock market signal. The necessary reduction in p_o to induce such a l is excessive, because the associated increase in $p_s(B)$ benefits mainly the low value firms.

These insights can be summarized as follows.

Proposition 1 *Any Pareto optimal PBE in which there is a positive probability of an IPO is either (i) pooling, with* $l = m = 1$ *and* $p_o = \hat{p}_o = k[h\overline{v}+(1-h)\underline{v}]$ *or (ii) semi–separating, with* $m = 1, l = l^*$ *and* $p_o^* < \hat{p}_o(l^*)$.

Proof: See Appendix.

Regarding existence of IPO equilibria, it is quite clear that such equilibria do not exist for excessively high values of a and f. Moreover, if the stock market signal is sufficiently imprecise, with $P(G|\overline{v})$ being close to $P(G|\underline{v})$ there is no rationale for going public. Conversely, with a and f being small and the signal sufficiently precise, there will exist IPO equilibria.

To illustrate some of the magnitudes involved, and to get some intuition for when IPO equilibria are likely to be pooling respectively semi–separating, let us consider a simple numerical example. As a starting point, we fix all parameters except a and h. Parameter values are: $f = 1/10, k = 6/5, \overline{v} = 2, \underline{v} = 1, Pr(G|\overline{v}) = 9/10, Pr(G|\underline{v}) = 1/10$. In Figure 1, we depict the resulting equilibria.

In region 1, where h is very low or a is very high, there is no trade of high quality firms at all. In the large region 2, a semi–separating equilibrium (SS) is the unique outcome. In regions 3 and

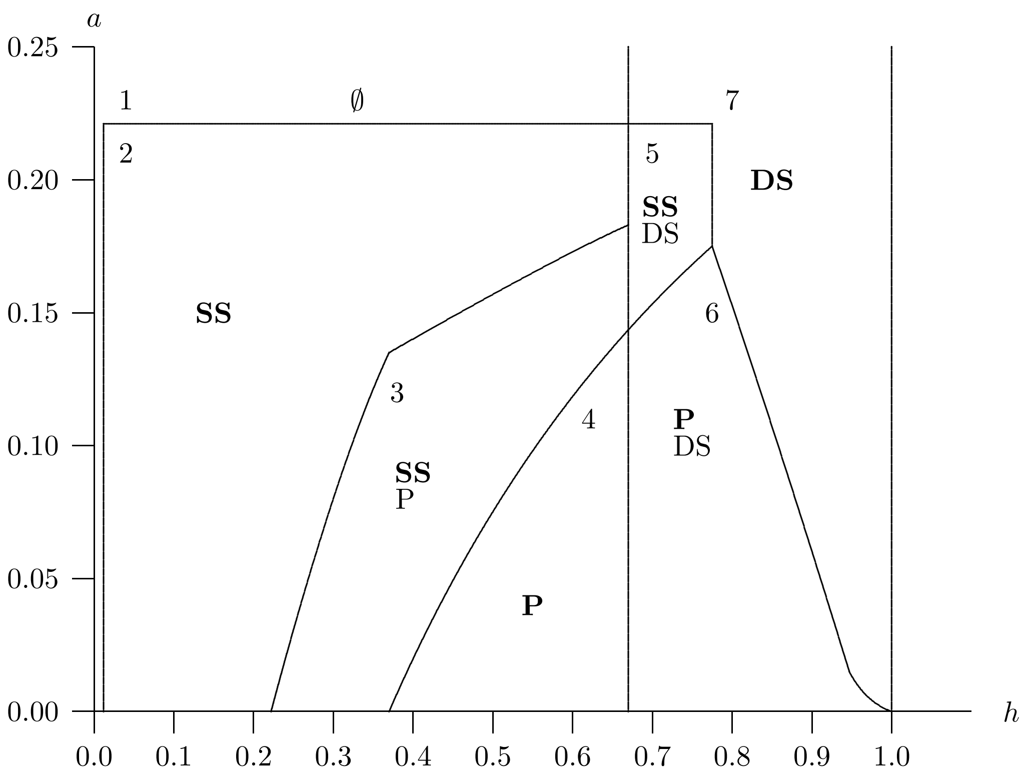

Figure 1: Pareto optimal equilibria

5, semi–separation is the equilibrium which is preferred by the high value founder. However, in region 3, low value founders prefer the pooling equilibrium in which both types go public (P), and in region 5 low value founders prefer the equilibrium in which both types make a direct sale (DS). Both types always go public in region 4, whereas both make a direct sale in region 7.

In the figure, boldface is used to identify which equilibrium is preferred by high value founders, as we think that this equilibrium is most plausible. To see why, consider e.g. the situation in region 6. Suppose a buyer expects either P or DS to be played. If the firm is of high value, the founder prefers the equilibrium in which he goes public; if the firm is of low value, the founder prefers the direct sale equilibrium. Understanding this, it is not unreasonable if the buyer upon seeing a direct sale is slightly sceptical. Why did the founder not go public given that there is such an equilibrium? Is he afraid of the market's judgement? Since the buyer earns zero profit in the direct sale equilibrium, only a slight doubt that something is wrong is sufficient to reject the offer. On the other hand, such doubts are not justified if the firm is taken public. Since this is the high value founder's preferred equilibrium, the question "why did the founder not make a direct sale?" has the reassuring answer that only a low value founder would wish to do so.

Table 1

		All IPOs	Pooling	Semi-sep.
r_o	mean	0.202	0.000	0.666
	std	0.520	0.000	0.763
	min	0.000	0.000	0.005
	max	6.169	0.000	6.169
	≥ 0	1.000	1.000	1.000
r	mean	0.021	0.024	0.015
	std	0.007	0.006	0.000
	min	0.000	0.000	0.015
	max	0.032	0.032	0.015
$\bar{r}_s$	mean	-0.048	-0.060	-0.021
	std	0.024	0.019	0.000
	min	-0.105	-0.105	-0.021
	max	0.000	0.000	-0.021

IPO initial return (r_o), run–up (r), and seasoned offer price reaction ($\bar{r}_s$). We have varied $Pr(G|\overline{v}) = \{0.60, 0.70, 0.80, 0.90\}$, $f \in [0.10, 0.49]$, $a \in [0.01, 0.49]$, and $h \in [0.01, 0.99]$ with increments of 0.01. The number of iterations is 800,000.

3.1 IPO Underpricing

The numerical example shows that semi–separating equilibria are not just a curious possibility. For some parameter values, the unique prediction of the model is a semi–separating equilibrium, while for other parameters pooling is the prediction. Hence, on average, in a large cross-section, IPO's should be underpriced in order to drive up the seasoned offer price.

The initial return from purchasing shares in an IPO is defined as

$$r_o := \frac{\hat{p}_o - p_o}{p_o}.$$

To get an illustration of the magnitudes implied by our model, we have simulated it for a variety of parameters. In Table 1 we report the resulting statistics.

Average intial return of around twenty per cent with a standard deviation of about fifty per cent is very similar to the numbers in actual European and US data (see e.g. Ritter (1987)). Notice also that, like reality, our model can produce very extreme levels of underpricing, an initial return of six hundred per cent being the highest.

A notable general feature of the model is that in a semi–separating equilibrium, $\pi(\overline{v})$ and $\pi(\underline{v})$ are independent of a. Thus, the founder does not mind the level of listing costs. The reason is that the listing costs keep out low value founders in exactly the same way that underpricing does.[8] In the model, the high value founder is indifferent between handing money to underwriters or handing them to buyers of his shares. Indeed, this may be one explanation why there is a negative relationship between the quality of the underwriters and the level of underpricing, as found by e.g. Jegadeesh, Weinstein and Welch (1993).

In some countries, but not in all, large firms on average underprice less than small firms do. Let us consider some of the size/price trade–offs in our model. First, the administrative listing costs are relatively smaller for large firms, so ceteris paribus they are more likely to end up in a pooling equilibrium in which there is no underpricing. However, in a semi–separating equilibrium there is a negative relationship between a and the level of underpricing, so in such an equilibrium large firms should underprice more. Second, the precision of the stock market's evaluation should be greater for large firms, as evidenced by the empirical negative relationship between firm size and the bid–ask spread. While a more precise signal makes semi–separation more likely, at the same time the amount of underpricing in a semi–separating equilibrium becomes smaller.

As mentioned in the introduction, the rationale for IPO underpricing in our model is reminiscent of the analysis by Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989). However, there is a subtle but important difference between our result and theirs. The earlier results on underpricing were derived in models in which only high value firms were thought to underprice their issues.[9] In our model, the amount of underpricing is unrelated to the value of the firm. In comparison with earlier work, some of our predictions regarding seasoned offers are therefore rather different and in better accordance with the facts, as we shall now demonstrate.

3.2 Seasoned offer dynamics

There is substantial empirical evidence on the determinants and characteristics of seasoned offers. Asquith and Mullins (1986) among others demonstrate that seasoned offers on average tend to follow a period of excess return, but that a substantial fraction of the seasoned offers are made by firms which have performed badly. (For a survey of the characteristics of seasoned offers, see Eckbo

[8] Essentially, we model both underpricing and listing costs as pure money burning. More realistically, the firm could improve the quality of underwriters by increasing a and increase the market liquidity by increasing the level of underpricing (as there would be more noise trade when many small owners want to cash in). We leave such extensions for future work.

[9] In Welch (1989) and Grinblatt and Hwang (1989) it is possible to obtain separating equilibria because the stock market signal can be fully revealing. Here, as in Allen and Faulhaber (1989), the signal is always imperfect. However, Allen and Faulhaber overlook the semi–separating equilibria of their model.

and Masulis (1995).) Jegadeesh, Weinstein and Welch (1993), who consider the seasoned offers in the years following an IPO, found that excess aftermarket returns in each of the two twenty–day periods after an IPO predict seasoned offers. In the literature, the excess return leading up to seasoned offers has been referred to as a "run–up."

It is quite easy to see that run–ups are predicted by our model, but that there is not necessarily a run–up prior to each seasoned offer. A firm always makes a seasoned offer after a good signal, but a good firm may not make an offer after a bad signal. Since $p_m(G) > p_m(B)$ the average pre–offer stock price must be higher for those firms which make a seasoned offer. More precisely, we can define the run–up as follows. Let the expected aftermarket stock price of a firm which makes an initial public offer be denoted

$$\bar{p}_m := Pr(B)p_m(B) + Pr(G)p_m(G), \tag{9}$$

where

$$Pr(S) := \frac{hPr(S|\overline{v}) + (1-h)lPr(S|\underline{v})}{h + (1-h)l}.$$

(Of course in equilibrium $\bar{p}_m = \hat{p}_o$.) The expected aftermarket price of a firm which eventually makes a seasoned offer can then be denoted

$$\bar{p}_m^{SO} = \begin{cases} \bar{p}_m & \text{, if } h_s(B) > h^*; \\ \dfrac{Pr(\underline{v}|B)Pr(B)p_m(B) + Pr(G)p_m(G)}{Pr(\underline{v}|B)Pr(B) + Pr(G)} & \text{, if } h_s(B) \leq h^*. \end{cases} \tag{10}$$

The natural definition of a run–up is

$$r := \frac{\bar{p}_m^{SO} - \bar{p}_m}{\bar{p}_m}.$$

At the same time, we know that there is a positive probability that the IPO is made by a low value firm, and low value firms make seasoned offers regardless of the signal.

Thus, we have established the following result.

Proposition 2 *(i) There is an expected run–up if and only if $h_s(B) < h^*$. (ii) There is a positive probability of a seasoned offer following a bad signal.*

In numerical examples the condition $h_s(B) < h^*$ condition almost always holds. Exceptions in the example depicted in Figure 1 occur only when a is very small (smaller than 0.015) and h is very large (larger than 0.95). Admittedly, the model tends to produce fairly small run–ups. In our simulations reported in Table 1, the average run–up is 2.1 per cent. The main explanation is that, under our assumptions low value firms choose to make the seasoned offer even if the signal is bad. In reality there could well be direct and indirect costs associated with revealing low quality in this way. An example of an indirect cost is that other sources of finance may dry up. If for some such reason only half the low value firms were to make a seasoned offer following a bad signal, the run–up would have increased by an order of magnitude.

Allen and Faulhaber (1989) and Welch (1989) predict a positive monotonic relationship between the amount of underpricing and the probability of a seasoned offer. Our model suggests that a positive relationship may be present in cross–section data, but that it should not be found in the part of the sample with significant underpricing. To see this, note that the probability of a seasoned offer given a pooling equilibrium is $hPr(G|\overline{v}) + (1-h)$ whereas the probability of a seasoned offer in a semi–separating equilibrium is $(hPr(G|\overline{v}) + (1-h)l^*)/(h + (1-h)l^*)$. Clearly, the latter expression is larger than the former. Thus, the fact that there is underpricing means that there is a greater likelihood of a seasoned offer. However, given some underpricing, the level of underpricing does not affect the probability of a seasoned offer. Remarkably, this is almost exactly the pattern discovered by Jegadeesh, Weinstein and Welch (1993).[10]

Jegadeesh, Weinstein and Welch (1993) have interpreted the run–up and the weak relationship between underpricing and seasoned offers as evidence against the sweet–taste theory of IPO underpricing. Our analysis shows that the opposite is true. These regularities are *supportive* of the theory that underpricing is a way to prop up seasoned offer prices.

A third regularity regarding seasoned offer prices is that the announcement of a seasoned offer entails a negative price reaction, and that this reaction is more negative when the run–up has been small [Asquith and Mullins (1986), Smith (1986)]. Again, this is exactly what our model predicts. Let

$$r_s(S) := \frac{p_s(S) - p_m(S)}{p_m(S)}$$

be the return associated with a seasoned offer announcement following a signal S. Since $p_m(G) = p_s(G)$, there is no negative reaction to a seasoned offer announcement for firms which have had a

[10]The probability of a seasoned offer in their sample is 15.6% for the lowest underpricing quintile (where there is no underpricing), and 21.2%, 21.4%, 21.7%, 23.9% for the next four quintiles. Admittedly, these numbers concern "unexplained" underpricing. An even better test of our hypothesis would split the sample according to whether there is no underpricing (pooling equilibrium) or substantial underpricing (semi–separating equilibrium), and then compute the "unexplained" underpricing for this latter group only.

large run–up. After a bad signal the price falls, as $p_s(B) \leq p_m(B)$ with the inequality being strict whenever $h_s(B) \leq h^*$.

Proposition 3 *(i) On average there is a negative seasoned offer announcement effect if and only if $h_s(B) \leq h^*$. (ii) After a good signal, the seasoned offer announcement effect is zero.*

Thus, in a large cross–section we would expect a negative seasoned offer announcement effect. From the above discussion we see that its size is

$$\bar{r}_s := Pr(B \cap \underline{v}|IPO)r_s(B) = \frac{(1-h)lPr(B|\underline{v})}{[hPr(G|\overline{v}) + (1-h)lPr(G|\underline{v})] + (1-h)lPr(B|\underline{v})} \frac{p_s(B) - p_m(B)}{p_m(B)}.$$

In Table 1 we report the result for a large number of simulations, and we see that the average is there 4.8 per cent, which is a bit high. On the other hand, as we have indicated above, our model may permit too many firms to make a seasoned offer after a bad signal. The fewer low value firms which make a seasoned offer after a bad signal, the less adverse is the announcement effect.

Finally, we would emphasize that conditional on some underpricing, the model predicts that the stock price reaction to seasoned offerings should be independent of the level of IPO underpricing. This too is consistent with the evidence in Jegadeesh, Weinstein and Welch (1993).

3.3 Stock Prices and IPO Activity

As shown by Loughran, Ritter and Rydqvist (1994), the number of IPOs tends to be pro–cyclical; i.e. more firms go public after a stock price increase than after a fall in stock prices. As we shall demonstrate, this is a prediction of our model too. The intuition is simple. As firm value increases, the gain from trade $kv - v = k(v-1)$ increases without affecting the cost of going public, a, which is fixed. Hence, at some price level v, high value firms will choose to go public rather than staying private.

Formally, let α be a shift parameter and write the founder's valuation as $v(\alpha)$. We assume that the fraction $\overline{v}/\underline{v}$ is independent of α. Then, we can prove the following result.

Proposition 4 *The high value founder's gain from going public, $\pi(\overline{v}(\alpha))$, is increasing in α.*

Proof: See Appendix.

Of course, our model is static, and so we should be cautious in making statements about the likely dynamics of stock markets. With this caveat, we think Proposition **??** suggests the following time-series: When market conditions are constant or worsening, low value firms are divested through direct sale and high value firms stay private. If stock prices increase enough, high value firms go public, and at least some low value firms switch from direct sale to public offer. As a result, IPO activity increases with stock prices.

Simulations reveal that our model can reproduce the dramatic swings in IPO activity witnessed in reality. Table 2 depicts a typical pattern.

Table 2

	Go public	Direct sale	Stay private
$\alpha = 0.5$	0.029	0.752	0.219
$\alpha = 1$	0.062	0.722	0.216
$\alpha = 2$	0.125	0.667	0.208
$\alpha = 3$	0.177	0.629	0.193

Frequency of a random seller's choice between going public, direct sale, and staying private as a function of the stock price level. We have varied $Pr(G|\overline{v}) = \{0.60, 0.70, 0.80, 0.90\}$, $f \in [0.10, 0.49]$, $a \in [0.01, 0.49]$, and $h \in [0.01, 0.99]$ with increments of 0.01. The frequency is the proportion of the total number of iterations which is 800,000.

3.4 Price/Earnings Ratios

Practitioners we have talked to claim that the price/earnings ratio is higher for firms which go public than for unlisted firms which make private sales of equity. Our simulations yield that outcome too. The reason is that a disproportionate number of the direct sales are made by low quality firms, and a disproportionate number of the IPOs are carried out by high quality firms. Table 3 reports transaction prices for direct sales and aftermarket prices for IPOs, and we see that on average the ratio of the latter to the former is about 1.36. Given that past earnings are indistinguishable for the two types, this means that if the P/E ratio is 11 in direct sales, then it should be around 15 for firms which choose to list.

However, the relationship depends crucially on parameter values. If h is generally quite high, adverse selection is a small problem and the typical P/E ratio will be higher in direct sales than in public offers (suppose for example that (a, h) always lie in the regions 4, 6 and 7 of Figure 1).

Of course, it is difficult to observe whether the P/E ratio changes as a result of listing (because previous prices are not listed). However, it is interesting to note that total firm value increases following an equity carve–out, as documented by Schipper and Smith (1986). In line with our

Table 3

	Direct sale	After-market prices All IPOs	Pooling	Semi-sep.
mean	1.540	2.104	2.003	2.337
std	0.483	0.190	0.133	0.000
min	1.200	1.572	1.572	2.337
max	2.400	2.340	2.340	2.337

Transaction prices in direct sales versus transaction prices in the after-market. We have varied $Pr(G|\overline{v}) = \{0.60, 0.70, 0.80, 0.90\}$, $f \in [0.10, 0.49]$, $a \in [0.01, 0.49]$, and $h \in [0.01, 0.99]$ with increments of 0.01. The number of iterations is 800,000.

theory, Schipper and Smith (1989) argue that the positive price reaction could be explained by the fact that firms which have favorable information regarding the performance of one of its divisions have more to gain by letting the division be monitored more closely by the stock market than do firms with less favorable information.[11]

3.5 Welfare Analysis

What are the welfare effects of public listing? The benefit is that more gains from trade are being realized. The cost is the administrative cost of listing. Looking again at Figure 1, it is obvious that the existence of the stock market is socially valuable in parameter regions 2, 3 and 4: In a semi–separating equilibrium, the high value founder and the buyers of stock are strictly better off than in a regime without a stock exchange, whereas the low value founder is indifferent. In a pooling equilibrium, both types of founder are better of, and the buyers are indifferent. However, in region 6 (focussing on the equilibrium favored by the high quality founder) welfare is reduced by the existence of the stock exchange. Essentially, the problem is that there are externalities between the two types of founder. The high value founder does not take into consideration the fact that a low value founder would be better off if both types were to make direct sales. And since the high type goes public, the low type's best response is to go public as well, neglecting in his turn the negative externality this course of action imposes on a high type. Since the total revenues are the same under a public offer and a direct sale, it follows that the existence of the IPO option generates a social loss of a.[12]

[11] For an alternative explanation, see Nanda (1991).

[12] We assume that there are no economic rents embodied in a, i.e. the participants in the financial sector earn their opportunity cost. If a fraction of a represents rents, the welfare measure improves accordingly.

In our simulations (not reported), the maximum welfare gain is 0.145 and the maximum welfare loss is 0.158, and never exceeds 10 per cent of firm value. The average welfare gain from trade when firms make IPOs is 0.011. Thus, the stock market's beneficial effect of increasing trade when adverse selection is severe (in the example $h < 2/3$) is almost wiped out by the wasteful signalling which it induces when adverse selection is less severe. Moreover, the probability that a firm goes public is quite low here, less than eight per cent. Of course, the estimates are highly sensitive to our guesses regarding a and k as well as to the informational content of stock prices. For example, as shown in Table 4, increasing the precision $Pr(G|\overline{v})$ from 0.6 to 0.9 improves the per IPO average net benefit from -0.005 to 0.013 and, more importantly, increases the frequency of IPO's from 0.005 to 0.15. Hence, we suggest that a main benefit from more informationally efficient stock markets is the increased propensity to list.

Table 4

Precision	Mean $E(\pi)$	Frequency Go public	Direct sale	Stay private
$Pr(G\|\overline{v}) = .60$	-0.005	0.005	0.775	0.220
$Pr(G\|\overline{v}) = .70$	0.003	0.026	0.759	0.215
$Pr(G\|\overline{v}) = .80$	0.011	0.067	0.730	0.203
$Pr(G\|\overline{v}) = .90$	0.013	0.150	0.623	0.227

Simulation of the net average social benefit of the stock market (as a fraction of pre-trade value) and a random seller's choice between going public, direct sale, and staying private for different levels of precision. We have varied $Pr(G \mid \overline{v}) = \{0.60, 0.70, 0.80, 0.90\}$, $f \in [0.10, 0.49]$, $a \in [0.01, 0.49]$, and $h \in [0.01, 0.99]$ with increments of 0.01. The number of iterations is 800,000.

Work regarding the social value of stock markets is scanty. A notable recent discussion is presented by Bresnahan, Milgrom and Paul (1992). However, these authors do not consider the costs and benefits associated with the reduction of adverse selection in the trade of shares. In light of our results, one of their main conclusions, that the improved liquidity of stock markets is unlikely to have had a great effect on the social value of the stock exchange, should perhaps be reassessed.

4 Final Remarks

To keep the theoretical analysis as simple as possible, we have treated a number of variables as being exogenous and independent. In a more realistic model, many of these variables would be endogenous. For example, subject to some regulation by the stock exchange, the issuer can decide the offer fraction f. If we allowed f to be endogenous in our model as it stands, it is quite clear that the regulatory minimum would be the unique outcome, as this maximizes the amount which can be cashed in after a good stock market signal.[13] In reality, the precision of the stock market signal is not independent of the fraction of shares sold through the IPO. If the volume of publicly traded shares is small, one cannot expect the signal to be very precise. Thus, a possible extension would be to treat explicitly the trade–off between obtaining a good seasoned offer price and keeping shares to be sold in the seasoned offer.

Central to our analysis is the founder's desire for a liquid market where the firm can be priced. An informative price requires that a sufficently large volume is dispersed among outside investors. There are many ways of achieving this objective which we have neglected here. First, underpricing the IPO may itself improve the precision of the market price [Chemmanur (1993)]. Second, the issuer can partially insure against IPO failure by dispersing shares prior to the IPO. Third, both the IPO and the pre-IPO selling activity may be primary distributions despite the seller's objective to sell the firm entirely.

It could also be worthwhile to endogenize the timing of seasoned offers. This would add a dimension to the signalling problem, as firms could signal high value by waiting to issue seasoned stock (see Nöldeke and van Damme (1990) for a model of endogenous timing in adverse selection problems).

We have assumed that the founders go public to rebalance their portfolios. Undoubtedly, many firms go public to finance growth. Although this case is not analyzed, the intuition of the model may apply. Inside information about the value of growth options can be a reason to obtain a market valuation before raising more equity from the market.

5 Appendix

Recall that l is the (endogenous) probability that a type $\underline{v}$ firm is taken public and m is the (endogenous) probability that a type $\overline{v}$ firm is take public. Thus, the probability of a public offer is given by

$$\gamma := hm + (1-h)l.$$

[13] Interestingly, the size of the IPOs themselves are often "the bare minimum needed to achieve an adequately liquid market" [Röell (1996)], an observation which accords well with our model.

5.1 Proof of Proposition 1.

Let us start by eliminating some strategies which cannot form part of a PBE. It is quite easy to see that there cannot be an equilibrium in which only one type of firm is taken public, i.e. there are no fully separating equilibria.

Lemma 2 *In any PBE such that $\gamma > 0, l > 0$ and $m > 0$.*

Proof: First, suppose $m = 0$. Then $p_s(S) = k\underline{v}$ for any S. As only type $\underline{v}$ firms go public, $p_o \leq k\underline{v}$. Since $a > 0$, $\pi(\underline{v}) < 0$ so $l = 0$. Suppose next that $m > 0$ and $l = 0$. From (**??**), we have $h_s(B) = h_s(G) = 1$, and (**??**) gives $p_s(B) = p_s(G) = k\overline{v}$. But then $\pi(\underline{v}) > \pi(\overline{v}) \geq 0$ contradicting the assumption that $l = 0$ ■

Intuitively, fully separating equilibria in which only good companies are taken public cannot arise because if the equilibrium is fully separating, the IPO itself is a certain signal of high value: If going public is profitable for high value founders when the stock market signal is uninformative, it must be even more profitable for low value founders who value their firm less.

Our next observation is that we may, without any further loss in generality, confine our attention to equilibria in which m is either zero or one.

Lemma 3 *Any PBE in which $0 < m < 1$ is weakly Pareto–dominated by some PBE in which $m = 1$.*

Proof: Consider first an equilibrium in which $1 > m > l$. Let $c := 1/m$. Since $h_s(S)$ only depends on l and m through the fraction m/l we may construct a new equilibrium with $m' = cm = 1$ and $l' = cl < 1$, keeping p_o (or its distribution) and $p_s(S)$ the same. This new equilibrium has the same payoff as the old one. Suppose instead $m < l < 1$. Let $d := 1/l$. This time, construct a new equilibrium with $m' = dm < 1$ and $l' = dl = 1$, keeping p_o (or its distribution) and $p_s(S)$ the same. Now we can do even better: Increasing m further, (l having attained its maximum value), we will raise $h_s(S)$ and thus $p_s(S)$, thereby improving the expected payoff for both types of seller. ■

From (**??**) we see that $h_s(B, p_o) < h^*$ only if $\lambda(p_o) > l^*$, where

$$l^* := \frac{h(1-h^*)Pr(B|\overline{v})}{(1-h)h^*Pr(B|\underline{v})}.$$

From (**??**) it follows that whether or not a high value firm makes a seasoned offer following a bad stock market signal depends on whether $\lambda(p_o)$ is above or below l^*.

What can we say about the offer price distributions g_l and g_m? Let $\mathcal{S}_i := \operatorname{supp} g_i = \{p|g_i(p) > 0\}$ denote the support of g_i. Our first observation is that no offer price can reveal the firm as being of low value.

Lemma 4 *In any equilibrium such that $\gamma > 0$, $\mathcal{S}_l \subseteq \mathcal{S}_m$.*

PROOF: Suppose not. Consider any $p \in \mathcal{S}_l \backslash \mathcal{S}_m$. Setting $p_o = p$ will thus reveal low value. Hence, the seasoned offer price is $k\underline{v}$ and in order to succeed, the initial offer price must be $p \leq k\underline{v}$. The net payoff from going public is hence at most $\pi(\underline{v}) = -a$ so the low value founder is better off making a direct sale. ■

Also, the scope for mixed strategies is limited.

Lemma 5 *For any PBE, let $\mathcal{P} = \{p|\lambda(p) \geq l^*\}$. If $\gamma > 0$, there exists a p' such that $\mathcal{P} \cap \mathcal{S}_l = p'$.*

PROOF: Suppose there are two different offer prices $\{p_1, p_2\} \subseteq \mathcal{P} \cap \mathcal{S}_l$. By Lemma **??**, $\{p_1, p_2\} \subseteq \mathcal{S}_l \cap \mathcal{S}_m$. Since equilibrium requires that both sellers are indifferent between the two offer prices, we must have $\pi(\overline{v}, p_1) = \pi(\overline{v}, p_2)$ and $\pi(\underline{v}, p_1) = \pi(\underline{v}, p_2)$. Recalling (**??**) and (**??**) and using the fact that $\lambda(p) \geq l^*$, the two equilibrium conditions imply

$$f(p_1 - p_2) = P(G|\overline{v})(1 - f)(p_s(G, p_2) - p_s(G, p_1)),$$

and

$$f(p_1 - p_2) = P(G|\underline{v})(1 - f)(p_s(G, p_2) - p_s(G, p_1)).$$

But $P(G|\overline{v}) > P(G|\underline{v})$ so these two equations cannot hold simultaneously. ■

Consider a set of equilibria which have $1 > \lambda(p) \geq l^*$ for all $p \in \mathcal{S}_m$. By Lemma **??**, the initial offer price, p_o is uniquely determined in each such equilibrium (i.e., there is no randomization over initial offer prices). Hence $\lambda(p_o) = l$. As it turns out we can Pareto–rank these equilibria. To see this, notice that semi–separating equilibria are characterized by the equation $\pi(\underline{v}) = 0$. When $l \geq l^*$, $p_s(B) = k\underline{v}$ and we see from (**??**) and (**??**) that the low type's zero–profit condition can be written

$$\pi(\underline{v}) = f(p_o - k\underline{v}) + Pr(G|\underline{v})(1 - f)(p_s(G) - k\underline{v}) - a = 0.$$

Thus, the semi-separating initial offer price is

$$p_o = k\underline{v} + \frac{1}{f}(a - (1-f)Pr(G|\underline{v})[p_s(G) - k\underline{v}]). \tag{11}$$

For any p_o define $l_e(p_o)$ as the unique solution to (**??**). (The solution is unique because $p_s(G)$ is decreasing in l.) Thus, any such semi–separating equilibrium is uniquely characterized by the initial offer price. We write such equilibrium outcomes as $(p_o, l_e(p_o))$. Since there is no randomization, the critical value of l is l^*. Let p_o^* be the (unique) solution to $l_e(p_o^*) = l^*$.

Lemma 6 *Any semi–separating equilibrium outcome $(p_o, l_e(p_o))$ satisfying $1 > l_e(p_o) > l^*$ is Pareto–dominated by the semi–separating equilibrium outcome (p_o^*, l^*).*

PROOF: In a semi-separating equilibrium, $\pi(\underline{v}) = 0$ regardless of p_o. Hence we seek the p_o which maximizes $\pi(\overline{v})$. Differentiation of $\pi(\overline{v})$ yields (see (**??**))

$$\frac{d\pi(\overline{v})}{dp_o} = \frac{\partial \pi(\overline{v})}{\partial p_s(G)} \frac{\partial p_s(G)}{\partial h_s(G)} \frac{dh_s(G)}{dp_o} + \frac{\partial \pi(\overline{v})}{\partial p_o}.$$

Since $\pi(\underline{v}) = 0$, we have that $l_e(p_o)$ must solve

$$h_s(G) = \frac{a + f(k\underline{v} - p_o)}{(1-f)k(\overline{v} - \underline{v})Pr(G|\underline{v})}.$$

Use (**??**) and the fact that

$$\pi(\overline{v}) = f(p_o - \overline{v}) + (1-f)Pr(G|\overline{v})(p_s(G) - \overline{v}) - a$$

to get

$$\frac{d\pi(\overline{v})}{dp_o} = f\left(1 - \frac{Pr(G|\overline{v})}{Pr(G|\underline{v})}\right) < 0,$$

where the inequality follows from the fact that $Pr(G|\overline{v}) > Pr(G|\underline{v})$. ■

As the proof indicates, it pays to lower the offer price down to p_o^*. At this point, if the offer price is lowered further, a new regime is entered, in which we may have that $h_s(B) \geq h^*$. Then both types can make seasoned offers even when the stock market signal is bad. Notice that p_o^* must represent underpricing. (Otherwise the other semi–separating equilibria with higher offer prices and l's could not have been acceptable to buyers.)

Could we ever have an equilibrium outcome in this regime Pareto–dominating the outcome $(p_o^*, l_e(p_o^*))$? The answer is no, as we shall now prove.

Lemma 7 *Any semi–separating equilibrium outcome* $(p_o, \lambda(p_o))$ *satisfying* $\lambda(p_o) < l^*$ *is Pareto–dominated by the semi–separating equilibrium outcome* (p_o^*, l^*).

PROOF: We proceed in two steps. First, we ask: How much below p_o^* must the initial offer price be reduced to induce some $\lambda(p_o) < l^*$? The condition for a semi–separating equilibrium with $\lambda(p_o) < l^*$ is that

$$\pi(\underline{v}) = f(p_o - k\underline{v}) + (1-f)[P(G|\underline{v})(p_s(G) - k\underline{v}) + P(B|\underline{v})(p_s(B) - k\underline{v})] - a = 0. \tag{12}$$

Let p_o' denote the highest offer price which satisfies this equation. Let $p_s'(G)$ and $p_s'(B)$ denote the corresponding seasoned offer prices. (Notice that a high p_o must come at the expense of a low expected seasoned offer price for the equality to hold.) Consistency requires that $p_s'(B) \geq \overline{v}$. To find the difference between $p_o^* - p_o'$, recall that p_o^* and the corresponding seasoned offer price $p_s^*(G)$ solves

$$f(p_o^* - k\underline{v}) + (1-f)P(G|\underline{v})(p_s^*(G) - k\underline{v}) - a = 0. \tag{13}$$

Since $h_s(B, p_o') > h_s(B, p_o^*)$ we must also have $h_s(G, p_o') > h_s(G, p_o^*)$ and hence $p_s'(G) > p_s^*(G)$. It follows that

$$f(p_o' - k\underline{v}) + (1-f)[P(G|\underline{v})(p_s^*(G) - k\underline{v}) + P(B|\underline{v})(\overline{v} - k\underline{v})] - a > 0. \tag{14}$$

Subtracting the left hand side of (**??**) from the left hand side of (**??**) yields after some manipulation

$$f(p_o' - p_o^*) = (1-f)[Pr(G|\underline{v})(p_s'(G) - p_s^*(G)) + Pr(B|\underline{v}(\overline{v} - k\underline{v})]. \tag{15}$$

This, then, is an expression for the loss associated with a lower initial offer price.

Then we ask: Can such a reduction in the initial offer price be justified by the higher expected seasoned offer price? The benefit to the high value founder of moving into a regime in which $p_s(B) > \overline{v}$ is

$$b := (1-f)[Pr(G|\overline{v})(p_s'(G) - p_s^*(G)) + (Pr(B|\overline{v})(p_s(B) - \overline{v}). \tag{16}$$

Thus, the reduction in the initial offer price is unjustified if $f(p_o' - p_o^*) > b$, or equivalently if

$$p_s'(G) - p_s'(B) - p_s^*(G) < \frac{Pr(B|\overline{v})\overline{v} - Pr(B|\underline{v})k\underline{v}}{Pr(G|\overline{v}) - Pr(G|\underline{v})}. \tag{17}$$

To evaluate this condition, notice that

$$p_s'(G) - p_s'(B) = k(h_s(G, p_o') - h_s(B, p_o'))(\overline{v} - \underline{v}).$$

Thus, we want to evaluate (**??**) at the initial offer price which maximizes $h_s(G) - h_s(B)$. Equivalently, we seek the level of λ which maximizes $h_s(G) - h_s(B)$ subject to $\lambda < l^*$. Differentiation, using (**??**), yields

$$\frac{\partial h_s(G)}{\partial \lambda} - \frac{\partial h_s(B)}{\partial \lambda} = \left(\frac{Pr(B)}{Pr(G)}\right)^2 - \frac{Pr(B|\underline{v})Pr(B|\overline{v})}{Pr(G|\underline{v})Pr(G|\overline{v})},$$

where we have written $Pr(S) := Pr(S|\overline{v})h + Pr(S|\underline{v})(1-h)\lambda$. Since $Pr(B)/Pr(G)$ is monotonically increasing in λ, it follows that $h_s(G) - h_s(B)$ is monotonically increasing in λ. Thus, the relevant seasoned offer prices become $p_s'(G) = \lim_{\lambda \to l^*} p_s(G) = p_s^*(G)$ and $\lim_{\lambda \to l^*} p_s(B) = \overline{v}$. Inserting back into (**??**) we see that the reduction in the initial offer price is unjustified, because the requirement

$$-\overline{v} < \frac{Pr(B|\overline{v})\overline{v} - Pr(B|\underline{v})k\underline{v}}{Pr(G|\overline{v}) - Pr(G|\underline{v})}$$

is equivalent to the statement

$$\overline{v} > Pr(G|\underline{v})\overline{v} + (1 - Pr(G|\underline{v})k\underline{v}$$

which is true because $\overline{v} > k\underline{v}$. ■

This completes the proof of Proposition **??**

5.2 Proposition 4

We restrict ourselves to the case in which $h < h^*$, and hence $\max\{p_d, \overline{v}\} = \overline{v}$. The proposition is proved first for a pooling equilibrium and then for a semi–separating equilibrium. Keep in mind that $\overline{v}/\underline{v}$ is independent of α, i.e.

$$\frac{d\overline{v}}{d\alpha}\frac{1}{\overline{v}} = \frac{d\underline{v}}{d\alpha}\frac{1}{\underline{v}}. \tag{18}$$

Consider a pooling equilibrium and suppose that $h_s(B) < h^*$. Differentiation in (??) then yields

$$\begin{aligned}
\frac{d\pi(\overline{v})}{d\alpha} &= (1-f)Pr(G|\overline{v})\left(k\left(h_s(G)\frac{d\overline{v}}{d\alpha} + (1-h_s(G))\frac{d\underline{v}}{d\alpha}\right) - \frac{d\overline{v}}{d\alpha}\right) \\
&+ f\left(k\left(h_s\frac{d\overline{v}}{d\alpha} + (1-h_s)\frac{d\underline{v}}{d\alpha}\right) - \frac{d\overline{v}}{d\alpha}\right).
\end{aligned}$$

The first term is the effect on the profit from the seasoned offer, and the second term the profit from the initial offer. Using equation (??) to substitute for $\underline{v}$ and rearranging, this can be written as

$$\begin{aligned}
\frac{d\pi(\overline{v})}{d\alpha} &= \frac{d\overline{v}}{d\alpha}\frac{1}{\overline{v}}((1-f)Pr(G|\overline{v})(p_s(G) - \overline{v})) - f(\overline{v} - p_o)) \\
&= \frac{d\overline{v}}{d\alpha}\frac{1}{\overline{v}}(\pi(\overline{v}) + a),
\end{aligned}$$

which is positive since $\pi(\overline{v}) + a > 0$.

It remains to check that α does not affect the choice between direct sale and public offer. The payoff to a high value firm from direct sale is $p_d - \overline{v}$. Differentiating using equation (??), we get

$$\frac{d(p_d(\overline{v}) - \overline{v})}{d\alpha} = \frac{d\overline{v}}{d\alpha}\frac{1}{\overline{v}}(p_d - \overline{v}).$$

Hence, given that the high value owner's pay-off from a direct sale is negative in the first place, improved stock market conditions is further discouragement.

Consider a semi-separating equilibrium. Recall that

$$p_o = k\underline{v} + \frac{1}{f}(a - (1-f)Pr(G|\underline{v})(p_s(G) - k\underline{v})).$$

Suppose it is optimal to set $p_o = p_o^*$ (this is the initial offer price which forces l down to l^*). Differentiation of $\pi(\overline{v})$ yields

$$\begin{aligned}
\frac{d\pi(\overline{v})}{d\alpha} &= (1-f)Pr(G|\overline{v})\Bigg[\frac{\partial p_s(G)}{\partial \overline{v}}\frac{d\overline{v}}{d\alpha} + \frac{\partial p_s(G)}{\partial \underline{v}}\frac{d\underline{v}}{d\alpha} + \\
&\quad \frac{\partial p_s(G)}{\partial h_s(G)}\frac{\partial h_s(G)}{pl^*}\left(\frac{\partial l^*}{\partial \overline{v}}\frac{d\overline{v}}{d\alpha} + \frac{\partial l^*}{\partial \underline{v}}\frac{d\underline{v}}{d\alpha}\right) - \frac{d\overline{v}}{d\alpha}\Bigg] \\
&+ f\Bigg[\frac{\partial p_o^*}{\partial \overline{v}}\frac{d\overline{v}}{d\alpha} + \frac{\partial p_o^*}{\partial \underline{v}}\frac{d\underline{v}}{d\alpha} + \frac{\partial p_o^*}{\partial p_s(G)}\Bigg(\frac{\partial p_s(G)}{\partial \overline{v}}\frac{d\overline{v}}{d\alpha} + \frac{\partial p_s(G)}{\partial \underline{v}}\frac{d\underline{v}}{d\alpha} + \\
&\quad \frac{\partial p_s(G)}{\partial h_s(G)}\frac{\partial h_s(G)}{\partial l^*}\left(\frac{\partial l^*}{\partial \overline{v}}\frac{d\overline{v}}{d\alpha} + \frac{\partial l^*}{\partial \underline{v}}\frac{d\underline{v}}{d\alpha}\right)\Bigg) - \frac{d\overline{v}}{d\alpha}\Bigg]
\end{aligned}$$

Note that

$$\frac{\partial l^*}{\partial \overline{v}}\frac{d\overline{v}}{d\alpha} + \frac{\partial l^*}{\partial \underline{v}}\frac{d\underline{v}}{d\alpha} = \frac{h(1-h)k(k-1)Pr(B|\overline{v})Pr(B|\underline{v})}{[(1-h)Pr(B|\underline{v})(\overline{v} - k\underline{v})]^2}\left(\overline{v}\frac{d\underline{v}}{d\alpha} - \underline{v}\frac{d\overline{v}}{d\alpha}\right),$$

which is equal to 0 by equation (**??**). Hence,

$$\begin{aligned}
\frac{d\pi(\overline{v})}{d\alpha} &= \frac{d\overline{v}}{d\alpha}\frac{1}{\overline{v}}\left[(1-f)Pr(G|\overline{v})(p_s(G) - \overline{v}) + a - f(\overline{v} - p_o)\right] \\
&= \frac{d\overline{v}}{d\alpha}\frac{1}{\overline{v}}[\pi(\overline{v}) + 2a],
\end{aligned}$$

which proves the proposition for $p_o = p_o^*$. ■

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