Properties of Actuarially-Fair and Pay-As-You-Go Health Insurance Schemes for the Elderly. An OLG Model Approach

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Abstract: The aged dependency ratio or ADR is growing at a fast pace in many countries. This fact causes stress to the economy and might create conflicts of interest between young and old. In this paper the properties of different health insurance systems for the elderly are analysed within an overlapping generations (OLG) model. The properties of actuarial health insurance and different variations of pay-as-you-go health insurance are compared. It turns out that the welfare properties of these contracts are heavily dependent on the economy's dynamic properties. Of particular importance is the magnitude of the rate of population growth relative to the interest rate. In addition it is shown that public health insurance is associated with an inherent externality resulting in a second-best solution.

1. Introduction

In many countries the population is ageing. This means that what is known as the "aged dependency ratio" or ADR is increasing. This ratio is defined as the number of people aged 65+ for every 10 people aged 15-64. The average for the OECD countries is that there are 4 people aged 65 and over for every 10 people aged 15 to 64. By the year 2025, there will be 8 people aged 65+ for every 10 aged 15-64 (Disney (1996)).

An increasing ADR causes stress to the economy and may create conflicts of interest between young and old generations. There are relatively fewer young people to pay for pensions and health care for the elderly. Studies for the US show that, on average, individuals 65 years old or older have four times the health care spending of younger people (Besley and Gouveia (1994, p. 213)). Nevertheless, Besley and Gouveia (1994, pp. 213-214) claim that the health care expenditure implications of ageing during the 1980s were smaller than might have been expected. However, they also conclude that the implications of ageing might be larger in the future as the proportion of elderly increases at a fast pace. Therefore, it seems to be an important task to study the long-term consequences of ageing populations for the sustainability and robustness of different health care (and pension) systems.

There are basically three types of health care system (Besley and Gouveia (1994)). In the first type there is (substantial) private financing and delivery. The only example of such a system seems to be provided by the US. In the second type there is public financing and (substantial) private delivery. Japan and many continental European countries have this kind of system. The third type is characterised by substantial public financing and delivery. This category includes the Scandinavian countries, and countries with a National Health Service (NHS), a set which includes the UK and the Southern European countries.

There is a wide spread across countries in the way public health is funded. In some countries it comes from earmarked taxes or earmarked social security fees. In other countries public health care is funded from general revenue. The US Medicare programme, which provides compulsory (and voluntary supplemental) health insurance for those aged 65 and over, is financed from payroll taxes on employers and employees. (However, the tax treatment of firms effectively subsidises employer-paid health insurance; see Jack and Sheiner (1997) for details.)

Voluntary health insurance may cause an uninsured problem[1]. Governments may provide a substitute for insurance by directly funding health care for the poor (e.g. Medicaid in the US) and the long-term sick. An alternative solution is to make insurance compulsory. Then there is universal access to health insurance as well as health care. Most Western countries seem to choose the latter approach; see, for example, Oxley and MacFarlan (1994).

There is a growing literature on the properties of different pension systems; see, for example, Blanchard and Fischer (1989), Hassler and Lindbeck (1998) and Meijdam and Verbon (1996). However, little research seems to have been undertaken with respect to the intergenerational effects of (a possibly growing) demand for health services by the elderly[2]. The purpose of this paper is to examine the properties of different types of health insurance within an overlapping generations model framework. Problems of intragenerational equity, moral hazard and adverse selection in health care systems are set aside in order to focus on compulsory health insurance for the elderly. This focus of the paper is motivated by the assumption that the ADR continues to grow in the OECD countries. Thus it seems to be an important task to examine the long-run properties of different systems for the financing of health care for the elderly.

In the model used in this paper individuals live for two periods. In the first period they consume non-health goods and supply labour. In the second period they are retired and consume both health goods and non-health goods. At each point in time there are two generations (young and old, respectively). A young person does not know his health status as an old person with certainty. Thus there is a role for a health insurance to play. The reference case here is actuarial private insurance. An alternative is pay-as-you-go (PAYG) health insurance financed by a proportional tax on labour income. This variation is reminiscent of the US Medicare system. Alternatively, the PAYG health insurance is funded from a proportional tax on labour plus pension income. Thus the insurance is financed from general revenue. This variation resembles the system used in Scandinavia and countries with an NHS. However, in the basic case considered in this paper, taxes can be considered as earmarked since the government's health care budget balances in each period.

The analysis is focused on the consequences of health insurance for a typical individual. Occasionally reference is made to the dynamic general equilibrium consequences of health insurance. However, the important but difficult question of how health insurance impacts on the economy's long-term stock of capital is left for future research. As is evident from Blanchard and Fischer (1989, Chapter 3) it is very difficult to determine the dynamic properties of an OLG-model economy. Often, very specific assumptions must be introduced in order to be able to derive meaningful results.

The paper is structured as follows. In Section 2 actuarial health insurance is introduced, and its properties are analysed. Section 3 introduces a pay-as-you-go fixed fee pension system. The properties of the two systems are compared and discussed in Sections 4 and 5. Section 6 is devoted to the effects of an increased ADR, while Section 7 contains a few concluding remarks.

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2. Actuarially fair health insurance for the elderly

Each individual lives for two periods. In the first period he consumes private goods and supplies (one unit of) labour. By assumption, he is healthy. In the second period he is retired, and his health is stochastic, as viewed from period 1. However, uncertainty is temporal in the sense that his true health state is revealed at the end of period 1 (i.e., before second-period decisions are made). This is the only kind of risk in the model. Consider an individual born at time t ($t=0,\ldots,T$). Viewed from time t, his expected present value utility is:

$$E_t\left[U_{t,t+1}\right]=u(c_{1,t},h^f)+\int_\Omega \gamma u\left[c_{2,t+1}(\theta),T,h_{2,t+1}(\theta)\right]dF(\theta) \tag{1}$$

where $u(.)$ is a smooth cardinal utility function, $c_{i,t}$ denotes consumption when young ($i=1$) and old ($i=2$), h^f denotes full health, γ is the discount factor, θ is used to model health status, $\theta\in\Omega=[\theta^l,\ \theta^f]$ with θ^l (θ^f) denoting the worst (best) health status, $h_{2,t+1}(\theta)$ denotes health goods consumption when old if state θ is realised, and $F(.)$ is a distribution function with support Ω. It is assumed that F(.) applies to all generations. The assumption that the utility function U(.) is separable is motivated by the fact that we want to draw on results derived by Blanchard and Fischer (1989) and others concerning the properties of OLG models.

The present value budget constraint is:

$$\begin{aligned}&w_t-\rho-c_{1,t}+\\ &R_{t+1}\left[y_{2,t+1}-c_{2,t+1}(\theta)-(1-\alpha_{t+1})h_{2,t+1}(\theta)\right]=0 \qquad ;\forall\theta\end{aligned} \tag{2}$$

where w_t is the real wage rate in period t, ρ is an insurance premium, $R_{t+1}=1/(1+r_{t+1})$, r_{t+1} is the market interest rate, $y_{2,t+1}$ denotes pension income, and $1-\alpha_{t+1}$ is the coinsurance rate, i.e. α_{t+1} is the fraction of the individual's total health services that the insurance covers. We do not consider here how the pension $y_{2,t+1}$ is financed. In order to avoid having to introduce two production sectors, it is assumed that the economy's single good is used for direct consumption as well as for improving health[3]. Finally, we assume that a large number of identical individuals are born at each point of time. Then the realised per capita health goods demand can be assumed to be equal to its expected value.

The actuarial-fairness constraint is:

$$R_{t+1}\int_\Omega \alpha_{t+1}h_{2,t+1}(\theta)dF(\theta)=R_{t+1}\alpha_{t+1}h_{t+1}^E=\rho \tag{3}$$

where a superscript E refers to an expected value. Here a uniform coinsurance rate is assumed. This is a reasonable assumption if insurers cannot observe individual true health status. It is assumed that competition forces insurers to invest their funds so as to achieve the market return, i.e. r_{t+1}.

Assume that the considered individual has arrived at time $t+1$ where the magnitude of θ is revealed. Then he maximises his utility subject to the relevant period $t+1$ budget constraint[4]. Assume that there is a well-behaved unique interior solution to this maximisation problem. Then, in principle, his demand functions for non-health and health goods, respectively, as a retired person can be written as follows:

$$c_{2,t+1}(\theta) = c_{2,t+1}(y_{t,t+1}, 1-\alpha_{t+1}, \theta)$$

$$h_{2,t+1}(\theta) = h_{2,t+1}(y_{t,t+1}, 1-\alpha_{t+1}, \theta) \tag{4}$$

where $y_{t,t+1} = y_{2,t+1} + w_t - \rho - c_{1,t}$, $c_{1,t}$ and ρ are fixed, the discount factor R_{t+1} is suppressed in order to simplify notation, and θ takes on a specific value. The demand functions are assumed to have the usual properties; for example, demand is assumed to be increasing (decreasing) in income (own price).

Using the demand functions and the two budget constraints in equation (2), expected present value utility at time t can be written as follows:

$$\begin{aligned} U^E_{t,t+1} &= E_t\left[U_{t,t+1}\right] = u\left[c_{1,t}, h^f\right] + \int_\Omega \gamma V\left[c_{2,t+1}(.), h_{2,t+1}(.), \theta\right] dF(\theta) = \\ &u[.] + E_t\left[\gamma v\left[y_{t,t+1}, 1-\alpha_{t+1,}\theta)\right]\right] \end{aligned} \tag{5}$$

where E_t refers to an expectations operator, and $v[.]$ is a variable indirect utility function; see Epstein (1975) for details. The individual is assumed to choose consumption as young and the health insurance contract so as to maximise expected present value utility. Let us write the insurance premium in the income argument $y_{t,t+1}$ in equation (5) as $\rho=\rho(c_{1,t}, \alpha_{t+1})$. Then the individual can be viewed as choosing from a menu of insurance contracts α_{t+1}, $\rho(.)$, while equation (3) can be interpreted as the equilibrium condition for the insurance market (per capita). Given this approach, two necessary conditions for an interior solution to the maximisation problem are that $c_{1,t}$ and α_{t+1} are chosen as follows:

$$\begin{aligned} &u_{c_{1,t}} - \gamma v^E_{y_{2,t+1}}\left[1 + \alpha_{t+1} h^E_{y_{t,t+1}}\right]^{-1} = 0 \\ &\gamma v^E_{y_{t,t+1}}\left[\alpha_{t+1}\left[h^E_{y_{t,t+1}}(\partial\rho/\partial\alpha_{t+1}) + h^E_{p_{2,t+1}}\right] - h^E_{2,t+1}\right] + \gamma E_t\left[v_{y_{t,t+1}}(.)h_{2,t+1}(.)\right] = 0 \end{aligned} \tag{6}$$

where a superscript E refers to an expected value, a subscript $c_{1,t}$ refers to a partial derivative with respect to consumption of non-health goods of young persons in period t, and a subscript

y (p) refers to a partial derivative with respect to income (price, i.e. 1 - α_{t+1}). The reader is also referred to equation (A.1) in the Appendix. The expression within brackets in the first line in equation (6) has been obtained by using equation (3) in order to eliminate $\partial\rho/\partial c_{1,t}$. The optimum consumption as a young person is such that the marginal utility of income as a young person is equal to the expected present value marginal utility of income as an old person, adjusted for the impact of consumption (or equivalently savings) while young on the cost of the insurance contract. The second line in equation (6) yields the condition for the optimal choice of a health insurance contract. This condition will be discussed further below.

In order to facilitate comparisons between different health insurance contracts, we will consider the effect on expected present value utility of small changes in the coinsurance rate. Substitution of equation (3) into equation (5) and differentiation with respect to α_{t+1}, using the first-order condition for optimal consumption when young, yields after some calculations:

$$dU_{t,t+1}^{E} = \gamma v_{y_{t,t+1}}^{E} \alpha_{t+1}\left[h_{p_{2,t+1}}^{Ec} - \operatorname{cov}(h_{2,t+1}, h_{y_{t,t+1}})\right](1+\alpha_{t+1}h_{y_{t,t+1}}^{E})^{-1} d\alpha_{t+1} + \gamma \operatorname{cov}(v_{y_{t,t+1}}, h_{2,t+1}) d\alpha_{t+1} \quad (7)$$

where a superscript c refers to a compensated price effect. The reader is referred to equations (A.2)-(A.4) in the Appendix for details.

Equation (7) contains the same terms as the corresponding expression for an atemporal model; see Jack and Sheiner (1997) for details. If health insurance is purchased at all, the optimal insurance contract is such that equation (7) is equal to zero (assuming throughout that the second-order conditions for an interior solution are satisfied). Thus at the optimum, small variations of ρ and α_{t+1} will not affect the individual's expected present value utility at time t. By assumption, the expected marginal utility of income is positive, health goods are normal, and the substitution effect of an increase in the coinsurance rate is negative. Hence a sufficient condition for an interior solution is that the two covariance terms in equation (7) are positive[5].

Assume that for any given coinsurance rate, healthier people (people experiencing high realised θ-values) consume less health goods, i.e. $\partial h_{2,t+1}/\partial\theta<0$. If the marginal propensity to spend on health goods is lower for healthier persons, i.e. $\partial h_{y_{t,t+1}} / \partial\theta < 0$, then the first covariance term in equation (7) is positive. The second covariance term in equation (7) is positive if the marginal utility of income is decreasing in θ, i.e. if $\partial v_{y_{t,t+1}} / \partial\theta < 0$ so that the marginal utility of income is lower for healthier people. If insurance is purchased, it can safely be claimed that the optimal coinsurance rate is strictly positive. If α_{t+1} were equal to unity so that the effective price of health services was equal to zero, then the individual would demand virtually an infinite amount of such services (unless the demand approached a finite level as the price approaches zero). Then there would be no equilibrium in the insurance market.

3. Pay-as-you-go health insurance

In this section (fixed fee) PAYG health insurance for the elderly is introduced. An individual born at time t maximises his present value utility, see equation (1), subject to the budget constraint:

$$w_t(1-\tau)-c_{1,t}+$$

$$R_{t+1}\left[y_{2,t+1}(1-\delta\tau)-c_{2,t+1}(\theta)-(1-\alpha_{t+1})h_{2,t+1}(\theta)\right]=0 \qquad ;\forall\theta \tag{8}$$

where τ is a tax on income imposed in order to finance health insurance for the elderly, $y_{2,t+1}$ denotes pension income, and δ is a dummy variable; δ=1 if retired people contribute to their own health insurance and δ=0 if not.

Let us consider a compulsory pay-as-you-go variation of the health care insurance system. The government's period $t+1$ (balanced) health insurance budget constraint is:

$$N_t \int_\Omega \alpha_{t+1}h_{2,t+1}(\theta)dF(\theta)=N_{t+1}\tau w_{t+1}+N_t\delta\tau y_{2,t+1} \tag{9}$$

where N_t is the number of (identical) individuals born at time t, i.e. N_t/N_{t+1} yields what is known as the aged dependency ratio (ADR). The government collects revenue by imposing a proportional tax τ on labour income and, possibly, also on pension income. The case δ=0 is similar to Medicare, where health insurance for the elderly is financed through payroll taxes. In the UK and the Scandinavian countries it is rather the case that δ=1. By assumption, the entire health tax revenue is spent on subsidising health care for the older generation. We allow for population growth by letting the number of individuals N_t born at time t to vary with t. Since the focus is on intergenerational issues, it is assumed that there are N_t identical individuals in each generation, and that there is full employment (N_t = L_t, where L_t is the aggregate period t demand for labour) for the currently young generation.

In this case, the individual's period $t+1$ demand function for health goods can be written as follows:

$$h_{2,t+1}(\theta) = h_{2,t+1}(y_{t,t+1}, 1-\alpha_{t+1}, \theta) \tag{10}$$

where $y_{t,t+1} = (1-\delta\tau)y_{2,t+1} + w_t(1-\tau) - c_{1,t}$, and the discount factor is once again suppressed.

Thus his expected present value utility at time *t* can be expressed as follows:

$$U^E_{t,t+1} = E_t\left[U_{t,t+1}\right] = u\left[(c_{1,t}, h^f)\right] + E_t\left[\gamma v\left[y_{t,t+1}, 1-\alpha_{t+1}, \theta\right]\right] \tag{11}$$

For a fixed insurance contract, first-order conditions for an interior solution to the utility maximisation problem include:

$$u_{c_{1,t}} - \gamma v^E_{y_{t,t+1}} = 0 \tag{12}$$

That is, the individual chooses the consumption level while young in such a way that the marginal utility of income while young is equal to the expected marginal utility of income when old. There is a difference between this condition and the one in the first line of equation (6). In the case of PAYG insurance the individual treats the insurance contract as fixed. Hence the individual ignores the impact of his savings on the cost for the government's insurance plan in equation (9).

Next, turning to the effects on expected present value utility of marginal changes in the tax and coinsurance rates one obtains:

$$dU^E_{t,t+1} = -\gamma v^E_{y_{t,t+1}}\left[w_t d\tau + R_{t+1} y_{2,t+1}\delta d\tau\right] + R_{t+1}\gamma E_t\left[\ v_{y_{2,t+1}}(.)h_{2,t+1}(.)\ \right]d\alpha_{t+1} \tag{13}$$

where the discount factor R is shown explicitly, for reasons to be explained below. Next multiply equation (9) by $1/g_{t+1}$, where $g_{t+1} = \eta_{t+1}w_{t+1}/w_t$ is one plus the growth of GDP, and $\eta_{t+1} = N_{t+1}/N_t$ is the inverse of the aged dependency ratio. Then differentiate the expression, and use it to eliminate $w_t d\tau$ from equation (13). After straightforward calculations, one arrives at the following huge expression:

$$\begin{aligned} dU^E_{t,t+1} &= \gamma v^E_{y_{t,t+1}}(R_{t+1} - g^{-1}_{t+1})h^E_{2,t+1}d\alpha_{t+1} - \gamma v^E_{y_{t,t+1}}(R_{t+1} - g^{-1}_{t+1})y_{2,t+1}d\tau + \\ &g^{-1}_{t+1}\gamma v^E_{y_{t,t+1}}\alpha_{t+1}h^E_{y_{t,t+1}}\left[dc_{1,t} + w_t d\tau + R_{t+1}y_{2,t+2}d\tau\right] + R_{t+1}g^{-1}_{t+1}\gamma v^E_{y_{t,t+1}}\alpha_{t+1}h^E_{p_{2,t+1}}d\alpha_{t+1} + \\ &\gamma R_{t+1}\operatorname{cov}(v_{y_{t,t+1}}, h_{2,t+1})d\alpha_{t+1} \end{aligned} \tag{14}$$

The income effects term shows up in equation (14) since the individual maximises utility given τ. That is, he fails to internalise the effects of his actions on the health insurance budget in equation (9). Equation (14) is more complicated than the one obtained in the case of actuarially fair insurance; see equation (7). This is so because the properties of the PAYG insurance contract depend on the market rate of interest as well as on the growth rates of GDP and population. If there is a unique interior solution, the optimal PAYG insurance contract must be such that equation (14) is equal to zero. Throughout it is assumed that the maximisation problem is well behaved so that the first-order approach is satisfactory.

A comparison of actuarially fair insurance and PAYG insurance is undertaken in Sections 4 and 5. However, in order to examine some of the intergenerational properties of PAYG health insurance, the following definition is introduced.

Definition: If $g_{t+1}-1<(>)r_{t+1}$, then the economy is said to be dynamically efficient[6] (inefficient). The economy is in a steady state if consumption per capita is constant over time. A steady state with g-1=η-1 = r, i.e. with the rate of growth of population equal to the market rate of interest, is Pareto optimal[7] (the golden rule).

Let us consider the introduction of PAYG insurance, i.e. evaluate equation (14) at $\alpha=\tau=0$. Then we arrive at the following result.

Proposition 1: *Consider a young individual living in a dynamically efficient (inefficient) economy. This individual will prefer (not prefer) a small PAYG health insurance scheme financed by a proportional tax on labour income plus pension income to a scheme financed by a tax on labour income. If the economy is in a golden rule steady state he will be indifferent between the schemes.*

This result follows directly from equation (14) with δ=1. The explanation for this Proposition is the fact that it pays to pay for insurance while young if the economy is growing fast. The return on savings in health insurance will be higher than the return on conventional savings. However, the mechanism behind this result is not identical to the one in operation in the case of a lump-sum subsidy to health insurance financed by a lump-sum tax on the young. Consider once again a small PAYG insurance.

Proposition 2: *Consider a lump-sum subsidy to health insurance financed by a lump-sum tax on the young (evaluated at $\alpha=\tau=0$). Given wages and interest rates, the currently young will lose (gain) from the considered lump-sum redistribution if the real wage rate is growing (decreasing) over time.*

In this case, equation (14) would have to be augmented by the term $-\gamma v^{E}_{y_{t,t+1}} d\Gamma +$ $(w_t / w_{t+1})\gamma v^{E}_{y_{t,t+1}} d\Gamma$, where $d\Gamma$ denotes the change in the lump-sum tax, and we have used the fact that $g^{-1}_{t+1}\eta_{t+1} = w_t / w_{t+1}$, and assumed that $\alpha_{t+1}=0$. Thus the lump sum $d\Gamma$ will not vanish from the expression unless the real wage rate is constant over time. The individual gains (loses) if the real wage rate falls (increases) over time. This reflects the fact that the shift is from a proportional tax on labour income, which may be changing over time, to a constant lump-sum tax. If $\alpha_{t+1}>0$ initially, there is also the usual endogenous adjustment in the demand for health goods, explaining the fact that Propositions 1 and 2 only refers to a small PAYG insurance.

4. A comparison of the two insurance contracts

In this section we compare the properties of actuarially fair health insurance and PAYG health insurance. In so doing we use throughout the assumptions that there are well-behaved interior solutions to the utility maximisation problems examined in Sections 2 and 3, and that individuals are identical. This set of assumptions will be denoted A1 in what follows.

Let us first examine a general property of PAYG insurance. To make this property transparent, let us assume that the economy is in a golden rule steady state (for simplicity with $\delta=0$). Then equation (14) reduces to:

$$dU^{E}_{t,t+1} = \gamma v^{E}_{y_{t,t+1}} \alpha_{t+1} h^{E}_{y_{t,t+1}} \left[1+\alpha_{t+1} h^{E}_{y_{t,t+1}}\right]^{-1} dc_{1,t} + \tag{15}$$

$$\gamma v^{E}_{y_{t,t+1}} \alpha_{t+1}\left[h^{Ec}_{p_{2,t+1}} - \operatorname{cov}(h_{2,t+1}, h_{y_{t,t+1}})\right](1+\alpha_{t+1} h^{E}_{y_{t,t+1}})^{-1} d\alpha_{t+1} + \gamma \operatorname{cov}(v_{y_{t,t+1}}, h_{2,t+1}) d\alpha_{t+1}$$

where the discount factor R is suppressed in order to facilitate comparisons with equation (7). In this case PAYG health insurance is actuarially fair since $g-1=\eta-1=r$ (and a proportional tax on a fixed supply of labour works like a lump-sum tax). Equation (15) contains the same terms as equation (7) plus an income effects term related to changes in $c_{1,t}$. This term is negative under the conditions stated in the Appendix; see equation (A.7). The term in question appears because the individual fails to internalise the effect of his first-period consumption decision on the health budget. This can be seen from a comparison of the first line in equation (6) and equation (12). We therefore arrive at the following result.

Proposition 3: *Given A1, assume that* g-1=η-1=r*, and that* δ=0*, i.e. that public insurance is actuarially fair. Such health insurance will result in a "second-best" solution (in comparison to the solution implied by eq. (7)*[8]*).*

As explored above, the second-best nature of the solution is due to the fact that individuals fail to internalise the impact of their actions on the public sector's health budget[9]. It might be noted that this second-best nature of the solution vanishes in an atemporal model of the kind considered by, for example, Blomqvist and Johansson (1996), and Jack and Sheiner (1997). In such a model all consumption decisions are taken conditional on the health insurance contract. Therefore, it does not matter whether the contract is designed by the individual or by the government. Things are different in a two-period model, where individuals both save and purchase private insurance (unless retired have quasi-linear utility functions so that health demand is independent of income). According to the maximisation problem considered in Section 2, individuals simultaneously choose insurance contract <u>and</u> consumption while young. This determines their disposable income when old and hence their (expected) demand for health goods. On the other hand, if there is a government plan of the kind considered in Section 3, individuals, just as is the case in the atemporal model, maximise expected utility conditional on the government plan. However, in an intertemporal model they thereby overlook or ignore the fact that the cost of the government plan depends on how much they save while young, i.e. on their disposable income when retired.

It might be argued that from the viewpoint of a single individual, the effect of his savings decision on the average cost of the government plan is negligible. When all individuals together act in this way, however, the resulting equilibrium will be inferior (in the sense of expected utility) to the optimum implicitly defined by equation (7). It might also be argued that it is a question of realism whether private insurance markets work in the way assumed in Section 2, where the cost of a contract was specified as a function of $c_{1,t}$ and α_{t+1}. It might be the case that it is more realistic to assume that individuals are offered contracts whose cost (explicitly) depends only on the coverage rate. Then an individual chooses consumption as a young person and insurance contract, viewing the premium as a function of the coverage rate, i.e. $\rho=\rho(\alpha_{t+1};c_{1,t})$, where $c_{1,t}$ is treated as a constant by the individual. However, competition forces the contracts α_{t+1}, $\rho(.)$ offered in the market to be such that insurers just break even in equilibrium. Thus equilibrium in the insurance market (per capita) is still given by equation (3). In this case private health insurance will result in a second-best solution that is parallel to the one in equation (15). First-order conditions for an interior solution to this second-best private contract problem are stated in equation (A.1') in the Appendix.

If the individual treats the insurance contract (whether private or public) as fixed he ignores the fact that the higher is his savings the higher is his demand for health goods as old. This seems to indicate that subsidising consumption of young persons might help to internalise the "externality" under consideration. In fact, the following result is obtained.

Proposition 4: *Given A1, g-1=η-1=r and δ=0, assume that consumption while young is subsidised at a rate* $\tau^c = \alpha_{t+1} h^E_{y_{t,t+1}} (1+\alpha_{t+1} h^E_{y_{t,t+1}})^{-1}$, *and that the subsidy is financed by a lump-sum tax. Then the optimal public health insurance contract will coincide with the optimal private contract implicitly defined by equation (7).*

See the Appendix for a proof. The Proposition will of course also hold for "second-best" private health insurance of the kind discussed above. In reality, it is not possible to discriminate between consumption while young and when old. Therefore, in practice, Proposition 4 must be interpreted as suggesting a tax on savings.

The question arises whether the second-best property of tax-financed public health insurance vanishes if supplementary actuarially fair insurance, as implicitly defined by equation (7), is allowed (while multiple, i.e. duplicate, private coverage is ruled out). Assume that initially $\alpha = \alpha^g$, where a superscript g refers to the PAYG system. Next, the individual is allowed to choose the utility maximising level of private coverage α^p. Then, borrowing the notation from Blomqvist and Johansson (1996), total coverage is $\alpha = \alpha^g + \alpha^p$. The following result is obtained.

Proposition 5: *Given A1, assume that g-1=η-1=r, δ=0, and that α^g=constant. Allowing supplementary actuarially fair private health insurance will not eliminate the second-best nature of the initial (PAYG) solution*[10].

This Proposition is proved by contradiction. Assume that α^g is such that the individual purchases supplementary private health insurance. The individual will not include the impact of his actions on the government's budget constraint (i.e. the new variation of eq. (9)) in his maximisation problem when he designs his private insurance contract. That is, he will maximise his expected present value utility given the tax rate τ, i.e. ignore any induced adjustments that his actions cause in the government's health budget. This means that the first-order conditions for an interior solution will look like equation (6) but with $\alpha = \alpha^p$. If the individual had recognised that his actions affect $\tau w_t g_{t+1} = \alpha^g h^E_{2,t+1}$, then $\alpha = \alpha^g + \alpha^p$ would show up in the first-order conditions. Assume that the two solutions are identical, i.e. yield the same levels of consumption, and the same premium and tax payments. Then, in terms of equation (6), $u_{c_{1,t}}, v^E_{y_{t,t+1}}, h^E_{y_{2,t+1}}, h^E_p$, and so on, would be the same in the two solutions. However, this is a contradiction since, in terms of equation (6), the equalities would hold with both $\alpha = \alpha^p$ and $\alpha = \alpha^g + \alpha^p$. Thus the individual's assumption that τ is independent of his actions means that the economy will end up in a kind of second-best solution. This fact does not mean that the government budget must show a surplus/deficit. Assume that the government correctly predicts the impact of the optimal private insurance contract on its budget. Then the tax rates can be set so as to balance the public sector's budget, conditional on individuals' choice of private insurance coverage.

5. Further comparisons between health insurance contracts

Thus far, different health insurance schemes have been compared under the assumption that the economy's growth rate is equal to the market rate of interest. In this section this assumption is relaxed. In order to be able to compare different schemes they must be evaluated at the same "point". For the sake of simplicity, we retain assumption A1, although the assumption that individuals are identical is not needed.

Firstly, let us consider the introduction of a "small" compulsory health insurance scheme for the elderly (evaluated at $\alpha=\rho=\tau=0$), which is financed by a proportional tax on labour income in the case of PAYG insurance (i.e. with $\delta=0$). Then we have the following result.

Proposition 6: *Assume that A1 holds. In a dynamically efficient economy, a newly born individual will prefer a small actuarial health insurance contract to a small PAYG contract financed by a proportional tax on labour income. If the economy satisfies the golden rule, the individual would be indifferent between the two contracts. If the economy is dynamically inefficient, the small PAYG health insurance will be preferred.*

This Proposition follows directly from a comparison of equations (7) and (14) evaluated at $\alpha=\rho=\tau=\delta=0$. However, it is a partial equilibrium result in the sense that wages and interest rates are kept constant. As is shown in the appendix, if we are close to a steady state with $1 + r = \eta$, the results hold in general equilibrium as well, provided we consider a marginal change in insurance contracts such that the economy's per capita stock of capital changes in a uniform way over time. In sharp contrast to an actuarial pension system, which leaves aggregate saving unchanged, see Blanchard and Fischer (1989, Chapter 3), a small actuarial health insurance increases total saving and capital accumulation provided individuals are sufficiently risk averse; see equations (A.7) and (A.8) in the Appendix. PAYG health insurance, on the other hand, has an ambiguous impact on saving and hence (reasonably) the stock of capital; see equation (A.9) in the Appendix. It might be noted that a decrease in the stock of capital is welfare improving if the economy is dynamically inefficient since the stock of capital is initially too high; see equation (A.12) in the Appendix and Blanchard and Fischer (1989) for details[11].

The following might also be noted. Given PAYG health insurance, the older generation in (say) period *t+1* would obviously gain from an (unanticipated) reduction in the coinsurance rate in period *t+1*. Thus the currently old generation has a strong incentive to advocate a lower "coinsurance" rate, i.e. a higher α for periods *t+1* onwards. The more "optimistic" the younger generation is with respect to population growth and/or income growth, the stronger is the position of the old in this inter-generational health insurance game. It is possible that a PAYG scheme (with $\alpha,\tau>0$) would be more sensitive to political games than an actuarial insurance system. The reason is the fact that more complex forces are in operation in the former system (when $\alpha,\tau,\rho>0$), as revealed by a comparison of equations (7) and (14). Thus it is much more difficult to correctly assess the effects of changes in the PAYG system than in the actuarial system.

Next, let us assume that the government determines the coverage rates α^g and α^p. In order to examine the welfare consequences of changes in α^p and α^g we will marginally increase one rate while marginally decreasing the other so as to keep α unchanged. Assume that in the initial situation there is only compulsory PAYG insurance so that $\alpha_{t+1}=\alpha_{t+1}^g$. Next, let us introduce a compulsory actuarial insurance. For the sake of simplicity, assume that $d\alpha_{t+1}=$ $d\alpha_{t+1}^g + d\alpha_{t+1}^p = 0$ with $d\alpha_{t+1}^p > 0$. Thus, the aggregate coinsurance rate is kept constant, i.e. there is simply a marginal shift from PAYG insurance to an actuarial one. Moreover, assume that PAYG insurance is financed by a proportional tax on labour income.

Proceeding in the same way as before, one finds that the considered policy experiment affects the individual's expected present value utility in the following way:

$$dU^{E}_{t,t+1} = -\gamma v^{E}_{y_{t,t1}}(d\rho + w_t d\tau) + E_t\left[\gamma v_{y_{2,t+1}}(.)h_{2,t+1}(.)\right]d\alpha_{t+1} =$$

$$-\gamma v^{E}_{y_{t,t+1}}\left[(R_{t+1} - g^{-1}_{t+1})h^{E}_{2,t+1}d\alpha^{p}_{t+1}\right] - \gamma v^{E}_{y_{t,t+1}} g^{-1}_{t+1}\alpha^{g}_{t+1}dh^{E}_{2,t+1} \tag{16}$$

where $dh^{E}_{2,t+1} = -h^{E}_{y_{2,t+1}}(dc_{1,t} + w_t d\tau + d\rho) = -h^{E}_{y_{t,t+1}}(u_{cc}/(u_{cc} + \gamma v^{E}_{yy}))(d\rho + w_t d\tau)$ from equations (A.7) and (A.9) in the Appendix, and the expressions in the final line are obtained by using the budget constraints for the private insurer and the government; see equations (3) and (9), respectively.

If $r_{t+1} \geq g_{t+1} - 1$, then $R_{t+1} \leq g^{-1}_{t+1}$, i.e. the first expression within brackets in the final line of equation (16) is non-negative. If the considered shift increases income, then $-dh^{E}_{2,t+1} > 0$ in equation (16). This is so because, by assumption, health services are a normal good. Then, the second term in (16) is negative.

In sum, we have the following result.

Proposition 7: *Consider PAYG insurance financed by a proportional tax on labour income. Given A1 and an unchanged overall coinsurance rate, a marginal shift from PAYG insurance to actuarial insurance will have an ambiguous impact on individual welfare (even if wages and interest rates are kept constant), unless the economy is in a golden rule steady state. If 1+r =g, then expected present value utility will be left unchanged by the considered marginal shift from compulsory PAYG insurance to compulsory actuarially fair private insurance.*

The last result in the Proposition follows from the fact that, for a fixed labour supply, the individual's disposable income is left unchanged if the tax rate τ on labour is decreased while the insurance premium ρ is increased in order to keep α unchanged (provided *1+r=g* so that both private and public insurance schemes are actuarially fair). Since disposable income and the coinsurance rate are left unchanged, the period one consumption $c_{1,t}$ and hence also the demand for health goods will remain constant.

6. An ageing population

In this section we consider the effects on the properties of our health insurance contracts of an ageing population. We model an ageing population simply as a small permanent decrease in the population growth parameter η, i.e. $d\eta$<0. This is equivalent to an increase in the aged dependency ratio. It is assumed that an increase in the ADR causes the market rate of interest to fall; see Blanchard and Fischer (1989, Chapter 3). For the sake of simplicity, we assume that the shift in η causes a shift of equal magnitude in the market interest rate, i.e. $dr = d\eta$.

In the case of actuarial insurance, it can easily be verified that a change in population growth has the following impact on expected present value utility at time t:

$$dU_{t,t+1}^{E} = dU_{t,t+1}^{E}(eq.(7)) - \gamma R_{t+1}^{-1} v_{y_{t+1}}^{E}\left[(s_t + \rho)(1-\alpha_{t+1} h_{y_{t,t+1}}^{E})\right] dR_{t+1} \tag{7'}$$

where $s_t = w_t - c_{1,t} - \rho$ denotes period t per capita savings net of investments in health insurance. If the initial contract is optimal, the first term in the right-hand side expression of equation (7') is equal to zero. Then, an increase (a decrease) in the rate of interest will increase (decrease) the individual's expected present value lifetime utility; recall that the young are net savers. Thus if an increase in the ADR causes the market rate of interest to fall, then the individual's welfare will go down. (In addition, for any fixed insurance contract, an increase in the ADR causing a fall in r will lower welfare.)

In the PAYG system (with, for the sake of simplicity, δ=0), the change in the population growth rate means that equation (14) changes as follows:

$$dU_{t,t+1}^{E} = dU_{t,t+1}^{E}(eq.14)) - \gamma R_{t+1}^{-1} v_{y_{t+1}}^{E}\left[s_t(1-g_{t+1}^{-1}\alpha_{t+1} h_{y_{t,t+1}}^{E})\right] dR_{t+1} +$$

$$\gamma v_{y_{t+1}}^{E}(g_{t+1}^{-1}/\eta_{t+1}) w_t \tau (1-\alpha_{t+1} h_{y_{t,t+1}}^{E}) d\eta_{t+1} \tag{14'}$$

Thus a sufficient condition for the individual's welfare to fall is that an increase in the ADR does not cause an increase in the interest rate.

Proposition 8: *Given A1, an increase in the aged dependency ratio (ADR) will adversely affect the welfare of incumbents of actuarial as well as PAYG insurance. In a golden rule steady state incumbents of small actuarial and PAYG contracts are affected in identical ways, provided* $d\eta=dr$.

The last part of the Proposition follows from a comparison of equations (7') and (14') evaluated at $\alpha=0$ and $g-1=\eta-1=r$. An increase in the economy's ADR would also have additional effects through adjustments over time of the stock of capital. These effects, however, are there regardless of (but their magnitude may be affected by) the presence or non-presence of health insurance.

The question arises of how a change in the interest rate would affect the optimal coinsurance rate chosen by a purchaser of actuarially fair insurance.

Proposition 9: *Assume that A1 holds and that $dc_{1,t}/dr_{t+1}<0$. Then an increase in the interest rate will induce the individual to choose a lower coinsurance rate.*

This result is proved in the Appendix. An increase in the return on investments in health insurance thus induces the individual to invest more, i.e. to purchase higher coverage. Proposition 9 also means that an increase in the ADR causing a fall in the market rate of interest induces individuals to choose a higher coinsurance rate. Similar forces are in operation in PAYG insurance. Thus, at least under the assumptions employed here, an increase in the ADR does not induce individuals to demand/require higher coverage from their health insurance.

7. Concluding remarks

This paper has analysed the properties of actuarially fair health insurance and PAYG health insurance. Unless the economy is in a steady state satisfying the golden rule and we consider (infinitesimally) small contracts, the two schemes have different properties. In fact, a newly born individual may prefer PAYG insurance to actuarially fair insurance if the economy grows sufficiently fast ($g=\eta>r$). However, if the population is ageing, it seems likely that actuarially fair insurance will become more attractive to the young. If an increase in the economy's ADR causes the market rate of interest to fall, at least under certain assumptions, individuals will choose higher coinsurance rates. Similar forces are in operation in the case of PAYG health insurance. This fact may make it easier to accept lower public coverage of health expenditures (provided an increase in the ADR induces a fall in interest rates). If so, it should ease the problems faced by public health care systems as the population grows older.

The paper has also demonstrated that shifting the tax burden between the young and the old in the case of PAYG health insurance has welfare consequences for the young, in general. Depending on the economy's growth rate and the rate of population growth, the currently young may gain or lose from changes in the way the insurance is financed. This property may make PAYG insurance sensitive to political "games" between, for example, young and old generations.

In the paper it has also been shown that in an intertemporal context public health insurance results in a kind of second-best solution (unless we consider an infinitesimally small insurance). The reason is the fact that individuals fail to internalise the impact of their decisions on the government's health budget. In some countries the public sector provides a basic level of health insurance but individuals are free to purchase supplementary private insurance. However, this paper has demonstrated that such mixed public/private insurance is unable to remove the second-best property inherent in public health insurance. These results might seem to indicate that public and mixed private/public health insurance schemes are inferior to strictly private ones. However, voluntary private insurance is associated with an adverse selection problem. This problem is eliminated by compulsory insurance schemes. On the other hand, the intragenerational incidence of public insurance might be such that there is a transfer from the poor to the wealthy. To illustrate this, wealthier people have longer survival times and they might also demand more sophisticated and expensive treatments than poorer people; see McClellan and Skinner (1997) for a discussion. There might also be a difference in administration costs between private and public insurers. For example, in the absence of perfect competition in insurance markets, the profitability motive will add to the cost of private insurance. On the other hand, lack of proper incentives to eliminate internal slacks (X-inefficiency) might be a cost driver in compulsory tax-financed health insurance. Thus the question of the overall superiority of one or the other insurance programme is still open.

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Appendix

The first-order conditions (6) in the main text can be written as follows:

$$u_{c_{1,t}} - \gamma v^{E}_{y_{2,t+1}}(1 + (\partial\rho / \partial c_{1,t})) = 0$$

$$- \gamma v^{E}_{y_{t,t+1}}(\partial\rho / \partial\alpha_{t+1}) + \gamma E_t\left[v_{y_{t,t+1}}(.)h_{2,t+1}(.)\right] = 0 \tag{A.1}$$

Using equation (3) one arrives at the variation specified in equation (6).

Differentiating equation (5) with respect to α_{t+1} and employing the first line in equation (A.1) yields:

$$dU^{E}_{t,t+1} = -\gamma v^{E}_{y_{t,t+1}}(\partial\rho / \partial\alpha_{t+1})d\alpha_{t+1} + \gamma E_t\left[v_{y_{t,t+1}}(.)h_{2,t+1}(.)\right]d\alpha_{t+1} \tag{A.2}$$

This expression can be written as follows:

$$dU^{E}_{t,t+1} = \gamma v^{E}_{y_{t,t+1}}\alpha_{t+1}\left[h^{E}_{y_{t,t+1}}(\partial\rho / \partial\alpha_{t+1}) + h^{E}_{p_{2,t+1}}\right]d\alpha_{t+1} + \gamma\,\mathrm{cov}(v_{y_{t,t+1}}, h_{2,t+1})d\alpha_{t+1} \tag{A.3}$$

where we have used the fact that $\rho = \alpha_{t+1}h^{E}_{2,t+1}(w_t + y_{2,t+1} - c_{1,t} - \rho, 1 - \alpha_{t+1})$ in equilibrium, the discount factor has been suppressed, and the covariance term follows from the fact that $E_t\left[v_{y_{t,t+1}}h_{2,t+1}\right] = v^{E}_{y_{t,t+1}}h^{E}_{2,t+1} + \mathrm{cov}(v_{y_{t,t+1}}h_{2,t+1})$.

Using $\rho = \alpha_{t+1}h^{E}_{2,t+1}(w_t + y_{2,t+1} - c_{1,t} - \rho, 1 - \alpha_{t+1})$ in order to solve for $\partial\rho/\partial\alpha_{t+1}$, equation (A.3) can be expressed in the following way:

$$dU^{E}_{t,t+1} = \gamma v^{E}_{y_{t,t+1}}\alpha_{t+1}\left[h^{E}_{2,t+1}h^{E}_{y_{t,t+1}} + h^{E}_{p_{2,t+1}}\right](1 + \alpha_{t+1}h^{E}_{y_{t,t+1}})^{-1}d\alpha_{t+1} +$$

$$\gamma\,\mathrm{cov}(v_{y_{t,t+1}}, h_{2,t+1})d\alpha_{t+1} =$$

$$\gamma v^{E}_{y_{t,t+1}}\alpha_{t+1}\left[h^{Ec}_{p_{2,t+1}} - \mathrm{cov}(h_{2,t+1}, h_{y_{t,t+1}})\right](1 + \alpha_{t+1}h^{E}_{y_{t,t+1}})^{-1}d\alpha_{t+1} + \tag{A.4}$$

$$\gamma\,\mathrm{cov}(v_{y_{t,t+1}}, h_{2,t+1})d\alpha_{t+1}$$

where a superscript c refers to a compensated price effect, and the final equality follows from the fact that $h^{E}_{p_{2,t+1}} = h^{Ec}_{p_{2,t+1}} - E_t\left[h_{2,t+1}h_{y_{t,t+1}}\right] = h^{Ec}_{p_{2,t+1}} - h^{E}_{2,t+1}h^{E}_{2,t+1} - \mathrm{cov}(h_{2,t+1}, h_{y_{t,t+1}})$.

In Section 4 a case is discussed where individuals choose from a menu of contracts α_{t+1}, $\rho=\rho(\alpha_{t+1};c_{1,t})$, where $c_{1,t}$ is treated as a constant by individuals. In this case the relevant first-order conditions for an interior solution read:

$$u_{c_{1,t}} - \gamma v^{E}_{y_{2,t+1}} = 0$$

$$-\gamma v^{E}_{y_{t,t+1}}(\partial\rho/\partial\alpha_{t+1}) + \gamma E_t\left[v_{y_{t,t+1}}(.)h_{2,t+1}(.)\right] = 0 \tag{A.1'}$$

The first-line expression in (A.1'), which contains the same terms as equation (12), differs from the corresponding expression in equation (A.1). The difference is due to the fact that (A.1') ignores the impact of the consumption (i.e. the savings) decision while young on the cost of the health insurance contract. If we examine the welfare effects of the contract structure behind (A.1'), it is straightforward to show that the "ignored" cost will show up in the same way as in equation (15) in the main text.

If consumption of young people is subsidised at a proportional rate, the first-order condition (12) will read:

$$u_{c_{1,t}} - \gamma v^{E}_{y_{2,t+1}}(1-\tau^{c}) = 0 \tag{A.5}$$

where we assume that the subsidy is financed by a lump-sum tax. The subsidy means that equation (15) will read:

$$dU^{E}_{t,t+1} = dU^{E}_{t,t+1}(eq.(15)) + \tau^{c} w_t(\partial\tau/\partial c_{1,t})dc_{1,t} - w_t(\partial\tau/\partial\Gamma^{c})d\Gamma^{c} - d\Gamma^{c} =$$

$$dU^{E}_{t,t+1}(eq.(15)) - d\Gamma^{c} \tag{A.6}$$

where $d\Gamma^c = \tau^c dc_{1,t}$. Thus if $\tau^{c} = \alpha_{t+1}h^{E}_{y_{t,t+1}}(1+\alpha_{t+1}h^{E}_{y_{t,t+1}})^{-1}$, the externality in equation (15) will net out. Moreover, using this optimal subsidy rate in equation (A.5), the equation will coincide with the first-line expression in equation (6). This proves Proposition 4. Proceeding in the same way as in equations (A.5)-(A.6) it is easily verified that a subsidy τ^c would induce purchasers of private contracts of the kind implicitly defined by equation (A.1') to internalise the externality under consideration.

To examine how saving is affected by private actuarial health insurance, let us assume that there is a fixed insurance contract. Then differentiate the first-order condition for optimal consumption while young, i.e. the analogue to equation (12), with respect to ρ and α_{t+1}, holding wages and interest rates constant. After some manipulation one obtains:

$$dc_{1,t} = \frac{-(\gamma v^{E}_{y_{t,t+1} y_{t,t+1}} d\rho + \gamma v^{E}_{y_{t,t+1}\alpha} d\alpha_{t+1})}{(u_{c_{1,t} c_{1,t}} + \gamma v^{E}_{y_{t,t+1} y_{t,t+1}})} \tag{A.7}$$

where the direct second-order derivatives are negative for a risk averse individual. A sufficient condition for $c_{1,t}$ to fall is that the expected marginal utility of income falls as α_{t+1} is increased; see equation (A.8) below. Then, in contrast to what is the case in a fully funded pension system, see Blanchard and Fischer (1989, Chapter 3), total savings will increase, i.e. $d(s_t+\rho)=-dc_{1,t}>0$.

In order to sign $v^{E}_{y_{2,t+1}\alpha}$ in equation (A.7), we proceed as follows:

$$v_{y\alpha} = v_{\alpha y} = (d/dy)v_{\alpha} = v_{yy}h + v_y h_y = (v_y h/y)\left[(v_{yy}y/v_y) + (h_y y/h)\right] =$$
$$(v_y h/y)(-\xi + \varepsilon) \tag{A.8}$$

This expression is negative if the coefficient of relative risk aversion ξ is larger than the income elasticity of demand ε for health services. Note that an increase in α is equivalent to a price reduction for health services. If $v_{y\alpha} < 0$ for all θ, it follows that $v^{E}_{y_{2,t+1}\alpha} < 0$.

Next, let us consider the PAYG system. Differentiating equation (12), one obtains:

$$dc_{1,t} = \frac{-(\gamma v^{E}_{y_{t,t+1} y_{t,t+1}} w_t d\tau + \gamma v^{E}_{y_{t,t+1}\alpha} d\alpha_{t+1})}{(u_{c_{1,t} c_{1,t}} + \gamma v^{E}_{y_{t,t+1} y_{t,t+1}})} \tag{A.9}$$

A sufficient condition for $dc_{1,t}$ to be negative is once again that the individual is sufficiently risk-averse. This means that the impact on savings of a marginal increase in the insurance is ambiguous. Recall that $ds_t=-(dc_{1,t}+w_t d\tau)$.

In order to examine the general equilibrium effects of health insurance, we must introduce firms. It is assumed that goods are produced using a linearly homogeneous production function $F(K_t, N_t)$, where K_t is the period t capital stock. The per capita production function is denoted $f(k_t)$, where k_t is the capital-labour ratio in period t (and per capita labour supply is set equal to unity). First-order conditions for profit maximisation are:

$$f(.) - f'(.)k_t = w_t$$
$$f'(.) = r_t \tag{A.10}$$

where a prime refers to a derivative with respect to the capital-labour ratio; see Blanchard and Fischer (1989, Chapter 3) for details. Goods market equilibrium requires that (in the case of actuarial insurance):

$$s_t + \rho = \eta_{t+1} k_{t+1} \tag{A.11}$$

In the PAYG variation, goods market equilibrium requires that:

$$s_t = \eta_{t+1} k_{t+1} \tag{A.11'}$$

In the main text general equilibrium effects through adjustments in real wages and interest rate were ignored. For the actuarial case, compare equation (7), these effects are as follows:

$$\gamma v^E_{y_{t,t+1}} \left[dw_t + R_{t+1}(s_t + \rho) dr_{t+1} \right] (1 - \alpha_{t+1} h^E_{y_{t,t+1}}) =$$

$$\gamma v^E_{y_{t,t+1}} \left[-f''(k_t) k_t dk_t + R_{t+1}(s_t + \rho) f''(k_{t+1}) dk_{t+1} \right] (1 - \alpha_{t+1} h^E_{y_{t,t+1}}) = \tag{A.12}$$

$$-\gamma v^E_{y_{t,t+1}} \left[f''(k_t) k_t dk_t - R_{t+1} \eta_{t+1} f''(k_{t+1}) k_{t+1} dk_{t+1} \right] (1 - \alpha_{t+1} h^E_{y_{t,t+1}})$$

where $f''(.) = \partial^2 f(.)/\partial k^2$, and we have used the first-order conditions for profit maximisation in equation (A.10) and the goods market equilibrium in equation (A.10). Assume that the economy is in a steady state so that the per capita stock of capital is constant over time, i.e. $k_t = k$ for all t. Consider a uniform change of the per capita stock of capital, i.e. $dk_t = dk$ for all t. If $\eta - 1 > (<) r$, then a permanent increase in the per capita stock of capital will decrease (increase) the individual's expected present value utility. If the economy is in a golden rule steady state, then his expected present value utility is left unchanged by the considered permanent change in the economy's per capita stock of capital. Similar forces are in operation in the case of PAYG health insurance.

In order to prove Proposition 9, totally differentiate equation (6). Suppressing the terms for the changes in the exogenous variables, one obtains:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \mathrm{ds_t} \\ \mathrm{d}\alpha_{\mathrm{t+1}} \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix} \tag{A.13}$$

where:

$a_{11} = u_{cc} A + (\gamma v^E_{yy} - u_c \alpha h^E_{yy})(1 + (\partial \rho / \partial c))$, $a_{12} = \gamma v^E_{yy} \rho' + \gamma v^E_{yp} + u_c (h^E_y - \alpha h^E_{yy} \rho' + \alpha h^E_{yp})$,

$a_{21} = -\gamma \alpha v^E_{yy} (h^{Ec}_p - \mathrm{cov}(h_y, h)) - \gamma \alpha v^E_y (d/dc)(h^{Ec}_p - \mathrm{cov}(h_y, h)) -$

$\mathrm{cov}(v_y, h) \alpha h^E_{yy} + A(d/dc)(\mathrm{cov}(v_y, h))$,

$$a_{22} = (\gamma v_y^E - \gamma v_{yy}^E \alpha\rho' - \gamma\alpha v_{yp}^E)(h_p^{Ec} - \text{cov}(h_y, h)) - \gamma\alpha v_y^E (h_{pp}^{Ec} + (d/d\alpha)(\text{cov}(h_y, h)))$$
$$+ \text{cov}(v_y, h)\rho' + A(d/d\alpha)(\text{cov}(v_y, h)),$$

$A = 1 + \alpha h_y^E$, and $\rho' = d\rho / d\alpha = h^E - \alpha h_y^E \rho' - \alpha h_{yp}^E$.

Note that time indices have been omitted in order to avoid unnecessary clutter. It is assumed that the determinant $D = a_{11}a_{22} - a_{12}a_{21}$ has a positive sign. This assumption follows from the second-order conditions for a maximum, see, for example, Varian (1992). In order to simplify the exposition, it is also assumed that a_{11}, $a_{12,}$ a_{21}, $a_{22} < 0$. (For example, this is the case if the covariance terms in equation (A.13) and the derivatives $h_{yy}^E, h_{pp}^E, h_{yp}^E, h_{py}^E, h_{pp}^{Ec}$, and h_{pc}^{Ec} are all approximately equal to zero, while the income effect h_y^E in a_{12} is not too large.) This set of assumptions is called Assumption AA.1.

Consider now an increase in the market rate of interest, i.e. differentiate equation (6) with respect to r. Denote by $\lfloor a_1^r dr, a_2^r dr \rfloor$ the resulting (transposed) right-hand side vector in equation (A.13). Assume that $a_1^r > 0, a_2^r < 0$. Given assumption AA.1 this is sufficient to ensure that consumption as young decreases, i.e. total savings increase, if the interest rate increases. In other words, it holds that $dc/dr = (1/D)(a_{22}a_1^r - a_{12}a_2^r) < 0$. Moreover, $d\alpha / dr = (1/D)(a_{11}a_2^r - a_{21}a_1^r) > 0$.

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Notes

[1] For discussions of moral hazard problems and adverse selection in health care systems, see, for example, Besley and Gouveia (1994), Cutler and Zeckhauser (1997), Pauly (1968, 1974) and Zeckhauser (1970).

[2]There are, however, papers on long-term care insurance and intergenerational relationships; see, for example, Zweifel and StrÜwe (1996).

[3] It is assumed that it is somehow possible to distinguish between purchases for direct consumption and purchases for health purposes (since c and h are associated with different effective prices).

[4] Prices and incomes are expressed as present values at time *t*, but this transformation has no bearing on the results derived here.

[5] Jack and Sheiner (1998, p. 210) point out that the endogeneity of the involved terms make interpretation of the covariance terms somewhat problematic.

[6] See Blanchard and Fischer (1992, pp. 103 and 147), and Hassler and Lindbeck (1998, pp. 18-19). A dynamically inefficient economy overaccumulates capital, i.e. its stock of capital exceeds its Pareto optimal level; see equation (A.12) in the Appendix.

[7] This result holds if we explicitly introduce the production sector of the economy; see equation (A.12) in the Appendix. The reader is referred to Blanchard and Fischer (1992, chapter 3) for details.

[8] In turn, one expects this solution to be second-best to the solution generated by state contingent insurance. The reader is referred to Besley (1989) for details.

[9]Of course, private and public insurance schemes might differ for a number of other reasons, for example, with respect to their administrative costs. In addition, if individuals are heterogenous, compulsory public insurance eliminates the adverse selection problem. The reader is referred to Pauly (1974) and Blomqvist and Johansson (1996) for further discussion.

[10] Blomqvist and Johansson (1996) have a similar claim. They claim that mixed private/public insurance will result in an equilibrium that is inferior to the solution obtained when the individual maximises expected utility subject to equations (3) and (9). The reader is also referred to Pauly (1974), Kaplow (1991), and to Selden (1997), who provides further references.

[11] As is evident from Blanchard and Fischer (1989, Chapter 3) it is very difficult to determine the dynamic properties of the economy. Often, very specific assumptions must be introduced. Therefore, in this paper we typically assume that real wages and interest rates are held constant.