Shareholder-Value Maximization and Tacit Collusion*

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Abstract

This paper shows that as long as the stock market has perfect foresight, some dividends are distributed, and incentives are paid more than once or are deferred, stock-related compensation packages are strong incentives for managers to support tacit collusive agreements in repeated oligopolies. The stock market anticipates the losses from punishment phases and discounts them on stock prices, reducing managers' short-run gains from any deviation. When deferred, stock-related incentives may remove all managers' short-run gains from deviation making collusion supportable at any discount factor. The results hold with managerial contracts of any length.

JEL CLASSIFICATION: D43, G30, J33, L13, L21.

KEYWORDS: CEO compensation, tacit collusion, oligopoly, delegation, managerial incentives, ownership and control, corporate governance.

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1 Introduction

In a highly discussed empirical study, Michael Jensen and Kevin Murphy (1990) showed that until the end of the '80s, and contrary to the predictions of agency theory, U.S. top-managers' compensation had on average a very low pay-performance sensitivity. Steven Kaplan (1994, 1998) found analogous results for other developed countries, such as Germany and Japan. These surprising findings led to concerns about the welfare implications of most common governance practices, since low-powered managerial incentives tend to soften product-market competition (e.g. Rajesh Aggarwal and Andrew Samwick, 1996; Giancarlo Spagnolo, 1996).

More recently, Brian Hall and Jeffrey Liebman (1998) have shown that the pay-performance sensitivity of U.S. top-managers' compensation has increased substantially in the last decade, mainly because of a widespread adoption of stock-related incentives, such as stock options plans. Stock-based managerial incentives are believed to be a powerful tool by which owners can motivate managers to work hard, to take risks, and to take into account the long-run effects of their choices (e.g. investments) on firms' profitability.[1] What about their effects on product markets? Does this trend towards stock-based incentives imply a more competitive attitude on the part of managers, so that concerns about tacit collusion and social welfare can be abandoned at least in the U.S.?

The results of this paper suggest that, unfortunately, this is not quite the case. Our model shows that as long as agents in financial markets have rational expectations and firms pay out dividends, most common stock-based managerial compensation plans greatly facilitate tacit collusion in long-run oligopolies. We find that stock-related compensation reduces managers' incentives to break any tacit agreement in any repeated oligopoly, and may make the joint monopoly agreement supportable at any level of the discount factor.

The phenomenon of tacit collusion in long-run oligopolies has been fruitfully studied in the last three decades within a discounted repeated games framework.[2] However, most classical supergame-theoretic analyses of collusion confined themselves to the standard assumption that firms maximize the discounted sum of expected per-period profits. In the real world many interacting factors affect firms' objective function, and consequently firms' ability to collude. Among these factors the most important are probably managerial incentives.[3]

A number of authors have already explored the strategic effects of delegating decision power to managers with preferences/incentives different from those of owners in oligopolies (e.g. John Vickers, 1985; Chaim Fershtman, 1985; Fershtman and Kenneth Judd, 1987; Steven Sklivas, 1987; Fershtman, Judd and Ehud Kalai, 1991; Michael Katz, 1991; David

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[1] See, for example, Sanjai Bhagat *et al.* (1985); Kevin Murphy (1985); Matthew Jackson and Edward Lazear (1991); Myron Scholes (1991); Bengt Holmström and Jean Tirole (1993).

[2] Classical references include James Friedman (1971); Robert Aumann and Lloyd Shapley (1976); Ariel Rubinstein (1979); Edward Green and Robert Porter (1984); Drew Fudenberg and Eric Maskin (1986); Julio Rotemberg and Garth Saloner (1986); Dilip Abreu (1986, 1988).

[3] This was recognized early by scholars interested in firm behavior. See Herbert Simon (1957); William Baumol (1958); Richard Cyert and James March (1963); Robin Marris (1964); Oliver Williamson (1964); Michael Jensen and William Meckling (1976).

Reitman, 1993). Most contributions to this literature focus on the strategic effects of managerial incentives in two-stages models, where owners simultaneously choose their managers' incentive schemes before a one-shot oligopolistic market interaction between manager-led firms. In Fershtman and Judd (1987) and Sklivas (1987) (FJS from now on), firm owners can precommit to a more aggressive market behavior by choosing the parameters of a managerial contract that is linear in profits and sales revenue. In the case of quantity competition, the simultaneous attempts to gain a strategic advantage by precommitting through managerial incentives offset one another and lead to higher output and lower profits than in the standard Cournot-Nash equilibrium.

In probably the closest paper in spirit to the present one, Reitman (1993) has shown that if one lets owners introduce stock options in an FJS-type model, these may curb managers' overly aggressive behavior and bring back the original Cournot-Nash equilibrium. This result is due to the non-linearity of stock options in stock price, which induces a discontinuity in managers' best response function and generates other equilibria than the FJS one. The Pareto-dominant among the symmetric equilibria corresponds to the no-delegation Cournot-Nash equilibrium, so that if managers coordinate on this equilibrium the "delegation Prisoner's Dilemma" identified by FJS disappears and, eventually, the ability to precommit through managerial incentives does not affect the outcome of the Cournot game.

We depart from previous work on strategic delegation in oligopolies by allowing for repeated interaction, so that tacit collusion can be analyzed with the tools of repeated games. One could say, therefore, that the results of this paper originate from the repetition of the FJS-Reitman model.

We focus on stock-based compensation plans as usually designed in the real world according to Stacey Kole's (1997) empirical findings. These plans are typically quite liquid (when not in cash, they have few restrictions on resale or transfers of shares) and pay managers stock-based bonuses for several consecutive years.

The pro-collusive effect that we identify is linked to the fact – forcefully stressed by Bengt Holmström and Jean Tirole (1993) – that the stock price incorporates additional information with respect to a firm's profits, information strictly related to the firm's future profitability. Incentive schemes based on stock price link managers' *present* compensation to the stock market's expectations about *future* firms' profitability. When a breach of a tacit collusive agreement occurs, a stock market with rational expectations anticipates the negative effect of the breach on firms' future profitability linked to the forthcoming price-quantity war, and immediately discounts it on stock price (for a real world example see e.g. Jonathan Laing, 1997). Because this effect occurs in the very same period in which a manager deviates from a collusive agreement, incentives linked to stock price directly reduce managers' short-run gains from deviation.

Furthermore, we find that when stock-based incentives are deferred, as they often are in reality, the first pro-collusive effect is reinforced by the fact that the already limited beneficial effect on the stock price of short-run profits from a unilateral deviation may be completely gone at the time when the manager receives the bonus. Then, the manager is left with no incentive whatsoever to deviate, which further stabilizes collusive agreements.

Interestingly, we also find that these pro-collusive effects of stock-based managerial incentives are not reduced, and may even be increased when managerial contracts are short-term.

Although this paper is close in spirit to Reitman (1993), the effects identified here are very different from the effect discussed there. Both the "expectations effect" and the "deferred incentives effect" are not linked to the non-linearity of stock options; they apply to any form of managerial compensation that is positively related to stock price. Further, the effects of managerial incentives discussed here have a strong impact on the equilibrium outcome of the oligopoly game. For example, in our model when owners delegate control to managers under deferred stock-related incentives, the joint monopoly outcome becomes supportable even when, without delegation, owners could not support any collusive agreement. Moreover, the results in this paper are not specific to Cournot competition; they extend to any other kind of repeated oligopoly.

We follow the literature on strategic delegation in assuming observable and binding managerial incentives. This assumption has been criticized on the ground of its robustness with respect to *secret* renegotiation (Mathias Dewatripont, 1988; Michael Katz, 1991). However, as is also made clear by Reitman (1993), for the case that we are focusing on this assumption is close to reality. The adoption of stock-based managerial incentive plans, such as stock options, normally requires shareholders' approval. Shareholders' approval must be obtained in open shareholders' meetings, and these make stock-based incentives and their renegotiation almost public information.[4]

Finally, the results of this paper are related to, but well distinct from those in Fershtman, Judd, and Kalai (1991), Michele Polo and Piero Tedeschi (1992), and Aggarwal and Samwick (1996). Fershtman, Judd, and Kalai (1991) obtain a full "folk theorem" for two-stage observable delegation games by using "target compensation functions" that award agents a fixed prize as long as managers keep principal's utility above a certain level. Michele Polo and Piero Tedeschi (1992) and Aggarwal and Samwick (1996) obtain cooperative outcomes in two-stage delegation games by allowing managerial contracts to be related to competing firms' profits. Here we work with repeated oligopoly models, instead, and we obtain full collusion at any discount factor with the empirically observed stock-related managerial incentive plans, which are not target compensation functions and which are conditional on the firm's own stock price only.

The rest of the paper is organized as follows: Section 2 presents the model; Section 3 discusses the pro-collusive effect linked to stock-market expectations; Section 4 considers deferred stock-based incentives; Section 5 discusses the length of managerial contracts; Section 6 extends and discusses the results; and Section 7 briefly concludes. All proofs are in the appendix.

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[4] Consider, for example, the recent world-wide discussions on the stock-option plan in Walt Disney's management's new compensation package. The results of this paper would be of interest even if secret renegotiation were possible, as the *costs* typically linked to contract renegotiation would still give commitment value to managerial incentives (see the discussion in Section 6.7).

2 The model

2.1 Product market

There are N symmetric firms, indexed by the subscript i. Market structure is a standard Cournot oligopoly (stage game) infinitely repeated in discrete time under complete and perfect information.

Let $\pi_i(q_i,q_{-i}) = P(q_i+q_{-i})q_i - c(q_i)$ denote firm i's static (stage-game's) profit function, where q_i represents firm i's output, q_{-i} the quantity produced by the other N-1 firms, $P(.)$ the inverse demand function and $c(.)$ firms' cost function.

We assume that the inverse demand function satisfies $P' < 0$ and $P'' \geq 0$, that profits are concave in firms' own output, and that marginal profits are decreasing in rivals' output, so that static reaction functions are continuous and downward sloping.

Let $\pi_i^N = \pi_i(q_i^N, q_{-i}^N)$ denote firm i's static (stage-game's) profits when firms produce the Cournot-Nash equilibrium output vector $q^N = (q_1^N, ..., q_n^N)$, $\pi_i^A = \pi_i(q_i^A, q_{-i}^A)$ denote owner i's static payoff from a stationary tacit agreement A to restrict production to the vector $q^A = (q_1^A, ..., q_n^A)$, and $\widehat{\pi}_i^A = \pi_i(\widehat{q}_i(q_{-i}^A), q_{-i}^A)$ denote his static payoffs from unilaterally deviating from A by producing the static best response output $\widehat{q}_i(q_{-i}^A)$. Analogously, $\pi_i^M = \pi_i(q_i^M, q_{-i}^M)$ will denote firm i's profits at the joint monopoly market outcome q^M, and $\widehat{\pi}_i^M = \pi_i(\widehat{q}_i(q_{-i}^M), q_{-i}^M)$ will denote static payoffs from unilaterally deviating from the joint monopoly collusive agreement.

Time is indexed by the superscript $t = 1, 2, 3...$ (the time superscript is absent when we refer to a representative period) and δ denotes the intertemporal discount factor common to all agents, owners and managers. We assume that at each point in time t each agent maximizes the discounted sum of expected monetary gains. So each owner i maximizes the discounted sum of firm i's expected profits $U_i^t = \sum_{\tau=1}^{\infty} \delta^\tau \, \pi_i^{t+\tau}$.

To simplify exposition we focus on stationary collusive agreements enforced by "unrelenting" trigger strategies, that is, by the threat of reverting to the non-cooperative Cournot-Nash equilibrium forever (Friedman, 1971).[5] Also, to make things more interesting, we assume throughout that the discount factor is too low for owners to support the joint monopoly collusive agreement in subgame-perfect equilibrium.

2.2 Financial market

We assume the following.

1. The stock market is perfectly informed, rational, and skilled in game theory: it fully understands equilibria selected in the product market.

2. The value of a firm (of its shares) in one period depends positively upon the discounted profit stream it is expected to generate and on the realized profits which

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[5] In Section 6.5 and Appendix 2 we show that the choice of more sophisticated strategies (e.g. finite length, "optimal," or renegotiation-proof punishment strategies) does not affect our conclusions. Also, at the cost of a more cumbersome exposition the results can easily be extended to encompass non-stationary collusive agreements.

have not yet been distributed as dividends (we assume no physical assets to simplify exposition).

3. At the end of each period realized profits are paid out to shareholders as dividends (but see Section 5.1).

Under these assumptions the price of one share of firm i, P_i^t, at the end of period t before period t's dividends are paid out is

$$P_i^t = V\left[\pi_i^t + E_t\left(\sum_{\tau=1}^{\infty} \delta^\tau\ \pi_i^{t+\tau}\right)\right],$$

where E_t is the expectation operator and V is a continuous and increasing function.

2.3 Managerial contracts

According to Kole (1997), most common stock-based managerial incentive plans are relatively liquid, such as stock options with stock appreciation rights (SARs) or share-performance cash bonuses. In most cases the effect of these incentive plans is deferred and distributed in time, probably to reduce the much advertised risk of costly managerial "short-termism" in investment choices.[6] For example, for stock options the typical vesting schedule includes a "wait to exercise" of 12 months for the first quarter of the award, after which the remainder of the award becomes available in equal installments over the next three years. We will focus mainly on the product-market effects of these more common stock-related incentive plans.[7] We assume the following:

1. Owners can delegate decision power (to profit-maximizing managers or) to managers under observable incentive contracts such that in each period managers receive a wage equal to their reservation wage – both of which we normalize to zero without loss of generality – plus an incentive payment linked to stock price, such as stock options or a cash bonus positively related to stock price.[8]

2. Managers are not required to keep firms' shares; the instant in which they receive their compensation they sell their shares or options in order to diversify their portfolio.

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[6] See, e.g. MC Narayanan (1985); Jeremy Stein (1989); Lucian Bebchuck and Lars Stole (1993); and John Bizjak *et al.* (1993).

[7] Restricted stock awards and stock options plans with restrictions on resale/transfer of the shares are used by a minority of firms. We discuss the product-market effects of these other incentives in Section 6.2 (managerial ownership).

[8] Alternatively, we could have followed Fershtman and Judd (1987) and Reitman (1993) in completely abstracting from the issue of managers' individual rationality constraint by assuming that managers' final compensation is some function $A(W+f(P))$, where W is a wage component and f an incentive component function of stock price. The parameter A can be freely set to reflect conditions on managers' labor market, as managerial behavior is driven by the marginal incentive component $f(P)$ only. The two assumptions are fully equivalent: they lead to identical results and they both simplify exposition by allowing us to focus exclusively on the incentive part of the compensation.

3. In periods in which the manager has no way of gaining from the stock-based part of the compensation, he is indifferent about available actions since they all lead to the same wage. We adopt the standard assumption that in such cases managers choose the action that maximizes the owner's objective function.

4. To skip straightforward comparisons between direct cost (managers' compensation) and benefits (higher collusive profits) of delegation, we assume that managers' reservation wage is smaller than or equal to owners' disutility of running the firm personally.

2.4 Useful benchmarks

Given the common discount factor, any collusive agreement A is sustainable in subgame-perfect equilibrium by owners or profit-maximizing managers as long as discounted expected profits from sticking to the agreement exceed expected profits from deviating, that is,

$$\frac{\pi_i^A}{1-\delta} \geq \widehat{\pi}_i^A + \frac{\delta \pi_i^N}{1-\delta}. \tag{1}$$

Let $\overline{A}$ denote the "most collusive" symmetric agreement that owners or profit-maximizing managers can support at the given discount factor, the one which makes (1) hold as an equality. We assumed that owners cannot support the joint monopoly collusive equilibrium, so that $\pi_i^{\overline{A}} < \pi_i^M$. Alternatively, one can rephrase (1) in terms of the minimum level of the discount factor $\underline{\delta}^A$ at which owners or profit-maximizing managers can support a given agreement A, that is,

$$\delta \geq \underline{\delta}^A = \frac{\widehat{\pi}_i^A - \pi_i^A}{\widehat{\pi}_i^A - \pi_i^N}. \tag{1'}$$

It is useful to state a simple lemma:

Lemma 1 *The Cournot-Nash equilibrium outcome (the equilibrium outcome of the stage game played by owners or profit-maximizing managers) is also a Nash equilibrium outcome of the stage game played by managers under incentive contracts positively related to stock price.*

The statement follows straightforwardly from the assumptions. In the static interaction, if all other managers choose the Cournot-Nash production level then a manager paid as a function of stock price cannot gain by choosing a different production level: any other choice will reduce the firm's profits, the stock price, and therefore the manager's compensation. This lemma makes sure that the reversion to the static Cournot-Nash equilibrium remains a credible punishment strategy when managers under stock-based compensation are running the firms, and allows us to study the effects of these incentives on firms' ability to collude by plugging managers' compensation function into condition (1).

3 Stock-related compensation, expectations, and collusion

3.1 The general case

Consider the following class of managerial incentive contracts linked to stock price:

Definition 1 ***Incentive contracts class A (ICA):*** *In each period* t *the manager of firm* i *receives a compensation positively related to the stock price* $f_i(P_i^t)$ *– where* f_i *is* any *monotone and strictly increasing function* – before *period* t *profits are paid out as dividends.*

Because under ICA-type contracts managers get paid before the distribution of dividends, the value of a share at the time when they receive their compensation is as in the example in Section 2.2. Then, the incentive compatibility condition for a stationary collusive agreement A to be supportable by the manager of firm i under a compensation package of the ICA type is

$$\frac{1}{1-\delta} f_i \left[V \left(\frac{\pi_i^A}{1-\delta} \right) \right] > f_i \left[V \left(\widehat{\pi}_i^A + \frac{\delta \pi_i^N}{1-\delta} \right) \right] + \frac{\delta}{1-\delta} f_i \left[V \left(\frac{\pi_i^N}{1-\delta} \right) \right], \tag{2}$$

where the inequality is strict because of assumption 3 in Section 2.3. We can now state the first result.

Proposition 1 *Suppose firms are led by managers under incentive contracts in the class ICA. Then, the minimum discount factor at which any collusive agreement can be supported in subgame-perfect equilibrium is strictly lower than when firms are led by owners or profit-maximizing managers. Conversely, for a given discount factor more profitable collusive agreements become supportable when firms are led by managers under incentive contracts in the class ICA.*

The intuition behind this result is the following. The short-run incentive to deviate from any collusive agreement is lower for a manager under an ICA-type contract than for a profit-maximizing one because the value of the shares of a firm that deviates from a collusive agreement does not increase as much as short-run profits in the period in which the deviation occurs. This is because, as noted by Holmström and Tirole (1993), the stock price contains more information than accounting profits, and in the case of a deviation the additional information is about the forthcoming punishment phase, that is, bad news. The stock market forecasts that the deviation will be followed by a production war leading to a period of low profits and adjusts firms' stock prices accordingly. Therefore a negative effect of the punishment phase occurs on (deviating and non-deviating) managers' compensation already in the *same* period in which the deviation occurs. In addition, expected stock price and related bonuses in the periods that follow the deviation are low because gains from deviation are distributed and per-period profits are depressed by the punishment phase. These effects make managers under stock-related compensation more prone to collude than owners or profit-maximizing managers.

3.2 Stock options

To make our result more concrete, we consider the case of the most popular type of managerial incentives related to stock price, namely stock option plans. Also, stock options are not strictly increasing functions of the stock-price, since for all strike prices above the stock price the value of the option is constant and equal to zero. Therefore we could not simply apply Proposition 1 to this case.

Definition 2 ***Incentive contracts class $A_{\underline{P}}$ (ICA$_{\underline{P}}$):*** *In each period t the manager receives the right to buy a number γ_i of shares at a predetermined price $\underline{P}_i$, both of which are constant across time periods,* before *period t profits are paid out as dividends.*

Again, because under these contracts managers get paid before the distribution of dividends, the value of a share at the time when they can cash their stock options includes that period's profits. Then the incentive compatibility condition for a stationary collusive agreement A to be supportable by the manager of firm i under a compensation package of the ICA$_{\underline{P}}$ type is

$$\begin{aligned}\frac{1}{1-\delta}\max\left\{\gamma_i\left[V\left(\frac{\pi_i^A}{1-\delta}\right)-\underline{P}_i\right],0\right\} &> \max\left\{\gamma_i\left[V\left(\widehat{\pi}_i^A+\frac{\delta\pi_i^N}{1-\delta}\right)-\underline{P}_i\right],0\right\}+ \\ &+\frac{\delta}{1-\delta}\max\left\{\gamma_i\left[V\left(\frac{\pi_i^N}{1-\delta}\right)-\underline{P}_i\right],0\right\},\end{aligned} \qquad (3)$$

where the inequality is strict because of assumption 3 in Section 2.3. In each period the stock options are "in the money" (valuable) if the price of the shares P_i^t at the end of the period is higher than $\underline{P}_i$. Then we can state a result analogous to Proposition 1.

Corollary 1 *Suppose firms are led by managers under incentive contracts in the class ICA$_{\underline{P}}$, with $\underline{P}_i < V\left(\frac{\pi_i^A}{1-\delta}\right)$, $\forall i$. Then, the minimum discount factor at which any collusive agreement delivering per-period profits π_i^A can be supported in subgame-perfect equilibrium is strictly lower (and, for a given discount factor, more profitable collusive agreements become supportable,) than when firms are led by owners or by profit-maximizing managers.*

Again, the intuition is that the short-run incentive to deviate from any collusive agreement is lower for a manager under an ICA$_{\underline{P}}$-type contract because the stock market forecasts the production war that follows a deviation and adjusts firms' stock price accordingly, anticipating the negative effects of the punishment during the same period in which the deviation occurs. One can also state the following corollary.

Corollary 2 *Suppose the repeated oligopoly game is played by managers under ICA$_{\underline{P}}$-type contracts. Then, the minimum discount factors at which any collusive agreement can be supported in subgame-perfect equilibrium is independent of γ_i and is maximized when $\underline{P}_i \leq V\left(\frac{\pi_i^N}{1-\delta}\right)$, $\forall i$.*

As before, a corresponding statement holds for the most profitable agreement supportable at a given discount factor. The corollary implies that the pro-collusive effect is stronger when the strike price is so generous that stock options are valuable whatever collusive equilibrium is chosen. It is easier to understand the intuition behind this conclusion by looking at condition (3). By increasing the strike price over $\underline{P}_i = V\left(\frac{\pi_i^N}{1-\delta}\right)$ owners reduce the value of stock options in periods in which they are "in the money," therefore they reduce managers' expected gains from cooperation (LHS of (3)) and one-period gains from deviation (first member of the RHS of (3)). However, managers' payoffs from the punishment phase (second member of the RHS of (3)) are not affected by such a change. Because the fall of gains from cooperation is protracted in time, it dominates the fall of one-period gains from deviation, making collusion harder to sustain. Instead, reductions of the strike price below $\underline{P}_i = V\left(\frac{\pi_i^N}{1-\delta}\right)$ increase both sides of condition (3) at the same rate and leave managers' incentive constraint unaffected.

4 Deferred stock-related compensation

4.1 The general case

Consider now a slightly different class of contracts, by which in each period managers receive their stock-related bonuses only after having distributed that period's profits as dividends.

Definition 3 ***Incentive contract class B (ICB):*** *In each period t the manager receives a compensation positively related to stock price $f_i(P_i^t)$ – where f_i is any monotone and strictly increasing function –* after *period t profits are paid out as dividends.*

Because managers get paid after the distribution of dividends, the value of the shares when they cash their options does not incorporate present profits π_i^t. When managers under ICB-type contract cash their bonuses the stock price is therefore:

$$P_i^t = V\left[E_t\left(\sum_{\tau=1}^{\infty}\delta^{\tau}\ \pi_i^{t+\tau}\right)\right].$$

Also, consider stock-related incentive plans such that managers can cash the bonuses only some time after having left the firm. This form of compensation is often introduced to avoid managers (who may be planning to leave the firm) taking actions against the long-run interest of shareholders in order to improve their short-run market valuation, and to maintain an incentive for managers close to retirement to work hard. To keep things simple we assume that managers face a constant per-period probability $(1-\eta)$ of leaving the firm (because of a take-over, say, or because they find a better job).

Definition 4 ***Incentive contract class C (ICC):*** *In each period the manager receives a wage, which we normalize to zero, and in the period after he stops working for the firm, say τ periods after he started, he receives additional compensation positively related to stock price $(1+r)_i^{\tau}\ f_i(P_i^{\tau}\ )$ – where f_i is any monotone increasing function and $\frac{1}{1+r} = \delta$.*

Then one can state the following result.

Proposition 2 *Suppose firms are led by managers under incentive contracts in the class ICB or ICC. Then, the joint monopoly collusive agreement can be supported in subgame-perfect equilibrium at any level of the discount factor.*

The intuition behind the proposition is somewhat analogous to that behind Proposition 1, but here the mechanism is taken at its extreme consequences. For a manager under contracts in class ICB or ICC there is no incentive whatsoever to deviate from collusion. After short-run profits from a deviation from a collusive agreement are paid out as dividends, the price of the shares of the deviating firm (and therefore its manager's compensation) depends *only* on stock market expectations about the firm's future profitability, and therefore it falls. Managers under contracts in the classes ICB and ICC incur a net loss when they deviate from a collusive agreement without ever being able to capture any of the short-run gains from deviating.[9]

4.2 Stock options

Consider now stock option plans with deferred realization.

Definition 5 ***Incentive contract class $B_{\underline{P}}$ (ICB$_{\underline{P}}$):*** *In each period t the manager receives the right to buy a number γ_i of shares at a predetermined price $\underline{P}_i$, both of which are constant across time periods,* after *period t profits are paid out as dividends.*

Because managers get paid after the distribution of dividends, the value of the shares when they cash their stock options does not incorporate present profits π_i^t. Also, consider stock option plans such that managers can exercise the options only some time after retirement. Again, we assume that in every period the manager faces a constant probability $(1-\eta)$ of leaving the firm.

Definition 6 ***Incentive contract class $C_{\underline{P}}$ (ICC$_{\underline{P}}$):*** *In each period the manager receives a flat wage, which we normalized to zero, and in the period after he stops working for the firm, say τ periods after he started, the manager receives a number of stock options $\gamma_i(1+r)^\tau$ – where $\frac{1}{1+r} = \delta$ – with a strike price $\underline{P}_i$, with γ_i and $\underline{P}_i$ constant across time periods.*

Then one can state what follows.

Corollary 3 *Suppose the repeated oligopoly game is played by managers under incentive contracts in the class ICB$_{\underline{P}}$ or ICC$_{\underline{P}}$, with $\underline{P}_i < V\left(\frac{\pi_i^M}{1-\delta}\right)$, $\forall i$. Then, the joint monopoly collusive agreement can be supported in subgame-perfect equilibrium at any level of the discount factor.*

The intuition behind this result is fully analogous to that behind Proposition 2.

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[9] Note that nothing changes if managers under ICC-type contracts know exactly when they will stop working for their firms. What is important is that, after managers have left, firms go on producing so that stock market expectations about future firms' profitability can influence the leaving managers' compensation through stock prices.

5 On the length of managerial contracts

In the previous sections managers where implicitly assumed to have an infinite horizon, as the firm. Although managers do tend to stay with one firm for long periods, in reality managerial contracts are seldom life-long. In this section we want to show that the results in the previous sections hold independently of the length of the explicit managerial contract. To do this, we assume here that managerial contracts last a finite number of periods T. In this case every T periods owners must decide whether to reconfirm the current manager and his incentive contract or to replace them. This situation can be modelled as a repeated game whose stage game is composed of several consecutive steps. To simplify exposition we assume that all managers' contracts last the same number of periods T and are signed (and expire) simultaneously. It will become clear below that the results of this section are not dependent on these simplifying assumptions. To make the model treatable and the results more clear-cut, we also assume that when owners or profit-maximizing managers are in control no collusive agreement can be supported.

Assumption δ : *For any collusive agreement* A*, it holds that* $\delta < \underline{\delta}^A = \frac{\widehat{\pi}_i^A - \pi_i^A}{\widehat{\pi}_i^A - \pi_i^N}$.

The stage game of the oligopoly supergame will now be composed of $T+1$ steps. The timing of a stage game beginning in a period t will be as follows.

Stage game t

- Step 1: Owners choose whether to delegate control and managers' incentive contracts.
- Steps 2 to $T+1$: All players observe the outcome of the previous step, then players in control choose output levels.

In other words, in Step 1 of each stage game each of the owners simultaneously decides whether to delegate or reconfirm control to a manager, and if he does it he also chooses the manager's incentive contract for the T following periods. If delegation occurs in Step 1, then starting from Step 2 the manager chooses output in T consecutive static Cournot market interactions; otherwise, the owner does it. This means that a new stage game will only begin every $T+1$ periods.

We can now state the following result.

Proposition 3 *Any collusive agreement supportable in subgame-perfect equilibrium by managers under life-long stock-related incentive contracts can be supported in equilibrium by managers under stock-related incentive contracts of any finite length* T.

That is, the pro-collusive effects linked to stock-market expectations and deferred incentives identified in Sections 3 and 4 apply independently of the length of managerial contracts. This is because, even when explicit managerial contracts last a finite number of periods, owners and managers are free to agree on *implicit* employment contracts with each other, which are long-term by definition (Bentley MacLeod and James Malcomson, 1989; Lorne Carmichael, 1989). On the side of managers, the negative effects of stock-based incentives on short-run gains from deviations highlighted in Sections 3 and 4 remain when managerial contracts are short-term. Regarding owners, they have no incentive to renege on implicit contracts that lead their manager to sustain collusion, since changes of management or incentives are observable and other firms' managers can react before any short-run gain from deviation can be realized. This is also why the proposition can easily be proved to hold when explicit contracts are not signed (do not expire) simultaneously or have different duration.

Note that the converse of the proposition is not true. In fact, the incentive compatibility conditions for the self-enforcing implicit contracts that replicate the results in the previous sections with short-term explicit managerial contracts can be less stringent than the incentive compatibility conditions with long-term explicit contracts. More formally:

Corollary 4 *If managers have stock-related incentives in the class ICA or in the class* $ICA^{\underline{P}}$ *with* $\underline{P}_i < V\left(\frac{\pi_i^N}{1-\delta}\right)$ $\forall i$*, then the shorter is the length* T *of the explicit managerial contracts, the smaller is the discount factor at which any collusive agreement can be supported (and, for a given discount factor, more profitable agreements become sustainable) in subgame-perfect equilibrium in the delegation supergame.*

Again, owners have no incentives to renege on the implicit managerial contracts, while the negative effect of stock-based incentives on managers' short-run gains from deviations highlighted in Sections 3 does not depend on the length of explicit managerial contracts. Moreover, with short-term contracts at the end of the stage in which a manager deviates he is fired and kept at his reservation wage forever. The threat of termination, with the loss of future stock-related bonuses it implies, has an additional pro-collusive effect that adds to that identified in Section 3. Because termination is closer in time the shorter is the length of explicit managerial contracts, the smaller is T the stronger is the pro-collusive effect.

6 Extensions and discussion

6.1 Alternative specifications of the model

6.1.1 *Dividend policy*

The pro-collusive effects identified in Sections 3 and 4 are driven by the fact that the stock-based incentives are paid to managers in several consecutive periods (ICA, ICB) or are deferred (ICB, ICC), and by assumption 3 in Section 2.2 by which all realized profits are paid out as dividends (so that the stock price at time t depends mostly (or only) on market expectations about firms' profitability in periods *after* t). While the time structure of stock-related incentives in our model reflects the evidence on most common real-world arrangements (Kole, 1997), the assumption that all profits are distributed as dividends is extreme; it has been made to simplify exposition and make results more clear-cut. In fact, this last assumption is not necessary for any of the pro-collusive effects of stock-based compensation identified above. It is easy to check that analogous results obtain as long as *some* profits are paid out as dividends *sometimes* (once is enough) after a deviation occurs. In a perfect information world profits distributed in past periods (distributed gains from deviation) do not enter the present firm's value, therefore future profits have a relatively larger weight than past profits in the determination of a firm's present stock price. This is enough for stock-related incentives to be pro-collusive: by substituting in conditions (2) and (3) one can immediately see that the *only* case in which the behavior of managers under stock-based incentives corresponds to that of profit-maximizing managers or owners, so that the pro-collusive effects disappear, is when firms *never* pay out any dividend *and* investors value a firm's retained profits as much as distributed ones. Both these conditions are generally not met in reality: most firms do pay out dividends, and any announcement of dividend cuts generates strong negative stock-price reactions (e.g. Franklin Allen and Roni Michaely, 1995). Of course, the less dividends are paid out, the closer are managers' and owners' objectives, and the weaker are the pro-collusive effects of stock-related compensation.

6.1.2 *Market structure*

The results above are robust to changes in modelling assumptions about market structure. It is straightforward to check that they apply to repeated oligopolies other than the Cournot type. All results and proofs are stated using only profit streams π_i^N, π_i^M, $\widehat{\pi}_i^M$, etc., with no direct reference to the specific strategic variables used in the product market. We can reinterpret the profit stream as deriving from any other repeated oligopoly (for example, setting $\pi_i^N = 0$ in the case of homogeneous good Bertrand competition), and note that all proofs continue to hold.

6.2 Profit sharing and managerial ownership

We have focused on liquid incentives related to stock price. What if we allow owners to choose managerial contracts that also incorporate a profit-sharing component or a requirement to retain for a minimum number of periods firm's shares received as a bonus?

Incentive contracts of the classes ICA, ICB, and ICC, whether continuous or discontinuous, make managers more prone to collude than owners. It is straightforward to check that any additional profit-sharing component leads managers to behave more like owners; it dilutes the pro-collusive commitment effect of stock-based incentives without bringing any countervailing benefit. Because of this, it is easy to show that in our model the choice to have a profit-sharing incentive component besides stock-related incentives is always dominated (see the section "Profit Sharing" in the appendix).

Analogous reasoning applies when managers are required to keep in their portfolios the shares they get as bonuses for a minimum amount of time (as in "stock-ownership plans"). If managers under contracts in the classes ICA or ICB keep the shares they receive each period, they will in time own an increasing fraction of the firm. This leads them to receive a larger and larger share of the profits realized in each period as dividends, with an effect on product-market behavior identical to that of a profit-sharing component increasing in time in the compensation package. The more shares the manager owns, the more dividends he gets, the more he behaves like an owner, the smaller is the set of collusive agreements he is willing to support, and the higher is the minimum discount rate at which he is willing to stick to any given collusive agreement. Summarizing:

Remark 1 *Profit-sharing incentives, restrictions on the resale or transfer of firm shares received as bonus, and, more generally, managerial ownership dilute the pro-collusive effects of stock-related compensation plans.*

6.3 Incentives linked to sales

In our oligopoly supergame owners can enforce tacit agreements to restrict output. Because in collusive equilibria output is given by the tacit agreement, colluding owners cannot gain strategic advantages (such as reductions in competing firms' output) by delegating control to managers under aggressive FJS-type incentives linked to sales revenue. Incentives linked to sales, though, may still play a role since they may affect owners' gains from deviations and payoffs in the punishment phase.

Consider the case analyzed in Section 5.1, with explicit managerial contracts of any time-length T, and suppose owners can also choose FJS-type incentive schemes linear in profits and sales. Then, in step 1 of any stage game an owner who expects other owners to choose pro-collusive stock-based managerial incentives may wish to deviate by choosing an aggressive FJS-type managerial contract increasing with sales. In the remainder of the supergame collusion would not be sustained, but the deviating owner would enjoy Stackelberg profits π_i^S for the first T periods after the deviation, and of course $\pi_i^S > \pi_i^N$. Normally it holds that $\pi_i^S < \pi_i^M$,[10] therefore if owners expect other owners to stick to a strategy profile prescribing the use of stock-related incentives that lead managers to support the joint monopoly agreement, they would lose strictly by deviating and choosing aggressive FJS-type incentives, whatever T and δ are. More generally (whether π_i^S is $>$, $=$, or $<$ than π_i^M), even when owners can choose FJS-type incentive schemes, any agreement to delegate control to managers under stock-related incentives leading to a collusive market price with per-period profits π_i^A remains supportable as long as $\pi_i^A > (1-\delta^T)\pi_i^S + \delta^T \pi_i^{FJS}$, where π_i^{FJS} denotes profits at Nash equilibrium of the FJS delegation game and $\pi_i^{FJS} < \pi_i^N$.[11] Of course, this is so because at the end of the stage game in which the owner deviates, T periods after the deviation, other owners can react and also optimally choose FJS-type incentives. Therefore, an owner's unilateral deviation is not profitable and collusion is supportable as long as

$$\frac{\pi_i^A}{1-\delta} > \frac{(1-\delta^T)}{1-\delta}\pi_i^S + \frac{\delta^T}{1-\delta}\pi_i^{FJS}.$$

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[10] We are not aware of any general study on the relation between Stackelberg profits and profits at the symmetric joint monopoly agreement, but in the simple examples we worked out we always obtained $\pi_i^S < \pi_i^M$.

[11] It is $\pi_i^{FJS} < \pi_i^N$ because with quantity competition, attempts to gain a strategic advantage through precommitment offset one another (e.g. Fershtmann and Judd, 1987; Sklivas, 1987).

Note that for small enough T this condition will be satisfied even for less profitable agreements, since the owners' positive short-run gains from deviation generated by the opportunity to choose FJS-type incentives (first member at the RHS) are outweighted by the lower profits they induce during the subsequent non-cooperative phase (second member on the RHS).

Finally, if owners can choose both "collusive" stock-based incentives and "aggressive" FJS-type incentives simultaneously, collusion can be further stabilized. To see this, consider a duopoly and the possibility of such "mixed" compensation contracts. Suppose the incentive part I_i of managers' per-period compensation can be composed of a FJS-type incentive scheme linear in per-period profits and in sales revenue (denoted by S_i), plus an additional stock-related bonus plan as in the previous sections. That is,

$$I_i = \rho_i \left(\alpha_i \pi_i + (1 - \alpha_i) S_i\right) + (1 - \rho_i) IC_i(P_i),$$

where $\mathrm{IC}_i(\mathrm{P}_i)$ can be chosen from the classes defined in Sections 3 and 4. Let α^{FJS} denote the equilibrium level of the parameter α of the classical FJS two-stage duopoly model. We get immediately the following result.

Proposition 4 *Even when $\pi_i^S > \pi_i^M$, and whatever T and δ are, any collusive agreement A delivering per-period profits π_i^A can be implemented by a mixed managerial contract with $\alpha = \alpha^{FJS}$, $IC_i(P_i) = ICA^{\underline{P}}$, $\underline{P}_i > \pi^{FJS}$, and $\rho > 0$ but small enough to satisfy the managers' incentive compatibility condition.*

A formal proof (which would be analogous to that of Proposition 3) is not needed, since the logic behind the proposition is straightforward. The point is that the pro-collusive effect of stock options identified in Section 3.2 remains when these are a part of a more complex managerial incentive scheme. In addition, when owners use the mixed contract described above, if in step 1 of a stage game an owner deviates optimally (sets $\rho_i = 1$), the competing manager reacts already in step 2 by maximizing the FJS-type part of his incentive scheme only, as his options are valueless whatever he does. Then, already from the step 2, instead of π_i^S the deviating manager obtains π_i^{FJS}. Therefore this mechanism, which is reminding of the one in Reitman (1993), further stabilizes collusion by ensuring that even an owner who deviates using FJS-type contracts incurs a direct loss in the same period in which he deviates.

6.4 Demand uncertainty

Many contributions to the literature on managerial incentives in oligopoly emphasize results obtained with demand uncertainty, both because uncertainty makes the model more realistic and because it leaves room for a function for managers. The managers' task is then to observe the realization of demand, which occurs after the delegation phase, and choose output using that information (e.g. Fershtman and Judd, 1987; Reitman, 1993).

Let θ denote the stochastic component of demand, and assume θ to be independently and identically distributed in time and its distribution to be common knowledge among agents. As in Julio Rotemberg and Garth Saloner's (1986) model, with demand uncertainty the expected losses from the punishment phase that disciplines the collusive agreement are constant in time, while short-run gains from deviation change together with the realization of the state of the world θ. Then, when the discount factor binds, most profitable collusive agreements must be conditioned on the per-period realization of the shock θ. Whether the supergame is played by owners or by managers, players can agree on a "collusive rule" $q^A(\theta) = (q_1^A(\theta), ..., q_n^A(\theta))$ mapping states of the world into firms' collusive output levels, and eventually into profits. The rule is chosen in order to ensure, given agents' discount factor and the expected punishment for deviations, that for each realization of θ the prescribed collusive output levels are such that the incentive constraint is satisfied for all players. The rule will therefore prescribe larger collusive output levels in good states of the world, when gains from deviations are larger. It is simple to check that in our model the introduction of demand uncertainty leaves the results unchanged. Of course, demand uncertainty adds to strategic uncertainty from the ex-ante point of view, so that we must substitute π_i^N, π_i^A, $\widehat{\pi}_i^A$, π_i^M,...etc., with the corresponding expected values $\pi_i^N(\theta)$, $\pi_i^A(\theta)$, $\widehat{\pi}_i^A(\theta)$, $\pi_i^M(\theta)$,...etc., in agents' incentive constraints. Also, when owners use stock options they will now choose a strike price conditional on the state of demand $\underline{P}_i(\theta)$, if θ can be contracted upon, or otherwise keep $\underline{P}_i$ below $\min_\theta \left\{\pi_i^M(\theta)\right\}$ in order to make collusion supportable in all states of demand. However, the logic behind our results goes through.

6.5 Alternative punishment strategies

We assumed that firms sustain collusive agreements by the threat of reverting to the static Nash equilibrium of the oligopoly game forever. Unrelenting trigger strategies are widely used in the literature because they satisfy the requirement of subgame perfection and they are easy to handle (both for researchers in models and for firms in markets). However, this kind of punishment is not optimal in repeated Cournot oligopolies (Abreu, 1986), and may be subject to ex-post renegotiation, which would undermine its credibility (e.g. Joseph Farrell and Maskin, 1989; Douglas Bernheim and Debraj Ray, 1989).

It is easy to check that all the results continue to hold when the threat used to enforce collusion is to revert to the static Nash equilibrium only for a finite number of periods, for example because the strength of the punishment is bounded by the finite costs of renegotiation (as in Andreas Blume, 1994, and Barbara McCutcheon, 1997).

More generally, the results concerning deferred stock-related incentives depend on managers being unable to capture any short-run gains from deviation. Therefore, all the results in Section 4 apply independent of what punishment strategies are used.[12]

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[12] Even in the case of ICC-type incentives, for which there is the chance that a punishment phase of finite length has passed at the time when the incentives are paid, the pro-collusive effect is independent of the shape of the punishment phase as long as the date at which the manager leaves the firm is uncertain. Of course if the punishment phase lasts one period only, as in Abreu's (1986) two-phase optimal punishments, and the ICC-type incentive is deferred for more than one period with certainty, then neither a deviation nor the punishment affect manager's compensation, and we are led to owners' incentive compatibility condition (condition (1), amended for the new punishment) by assumption 3 in Section 2.3. However, in this case

What if managers have contracts in the class ICA or ICA$^{\underline{P}}$ and there are no renegotiation costs? In Appendix 2 we analyze the case of long-term stock-option plans and find that the results in Section 3 can be extended both to the case of Abreu's (1986) two-phase optimal punishments and to that of Eric van Damme's (1989) "repentance" renegotiation-proof strategies.

6.6 Renegotiation of managerial contracts

In the introduction we explained that stock-related incentives are less subject than other types of incentive to secret renegotiation, because they require shareholders' approval which is given in public shareholders' meetings. If this were not the case, the results of the model would still be of interest for several reasons.

Even if we assume concentrated ownership, so that public shareholders' meetings are not required to renegotiate managers' compensation, the cost of renegotiation may be substantial for owners; and renegotiation costs give commitment value to managerial incentives. There will typically be direct costs of the bilateral bargaining process between managers and owners, even if there are no information asymmetries (Luca Anderlini and Leonardo Felli, 1998). When third parties (for example debtholders) have seats on the board, the bargaining process becomes trilateral and bargaining costs increase. Moreover, interlocked directors and large finance-providers with industry-wide interests will oppose any renegotiation of managerial contract that leads to a market war (Spagnolo, 1996, 1998).

Finally, suppose secret renegotiation were possible and costless. The results would still be of substantial interest. Many economists believe that the incentive compatibility conditions for tacit collusion, inequality (1) in our model, are easily satisfied in most real-world oligopolistic industries (e.g. Carl Shapiro, 1989). If this is true, then if tacit collusion is not present in *all* oligopolistic industries it is only because of coordination failures.[13] Then, owners would not have incentives to renegotiate managers' contracts, and the pro-collusive effects of incentives related to stock price would be to further stabilize tacit collusion and to facilitate coordination.[14]

7 Concluding remarks

We are not arguing here that the effect on tacit collusion is the only force driving firms' adoption of managerial compensation plans related to stock price. As in most previous work on the strategic effects of delegation, to make the model treatable we had to abstract from many important issues, particularly from that of managerial moral hazard (just as most of the literature on moral hazard abstracts from the strategic effects of incentive

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owners can simply restrict the choice of incentives to the classes ICA and ICB.

[13] Fines from competition authorities seem much too small to deter collusion (e.g. McCutcheon, 1997).

[14] CEOs have typically a more homogeneous background than shareholders, they are professionals with similar educations and careers, and a common background is the best known among the factors that facilitate coordination.

contracts). When managers' moral hazard is brought into the picture many other beneficial effects of these incentives emerge.

However, we believe that in the imperfectly competitive real world, the pro-collusive effect of these incentives is one important reason behind their success. In the end, shareholders are satisfied when their managers' incentive schemes lead to higher stock prices, regardless of whether this is achieved through higher effort or through more effective collusion.

8 Appendices

8.1 Appendix 1: Proofs

Proof of Lemma 1 When profits increase (decrease) in one period and everything else remains equal, the stock price increases (decreases) too. Therefore – with regard to the stage game – the stock price function is a monotone transformation of the profit function. A managerial compensation function increasing with stock price is a further monotone transformation of the profit function; consequently, managers' objective function is a monotone transformation of owners' objective function. The set of Nash equilibria of a game is not affected by monotone transformations of payoff functions, as these generate ordinally equivalent games. The statement follows. **Q.E.D.**

Proof of Proposition 1 At $\delta = \underline{\delta}^A$ condition (1) is satisfied as an equality so

$$\frac{\pi_i^A}{1-\underline{\delta}^A} = \widehat{\pi}_i^A + \frac{\underline{\delta}^A}{1-\underline{\delta}^A}\pi_i^N, \Longrightarrow V\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right) = V\left(\widehat{\pi}_i^A + \frac{\underline{\delta}^A}{1-\underline{\delta}^A}\pi_i^N\right),$$

by definition (1′). Substituting into condition (2) we obtain

$$\frac{1}{1-\underline{\delta}^A} f_i\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)\right] > f_i\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)\right] + \frac{\underline{\delta}^A}{1-\underline{\delta}^A} f_i\left[V\left(\frac{\pi_i^N}{1-\underline{\delta}^A}\right)\right],$$

which after a few algebraic manipulations becomes

$$\underline{\delta}^A f_i\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)\right] > \underline{\delta}^A f_i\left[V\left(\frac{\pi_i^N}{1-\underline{\delta}^A}\right)\right],$$

which is always satisfied. Because the inequality is strict, by continuity, perturbing the discount factor around $\underline{\delta}^A$ we can find a continuum of discount factors lower than $\underline{\delta}^A$ at which such a condition is satisfied but (1) is not. This reasoning applies to any stationary collusive agreement A and to each firm i. This proves the statement regarding the discount factor.

Conversely, given agents' discount factor, at the most collusive agreement that owners can support, delivering $\overline{\pi}_i^A$, we have $V\left(\frac{\overline{\pi}_i^A}{1-\delta}\right) = V\left(\widehat{\overline{\pi}}_i^A + \frac{\delta}{1-\delta}\pi_i^N\right)$, and substituting into condition (2) we obtain

$$\frac{1}{1-\delta} f_i\left[V\left(\frac{\overline{\pi}_i^A}{1-\delta}\right)\right] > f_i\left[V\left(\frac{\overline{\pi}_i^A}{1-\delta}\right)\right] + \frac{\delta}{1-\delta} f_i\left[V\left(\frac{\pi_i^N}{1-\delta}\right)\right],$$

that leads to

$$f_i\left[V\left(\frac{\overline{\pi}_i^A}{1-\delta}\right)\right] > f_i\left[V\left(\frac{\pi_i^N}{1-\delta}\right)\right],$$

which is always true. By continuity, perturbing profits around $\overline{\pi}_i^A$ we can find a continuum of higher collusive profit streams which satisfy this condition but not condition (1). This reasoning applies to any stationary collusive agreement and to each firm i. The statement regarding the equilibrium set follows. **Q.E.D.**

Proof of Corollary 1 In the non-trivial case in which $\underline{P}_i < V\left(\frac{\pi_i^A}{1-\delta}\right)$, manager i's incentive compatibility constraint (2) becomes

$$\begin{aligned}\frac{1}{1-\delta}\gamma_i\left[V\left(\frac{\pi_i^A}{1-\delta}\right)-\underline{P}_i\right] &> \gamma_i\left[V\left(\widehat{\pi}_i^A+\frac{\delta\pi_i^N}{1-\delta}\right)-\underline{P}_i\right]+\\ &+\frac{\delta}{1-\delta}\max\left\{\gamma_i\left[V\left(\frac{\pi_i^N}{1-\delta}\right)-\underline{P}_i\right],0\right\}.\end{aligned}$$

Evaluating (1) at $\delta=\underline{\delta}^A$ we obtain

$$\frac{\pi_i^A}{1-\underline{\delta}^A}=\widehat{\pi}_i^A+\frac{\underline{\delta}^A}{1-\underline{\delta}^A}\pi_i^N, \Longrightarrow V\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)=V\left(\widehat{\pi}_i^A+\frac{\underline{\delta}^A}{1-\underline{\delta}^A}\pi_i^N\right).$$

Substituting from this equality into the previous inequality and simplifying we obtain

$$\gamma_i\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)-\underline{P}_i\right]>\max\left\{\gamma_i\left[V\left(\frac{\pi_i^N}{1-\underline{\delta}^A}\right)-\underline{P}_i\right],0\right\}.$$

By inspection, for any strike price $\underline{P}_i < V\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)$ and number of options $\gamma_i \neq 0$ this condition holds as a *strict* inequality. By continuity, perturbing the discount factor around $\underline{\delta}^A$ we can find a continuum of discount factors lower than $\underline{\delta}^A$ (of more collusive agreement, i.e., $\pi_i \geq \pi_i^A$) at which such a condition is satisfied but (1) is not. This reasoning applies to any stationary collusive agreement A and to each firm i.

Conversely, given the discount factor, at the most collusive agreement owners can support, delivering firm profits $\pi_i^{\overline{A}}$, we have $V\left(\frac{\pi_i^{\overline{A}}}{1-\delta}\right)=V\left(\widehat{\pi}_i^{\overline{A}}+\frac{\delta}{1-\delta}\pi_i^N\right)$, and substituting in the condition above we obtain

$$\gamma_i\left[V\left(\frac{\pi_i^{\overline{A}}}{1-\delta}\right)-\underline{P}_i\right]>\max\left\{\gamma_i\left[V\left(\frac{\pi_i^N}{1-\delta}\right)-\underline{P}_i\right],0\right\},$$

which holds as a strict inequality for any $\underline{P}_i < V\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)$ and $\gamma_i \neq 0$. By continuity, perturbing profits around $\pi_i^{\overline{A}}$ we can find a continuum of collusive profit streams which satisfy this condition but not condition (1). This line of reasoning applies to any stationary collusive agreement and to each firm i. The statement follows. **Q.E.D.**

Proof of Corollary 2: Consider first the case $V\left(\frac{\pi_i^N}{1-\delta}\right) \leq \underline{P}_i < V\left(\frac{\pi_i^A}{1-\delta}\right)$. Condition (3) becomes

$$\frac{1}{1-\delta}\gamma_i\left[V\left(\frac{\pi_i^A}{1-\delta}\right)-\underline{P}_i\right]>\gamma_i\left[V\left(\widehat{\pi}_i^A+\frac{\delta\pi_i^N}{1-\delta}\right)-\underline{P}_i\right], \tag{4}$$

or, equivalently,

$$\delta\left[V\left(\widehat{\pi}_i^A+\frac{\delta\pi_i^N}{1-\delta}\right)-\underline{P}_i\right]>V\left(\widehat{\pi}_i^A+\frac{\delta\pi_i^N}{1-\delta}\right)-V\left(\frac{\pi_i^A}{1-\delta}\right).$$

Then, a manager under an ICA$_{\underline{P}}$-type contract with strike price $\underline{P}_i$ is willing to support any given collusive agreement A as long as

$$\delta > \underline{\delta}^A_{ICA_{\underline{P}}} = \frac{V\left(\widehat{\pi}_i^A + \frac{\underline{\delta}^A_{ICA_{\underline{P}}}\pi_i^N}{1-\underline{\delta}^A_{ICA_{\underline{P}}}}\right) - V\left(\frac{\pi_i^A}{1-\underline{\delta}^A_{ICA_{\underline{P}}}}\right)}{V\left(\widehat{\pi}_i^A + \frac{\underline{\delta}^A_{ICA_{\underline{P}}}\pi_i^N}{1-\underline{\delta}^A_{ICA_{\underline{P}}}}\right) - \underline{P}_i}.$$

By inspection, $\underline{\delta}^A_{ICA_{\underline{P}}}$ is independent of γ_i and is increasing with $\underline{P}_i$. Analogously, the upper bound of the collusive profit streams supportable by a manager under ICA$_{\underline{P}}$-type contracts is $\pi_i^{\overline{ICA}_{\underline{P}}}$, where

$$V\left(\frac{\pi_i^{\overline{ICA}_{\underline{P}}}}{1-\delta}\right) - (1-\delta)V\left(\widehat{\pi}_i^{\overline{ICA}_{\underline{P}}} + \frac{\delta\pi_i^N}{1-\delta}\right) = \delta\underline{P}_i.$$

Take the upper bound $\pi_i^{\overline{ICA\prime}}$ which satisfies the equality above at a given strike price $\underline{P}'_i$. A reduction in $\underline{P}_i$ makes the condition satisfied as a strict inequality, moving the upper bound to a higher profit level. So $\pi_i^{\overline{ICA}_{\underline{P}}}$ is a decreasing function of $\underline{P}_i$.

Consider now the case of $\underline{P}_i < V\left(\frac{\pi_i^N}{1-\delta}\right)$. The incentive compatibility condition becomes

$$\begin{aligned}\frac{1}{1-\delta}\gamma_i\left[V\left(\frac{\pi_i^A}{1-\delta}\right) - \underline{P}_i\right] \quad > \quad & \gamma_i\left[V\left(\widehat{\pi}_i^A + \frac{\delta\pi_i^N}{1-\delta}\right) - \underline{P}_i\right] + \\ & + \frac{\delta}{1-\delta}\gamma_i\left[V\left(\frac{\pi_i^N}{1-\delta}\right) - \underline{P}_i\right].\end{aligned}$$

The minimum level of the discount factor at which the manager can support collusion becomes

$$\underline{\delta}^A_{ICA_{\underline{P}}} = \frac{V\left(\widehat{\pi}_i^A + \frac{\underline{\delta}^A_{ICA_{\underline{P}}}}{1-\underline{\delta}^A_{ICA_{\underline{P}}}}\pi_i^N\right) - V\left(\frac{\pi_i^A}{1-\underline{\delta}^A_{ICA_{\underline{P}}}}\right)}{V\left(\widehat{\pi}_i^A + \frac{\underline{\delta}^A_{ICA_{\underline{P}}}}{1-\underline{\delta}^A_{ICA_{\underline{P}}}}\pi_i^N\right) - V\left(\frac{\pi_i^N}{1-\underline{\delta}^A_{ICA_{\underline{P}}}}\right)},$$

and the condition that identifies the most collusive agreement supportable at the given discount factor $\pi_i^{\overline{ICA}_{\underline{P}}}$ becomes

$$V\left(\frac{\pi_i^{\overline{ICA}_{\underline{P}}}}{1-\delta}\right) = (1-\delta)V\left(\widehat{\pi}_i^{\overline{ICA}_{\underline{P}}} + \frac{\delta\pi_i^N}{1-\delta}\right) + \delta V\left(\frac{\pi_i^N}{1-\delta}\right).$$

The last two equalities are both independent of γ_i and $\underline{P}_i$. All this holds for every firm i and the statement follows. **Q.E.D.**

Proof of Proposition 2: Consider first the class ICB. The incentive compatibility condition for a stationary collusive agreement A to be respected by the manager of firm i under ICB-type contracts is

$$\frac{1}{1-\delta}f_i\left[V\left(\frac{\delta\pi_i^A}{1-\delta}\right)\right] > f_i\left[V\left(\frac{\delta\pi_i^N}{1-\delta}\right)\right] + \frac{\delta}{1-\delta}f_i\left[V\left(\frac{\delta\pi_i^N}{1-\delta}\right)\right],$$

or, equivalently,

$$f_i\left[V\left(\frac{\delta\pi_i^A}{1-\delta}\right)\right] > f_i\left[V\left(\frac{\delta\pi_i^N}{1-\delta}\right)\right],$$

which is *always* satisfied, at any discount factor, for any agreement A, and for every firm i.

Consider now contracts in the class ICC. The expected flow of earnings for the manager of firm i in any period t in which he is running the firm is

$$\begin{aligned} E_t(W) &= E_t\left\{\delta(1-\eta)(1+r)f_i(P_i^{t+1}) + \delta^2\eta(1-\eta)(1+r)^2 f_i(P_i^{t+2}) + \right. \\ &\left. +\delta^3\eta^2(1-\eta)\gamma_i(1+r)^3 f_i(P_i^{t+3}) + ...\right\}. \end{aligned}$$

As long as the manager sticks to a stationary collusive agreement delivering per period profits π_i^A we have $E_t\left[P^{t+\tau}\right] = V\left(\frac{1}{1-\delta}\pi_i^A\right)$, $\forall\tau > 0$. If the manager deviates in any period t we have $E_t\left[P^{t+\tau}\right] = V\left(\frac{1}{1-\delta}\pi_i^N\right)$, $\forall\tau > 0$. Because $V\left(\frac{1}{1-\delta}\pi_i^A\right) > V\left(\frac{1}{1-\delta}\pi_i^N\right)$ is always satisfied, whatever the discount factor δ the manager always finds it convenient not to deviate from the agreement. This applies to any agreement A and firm i, and the statement follows. **Q.E.D.**

Proof of Corollary 3: Consider first the class ICB$_{\underline{P}}$. The incentive compatibility condition for a stationary collusive agreement A to be respected by the manager of firm i under ICB$_{\underline{P}}$-type contracts becomes

$$\begin{aligned} \frac{1}{1-\delta}\max\left\{\gamma_i\left[V\left(\frac{\delta\pi_i^A}{1-\delta}\right) - \underline{P}_i\right], 0\right\} &> \max\left\{\gamma_i\left[V\left(\frac{\delta\pi_i^N}{1-\delta}\right) - \underline{P}_i\right], 0\right\} + \\ &+\frac{\delta}{1-\delta}\max\left\{\gamma_i\left[V\left(\frac{\delta\pi_i^N}{1-\delta}\right) - \underline{P}_i\right], 0\right\}, \end{aligned}$$

or, equivalently,

$$\begin{cases} V\left(\frac{\delta\pi_i^A}{1-\delta}\right) > \underline{P}_i, & \text{for } \underline{P}_i \geq V\left(\frac{\delta\pi_i^N}{1-\delta}\right), \\ V\left(\frac{\delta\pi_i^A}{1-\delta}\right) > V\left(\frac{\delta\pi_i^N}{1-\delta}\right), & \text{for } \underline{P}_i \leq V\left(\frac{\delta\pi_i^N}{1-\delta}\right). \end{cases}$$

Because owners always choose $\underline{P}_i < V\left(\frac{\delta\pi_i^A}{1-\delta}\right)$ this condition is *always* satisfied. This holds at any discount factor, for any agreement A, and for every firm i.

Consider now contracts in the class ICC. The amount of stock options given to a manager who stops working for the firm τ periods after he started is $\gamma_i(1+r)^\tau$. The expected flow of earnings for the manager of firm i in any period t in which he is running the firm is then

$$\begin{aligned} E_t(W) &= E_t\left\{\delta(1-\eta)\gamma_i(1+r)(P_i^{t+1} - \underline{P}_i) + \delta^2\eta(1-\eta)\gamma_i(1+r)^2(P_i^{t+2} - \underline{P}_i) + \right. \\ &\left. +\delta^3\eta^2(1-\eta)\gamma_i(1+r)^3(P_i^{t+3} - \underline{P}_i) + ...\right\}. \end{aligned}$$

As long as the manager sticks to a stationary collusive agreement delivering per period profits π_i^A we have $E_t\left[P^{t+\tau}\right] = V\left(\frac{1}{1-\delta}\pi_i^A\right)$, $\forall\tau > 0$. When owners choose $V\left(\frac{\pi_i^N}{1-\delta}\right) \leq \underline{P}_i < V\left(\frac{\pi_i^A}{1-\delta}\right)$, manager i's expected payoff function reduces to

$$E_t(W) = \begin{cases} \gamma_i \left[V\left(\frac{\pi_i^A}{1-\delta}\right) - \underline{P}_i \right] & \text{if } q_i^t \leq q_i^A \ \ \forall t < \tau, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, whatever the discount factor δ, the manager always finds it convenient not to deviate from the agreement. This applies to any agreement A and firm i. The statement follows. **Q.E.D.**

Proof of Proposition 3: Consider the following strategy profile for the delegation supergame.

Each owner's strategy: "*Delegate control to a manager under any kind of stock-related incentive contract (among those defined in Sections 3 and 4) of finite length T such that, if the same contract had infinite length (if $T \to \infty$), the manager would be willing to support the collusive agreement A delivering per-period profits π_i^A; at the beginning of each of the following stage games (in periods $t+T$, $t+2T$, etc.), reconfirm the manager and the contract for one more stage if all other owners have done so in the past and no manager has ever deviated from equilibrium strategies; fire the manager and choose the Cournot-Nash output level (or hire managers under short-term profit-sharing contracts at their reservation wage) forever otherwise.*

Each manager's strategy: "*Respect the collusive agreement A at all steps of each stage game as long as all owners have delegated/reconfirmed managers with the above incentive contracts in the first step of each past stage game and no manager has ever deviated from the collusive agreement; maximize the firm's static profits forever otherwise.*"

Let us check for unilateral deviations to see whether this strategy profile is a subgame-perfect equilibrium independently of the length of the managerial contract T and of the discount factor δ.

Owners: In step 1 of each stage game an owner can choose to deviate unilaterally from the equilibrium strategy profile by not delegating control, by choosing a different contract, or by replacing the manager. However the owner deviates, and whatever T and δ are, the deviation is observed by the managers of the competing firms who, following equilibrium strategies, start maximizing firms' profits already from Step 2. Therefore, owners' expected payoff from deviation is the discounted flow of Cournot Nash profits. If an owner sticks to equilibrium strategies, his manager continues to support collusion in the product market game, and profits are above the Cournot-Nash level. It follows that owners lose strictly by deviating unilaterally from the strategy profile above, whatever the length of the contract and the discount factor are.

Managers: Managers can deviate from equilibrium in any step of each stage game. Whatever T and δ are, if a manager has a contract in the class ICB, ICC, ICB$^{\underline{P}}$ or ICC$^{\underline{P}}$, he gains nothing by deviating because short-run profits are distributed before he gets his stock-related bonuses (see the Proof of Proposition 2). If the manager has a contract in the class ICA or ICA$^{\underline{P}}$, in the period in which he deviates his stock-related bonus does increase in value. However, starting from the following period other managers maximize static firm profits and the value of his stock related bonus falls. And at the end of the stage game in which he deviated he is fired, so in all periods after that stage game he receives only his reservation wage (that was normalized to zero). It follows that the most profitable deviation is the one that occurs in Step 2 of a stage game. Consider the case of ICA contracts. If explicit contracts are of any finite length T, a manager's no deviation condition is

$$\frac{1}{1-\delta} f_i \left[V\left(\frac{\pi_i^A}{1-\delta}\right)\right] > f_i \left[V\left(\widehat{\pi}_i^A + \frac{\delta \pi_i^N}{1-\delta}\right)\right] + \frac{\delta(1-\delta^{T-1})}{1-\delta} f_i \left[V\left(\frac{\pi_i^N}{1-\delta}\right)\right].$$

Because for any finite T and at any δ it holds $\frac{\delta(1-\delta^{T-1})}{1-\delta} < \frac{\delta}{1-\delta}$, the RHS of this condition is smaller than the RHS of condition (2), and the condition is always satisfied when (2) is. Since equilibrium strategies prescribe owners to use stock-related incentive contract such that, if they had infinite length, the manager would be willing to support the joint monopoly collusive agreement, if in equilibrium managers have ICA-type contracts condition (2) will be satisfied, and so will be the inequality above. The same reasoning holds when owners choose a contract in the class ICA$^{\underline{P}}$. The statement follows. **Q.E.D.**

Proof of Corollary 4: Consider the case of managers under ICA contracts of length T. Managers stick to an agreement A as long as the inequality in the proof of Proposition 3 holds:

$$\frac{1}{1-\delta} f_i \left[V\left(\frac{\pi_i^A}{1-\delta}\right)\right] > f_i \left[V\left(\widehat{\pi}_i^A + \frac{\delta \pi_i^N}{1-\delta}\right)\right] + \frac{\delta(1-\delta^{T-1})}{1-\delta} f_i \left[V\left(\frac{\pi_i^N}{1-\delta}\right)\right].$$

Because $\frac{\delta(1-\delta^{T-1})}{1-\delta}$ is increasing in T, the RHS of the inequality is increasing in T. It follows that the condition becomes more stringent the larger T is. Analogous reasoning holds for contracts in the class ICA$^{\underline{P}}$ with $\underline{P}_i < V\left(\frac{\pi_i^N}{1-\delta}\right)$ $\forall i$. **Q.E.D.**

Profit-sharing: Suppose owners can choose the parameters α_i, γ_i and $\underline{P}_i$ of managers' compensation package. We show here that it is a dominant strategy for each owner to maximize managers' ability to support collusive agreements by choosing $\alpha_i = 0$.

Let us normalize the parameters and restrict attention to the case in which $\gamma_i = (1-\alpha_i)$ and $\alpha_i \leq 1$, which encompasses all economically relevant cases. Under such a contract, the incentive compatibility condition for a stationary collusive agreement A to be respected by the manager of firm i becomes

$$\frac{1}{1-\delta}\left\{\alpha_i \pi_i^A + (1-\alpha_i)\left[V\left(\frac{\pi_i^A}{1-\delta}\right) - \underline{P}_i\right]\right\} \geq$$

$$\geq \alpha_i \widehat{\pi}_i^A + (1-\alpha_i)\left[V\left(\widehat{\pi}_i^A + \frac{\delta\pi_i^N}{1-\delta}\right) - \underline{P}_i\right] +$$

$$+\alpha_i \frac{\delta \pi_i^N}{1-\delta} + (1-\alpha_i)\frac{\delta \max\left[V\left(\frac{\pi_i^N}{1-\delta}\right) - \underline{P}_i, 0\right]}{1-\delta},$$

which can be rearranged as

$$(1-\alpha_i)\left\{\frac{V\left(\frac{\pi_i^A}{1-\delta}\right)-\underline{P}_i}{1-\delta}-V\left(\widehat{\pi}_i^A+\frac{\delta\pi_i^N}{1-\delta}\right)+\underline{P}_i\right.$$

$$\left.-\frac{\delta\max\left[V\left(\frac{\pi_i^N}{1-\delta}\right)-\underline{P}_i,0\right]}{1-\delta}\right\}+\alpha_i\left\{\frac{\pi_i^A}{1-\delta}-\widehat{\pi}_i^A-\frac{\delta\pi_i^N}{1-\delta}\right\}\geq 0. \tag{5}$$

It is evident that condition (5) is a linear combination of conditions (1) and (3), with α_i and $(1-\alpha_i)$ as weights. If owners decide to delegate, they can choose the value of two parameters in their managers' compensation package, α_i and $\underline{P}_i$. Given a level of $\underline{P}_i$, when $\alpha_i = 1$ managers have exactly the same incentives as owners (only condition (1) matters), and when $\alpha_i = 0$ we are in the case of Section 3.2.

Evaluating condition (5) at the most collusive profit stream which owners can sustain, we obtain that the first term of the LHS is strictly positive (by the Proof of Proposition 1) and the second is zero (by definition). This means that as long as $\alpha_i < 1$, condition (5) is satisfied as a strict inequality and managers under this contract can support more collusive agreements than owners. However, because the second term on the LHS is negative, as long as $\alpha_i > 0$ managers under ICA-type contracts will still be able to support more collusive agreements. In the Proof of Corollary 2 we have shown that the member within the first graph parentheses on the LHS of (5) is *always* larger than the content of the second graph parentheses. It follows that owners maximize managers' ability to support collusive agreements by choosing $\underline{P}_i \leq V\left(\frac{\pi_i^N}{1-\delta}\right)$ and $\alpha_i = 0$. Because contracts are observable, managers collude only if all owners delegate under suitable incentive contracts, and can react if an owner deviates from the agreed strategies. Because owners cannot lose by delegating control, while they gain strictly when more profitable collusive agreements become supportable in equilibrium, each owner's dominant strategy is to set $\alpha_i = 0$. **Q.E.D.**

8.2 Appendix 2: Long-term contracts with alternative punishment strategies

8.2.1 Optimal punishments

The results of Section 3 can be extended to the case in which players use two-phase optimal punishment strategies (Abreu, 1986). Consider the case of stock options.

Proposition 5 *When the repeated oligopoly game is played by managers under* $ICA_{\underline{P}}$*-type contracts with* $\underline{P}_i < V\left(\frac{\pi_i^A}{1-\delta}\right)$, $\forall i$, *then the minimum discount factor at which any collusive agreement delivering per-period profits* π_i^A *can be supported in subgame-perfect equilibrium by two-phase symmetric optimal punishments (as defined in Abreu, 1986) is strictly lower (and, for a given discount factor, more profitable collusive agreements become supportable,) than when firms are led by owners or by profit-maximizing managers.*

Proof: By Theorem 15 in Abreu (1986), producing for one period a symmetric vector $q^{PA} > q^N$ delivering profits $\pi_i^{PA} = \pi_i(q^{PA})$, with $\pi_i^{PA} < \pi_i^N < \pi_i^A$, and then going back to the collusive output is an optimal two-phase symmetric punishment. The punishment is able to support a symmetric stationary collusive agreement A to restrict production to the vector q^A if the system

$$\begin{cases} \delta(\pi_i^A - \pi_i^{PA}) = \widehat{\pi}_i^{PA} - \pi_i^{PA} \\ \widehat{\pi}_i^A - \pi_i^A \geq \delta(\pi_i^A - \pi_i^{PA}), \end{cases} \tag{OP}$$

is satisfied, where as usual $\widehat{\pi}_i^{PA} = \pi_i(\widehat{q}_i(q_{-i}^{PA}), q_{-i}^{PA})$ are firm i's profit from deviating from the prescribed (punishment) equilibrium path and choosing a best response to other firms' output vector q_{-i}^{PA}.

Denote by $\underline{\delta}_{OP}^A$ the minimum level of the discount factor at which owners can support the collusive agreement A using a symmetric two-stage optimal punishment. This level is defined by the equality

$$\widehat{\pi}_i^A - \pi_i^A = \underline{\delta}_{OP}^A(\pi_i^A - \pi_i^{PA}) = \widehat{\pi}_i^{PA} - \pi_i^{PA}.$$

By Theorems 14 and 18 in Abreu (1986), the symmetric two-stage punishment strategy which delivers profits $\pi_i^{PA} < \pi_i^N$ for one period after a deviation and then reverts to the collusive agreement A is the unique symmetric optimal punishment; further, it is a globally optimal punishment if $\underline{\delta}_{OP}^A$ is not too low. From the definition of $\underline{\delta}_{OP}^A$ we have

$$\widehat{\pi}_i^A - \pi_i^A = \underline{\delta}_{OP}^A(\pi_i^A - \pi_i^{PA})$$

or, equivalently,

$$\frac{\underline{\delta}_{OP}^{A\ 2}\pi_i^A}{1 - \underline{\delta}_{OP}^A} + \widehat{\pi}_i^A - \pi_i^A = \underline{\delta}_{OP}^A(\pi_i^A - \pi_i^{PA}) + \frac{\underline{\delta}_{OP}^{A\ 2}\pi_i^A}{1 - \underline{\delta}_{OP}^A},$$

that simplifies to

$$\frac{\pi_i^A}{1 - \underline{\delta}_{OP}^A} = \widehat{\pi}_i^A + \underline{\delta}_{OP}^A\pi_i^{PA} + \frac{\underline{\delta}_{OP}^{A\ 2}\pi_i^A}{1 - \underline{\delta}_{OP}^A}. \tag{6}$$

Consider now managers' incentive compatibility constraint to support the joint monopoly agreement under ICA when optimal two-phase punishments are used and the discount rate is $\underline{\delta}_{OP}^A$.

First we check that these strategies are subgame-perfect for managers too. With $\gamma_i = 1$ in the period after a deviation, if a manager sticks to the agreed strategies he expects discounted payoffs

$$S^{OP} = \max\left\{V\left(\pi_i^{PA} + \frac{\underline{\delta}_{OP}^A\pi_i^A}{1 - \underline{\delta}_{OP}^A}\right) - \underline{P}_i, 0\right\} +$$

$$+\frac{\underline{\delta}_{OP}^A}{1 - \underline{\delta}_{OP}^A}\left[V\left(\frac{\pi_i^A}{1 - \underline{\delta}_{OP}^A}\right) - \underline{P}_i\right].$$

If he deviates and causes the other manager to restart the punishment phase he expects

$$\begin{aligned} D^{OP} = \max & \left\{ V\left(\widehat{\pi}_i^{PA} + \underline{\delta}_{OP}^{A} \pi_i^{PA} + \frac{\underline{\delta}_{OP}^{A}{}^{2} \pi_i^{A}}{1-\underline{\delta}_{OP}^{A}} \right) - \underline{P}_i, 0 \right\} + \\ & + \underline{\delta}_{OP}^{A} \max \left\{ V\left(\pi_i^{PA} + \frac{\underline{\delta}_{OP}^{A} \pi_i^{A}}{1-\underline{\delta}_{OP}^{A}} \right) - \underline{P}_i, 0 \right\} + \\ & + \frac{\underline{\delta}_{OP}^{A}{}^{2}}{1-\underline{\delta}_{OP}^{A}} \left\{ V\left(\frac{\pi_i^{A}}{1-\underline{\delta}_{OP}^{A}} \right) - \underline{P}_i \right\}, \end{aligned}$$

and $S^{OP} - D^{OP} = 0$ by the definition of owners' optimal punishment (OP). So managers have no incentives to deviate ex post.

Now, with owners' optimal punishment strategies, managers' incentive compatibility condition becomes

$$\frac{1}{1-\underline{\delta}_{OP}^{A}} \left[V\left(\frac{\pi_i^{A}}{1-\underline{\delta}_{OP}^{A}} \right) - \underline{P}_i \right] \geq$$

$$\begin{aligned} \geq \; & \left[V\left(\widehat{\pi}_i^{A} + \underline{\delta}_{OP}^{A} \pi_i^{PA} + \frac{\underline{\delta}_{OP}^{A}{}^{2} \pi_i^{A}}{1-\underline{\delta}_{OP}^{A}} \right) - \underline{P}_i \right] \\ & + \underline{\delta}_{OP}^{A} \max \left\{ V\left(\pi_i^{PA} + \frac{\underline{\delta}_{OP}^{A} \pi_i^{A}}{1-\underline{\delta}_{OP}^{A}} \right) - \underline{P}_i, 0 \right\} \\ & + \frac{\underline{\delta}_{OP}^{A}{}^{2}}{1-\underline{\delta}_{OP}^{A}} \left[V\left(\frac{\pi_i^{A}}{1-\underline{\delta}_{OP}^{A}} \right) - \underline{P}_i \right]. \end{aligned} \tag{7}$$

By equality (6) the first term in squared brackets on the RHS of (7) equals the content of the square brackets in the term at the LHS and in the third term at the RHS of (7), and is strictly larger than the content of the squared bracket in the second term at the RHS of (7). It follows that (7) is satisfied as a strict inequality and, by continuity, there will be a sequence of discount factors lower than $\underline{\delta}_{OP}^{A}$ at which (7) is still satisfied. This applies to any firm i and collusive agreement A. For a fixed δ the same line of reasoning proves that the set of supportable collusive agreements is larger for managers under ICA-type incentives when optimal two-phase punishments are used. For incentives linear in stock price, substitute $\underline{P}_i = 0$ and note that the reasoning above continues to hold. The statement follows. **Q.E.D.**

The intuition is analogous to that behind Proposition 1 and Corollary 1. The only difference is that here the negative effect of the punishment is concentrated in the period immediately following the deviation.

8.2.2 Renegotiation-proof strategies

Consider the case in which firms use renegotiation-proof strategies (in the sense of Farrell and Maskin, 1989). For simplicity, let us focus on a duopoly ($i \in \{1, 2\}$), so that we can consider renegotiation-proof punishment strategies of the kind proposed by van Damme (1989) for the repeated Prisoner's Dilemma. In our Cournot model, these strategies can be defined as follows:

Punishment strategies "R":

Phase 1: Stick to the collusive output level as long as the other firm did the same in the past; if the other firm deviates, then start Phase 2;

Phase 2: Produce the full monopoly output q^M as long as the other firm's output is positive; if for (say) one period the other firm's output is zero, restart Phase 1 in the following period.[15]

Then we can state the following corollary.

Proposition 6 *When the repeated oligopoly game is played by managers under* $ICA_{\underline{P}}$*-type contracts with* $\underline{P}_i < V\left(\frac{\pi_i^A}{1-\delta}\right)$, $\forall i$, *then the minimum discount factor at which any collusive agreement delivering per-period profits* π_i^A *can be supported in subgame-perfect equilibrium by renegotiation-proof strategies R is strictly lower (and, for a given discount factor, more profitable collusive agreements become supportable,) than when firms are led by owners or by profit-maximizing managers.*

Proof: To support a collusive agreement in subgame-perfect equilibrium, the strategy profile must also satisfy the deviating firm's incentive constraint, that is, it must be convenient for the firm which has deviated to repent after the deviation so that

$$0 + \frac{\delta \pi_i^A}{1-\delta} \geq \frac{\pi_i(\widehat{q}_i(q^M), q^M)}{1-\delta}.$$

Assume this condition is satisfied. Let $\underline{\delta}_R^A$ denote the minimum level of the discount factor at which the joint monopoly collusive agreement is supportable by owners using these renegotiation-proof strategies with one-period "repentance," where – assuming $\pi_i^A \underline{\delta}_R^A \geq \pi_i(\widehat{q}_i(q^M), q^M)$ – $\underline{\delta}_R^A$ is defined by the equality

$$\frac{\pi_i^A}{1-\underline{\delta}_R^A} = \widehat{\pi}_i^A + \frac{\underline{\delta}_R^{A\,2} \pi_i^A}{1-\underline{\delta}_R^A}. \tag{R}$$

First we check that strategies R are subgame-perfect for managers too. In the period after a deviation, if the manager of firm i who deviated sticks to the agreed punishment strategies R, he gets expected payoffs

$$S_i^R = \max\left\{ V\left(\frac{\underline{\delta}_R^A \pi_i^A}{1-\underline{\delta}_R^A}\right) - \underline{P}_i, 0 \right\} +$$

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[15] These strategies are weakly renegotiation-proof (and strongly renegotiation-proof for the joint monopoly collusive agreement) because after firm i deviates it is supposed to "give a premium" to the other firm j in order to restart cooperation. By not producing for one period, firm i earns zero profits while it makes firm j's profits increase over the level of the joint monopoly collusive agreement. Such a premium makes it more profitable for firm j to insist on the agreed punishment strategies rather than to renegotiate them toward new collusive outcomes.

$$+\frac{\underline{\delta}_R^A}{1-\underline{\delta}_R^A}\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i\right],$$

while if deviating he causes the other manager to restart the punishment and obtains expected payoffs

$$D_i^R=\max\left\{V\left(\pi_i(\widehat{q}_i(q^M),q^M)+\frac{\underline{\delta}_R^{A2}\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i,0\right\}+$$
$$+\underline{\delta}_R^A\max\left\{V\left(\frac{\underline{\delta}_R^A\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i,0\right\}+$$
$$+\frac{\underline{\delta}_R^{A2}}{1-\underline{\delta}_R^A}\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i\right].$$

Consider now the incentive not to deviate, the difference

$$S_i^R-D_i^R=\max\left\{V\left(\frac{\underline{\delta}_R^A\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i,0\right\}-\max\left\{V\left(\pi_i(\widehat{q}_i(q^M),q^M)+\frac{\underline{\delta}_R^{A2}\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i,0\right\}$$
$$+\underline{\delta}_R^A\left\{\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i\right]-\max\left\{V\left(\frac{\underline{\delta}_R^A\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i,0\right\}\right\}.$$

The difference between the first two members on the RHS is positive because when defining R we assumed $\pi_i^A\underline{\delta}_R^A\geq\pi_i(\widehat{q}_i(\overline{q}_j),\overline{q}_j)$; the third member on the RHS is strictly positive by inspection, therefore $S_i^R-D_i^R>0$.

On the other hand, the manager of firm j, who did not deviate, by sticking to the agreed strategies in the period after a deviation gets expected payoffs:

$$S_j^R=\left[2\pi_j^M+\underline{\delta}_R^A V\left(\frac{\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i\right]+$$
$$+\frac{\underline{\delta}_R^A}{1-\underline{\delta}_R^A}\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i\right],$$

and any deviation from R will cause him a loss.

Then, with punishment strategies R the managers' incentive compatibility condition becomes

$$\frac{1}{1-\underline{\delta}_R^A}\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i\right]\geq$$

$$\geq\left[V\left(\widehat{\pi}_i^A+\frac{\underline{\delta}_R^{A2}\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i\right] \tag{8}$$
$$+\underline{\delta}_R^A\max\left\{V\left(\frac{\underline{\delta}_R^A\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i,0\right\}$$
$$+\frac{\underline{\delta}_R^{A2}}{1-\underline{\delta}_R^A}\left[V\left(\frac{\pi_i^A}{1-\underline{\delta}_R^A}\right)-\underline{P}_i\right].$$

By equality (R) the first term in squared brackets on the RHS of (8) equals the content of the squared brackets in the term on the LHS and in the third term on the RHS of (8), and is strictly larger than the content of the squared brackets in the second term on the RHS of (8). It follows that (8) is satisfied as a strict inequality and, by continuity, there will be a sequence of discount factors lower than $\underline{\delta}_R^A$ at which (8) is still satisfied. This reasoning applies to any collusive agreement other than the joint monopoly one. For incentives linear in stock price substitute $\underline{P}_i = 0$, and note that the reasoning above continues to hold. The statement follows. **Q.E.D.**

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