Human Capital, R&D and Economic Growth*

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Abstract

People go to school and firms do R&D. These activities result in human capital accumulation and new ideas and technologies which make economies grow. We try to capture the interaction between human capital and R&D by allowing for endogenous human capital accumulation in an economy where the number of products and technologies expands because profit maximizing entrepreneurs do R&D. We find that, in the absence of scale effects, long run growth is determined by the capacity to accumulate human capital. A relative lack of R&D capital causes the economy to grow slowly during its transition to the steady state, while a relative abundance of R&D capital gives high growth rates during transition.

Keywords: human capital, R&D, endogenous growth, scale effects, transitional dynamics.
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1 Introduction

People go to school and firms do research and development (R&D). Insufficient investment in either of these activities is brought forth as an explanation for slow growth in both rich and poor countries. The purpose of this paper is to analyse how human capital accumulation and the development of new products and technology through investments in R&D interact in the determination of economic growth by providing incentives for each other. The basic idea is that the incentive to accumulate human capital is affected by the introduction of new products and technologies. The accumulation of human capital also affects the rate of return to investments in R&D. As we will show, the effects go via the relative returns to investments in human capital and R&D. This interaction has largely been left out in previous work which has focused either on human capital accumulation *or* on R&D as engines of endogenous growth, e.g. Romer (1990), Grossman and Helpman (1991), Segerstrom (1991), Aghion and Howitt (1992), Lucas (1988), and Stokey (1988, 1991). By merging these two theoretical frameworks, our ambition is to construct a model of economic growth based on human capital accumulation and R&D which should ideally manage to produce predictions in line with the findings of the empirical growth literature.

Empirical studies of economic growth have introduced a variety of explanatory variables to account for cross-country variation in growth performance, e.g. enrolment rates, human capital stock proxies, labour force participation rates, fertility, private and public investment rates, foreign direct investment, equipment investment, revolutions and coups, inflation, black market exchange rate premium, market size, and number of people employed in R&D activities (see e.g. Barro 1991, DeLong and Summers 1991, Backus, Kehoe and Kehoe 1992, Levine and Renelt 1992, Mankiw, Romer, and Weil 1992, and Blomström, Lipsey and Zejan 1994). The results from these studies can be summarized in three key areas of importance for growth, namely, human capital, investments, and variables related the business environment. In particular, empirical studies have found that a country tends to grow rapidly if it is poor relative to its human capital endowment. Most studies also find savings and investment rates (whether in physical or in human capital) to be positively correlated with growth.[1]

[1] Mankiw, Romer, and Weil (1992) find that as much as half of the cross-country variation in the growth rate of gross domestic product per worker can be accounted for by differences in human capital investment (enrolment in secondary school/per working population), investments in physical capital (investment/GDP), population growth, and differences in initial income.

Furthermore, there is evidence that the increased participation in the labour force is associated with growth. Measures trying to capture the freedom to engage in and enjoy the fruits of economic activity are also found to be positively correlated with growth. However, despite ample efforts to verify the existence of scale effects, there seems to be little empirical evidence of a positive effect on growth of either country size, size of the R&D sector or of other types of scale effects predicted by most R&D-based endogenous growth models.

In developing a model which recognizes that human capital accumulation and investment in R&D take place at the same time, we are inspired by the versions of the model of technological progress with expanding varieties and the Uzawa-Lucas human capital model developed in Barro and Sala-i-Martin (1995). The implications for growth are similar to those found when technological progress and human capital are analysed separately, but by assuming that there are decreasing returns to human capital both in final output production and in schooling, the interaction between human capital and R&D generates some interesting implications for the transitional dynamics. Furthermore, elimination of scale effects, in order to comply with the empirical evidence, has implications for the determinants of long run economic growth.

Growth models with R&D based technological progress in which the stock of human capital is fixed and exogenously determined, rely on scale effects to generate endogenous growth. In such models, the steady state growth rate, which is driven by the advances in technology, is higher the larger the scale of the economy, i.e. the larger the stock of human capital or the more resources spent on R&D.[2] A recent exception is Young (1998) where long run growth is driven by improvements in product quality and is independent of scale. In our model, which also does not exhibit scale effects, we find that the economy's long run growth rate of output is the same as the rate of introduction of new products and technologies. However, the growth rate of the economy is ultimately determined by the capacity to accumulate human capital, since the accumulation of human capital drives the incentives for investing in R&D.

In the Uzawa-Lucas model, output is produced from human and physical capital. Interaction between the two forms of capital accumulation results in an imbalance effect between human

[2] See Jones (1995) for a discussion of scale effects in R&D-models.

capital and physical capital. This imbalance effect implies that consumption and broad output[3] grow faster, the higher the ratio of human capital to physical capital in relation to some steady state ratio. In this paper, we replace accumulation of physical capital by investments in a profit maximizing R&D-sector which result in what we will call R&D capital. Accumulation of R&D capital in this model has much in common with accumulation of physical capital in that both require that individuals forgo consumption of final output.

The interaction between human capital and R&D capital in our model generates an analogous imbalance effect for consumption growth, but an imbalance effect in the opposite direction for output growth compared to the Uzawa-Lucas model. We find that the growth rate of consumption is higher the higher the ratio of human capital to R&D capital relative to some steady state ratio. However, we find that the growth rate of output is lower the higher the ratio of human capital to the stock of R&D capital in excess of the steady state ratio. A relative lack of R&D capital implies poor growth because accumulation of human capital is slow when human capital is relatively abundant. Abundance of R&D capital, on the other hand, leads to rapid growth since it implies high rates of return to human capital and thereby provides a strong incentive for rapid human capital accumulation.

The paper is organized as follows. Section 2 develops the model. The steady state is characterized in Section 3 and the dynamics of the model are described in Section 4. Section 5 discusses sensitivity of the qualitative results to changes in parameter values. Section 6 concludes.

2 The Model

There are individuals, firms and entrepreneurs. Individuals derive utility from consumption of a final good, and spend their time working in the firms or accumulating human capital. When they work, the individuals earn a wage which is spent either on consumption or lent to entrepreneurs in return for interest. Firms produce final output with human capital and

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[3] Broad output is measured as the output of the final output sector plus the value (in terms of final output) of accumulated human capital, see Barro and Sala-i-Martin (1995). In the Uzawa-Lucas model, the imbalance effect on final output growth implies that output grows faster when the ratio of human capital to physical capital is not in the steady state.

intermediate inputs. These intermediates can be technologies, machines, processes, a marketing model or any other type of input or idea of how production can be more efficient. In the introduction we spoke of R&D capital. We can think of this R&D capital as the stock of blueprints for how to produce the intermediates. The wider the variety of intermediates the firm has access to the better, although more intermediates will increase the demand for human capital, since a worker cannot work with an ever increasing number of intermediates without facing a reduction in the marginal product of new intermediates. Firms rent human capital from the households and buy intermediates from the entrepreneurs.

The entrepreneurs do research to come up with ideas that they develop into blueprints for new intermediates. This research and development activity consumes final output which the entrepreneur obtains by selling shares in his future profits to the households. As opposed to the modelling of R&D in e.g. Romer (1990), labour or human capital is not used in the R&D process. Furthermore, the R&D process is deterministic in the sense that the entrepreneur is sure to come up with a blueprint for a new intermediate if he uses the right amount of final output on experimentation to make a prototype. Once the entrepreneur has developed a new type of intermediate, he can convert final output into intermediates at zero cost, that is he can buy one unit of final output at cost one, turn it costlessly into an intermediate and then sell it back to the firm. The entrepreneur has monopoly power over the sale of his intermediate and can thus charge a price above marginal cost. This allows him to cover the R&D costs and pay dividends to the shareholders of his enterprise. In a setting where the entrepreneurs spend the R&D cost in order to come up with blueprints for new products which in principle could be produced without set up costs by anyone with access to the blueprint, the monopoly power of the entrepreneur needs to be legally protected through patent or otherwise. However, the entrepreneur's R&D cost could also be interpreted in terms of set up costs which are incurred for anyone interested in producing a type of intermediate. In this case, the monopoly power need not be protected, since the fixed costs assure that no entrepreneur will take up production of an already existing intermediate, since that would give a certain loss.

2.1 The firm:

The production of final output, Y_t, takes place in a representative competitive firm, which has the following production function:

$$Y_t = A\int_0^{N_t} \left[h_t^{1-\alpha} X_{jt}^{\alpha}\right] dj, \quad \alpha \in (0,1). \tag{1}$$

A is the general level of technology which can be thought of as being a function of institutional factors like government behaviour, the maintenance of law and order, property rights etc. Production requires a continuum of different types of intermediates. N_t is the number of intermediates that have been introduced at time *t,* i.e. the stock of R&D capital. X_j is the quantity of the j:th type of intermediate. For each type of intermediate, the firm requires an amount of human capital, h_t. This specification of a production process which benefits from the variety of intermediate inputs builds on Ethier (1982), Romer (1987, 1990), Barro and Sala-i-Martin (1995) and others.[4] The intermediates can as mentioned previously be different machines, marketing technologies, organizational forms etc. That is, anything which adds to the production capacity of the firm. The total stock of human capital employed in final output production is aggregated over all *L* individuals employed in the firm. These workers split their time between working and going to school. The total human capital, H_t, employed in the firm is thus:

$$H_t = h_t N_t = \sum_{i=1}^{L} (1 - u_{it}) m_{it} , \tag{2}$$

where u_{it} is share of time individual *i* spends in school and m_{it} is his human capital.

The production function formulated in equation (1), differs from the production function with expanding varieties which have been employed in previous growth models with technological progress, (e.g. Romer, 1987, 1990, Barro and Sala-i-Martin, 1995). The difference stems from the division of firm human capital, *H*, with the number of intermediates, *N*. This implies that we are assuming that it is the amount of human capital per type of intermediate that matters in production. We can think of the intermediates as ingredients that need to be added in different processes and that each of these processes requires human capital. At the same time, adding new ingredients and, hence, processes, still benefits production since it provides an alternative

[4] Ethier's production function was, in turn, an application of the formulation of consumer preferences over a variety of goods developed in Spence (1976) and Dixit and Stiglitz (1977) .

to increasing the quantity of each intermediate. The exponent α on X under the integral indicates that there are decreasing returns to the quantity of each intermediate. As we shall see, N in the denominator in combination with the integral results in overall decreasing returns to the number of intermediates, N, just as to the quantity, X, of each intermediate. In equilibrium, production depends on the total number of intermediates, NX, employed in the firm. NX is then analogous to the capital stock in the Uzawa-Lucas model.

If we assume that the quantity employed is the same across intermediates, we can write the production function in reduced form as in expression (3). This will hold in equilibrium when all different types of intermediates enter symmetrically into the production function, provided that it is assumed that the R&D technology and the R&D cost are the same for all entrepreneurs.

$$Y_t = AH_t^{1-\alpha} N_t^{\alpha} X_t^{\alpha} . \tag{3}$$

This shows that output of the firm depends on the total quantity of intermediates used, NX, rather than on the particular types employed. Our formulation of the production function does away with the scale effects present in many models of endogenous technological progress. It also makes sure that the growth rate is constant in the steady state.

The firm's demand for intermediates and human capital is determined by profit maximization with respect to quantities of X_{jt} and H_t at each point in time, when the wage, w_t, and prices, p_{jt}, are taken as given.

$$\max_{X_{jt}, H_t} \Pi_t = Y_t - w_t H_t - \int_0^{N_t} \left[p_{jt} X_{jt} \right] dj . \tag{4}$$

Profit maximization implies the following demand function for X_{jt}.:

$$X_{jt} = \left(\alpha A / p_{jt} \right)^{1/(1-\alpha)} \frac{H_t}{N_t}, \forall j . \tag{5}$$

Profit maximization also requires that the marginal product of human capital equal the wage:

$$(1-\alpha)AH_t^{-\alpha}N_t^{\alpha-1}\int_0^{N_t}\left[X_{jt}^{\alpha}\right]dj = w_t\,. \tag{6}$$

Equation of marginal products to the price of inputs assures zero profits for the final output producing firms since there are constant returns to scale.

2.2 The entrepreneur

The incentive to do R&D, comes from the monopoly rents the entrepreneur can extract due to the monopoly power he enjoys in the sale of his intermediates. The marginal cost of supplying intermediates to firms is one unit of final output. That is to say, once the entrepreneur has spent η units of final output on R&D to aqcuire the knowledge of how to make the new intermediate out of final output, he can turn final output into intermediates at zero cost. The entrepreneur covers the R&D cost by selling shares in his future profit to consumers. The net present value of doing R&D at time *t* is:

$$V_{jt} = -\eta + \int_t^{\infty}\left[X_{jv}(p_{jv}-1)e^{-\int_t^v r(s)ds}\right]dv\,. \tag{7}$$

The net present value function, (7), shows that the fixed cost, η, involved in introducing a new intermediate only can be covered if the sales price, p_j, exceeds the marginal cost of production, one, for at least part of the time after the date of introduction t. The entrepreneur discounts his future profits by the interest rate *r*, which is the rate of return consumers demand on their investments. The entrepreneur prices the X_{jt} optimally, subject to the demand function of the final goods producers. Substituting the demand function for X_{jt} in equation (5) into equation (7) gives:

$$V_{jt} = -\eta + \int_t^{\infty}\left[\left(\alpha A / p_{jv}\right)^{1/(1-\alpha)}\frac{H_v}{N_v}(p_{jv}-1)e^{-\int_t^v r(s)ds}\right]dv\,. \tag{8}$$

Since the entrepreneur takes human capital and the number of intermediates previously introduced as given at each point in time, maximization of V_{jt} with respect to p_j in each instant will yield a solution for the equilibrium price of an intermediate:

$$p_{jv} = p = 1/\alpha > 1,$$

which, as we can see, depends neither on time nor on the type of intermediate. The reason being that the R&D cost is time and type invariant and that the intermediates enter symmetrically in the production of final output. The closer α is to one, the smaller the mark up, since $\alpha=1$ would imply that the intermediates were perfect substitutes and there would be no room for monopoly pricing.

We can now determine the quantity that will be produced of each intermediate by substituting the equilibrium price into the expression for intermediate demand, equation (5):

$$X_{jt} = X_t = \frac{H_t}{N_t} A^{1/(1-\alpha)} \alpha^{2/(1-\alpha)}. \tag{9}$$

The quantity produced is the same for all intermediates and changes over time at the same rate as the ratio of the firm's stock of human capital to the number of intermediates introduced. If we substitute (9) into (6) we get the following expression for the equilibrium wage:

$$w_t = w = A^{\frac{1}{1-\alpha}} \left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{1+\alpha}{1-\alpha}} \right) > 0. \tag{10}$$

The wage will thus be fixed and depend only on the general level of technology A in final output production and on the degree of decreasing returns to intermediates, α. As we can see, the wage does not depend on the availability of human capital. The reason for this is that in equilibrium, the marginal return to human capital is constant. Increasing the human capital employed in production leads to decreasing direct marginal returns, but more H relative to intermediates induces the firm to demand a higher quantity X of each intermediate which raises the marginal return to human capital by an offsetting amount.

Substituting (9) and price, p, into (8) reduces the net present value of doing R&D to:

$$V_{jt} = -\eta + \kappa \int_t^{\infty} \left[\frac{H_v}{N_v} e^{-\int_t^v r(s)ds} \right] dv \tag{11}$$

where

$$\kappa = \frac{w}{p} = A^{\frac{1}{(1-\alpha)}} \left(\alpha^{\frac{(1+\alpha)}{(1-\alpha)}} - \alpha^{\frac{2}{(1-\alpha)}} \right) > 0. \tag{12}$$

If we assume that there is free entry into the R&D-sector, that is, anyone can become an entrepreneur, the net present value of doing R&D will be driven down by new entrants. Were net present value less than zero, there would be no R&D going on in equilibrium. In an equilibrium with R&D activity going on, we therefore have the following zero-profit condition

$$\eta \kappa^{-1} = \int_t^{\infty} \left[\frac{H_v}{N_v} e^{-\int_t^v r(s)ds} \right] dv . \tag{13}$$

Since the left-hand side of expression (13) is constant, we can differentiate with respect to time, *t*, to get

$$0 = \frac{H_t}{N_t} - r_t \int_t^{\infty} \left[\frac{H_v}{N_v} e^{-\int_t^v r(s)ds} \right] dv . \tag{14}$$

Equations (13) and (14) can be manipulated to give us an expression for the highest interest rate an entrepreneur is willing to pay in return for the η units of final output he needs to borrow for his project:

$$r_t = \frac{H_t}{N_t} \frac{1}{\eta} \kappa . \tag{15}$$

This interest rate depends positively on the ratio of human capital to intermediates and negatively on the R&D cost. Using the definition of κ in (12), it can also be shown that the interest rate depends positively on the level of technology in final output production. The dependence of the interest rate on α is not monotonous. For small α's the interest rate increases with α, whereas the relation is negative for large α's.[5]

2.3 The individual

The L representative individuals of the economy part their time between working to earn a wage, denominated in final output, and going to school to accumulate human capital. Individuals consume some of their wage directly and lend some of to entrepreneurs in return for future interest. The individuals, who all have the same preferences and talents, derive their utility from consumption of the final good. The individual's lifetime utility is:

$$U = \int_0^\infty U(c_t)e^{-\rho t}\,dt, \tag{16}$$

where the stream of consumption is discounted with the rate of time preference, ρ, which measures how much the individual favours consumption today over consumption tomorrow. The higher the value of ρ, the less patient is the individual. We assume that $U(c_t)$ takes the following form:

$$U(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}, \tag{17}$$

where $1/\theta$ is the intertemporal elasticity of substitution. The higher the value of θ, the more the individual will dislike deviations from a smooth consumption path.

The individual maximizes lifetime utility subject to his intertemporal budget constraint and subject to the evolution of his human capital stock. The intertemporal budget constraint is:

$$\dot{a}_t = w_t(1-u_t)m_t + r_t a_t - c_t, \tag{18}$$

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5 How large α has to be for the relation of r and α to be negative, depends on A.

where w_t is the equilibrium wage, which we found to be constant in the section 2.2, 1-u_t is the share of time dedicated to working, m_t is the individual's endowment of human capital and r_t the return on accumulated assets, a_t, i.e. the accumulated amount of final output the individual has lent to entrepreneurs.

The stock of human capital evolves in the following way:

$$\dot{m}_t = B(u_t m_t)^{\beta} \overline{m}_t^{1-\beta} - \delta m_t, \quad \beta \in (0,1). \tag{19}$$

B is a constant representing the state of technology in the educational sector of the economy. u_t is the share of time or the fraction of human capital dedicated to learning activities, m_t is the individual's human capital stock, $\overline{m}_t$ is the average endowment of human capital of the inhabitants in the economy, and δ is the rate of depreciation of human capital. This depreciation rate could be thought of as the rate at which knowledge becomes obsolete due to technological progress. For simplicity, we shall assume a constant rate of depreciation. B has to be larger than the rate of depreciation, otherwise m declines even if the individual spends all his time in school. This is not the case if the average human capital stock exceeds that of the individual, but since all individuals are alike and the individual and the average human capital stock are equal in equilibrium, we assume that B>δ.

Production of human capital depends both on the individual's input of human capital into the learning process, $u_t m_t$, and on the average endowment of human capital in the economy. This dependence on the average endowment of human capital can be interpreted as the effect of the skills of teachers and colleagues on the individual's learning process. Holding constant the individual input into the learning process, an improvement in the average level of human capital would speed up the learning process of the individual. There is, however, decreasing returns both to the individual's input of human capital and to the average endowment of human capital. When the individual decides how much effort to dedicate to learning, he regards the average stock of human capital as exogenous, that is, he will not take into account the fact that his own decision to accumulate human capital will affect the growth in the average stock of human capital. This external effect of human capital is likely to make the decentralized equilibrium deviate from the social optimum as a result of too little investment in human capital.

The utility maximizing individual will choose how much to consume and how much of his time to spend in school. That is, c_t, and, u_t are his control variables. The state variables he worries about are human capital, m_t and assets, a_t.

We write the individual's present value Hamiltonian:

$$\max_{c_t, u_t} \mathbf{H}(t) = \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t} + \lambda_t \left[w(1-u_t)m_t + r_t a_t - c_t \right] + \mu_t \left[B(u_t m_t)^{\beta} \overline{m}_t^{1-\beta} - \delta m_t \right]. \quad (20)$$

The first order conditions are:

$$\mathrm{H}_c = c_t^{-\theta} e^{-\rho t} - \lambda_t = 0, \quad (21)$$

$$\mathrm{H}_u = -\lambda_t w m_t + \mu_t \beta B u_t^{\beta-1} m_t^{\beta} \overline{m}_t^{1-\beta} = 0 \quad (22)$$

and:

$$\mathrm{H}_a = \lambda_t r = -\dot{\lambda}_t, \quad (23)$$

$$\mathrm{H}_m = \lambda_t w(1-u_t) + \mu_t (\beta B u_t^{\beta} m_t^{\beta-1} \overline{m}_t^{1-\beta} - \delta) = -\dot{\mu}_t, \quad (24)$$

which imply

$$\frac{\dot{\lambda}_t}{\lambda_t} = -r_t, \quad (25)$$

$$\frac{\dot{\mu}_t}{\mu_t} = -\frac{\lambda_t}{\mu_t} w(1-u_t) - (\beta B u_t^{\beta} m_t^{\beta-1} \overline{m}_t^{1-\beta} - \delta). \quad (26)$$

The transversality conditions are:

$$\lim_{t\to\infty}\left[\lambda(t)a(t)\right]=0, \tag{27}$$

$$\lim_{t\to\infty}\left[\mu(t)m(t)\right]=0. \tag{28}$$

The first order conditions (21) and (25) yield a growth rate of consumption of

$$\frac{\dot{c}_t}{c_t}=\frac{1}{\theta}(r_t-\rho), \tag{29}$$

which implies that the growth rate of consumption depends on the difference between the interest rate and the rate of time preference, and on the intertemporal elasticity of substitution in the same fashion as in the Ramsey model.

2.4 Equilibrium

In order to solve for the rate of change of the time share spent in school, we manipulate (22) to get:

$$\frac{\lambda_t}{\mu_t}=\beta B u_t^{\beta-1}w^{-1}, \tag{30}$$

where we have taken into account that $m_t=\overline{m}_t$, in equilibrium, since all individuals are alike. Expression (30) is the inverse of the shadow price of human capital in terms of final output.

Taking logs and time derivatives of the expression in (30) gives:

$$\frac{\dot{\lambda}_t}{\lambda_t}-\frac{\dot{\mu}_t}{\mu_t}=(\beta-1)\frac{\dot{u}_t}{u_t}. \tag{31}$$

We can thus derive the following expression for the rate of change in the share of time spent in school by substituting (25) and (26) into (31) and rearranging:

$$\frac{\dot{u}_t}{u_t} = \frac{1}{1-\beta}\left(r_t - \beta B u_t^{\beta-1} + \delta\right). \tag{32}$$

Equation (32) tells us that the change in the time share spent in school will depend on the interest rate, i.e. the rate of return offered on loans to R&D activity and on the rate of return to additional human capital. If the interest rate is high today, that is, if it exceeds the marginal rate of return to the individual of an additional unit of human capital, there will be investments in R&D. The individual can foresee that this will lead to an expansion in the number of intermediates and will therefore increase his schooling effort to meet the increased demand for human capital. If, on the other hand, the interest rate is lower than the rate of return to additional human capital, the individual will initially spend most of his time accumulating human capital, but the time share will decline until the gap between the interest rate and the return to human capital is closed.

Equilibrium in the asset market will close the model. Asset market equilibrium requires the increase in the population's total assets to be equal to the total amount of resources spent on R&D by the entrepreneurs at each point in time. Since all individuals are identical, this yields the following asset market equilibrium condition:

$$L\dot{a}_t = \eta \dot{N}_t. \tag{33}$$

It also follows that the present value of total assets must equal the present value of future profit flows from all existing intermediates:

$$L a_t = \eta N_t. \tag{34}$$

The asset market equilibrium conditions, the equilibrium quantity demanded of each intermediate, (9), the equilibrium wage, (10), the interest rate, (15), and the individual’s intertemporal budget restriction, (18), can be used to show that all final goods produced are either consumed, used to produce intermediates or consumed in the R&D process, i.e.:

$$Y_t = c_t L + N_t X_t + \eta \dot{N}_t. \tag{35}$$

The asset market equilibrium conditions and the individual's intertemporal budget constraint (18), can be used to find an expression for the evolution of the number of intermediates

$$\dot{N}_t = \frac{L}{\eta}\left(w(1-u_t)m_t + r_t \frac{\eta}{L} N_t - c_t \right). \tag{36}$$

Since the interest rate paid by the entrepreneurs is that received by the individuals, we can substitute the expression for the interest rate, (15), into the above motion equations, (29), (32) and (36), and describe the economy as a system of four differential equations in the level of consumption, c_t, the human capital stock, m_t, the number of intermediates, N_t, and the share of time spent in school, u_t.

$$\frac{\dot{c}_t}{c_t} = \gamma_c = \frac{1}{\theta}\left(\frac{L}{\eta} \kappa(1-u_t)\frac{m_t}{N_t} - \rho \right), \tag{37}$$

$$\frac{\dot{u}_t}{u_t} = \gamma_u = \frac{1}{1-\beta}\left(\frac{L}{\eta} \kappa(1-u_t)\frac{m_t}{N_t} - \beta B u_t^{\beta-1} + \delta \right), \tag{38}$$

$$\frac{\dot{m}_t}{m_t} = \gamma_m = B u_t^{\beta} - \delta , \tag{39}$$

$$\frac{\dot{N}_t}{N_t} = \gamma_N = \frac{L}{\eta}(w+\kappa)(1-u_t)\frac{m_t}{N_t} - \frac{L}{\eta}\frac{c_t}{N_t}. \tag{40}$$

3 Steady State

Solving the above system involves defining what we mean by steady state in this model. A steady state situation requires that the individuals' allocation of time between school and work does not change over time, that is, $\dot{u}$ has to be zero. Equation (39) shows that a constant u implies that the growth rate of m is also constant in the steady state. Furthermore, since equation (38) implies that the ratio of human capital to the number of intermediates has to be constant, we know that the growth rate of the number of intermediates is also constant and equal to the growth rate of human capital in steady state. It follows that consumption grows at the same rate and that the ratio of consumption to the number of intermediates is constant in steady state.

In order to analyse the steady state and the transitional dynamics, we transform the system into a system of three differential equations in variables which are all constant in the steady state. We do this by forming the state-like variable *z*, which is the ratio of individual human capital to the number of intermediates and the control-like variable *Q*, which is the ratio of consumption to the number of intermediates. Together with equation (38), these form the following dynamic system:

$$\gamma_u = \frac{\dot{u}_t}{u_t} = \frac{1}{1-\beta}\left(\frac{L}{\eta}\kappa(1-u_t)z_t - \beta B u_t^{\beta-1} + \delta\right), \tag{38}$$

$$\gamma_z = \frac{\dot{z}_t}{z_t} = \frac{\dot{m}_t}{m_t} - \frac{\dot{N}_t}{N_t} = Bu_t^{\beta} - \frac{L}{\eta}(w+\kappa)(1-u_t)z_t + \frac{L}{\eta}Q_t - \delta\,, \tag{41}$$

$$\gamma_Q = \frac{\dot{Q}_t}{Q_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{N}_t}{N_t} = \frac{L}{\eta}\left(\left(\frac{1}{\theta}-1\right)\kappa - w\right)(1-u_t)z_t - \frac{1}{\theta}\rho + \frac{L}{\eta}Q_t\,. \tag{42}$$

We can now look for a steady state, *(u*,z*,Q*),* in which the growth rates of *u*, and our new variables, *z* and *Q,* are zero.

$$\gamma_u^* = \frac{1}{1-\beta}\left(\frac{L}{\eta}\kappa(1-u^*)z^* - \beta B {u^*}^{\beta-1} + \delta\right) \equiv 0 \tag{43}$$

$$\gamma_z^* = -\frac{L}{\eta}(w+\kappa)(1-u^*)z^* + B{u^*}^{\beta} + \frac{L}{\eta}Q^* - \delta \equiv 0 \tag{44}$$

$$\gamma_Q^* = \frac{L}{\eta}\left(\left(\frac{1}{\theta}-1\right)\kappa - w\right)(1-u^*)z^* + \frac{L}{\eta}Q^* - \frac{1}{\theta}\rho \equiv 0 \tag{45}$$

We cannot solve for the steady state explicitly, but we can derive expressions for the steady state values of z and Q in terms of u^*, and we can then show that the steady state value of u lies between zero and one for plausible parameter values. Manipulation of (44) and (45) using the expressions for w and κ, results in the following expressions for the steady state values of z and Q:

$$z^* = \frac{\theta\left(Bu^{*\beta} - \delta\right) + \rho}{\frac{L}{\eta}\kappa(1-u^*)}, \tag{46}$$

$$Q^* = \frac{\eta}{L}\left(\left(\frac{\theta}{\alpha} + \theta - 1\right)\left(Bu^{*\beta} - \delta\right) + \left(1 + \frac{1}{\alpha}\right)\rho\right). \tag{47}$$

We can ensure that both z^* and Q^* are non-negative by invoking the transversality condition. This condition rules out the possibility that consumption should decline in steady state, i.e.:

$$\gamma_c^* = \frac{1}{\theta}\left(\frac{L}{\eta}\kappa(1-u^*)z^* - \rho\right) \geq 0. \tag{48}$$

Since we know that the steady state growth rate of consumption must equal that of human capital, this condition implies that:

$$Bu^{*\beta} - \delta = \frac{1}{\theta}\left(\frac{L}{\eta}\kappa(1-u^*)z^* - \rho\right) \geq 0.$$

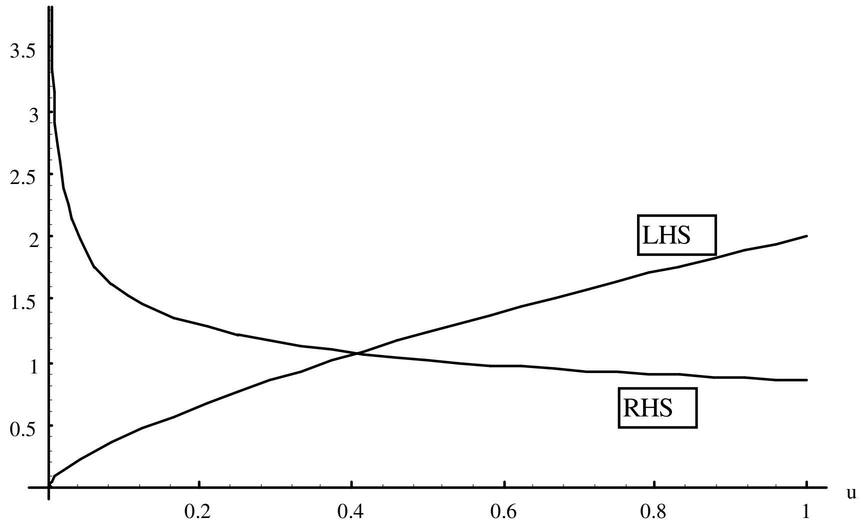
Since both z^* and Q^* are increasing functions of u we also know that the steady state is unique if there is a unique u^*.

By manipulating (43)-(45) we can derive the following expression which determines the value of u^*:

$$\theta u^{*\beta} = \beta u^{*\beta-1} + \frac{(\theta-1)\delta - \rho}{B}. \tag{49}$$

Figure 1 depicts a graphical solution for *u* for a set of base line parameter values.

Figure 1. *The determination of the steady state time share spent in school, u*.*



LHS and RHS are depicted for the parameter values: θ=2, β=0.7, δ=0.05, ρ=0.03 and B=0.131. For this choice of parameter values, the steady state growth rate is 2 per cent.

To obtain a solution, we need to assign values to the parameters in the equality, but we know that a unique u solves this equation since the left hand side (LHS) is a monotonically increasing function of u, and the right hand side (RHS) is a monotonically decreasing function of u. When u is increased from zero to one, the LHS starts out at zero while the RHS starts at infinity. We can be sure that the functions cross somewhere before u takes on the value one, if LHS>RHS for u=1. This condition will hold when $\delta<B$ provided that:

$$\theta > \frac{\beta B - \delta - \rho}{B - \delta}. \tag{50}$$

3.1 The steady state growth rate

Having ensured the uniqueness of the steady state, we go on to analysing the dynamics around this steady state. First, we determine the steady state growth rate.

In steady state, consumption, the number of intermediates, and human capital all grow at the same rate. Hence, the steady state growth rate of these variables is given by the growth rate of human capital in steady state:

$$\gamma_c^* = \gamma_N^* = \gamma_m^* = Bu^{*\beta} - \delta\,, \tag{51}$$

where u^* is determined by (49). What about the growth rate of final output? We can rewrite equilibrium output (3) as:

$$Y_t = A\Big(L\big(1-u_t\big)\Big)^{1-\alpha} z_t^{1-\alpha} N_t X_t^{\alpha}\,.$$

From this equation we can see that in steady state, when *u, z* and *X* are all constant, final output grows thanks to the expansion of the number of intermediates. The steady state growth rate of final output is, thus, equal to the steady state growth rate of consumption, human capital and of the number of intermediates. The incentive of the entrepreneurs to expand the number of intermediates rises when the interest rate is pushed up by the growth in the human capital stock which, in turn, results from the individual's decision to go to school. In making the choice between going to school and working, the individual weighs the benefit of being more educated in the future against the possibility of saving to earn future rent or consuming today.

In the end, the steady state growth rate is higher the larger the share of time dedicated to education. By taking the derivative of the growth rate with respect to each of the parameters and by differentiating (49) with respect to u and each of the parameters, we can determine how the growth rate depends on the parameters β, θ, δ, ρ and B. The results are presented in equations (52)-(56), which show that the growth rate is higher the more efficient the educational system (i.e. the higher the technology parameter *B)*, the smaller the rate of depreciation of human capital, the smaller the rate of time preference, ρ, and the larger the intertemporal elasticity of substitution, $1/\theta$. Hence, the steady state growth rate depends on the parameters of the demand function, in a fashion similar to the neoclassical growth model: the more patient and the less keen on consumption smoothing the consumers are, the faster the economy grows in the steady state.

The dependence of the steady state growth rate on the degree of diminishing returns to the individual's investment in human capital, β, is ambiguous. For small time shares invested in schooling, higher β leads to slower steady state growth, while growth increases with β for a sufficiently large equilibrium time share spent in school. For sufficiently large u, the positive effect on growth of an upward shift in β, is the result of an increase in the equilibrium u. More time spent in school results in more rapid human capital accumulation. For small time shares, the increase in the schooling effort does not result in enough human capital accumulation to offset that less effort is used to produce goods.

$$\frac{\partial\gamma^*}{\partial B} = u^{*\beta} + B\beta u^{*\beta-1}\frac{du^*}{dB} = \frac{u^{*\beta-1} + (\theta-1)u^{*\beta}}{(1-\beta)u^{*-1}+\theta} > 0, \tag{52}$$

$$\frac{\partial\gamma^*}{\partial\delta} = B\beta u^{*\beta-1}\frac{du^*}{d\delta} - 1 = -\frac{1+(1-\beta)u^{*-1}}{(1-\beta)u^{*-1}+\theta} < 0, \tag{53}$$

$$\frac{\partial\gamma^*}{\partial\theta} = B\beta u^{*\beta-1}\frac{du^*}{d\theta} = -\frac{Bu^{*\beta}-\delta}{(1-\beta)u^{*-1}+\theta} < 0, \tag{54}$$

$$\frac{\partial\gamma^*}{\partial\rho} = B\beta u^{*\beta-1}\frac{du^*}{d\rho} = -\frac{1}{(1-\beta)u^{*-1}+\theta} < 0, \tag{55}$$

$$\frac{\partial\gamma^*}{\partial\beta} = Bu^{*\beta}\ln u^* + \beta Bu^{*\beta-1}\frac{du^*}{d\beta} = Bu^{*\beta}\frac{\ln u^*+1}{1-\beta+\theta u^*} \begin{matrix} > \\ < \end{matrix} 0 \quad if \quad u^* \begin{matrix} > \\ < \end{matrix} e^{-1}. \tag{56}$$

4 Dynamics

Having established that there is a unique steady state, we need to find out whether the economy actually converges towards the steady state or not. Ideally we would like to make a global analysis of the dynamics. That is, we want to know the shape, slope, and position of the stable arm, if such a stable arm exists, and we want to know how fast the economy converges towards the steady state along the stable arm. Since we are unable to reduce the system to a two-variable problem, we cannot display the dynamics using phase diagrams. We could analyse the system globally with the time elimination method developed in Mulligan(1991). As long as the steady state is unique, such an analysis would generate the same qualitative results as a local analysis of the system when linearized around the steady state, see Mulligan and Sala-i-Martin (1993). We therefore restrict the analysis to the much easier linearization method. We display the qualitative behaviour of the model for a set of base line parameter values.

4.1 Stability and the slope and position of the stable arm

In order to find the slope and position of the stable arm, we make a first order Taylor expansion around the steady state which yields the following linear system:

$$\dot{u}_t = \left(-\frac{1}{1-\beta}\frac{L}{\eta}\kappa u^* z^* + B\beta u^{*\,\beta-1}\right)\left(u_t - u^*\right) + \frac{1}{1-\beta}\left(\frac{L}{\eta}\kappa\left(1-u^*\right)u^*\right)\left(z_t - z^*\right), \quad (57)$$

$$\begin{aligned}\dot{z}_t = {} & \left(\frac{L}{\eta}(\kappa + w)z^{*2} + B\beta u^{*\,\beta-1}z^*\right)\left(u_t - u^*\right) - \left(\frac{L}{\eta}(\kappa + w)\left(1-u^*\right)z^*\right)\left(z_t - z^*\right) + \\ & \frac{L}{\eta}z^*\left(Q_t - Q^*\right)\end{aligned} \quad (58)$$

$$\begin{aligned}\dot{Q}_t = {} & -\frac{L}{\eta}\left(\kappa\left(\frac{1}{\theta}-1\right) - w\right)z^* Q^*\left(u_t - u^*\right) + \frac{L}{\eta}\left(\kappa\left(\frac{1}{\theta}-1\right) - w\right)\left(1-u^*\right)Q^*\left(z_t - z^*\right) + \\ & \frac{L}{\eta}Q^*\left(Q_t - Q^*\right).\end{aligned} \quad (59)$$

The linearized system can be written on matrix form:

$$\begin{bmatrix}\dot{u}\\ \dot{z}\\ \dot{Q}\end{bmatrix} = A\begin{bmatrix}u_t\\ z_t\\ Q_t\end{bmatrix} - k\,, \quad (60)$$

where A is a 3x3-parameter matrix and k is the vector of parameters:

$$k = A\begin{bmatrix}u^*\\ z^*\\ Q^*\end{bmatrix}. \quad (61)$$

Since we have two control variables and one state variable, a stable system requires that all the eigenvalues of the matrix A are negative, while a saddle path stable system requires that *A* has one negative and two positive eigenvalues. If the system is saddle path stable we also know that the slope of the stable arm at the steady state is determined by the eigenvector, $\boldsymbol{v}$, associated with the negative eigenvalue, ε, and that the speed at which the economy converges toward the steady state is determined by the size of the negative eigenvalue, ε.

In order to determine whether the system is stable, saddle path stable, or unstable we use some results from linear algebra: *The sum of the eigenvalues of a square matrix equals the trace of the matrix, and the product of the eigenvalues of the matrix equals the determinant of the matrix.*

The determinant of A is equal to:

$$Det(A) = r^*(r^* + \delta)\frac{\left(1-\beta+\theta u^*\right)}{\kappa\theta(\beta-1)}\left(r^*\left(\kappa\left(1-\frac{1}{\theta}\right)+w\right)+\frac{\rho}{\theta}(\kappa-w)+w\beta\right). \tag{62}$$

It can be shown that the determinant of A is negative if the steady state growth rate is non-negative and if

$$\theta > \frac{\rho w + \kappa\gamma^*}{w(\beta+\rho+\gamma^*)+\kappa(\rho+\gamma^*)}. \tag{63}$$

The trace of A is equal to:

$$Tr(A) = -\frac{(r^*-\rho)}{\theta} - r^*\frac{\beta}{1-\beta} - \delta < 0. \tag{64}$$

This implies that there must be either one or three negative eigenvalues. Thus, we know that the system is either stable or saddle path stable. A positive trace is a sufficient, but not necessary condition for saddle path stability. Since the trace of A is negative we need further conditions to determine whether our system is stable or saddle path stable. The numerical analysis in section 5 shows that the system is saddle path stable for the parameter ranges tested.

Assuming that the conditions for saddle path stability are satisfied, we can now find the position of the stable arm and derive how the time share allocated to education, u, and the ratio of consumption to intermediates, Q, change as the economy moves towards the steady state. We are also able to study the transitional behaviour of growth rates of the variables in the model. It is convenient to relate the changes to a measure of how far the economy is from its steady state position. We measure this distance in terms of $d=ln(z_t/z^*)$.

The stable arm is defined by:

$$\begin{bmatrix} u_t \\ z_t \\ Q_t \end{bmatrix} = \begin{bmatrix} u^* \\ z^* \\ Q^* \end{bmatrix} + s \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \tag{65}$$

where

$$s = \frac{z^*\left(e^d - 1\right)}{v_2}. \tag{66}$$

4.2 Choice of baseline parameter values

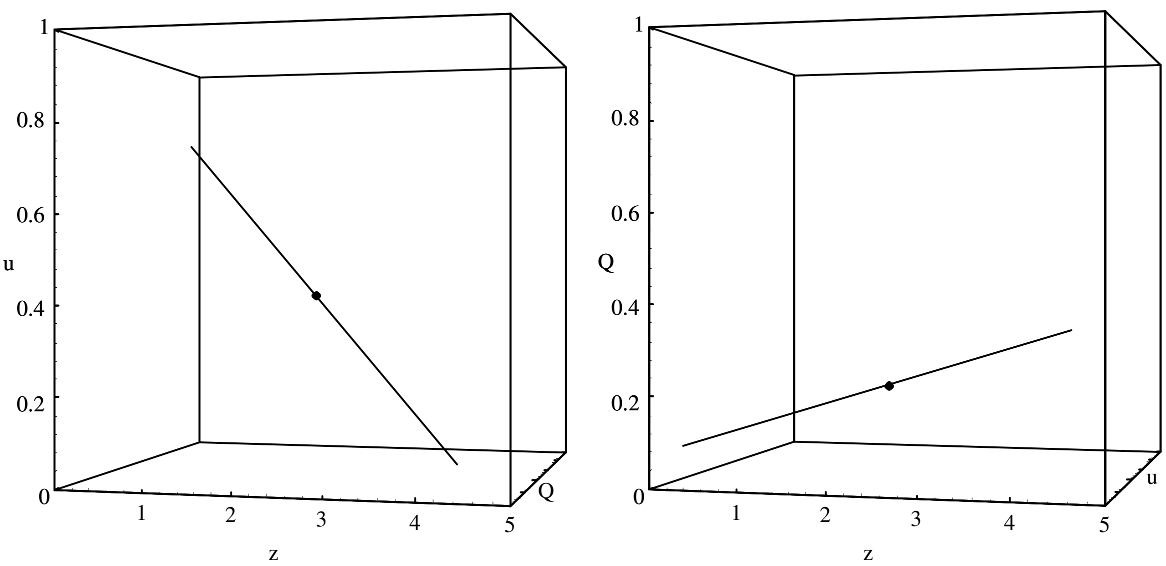
The parameter values in this baseline case are chosen to calibrate the growth rate to two percent in order to make comparisons to reality less far fetched. For some parameters this choice is guided by empirical findings, while other parameter values are set arbitrarily. The baseline value for the inverse of the intertemporal elasticity of substitution, θ, is set to 2, which is lower than what has been suggested by recent attempts to estimate, θ. Acemoglu and Scott (1994) estimate a θ between 10 and 30, and Hall (1988) find a θ of at least 5. The choice of θ=2 is, however, more in line with earlier calibrations of growth models, e.g. Mulligan and Sala-i-Martin, (1993) and Jones, Manuelli and Rossi (1993). The rate of time preference, ρ, which is set to 0.03, is also set in line with previous work on growth. The depreciation rate of human capital, δ, is set to 0.05, which is in line with estimates by Mincer and Ofek (1982). It is often argued that production of human capital, education, is more human capital intensive than other production. In line with this argument, β is set to 0.7, while α is set to 0.5. This α is higher than the commonly used α of 0.3, in calibrations of models with accumulation of physical capital and a fixed labour force. The motivation is that we think of the intermediates as a wider concept than physical capital. In order for the steady state growth rate to be 0.02, the chosen baseline parameter values require a technology parameter, B, of 0.131. The R&D cost, η, the size of the population, L, and the technology parameter A, are set arbitrarily to 1.

4.3 Qualitative results in the base line case

The linearized stable arm defined in (65) is depicted in three dimensions in the boxes in Figures 2a and 2b. There is a negative relation between the time share spent in school and the ratio of human capital to intermediates, and a positive relation between the ratio of consumption to intermediates and the ratio of human capital to intermediates. It should be noted again that this relation between u, z and Q, holds only for a region close to the steady state.

Figure 2 *The linearized stable arm and the steady state depicted in (u,z,Q)-space.*

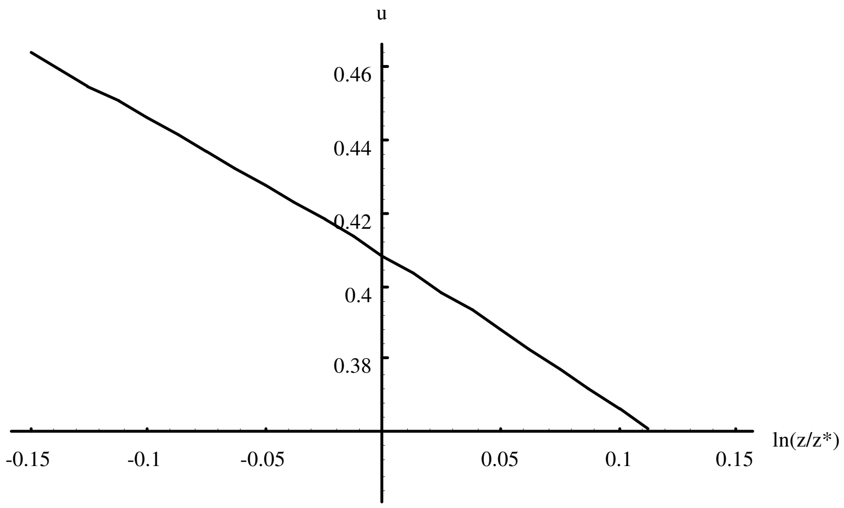
a *b*



The stable arm and the steady state position are computed for: θ=2, β=0.7, δ=0.05, ρ=0.03, B=0.131, α=0.5, η=1, L=1 and A=1. For this choice of parameter values, the steady state growth rate is 2 per cent

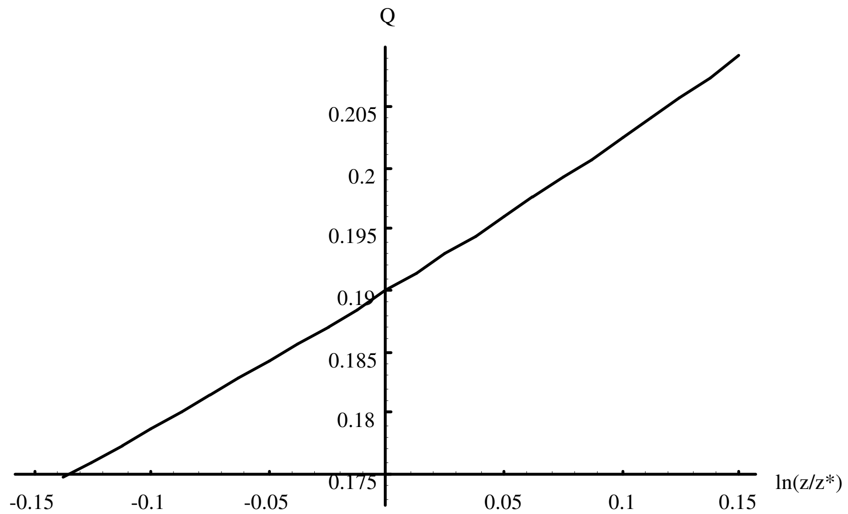
The graphs in Figures 3 and 4 show how u and Q change as the ratio of human capital to intermediates, z, approaches its steady state position. If the economy starts out with a relative scarcity of human capital, that is $z<z^*$, individuals spend a large share of their time accumulating human capital. Because they have little time left over for work, their wage earnings are small and they can therefore only afford to consume relatively little in relation to the steady state ratio of consumption to intermediates. As the economy moves towards the steady state, the time share spent in school gradually declines and the level of consumption increases. If, on the other hand, the economy is relatively rich in human capital, u increases and Q decreases as the steady state is approached. We can understand these slopes by taking a look at equation (32) which tells us that the change in the time share spent in school depends on the interest rate, i.e. the rate of return offered on loans to R&D activity and on the rate of return to additional human capital. If the economy has a relative scarcity of human capital, that is $z<z^*$, the rate of return to human capital is high in relation to the interest rate. The individual thus chooses to invest so much in schooling that he has little time left to work in order to earn wage income. The individual chooses to give up consumption in order to accumulate human capital and the ratio of consumption to intermediates is therefore small, despite very small investments in R&D. As the rate of return to human capital decreases and the interest rate increases, the share time spent in school is reduced and investments in R&D are increased. Although the individual's incentive to invest in R&D increases, the increased wage earnings, resulting from his enlarged human capital stock and from a larger time share spent working, allows the individual to choose an increasing ratio of consumption to R&D capital as the gap between the interest rate and the return to human capital investments is closed.

Figure 3*. The choice of the time share spent in school, u, as a function of the position of the ratio of human capital to intermediates, z, relative to the steady state position, z*.*



Computed for the parameters: θ=2, β=0.7, δ=0.05, ρ=0.03, B=0.131, α=0.5, η=1, L=1 and A=1. For this choice of parameter values, the steady state growth rate is 2 per cent.

Figure 4. *The choice of the ratio of consumption to intermediates, Q,, as a function of the position of the ratio of human capital to intermediates, z, relative to the steady state position, z*.*



Computed for the parameters: θ=2, β=0.7, δ=0.05, ρ=0.03, B=0.131, α=0.5, η=1, L=1 and A=1. For this choice of parameter values, the steady state growth rate is 2 per cent.

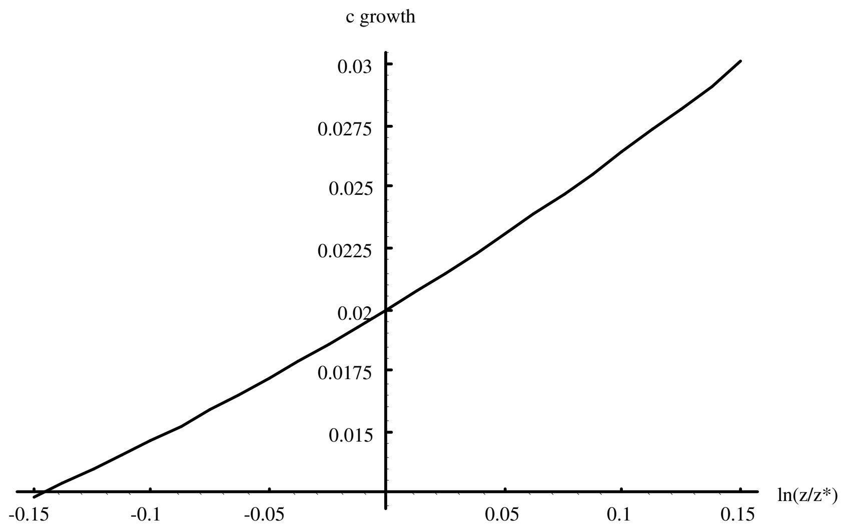
We are now interested in how the transitional behaviour of the growth rates of consumption, human capital, number of intermediates, and output relate to the distance of the economy to its steady state position. We take a particular interest in how imbalances between human capital and the number of intermediates affect the growth rates in the economy. The transitional behaviour of growth rates is shown in Figures 5 to 8, which display how the growth rates change in relation to distance of the state variable from its steady state position.

Due to the linearization around the steady state, the relations shown in the figures only hold close to the steady state, i.e. close to *ln(z/z*)=0*. A *ln(z/z*)*-value of 0.1 implies that the state variable is ten percent larger than its steady state value.

4.3.1 The growth rate of consumption.

If the economy initially has a relative scarcity of human capital, that is $z < z^*$, the return to introducing new intermediates is low to start with. The return to human capital, however, is high and individuals therefore accumulate human capital. This causes the return to intermediates to increase. This increase in the interest rate causes the growth rate of consumption to increase during the transition to the steady state. Should the economy instead have a relatively large stock of human capital at the outset, the interest rate, and thus the growth rate of consumption, are high initially and then fall as the economy approaches its steady state. This imbalance effect is similar to the imbalance effect between human capital and physical capital found in the Uzawa-Lucas model.

Figure 5. *The transitional behaviour of the growth rate of consumption, γ_c, as a function of the position of the ratio of human capital to intermediates, z, relative to the steady state position, z*.*

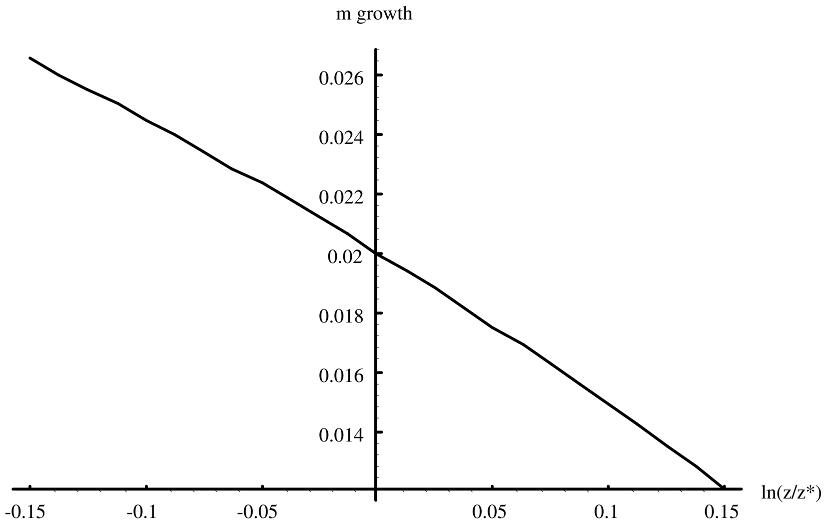


Computed for the parameters: θ=2, β=0.7, δ=0.05, ρ=0.03, B=0.131, α=0.5, η=1, L=1 and A=1. For this choice of parameter values, the steady state growth rate is 2 per cent.

4.3.2 The growth rate of human capital.

The growth rate of the stock of human capital is determined by how much time is spent in school. If the economy has a relative scarcity of human capital at the outset, the growth rate of human capital is initially high and then subsequently declines towards the steady state growth rate of human capital. If the economy is very abundant in human capital, the share of time spent in school is very small and the growth rate of human capital may therefore initially be negative, since the rate of depreciation of human capital may exceed the rate at which human capital is formed. The growth rate of human capital increases gradually as the steady state ratio of human capital to intermediates is reached.

Figure 6. *The transitional behaviour of the growth rate of human capital,* γ_m *,as a function of the position of the ratio of human capital to intermediates, z, relative to the steady state position, z*.*

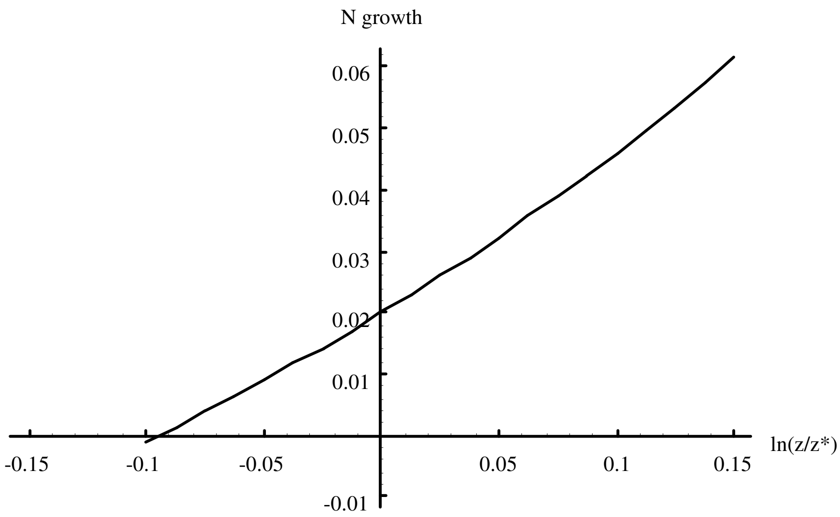


Computed for the parameters: θ=2, β=0.7, δ=0.05, ρ=0.03, B=0.131, α=0.5, η=1, L=1 and A=1. For this choice of parameter values, the steady state growth rate is 2 per cent.

4.3.3 The growth rate of the number of intermediates.

The growth rate of *N* depends on the willingness of individuals to invest in R&D. This willingness depends on the interest rate and on the individuals allocation of time between work and school. As we have seen above, time allocation depends on the relative rates of return to R&D and human capital. We know from Figure 4 that the ratio of consumption to intermediates increases as the economy moves from a situation of relative human capital scarcity towards the steady state. Thus, the number of intermediates must grow at a slower rate than consumption during the transition. If, on the other hand, the economy is relatively human capital abundant, the speed at which new intermediates are introduced will be higher than the growth rate of consumption, since the ratio of consumption to intermediates, *Q,* decreases as the economy moves closer to the steady state.

Figure 7. *The transitional behaviour of the growth rate of the number of intermediates,* γ_N *,as a function of the position of the ratio of human capital to intermediates, z, relative to the steady state position, z*.*



Computed for the parameters: θ=2, β=0.7, δ=0.05, ρ=0.03, B=0.131, α=0.5, η=1, L=1 and A=1. For this choice of parameter values, the steady state growth rate is 2 per cent.

4.3.4 The growth rate of output per capita.

So far we have not discussed what happens to output in the model. If the equilibrium expressions for the demand for intermediates and the price of intermediates are substituted into the production function of final output, we can write final output per capita, *y,* as:

$$y_t = \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}}\left(1-u_t\right)m_t . \tag{67}$$

Taking logs and time derivatives, we can write the growth rate of output as:

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{m}_t}{m_t} + \frac{\dot{\overline{\left(1-u_t\right)}}}{1-u_t}. \tag{68}$$

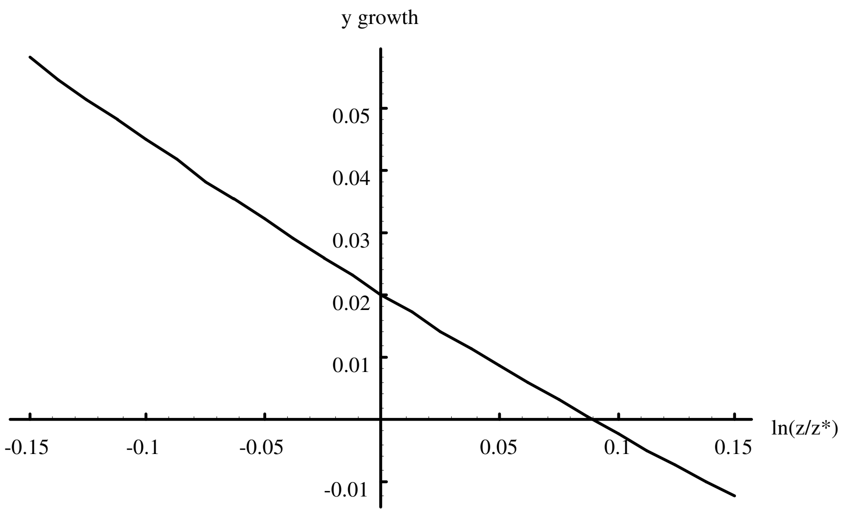
The growth in output per capita thus depends positively on the growth rate of human capital, but is, of course, also affected by changes in the time share dedicated to work. The more human capital employed in production, the higher is output. Given how *m* and *u* evolve as the steady state is approached, final output per capita grows fast, but at a decreasing rate if the economy starts out with a relative scarcity of human capital, and very slowly if the economy initially is relatively abundantly endowed with human capital. That is, a situation with plenty of human capital and little R&D capital results in lower than steady state output growth, whereas plenty of R&D capital and little human capital give higher than steady state output growth. The reason is that an economy which is rich in R&D capital has a strong incentive for accumulation of human capital and a potential for output growth through the reallocation of time from school to work. If, on the other hand, the economy has little R&D capital, individuals work most of their time and although there is a strong incentive for investments in R&D, the effect on output is small. The reason is that the increase in the number of intermediates puts a downward pressure on the interest rate which raises the relative return to human capital. This causes individuals to spend more time in school. This effect reduces the growth rate of output. The imbalance effect between human capital and R&D capital thus goes in the opposite direction from the imbalance effect on consumption. Our imbalance effect for output is also opposite to the imbalance effect for broad output found in the Uzawa-Lucas model.

The concept of broad output was mentioned in the introduction. As defined in Barro and Sala-i-Martin (1995), broad output is the output produced in the final output sector plus the value, in terms of consumption goods, of the accumulation of human capital. In this context, per capita broad output, O, is:

$$O_t = y_t + \frac{\mu_t}{\lambda_t} B u_t^{\beta} m_t . \tag{69}$$

It can be shown that the qualitative results that are presented for final output are valid also for this concept of broad output. We will therefore not present separate results for broad output.

Figure 8. *The transitional behaviour of the growth rate of output,* γ_Y *,as a function of the position of the ratio of human capital to intermediates, z, relative to the steady state position, z*.*



Computed for the parameters: θ=2, β=0.7, δ=0.05, ρ=0.03, B=0.131, α=0.5, η=1, L=1 and A=1. For this choice of parameter values, the steady state growth rate is 2 per cent.

The direction of the imbalance effect on output in our model leads the thought to the idea gap discussed in Romer (1993). Romer argues that lack of ideas, i.e. knowledge of what and how things can be done, is more important than lack of machines, object or even skills when one tries to understand why some developing countries grow slowly or not at all. While ideas generate demand for other factors of production, people or firms cannot demand things or services they do not know of. The model developed here, a relative lack of R&D capital which could be interpreted as idea gap, implies poor growth because accumulation of human capital is slow. Abundance of R&D capital, on the other hand, leads to rapid growth as a result of rapid human capital accumulation inspired by a high rate of return and because individuals reallocate time from going to school to working in the final output producing sector.

4.4 Speed of convergence

The previous analysis showed how decision variables and growth rates change as the economy moves towards its steady state. This section focuses on how fast these changes take place. In order to analyse the speed of convergence towards the steady state, we log-linearize the system around the steady state:

$$\frac{\partial \ln u_t}{\partial t} = \gamma_u = \left(-\left(\frac{1}{1-\beta}\right)\frac{L}{\eta}\kappa z^* u^* + \beta B u^{*\beta-1} \right)\ln\left(u/u^*\right) + \left(\frac{1}{1-\beta}\right)\frac{L}{\eta}\kappa z^*\left(1-u^*\right)\ln\left(z/z^*\right), \tag{70}$$

$$\frac{\partial \ln z_t}{\partial t} = \gamma_z = \left(\frac{L}{\eta}(w+\kappa) z^* u^* + \beta B u^{*\beta} \right)\ln\left(u/u^*\right) - \frac{L}{\eta}(w+\kappa) z^*\left(1-u^*\right)\ln\left(z/z^*\right) + \frac{L}{\eta} Q^* \ln\left(Q/Q^*\right), \tag{71}$$

$$\frac{\partial \ln Q_t}{\partial t} = \gamma_Q = -\frac{L}{\eta}\left(\left(\frac{1}{\theta}-1\right)\kappa - w\right) z^* u^* \ln\left(u/u^*\right) + \left(\frac{L}{\eta}\left(\left(\frac{1}{\theta}-1\right)\kappa - w\right) z^*\left(1-u^*\right)\right)\ln\left(z/z^*\right) + \frac{L}{\eta} Q^* \ln\left(Q/Q^*\right). \tag{72}$$

which can be written as

$$\begin{bmatrix} \dfrac{\partial \ln u_t}{\partial t} \\ \dfrac{\partial \ln z_t}{\partial t} \\ \dfrac{\partial \ln Q_t}{\partial t} \end{bmatrix} = \hat{A} \begin{bmatrix} \ln\left(u_t/u^*\right) \\ \ln\left(z_t/z^*\right) \\ \ln\left(Q_t/Q^*\right) \end{bmatrix}, \tag{73}$$

where $\hat{A}$ is a 3x3 parameter matrix with the same eigenvalues as the matrix A in (60). The solution to the system is then:

$$\ln u_t = \left(1-e^{\varepsilon t}\right)\ln u^* + e^{\varepsilon t} \ln u(0), \tag{74}$$

$$\ln z_t = \left(1-e^{\varepsilon t}\right)\ln z^* + e^{\varepsilon t} \ln z(0), \tag{75}$$

$$\ln Q_t = \left(1-e^{\varepsilon t}\right)\ln Q^* + e^{\varepsilon t} \ln Q(0), \tag{76}$$

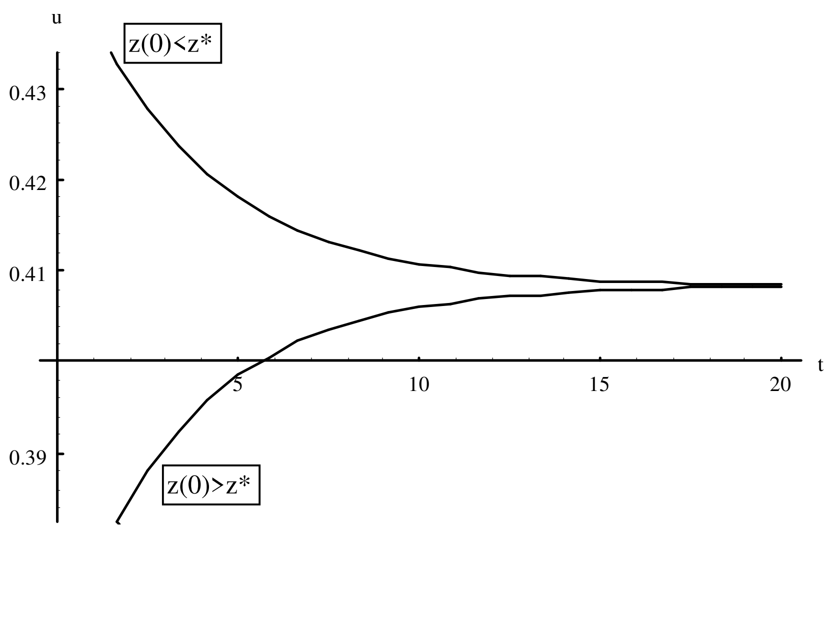
where ε is the negative eigenvalue of the matrix $\hat{A}$ and -ε is the speed of convergence. (*u(0), z(0), Q(0))* is the position on the stable arm at which the economy starts out at time *t=0*. If we rearrange equation (75) we obtain an expression for how the economy's position in relation to the steady state, *d*, evolves over time, given the position at the outset:

$$d = \ln\left(z_t / z^*\right) = \ln\left(z(0)/z^*\right)e^{\varepsilon t}. \tag{77}$$

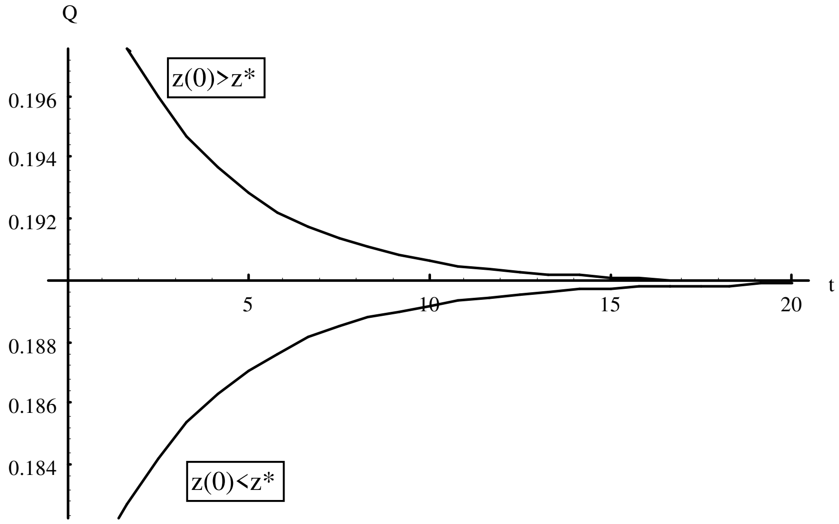
This expression, together with expressions (65) and (66), allows us to display graphically in Figures 9a to 9f how decision variables and growth rates converge towards their steady state values over time, when the economy starts out in a situation where the ratio of human capital to intermediates is half its steady state value. In the baseline case, it takes about 2.5 years to get half-way to the steady state. This implies that the speed of convergence, -ε, is 0.28. To determine how the speed of convergence depends on the parameters of the model, requires us to solve explicitly for the eigenvalues of matrix A. Since, we have not been able to do so, the dependence of the convergence speed on parameter values will only be analysed numerically.

Figure 9. *The convergence over time of the time share spent in school, u, the ratio of consumption to intermediates, Q, and of growth rates.*

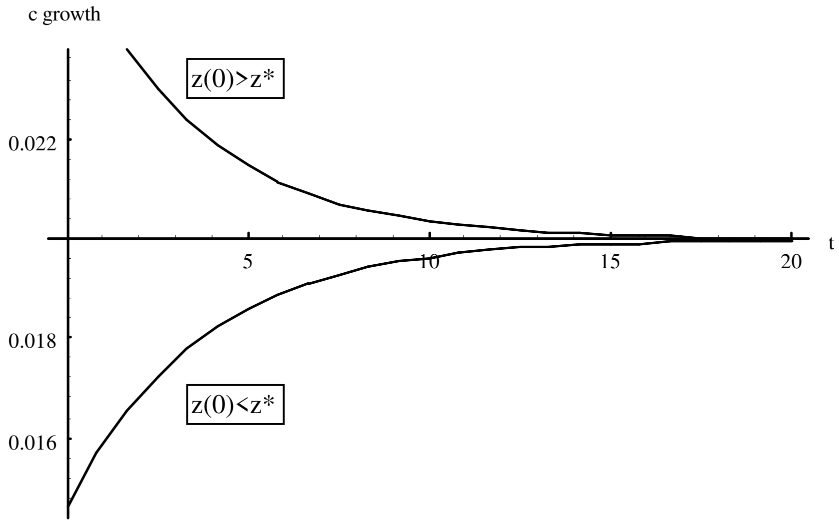
a Time share spent in school, u.



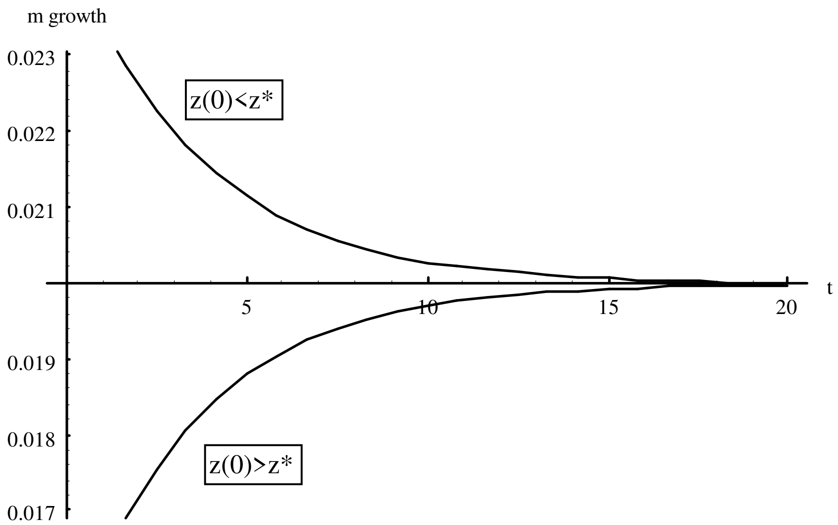
b The ratio of consumption to intermediates, Q.



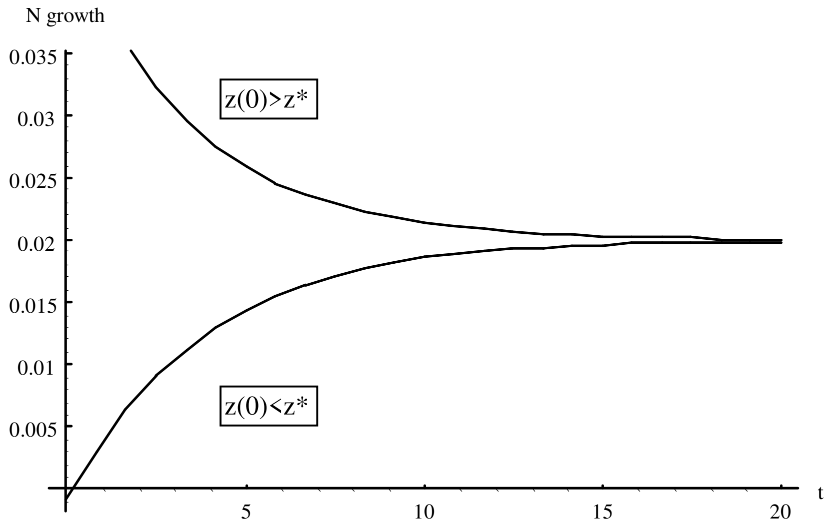
c The growth rate of consumption, γ_{c}.



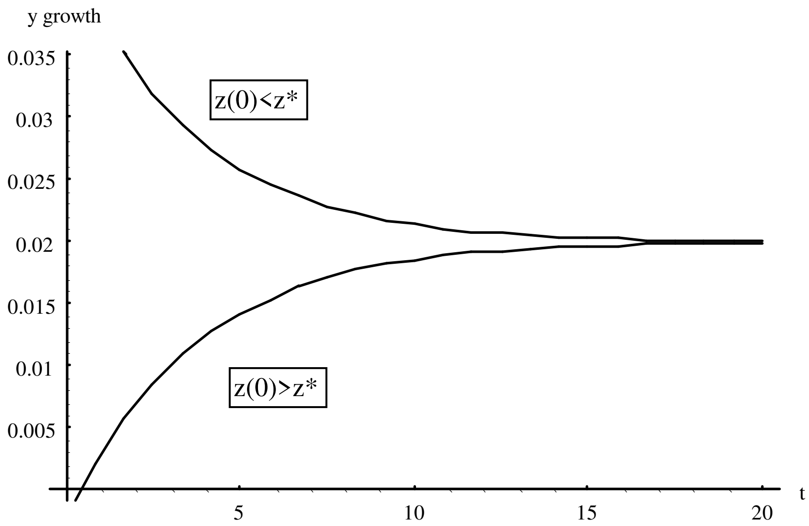
d The growth rate of human capital, γ_{m}



e The growth rate of the number of intermediates, γ_N



f The growth rate of output, γ_Y



5 Sensitivity analysis

As shown above, the steady state growth rate depends on the parameters of the demand function and on the parameters in the motion equation of human capital. In this section we examine how the position of the steady state, the slope of the stable arm, and the convergence speed depend on the parameters of the model. We present the results of a sensitivity analysis of the steady state and the dynamics. It turns out that the model behaves qualitatively similar for the ranges of parameter values tested.

The qualitative results of the numerical simulations to compute the steady state, the slope of the stable arm, the convergence speed, and the steady state growth rates for different parameter values, are summarized in Table 1, while the full results are presented in Table 2. Apart from showing how these depend qualitatively and quantitatively on changes in parameter values, the numerical simulations also show that the dynamics around the steady state exhibit saddle path stability, that is, there is one negative and two positive eigenvalues, for the ranges of parameter values tested here. The results reported in Tables 1 and 2 are obtained in numerical simulations in which one parameter at a time is allowed to deviate from its baseline value, and we can thus say nothing about where the economy ends up if all parameters are altered at the same time.

5.1 Qualitative results of the sensitivity analysis

Table 1 presents the effects on steady state values and the convergence rate of allowing one parameter at a time to vary from the lower to the upper bound of a given interval. An arrow pointing upward (downward) means that the value of the particular variable increases (decreases) as the parameter value is increased. Column 1 reports the parameter and the range tested, column 2 reports the effect on the steady state growth rate of a change in the parameter. In column 3, the effect on the number of years it takes for the economy to get half-way to its steady state is reported, columns 4-6 report the effects on the steady state values of u, z and Q, and columns 7-8 report how a change in the parameter affects the elasticities of u and Q with respect to deviations of the ratio of human capital to intermediates, z, from the steady state position.

The results displayed in column 2, are consistent with the findings in Section 3.1 on the steady state growth rate, that is, the steady state growth rate depends on the parameters of the demand function and of the parameters of the human capital accumulation function. The more patient and the less keen the individual is on consumption smoothing, the faster the steady state growth rate. The economy also grows faster in steady state, the higher the level of technology in the educational sector and the lower the rate of depreciation of human capital. As mentioned before, there is an ambiguity with respect to the degree of decreasing returns to human capital in the learning process.

In column 3, we can see that the speed of convergence towards the steady state depends on the same parameters as the steady state growth rate, with the addition of the elasticity of the final output production function with respect to intermediates. The economy moves faster (half life is shorter) towards the steady state, the less keen the individual is on consumption smoothing and the more favourable the environment for human capital accumulation (that is the higher the level of technology). If the individual does not care very much about a smooth consumption path, he is willing to experience rapid changes in consumption levels along the transition to the steady state. Convergence towards steady state is on the other hand slower if the elasticity of output with respect to intermediates is high.

The position of the steady state is also affected by the parameters of the model. While the time share dedicated to education is affected only by demand and human capital accumulation related parameters, the steady state positions of the ratio of human capital to intermediates, z, and the ratio of consumption to intermediates are affected also by the parameters of the final output sector and the parameters pertaining to the R&D activity. In column (4), we see that people go to school less in steady state, the more keen they are on a smooth consumption path and the less patient they are. A high level of technology in the educational sector and a low rate of depreciation reduces the educational effort needed to keep up a certain level of accumulation, whereas a high β makes high educational efforts worthwhile since a high β implies that the degree of decreasing returns to educational effort is low.

The steady state ratio of human capital to intermediates in column (5), is higher the higher the level of technology in the educational sector and the lower the rate of depreciation of human capital. Both these parameters determine how costly it is to accumulate human capital, and the more costly it is, the lower will be the steady state ratio of human capital to intermediates. If the population is large, low human capital per worker is compensated by the large number of workers and thus the steady state ratio of human capital to intermediates decreases with the size of the labour force. This result is, of course, directly linked to the assumption that firms care about the total amount of human capital employed and not on how it is distributed among the workers. The R&D cost is positively related to the ratio of human capital to intermediates, since a high R&D cost makes the cost of intermediates high relative to the cost of human capital.

The steady state ratio of consumption to intermediates in column (6), could be interpreted as the proportion of material wealth consumed at each moment. If the individual likes a smooth consumption path and if he is impatient, the ratio of consumption to intermediates is high. If the level of technology in the educational sector is high, the individual chooses to have a larger share of his wealth denominated in human capital, and consumption as a share of wealth denominated in intermediates is therefore high. The same kind of reasoning applies to the rate of depreciation, the size of the work force and the R&D cost.

Table 1. *Qualitative results of sensitivity analysis*

(1) Parameter (range tested)	**(2) Steady state growth rate**	**(3) Half life of distance to Steady state**	**(4) u***	**(5) z***	**(6) Q***	**(7) (du/u) / (dz/z)**	**(8) (dQ/Q) / (dz/z)**
θ (0.5-100)	↘	↗	↘	↘↗	↗	↘	↗
ρ (0.001-0.05)	↘	↗	↘	↘	↗	↘	↘
B (0.01-100)	↗	↘	↘	↗	↗	↘	↗
β (0.001-0.999)	↘↗	↗↘	↗	↘↗	↘↗	↘	↘↗
δ (0-0.075)	↘	↗	↗	↘	↘	↘↗	↘
L (0.01-100)	→	→	→	↘	↘	→	→
η (0.01-100)	→	→	→	↗	↗	→	→
α (0.001-0.999)	→	↗	→	↘↗	↘	↘	↘
A (0.05-100)	→	→	→	↘	→	→	→

→=a change in the parameter has no effect, ↗=an increase in the parameter results in a monotonous increase in the variable, ↘=an increase in the parameter results in a monotonous reduction in the variable, ↘↗=raising the parameter first leads to a decline and then to an increase in the variable, ↗↘=raising the parameter first leads to an increase and then to a decline in the variable.

5.2 Quantitative results of the sensitivity analysis

In Table 2, we can go further and see how sensitive the steady state and the dynamics are to parameter changes. We have added a column, column (2), which reports the value of the parameter being tested. It can be noted that altering the parameters of the demand function, θ and δ, does not seem to have very large effect on the steady state and the dynamics.

Table 2. Quantitative results of sensitivity analysis

(1) Parameter	(2) Parameter value	(3) Steady state growth	(4) Half life of distance to Steady State	(5) u*	(6) z*	(7) Q*	(8) (du/u) / (dz/z)	(9) (dQ/Q) / (dz/z)
θ	0.5	0.048	1.520	0.659	2.530	0.114	-0.846	0.249
	1	0.032	2.057	0.513	2.037	0.154	-0.894	0.490
	2	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	3	0.015	2.670	0.365	1.866	0.208	-1.021	0.671
	5	0.010	2.877	0.325	1.860	0.226	-1.075	0.715
	10	0.005	3.084	0.291	1.868	0.243	-1.130	0.752
	100	0.001	3.339	0.257	1.892	0.263	-1.199	0.790
ρ	0.001	0.031	2.167	0.502	2.017	0.157	-0.944	0.642
	0.005	0.029	2.213	0.488	1.922	0.162	-0.947	0.638
	0.01	0.027	2.268	0.472	1.965	0.167	-0.952	0.635
	0.03	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	0.05	0.013	2.642	0.350	1.862	0.214	-1.000	0.611
B	0.01	0.005	4.210	0.428	1.127	0.116	-0.870	0.593
	0.131	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	0.5	0.197	0.437	0.365	10.671	1.074	-1.131	0.657
	1	0.437	0.208	0.357	22.486	2.273	-1.151	0.660
	5	2.355	0.040	0.351	116.931	11.864	-1.167	0.663
	10	4.753	0.020	0.351	234.979	23.853	-1.169	0.663
	100	47.914	0.002	0.350	2359.820	239.658	-1.171	0.664
β	0.001	0.080	2.675	0.000	3.036	0.489	-0.381	0.634
	0.05	0.059	3.308	0.028	2.450	0.387	-0.388	0.631
	0.3	0.027	4.637	0.172	1.635	0.226	-0.467	0.617
	0.5	0.021	4.054	0.291	1.611	0.193	-0.611	0.613
	0.7	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	0.95	0.024	0.336	0.549	2.776	0.211	-5.519	0.665
	0.999	0.025	0.006	0.576	3.053	0.217	-271.455	0.678
δ	0	0.049	0.299	0.245	21.406	2.535	-0.756	0.648
	0.025	0.025	0.307	0.251	20.782	2.441	-0.748	0.647
	0.05	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	0.075	0.004	3.881	0.488	1.210	0.112	-0.762	0.576

Baseline results are reported in bold print.

Changes in B, on the other hand, have large effects on growth, speed of convergence, and on the steady state ratios of human capital to intermediates and consumption to intermediates. β also has a marked impact on growth and convergence speed. The individuals allocation of time between work and school, u*, is more sensitive to changes in the degree of diminishing returns to schooling effort, β, than to any other parameter changes. Whereas the size of the work force, L, the R&D cost, the elasticity with respect to intermediates and the level of technology in final output production have no or little impact on steady state growth, convergence speed and educational effort, they are influential when it comes to determining the whereabouts of the steady state.

Table 2 continued

(1) Parameter	(2) Parameter value	(3) Steady state growth	(4) Half life of distance to Steady State	(5) u*	(6) z*	(7) Q*	(8) (du/u) (dz/z)	(9) (dQ/Q) (dz/z)
L	0.01	0.020	2.470	0.408	189.210	18.992	-0.973	0.622
	0.5	0.020	2.470	0.408	3.784	0.380	-0.973	0.622
	1	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	2	0.020	2.470	0.408	0.946	0.095	-0.973	0.622
	5	0.020	2.470	0.408	0.378	0.038	-0.973	0.622
	10	0.020	2.470	0.408	0.189	0.019	-0.973	0.622
	100	0.020	2.470	0.408	0.019	0.002	-0.973	0.622
η	0.01	0.020	2.470	0.408	0.019	0.002	-0.973	0.622
	0.5	0.020	2.470	0.408	0.946	0.095	-0.973	0.622
	1	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	2	0.020	2.470	0.408	3.784	0.380	-0.973	0.622
	5	0.020	2.470	0.408	9.461	0.950	-0.973	0.622
	10	0.020	2.470	0.408	18.921	1.899	-0.973	0.622
	100	0.020	2.470	0.408	189.210	18.992	-0.973	0.622
α	0.001	0.020	0.184	0.408	120.023	70.016	-0.062	0.990
	0.05	0.020	1.136	0.408	3.413	1.449	-0.410	0.894
	0.3	0.020	2.163	0.408	1.580	0.283	-0.835	0.700
	0.5	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	0.7	0.020	2.656	0.408	2.975	0.150	-1.060	0.569
	0.95	0.020	2.808	0.408	17.484	0.124	-1.133	0.523
	0.999	0.020	2.831	0.408	873.803	0.120	-1.144	0.515
A	0.5	0.020	2.470	0.408	7.568	0.190	-0.973	0.622
	1	**0.020**	**2.470**	**0.408**	**1.892**	**0.190**	**-0.973**	**0.622**
	2	0.020	2.470	0.408	0.473	0.190	-0.973	0.622
	5	0.020	2.470	0.408	0.076	0.190	-0.973	0.622
	10	0.020	2.470	0.408	0.019	0.190	-0.973	0.622
	100	0.020	2.470	0.408	0.000	0.190	-0.973	0.622

Baseline results are reported in bold print.

6 Discussion and conclusion

This paper has analysed an economy in which individuals accumulate human capital and invest in R&D which is carried out by entrepreneurs. In the model we have presented, output per capita of an economy is determined by the stock of human capital and the time share spent working. We find that the long run growth rate of an economy is determined by the capacity of the economy to accumulate human capital and by the preferences of individuals. It follows that if an economy is poor in output per capita terms relative to its stock of human capital, it will grow fast if the reason for low output per capita is that the citizens are spending their time accumulating human capital instead of working. In such a situation, the country has a strong output growth potential which is realized once people reallocate time from school to work. Our model thus implies that changes in participation rates should have explanatory power for growth. In line with other growth models, we find that steady state growth increases with patience and decreases with the preference for smooth consumption. Apart from preferences, the determinants for long run growth are the efficiency of the educational system and the rate of depreciation of human capital. Our model also predicts a positive correlation between longrun growth and the rate of investment in the R&D sector, as well as between output growth and investments in human capital.

Furthermore, we find that a relative lack of ideas or R&D capital, i.e. a ratio of human capital to intermediates in excess of the steady state ratio, will cause the economy to grow slowly during its transition to the steady state, while a relative abundance of R&D capital gives high growth rates during transition. This asymmetric imbalance effect can be interpreted as being the effects of an idea gap. Abundance of ideas provides a strong incentive for investments in other factors of production, which leads to growth. Lack of ideas, on the other hand, is an incentive for investments in ideas. The growth in the stock of ideas stimulate people to reallocate effort from producing output to accumulation of factors. This reallocation reduces the output growth effect induced by growth in the stock of ideas.

Although our model has little to say about why or how an economy would end up in a situation where R&D capital and human capital are out of balance, our results offer some insight as to why human capital rich economies such as many European economies can experience poor growth. Political decisions which affect the efficiency of the educational system, are examples of events which may cause the steady state to shift. Such a steady state shift would render the ratio of human capital to R&D capital out of balance. If such a political measure had the effect of worsening the efficiency of the educational system, it would put the economy in a situation with a relative human capital abundance. People would shift from schooling to working, which would give a short run bost in output, but the economy would then grow slowly. Were the measure positive for the efficiency of schooling, the effect would be the opposite and lead to rapid growth in transition and a higher steady state growth rate.

An implication of this imbalance effets is that there is a possibility both of convergence and non-convergence in per capita output levels over time. If the citizens of two economies have the same preferences and if all the technology parameters are the same for the two countries, history will determine whether per capita output of the two economies converge over time or not. If history has endowed the two economies with the same ratio of human capital to R&D capital, they will follow the same growth path and initial income differences will be permanent. However, if history for some reason has endowed the poorest of the two economies with a relative scarcity of human capital and the richer economy with a relative abundance of human capital, output levels will converge over time. The poor economy may get closer or catch up completely, but it could also surpass the rich economy. In the case where the initially rich economy is also the one scarcely endowed with human capital, the output gap will widen. If technologies are different, long run growth rates will differ and income gaps will widen. In this case, the economy with superior capacity to accumulate human capital will always eventually be the richest economy, whether it was rich or poor to start with.

The reason for catching up or surpassing in this model is that the citizens of the catcher up (or surpasser) economy have a stronger incentive to accumulate human capital than the citizens of the leader and laggard economies. This will be the case if the catcher up economy has a relative abundance of R&D capital which makes the relative return to human capital investments large. Because of the high returns to investment in human capital and, hence, large time share in spent in school, a catcher up (surpasser) economy will initially have a small output per capita relative to its human capital endowment. This result finds support in the empirical literature.

The numerical simulations carried out in section 5 indicate that the transition to the steady state growth rate is rapid, in comparison with the empirical findings of rates of convergence of about 2 per cent per year. Our simulations indicate a half life of at the most 5 years in comparison to a half life of about 34 years which corresponds to a speed of convergence of 2 per cent. This means that growth rates will deviate from the steady state growth rate only for brief periods of time, making the likelihood of catching up small unless we assume that economies for some exogenous reason are frequently pushed away from their steady states.

Apart from rendering the model interesting transitional dynamics, the R&D sector is of limited importance for growth in the long run. This is due to the absence of scale effects. Human capital accumulation is the real source of growth in the model. The individual's possibility to choose between human capital accumulation and investments in R&D as alternative ways of influencing future income is what drives the dynamics of the model, but in the steady state, the individual's savings, i.e. investments in R&D, increase at the same rate as he accumulates human capital. Production of output in the model depends on the total amount of intermediates in production, NX, which means that the number of intermediates and the quantity of each intermediate are, in fact, perfect substitutes. An implication of this is that it would be possible to sustain growth by expanding the quantity of each intermediate instead of expanding the number of intermediates. Although this would violate the zero profit condition of the entrepreneurs, a social planner could actually improve growth by forbidding R&D, thereby stopping the expansion of N and instead increasing the use of each type of intermediate. This implication which is present, albeit not discussed, in Jones (1995) appears to be a consequence of there being no scale effects from R&D. It is clear that the importance of R&D and entrepreneurship for economic growth, in the absence of scale effects, deserves more attention.

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