Labor- and Product-Market Structure and Excess Labor

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Abstract

This study analyzes under what labor- and product-market structures a firm may hire more labor than needed to produce its profit maximizing output. Three labor-market structures are studied: (1) *decentralized* (firm-specific unions), (2) *one-sided centralization* (central union and several firms), and (3) *centralized* (central union and employers' association). Excess labor is explained by the risk-sharing motive that in the model exists between the risk-averse workers and the risk-neutral firm owner. Labor may be excessively hired in any of the labor-market structures and under a wide range of product-market structures; duopoly, oligopoly etc. *Journal of Economic Literature* Classification Numbers: J21, J51, and L11.

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1. Introduction

This paper explains why a firm may hire more labor than it needs to produce its profit maximizing quantity. It also analyzes under what labor- and product-market structures excess labor is hired. The analysis is carried out within the framework of a standard wage-bargaining model. In the model, the product-market demand is known but the size of the union will typically be larger than the number of available jobs. The only uncertainty is therefore faced by the individual worker who doesn't know in advance if he will be employed or not. Workers has limited access to income insurance and it turns out that hiring of excess labor can be realized as a consequence of the risk-sharing motive that exists between the risk-averse workers and the risk-neutral firm owner (see Parsons (1986) for a more detailed discussion). The employment prescribed by a Pareto efficient agreement over wages and employment between the firm and the union is higher than the employment prescribed by the firm's labor-demand curve at the same wage. This is to be viewed as additional protection for the workers compared to a situation where the firm unilaterally determines the employment. The size of this additional employment and its use in production depend on the structures of the labor and product markets. It is these relationships that are investigated in this study.

The timing in the model is the following: first there is simultaneous bargaining over *labor contracts* (wages and employment) between one or several firms and firm-specific unions or the corresponding central organizations. Prior to the bargaining all firms and unions share a common belief over the future agreements. This stage will be called the *labor-market stage*. Thereafter the firms compete in a Cournot market for their products. They simultaneously make their production decisions without being informed about the labor contracts of the other firms and subject to the restrictions set out by their labor contracts and the belief discussed above. This stage will be called the *product-market stage*. Three different labor-market structures are studied; decentralized bargaining, one-sided centralized bargaining, and centralized bargaining.

Throughout the study, firms are assumed to be identical and workers to have no mobility between firms, to have identical well-behaved preferences, and to be divided into equally large firm-specific unions. The central- or firm-specific union maximizes the joint utility of its members. The firm or employers' association maximizes profits or joint profits, respectively. The bargaining is modelled by way of the Nash bargaining solution and it is assumed to be over both wages and employment. This assumption is made because it makes the bargaining outcome Pareto efficient. We believe that it is reasonable to expect the outcome of a bargaining between two rational parties to be Pareto efficient. Moreover, it can be shown that in the context of the model, it is in the interest of the union to bargain over both wages and employment. For more exhaustive discussions of bargaining over wages versus bargaining over wages and employment, see Farber (1986), Flanagan *et al.* (1993), and Andrews and Simmons (1995).

Labor is assumed to be indivisible, either a worker is employed and supplies one unit of labor or he is unemployed. If the labor contract specifies a labor force smaller than the size of the union, then every worker has equal probability of being employed. Unemployed workers receive an unemployment benefit paid by the government.[1]

After the labor-market stage the firms enter the product-market stage where they compete in quantities in a market with a known inverse demand function. Every labor contract is assumed to be private information to the firm and union in question (this assumption is discussed further in Section 6). The production technology is a constant returns-to-scale technology using only labor as input.

The individual firm is assumed to have *free disposal* of its contracted labor force. The firm's expectation over its competitors' output decisions is contingent on its belief over their labor contracts. If every firm and union anticipates the labor contracts correctly, i.e., if expectations are rational, every labor contract will be a "best reply" to all the other contracts and the produced quantities will constitute an equilibrium in the product market. In the following we will call a situation where both markets are in equilibrium a *labor-market equilibrium* or just an *equilibrium*. If the hired labor force of a firm is greater than needed to produce the profit maximizing quantity, the firm will not produce on the boundary of its production-possibility set and some or all workers will be told to "shirk."

In the decentralized setting, every firm-specific union negotiates with "its" firm and the unique equilibrium vector of labor contracts is symmetric. Excess labor may be hired under any product-market structure depending on the degree of workers' risk aversion and the size of the unemployment benefit. The higher the degree of risk aversion and the lower the unemployment benefit, the higher is the largest number of firms that is consistent with excess employment in equilibrium.

The second case is one-sided centralization. Here, the firm-specific unions join together into one central union that negotiates simultaneously with every firm. Labor contracts are private information and this allows the central union to treat every bargaining as a separate problem since the firms' output decisions

[1] In the model, if a labor contract specifies less than full employment then it is not in the interest of the union to share the available jobs equally among its members, i.e., to create part-time jobs. This is because the members would then lose the unemployment benefit paid by the government, and the total amount of money received would be smaller. The elimination of the risk of being unemployed can not compensate for this loss. Moreover, in the model this is true for any income-insurance system financed by the government and in which the benefit is a non-decreasing function of the wage.

are contingent on their belief and not on the actual labor contracts. In every bargaining the incentives of the central union coincide with the incentives of the firm-specific union and, hence, the labor-market equilibrium is identical to the decentralized labor-market equilibrium.

The last labor-market structure to be analyzed is the fully centralized structure. Here, the firm owners join together into an employers' association that bargains with the central union over a central labor contract specifying the same wage and employment in all firms. The employers' association maximizes joint profit conditional on that its members cannot collude in the product market but will compete as an one-shot oligopoly. It turns out that the central labor contract isn't a continuous function of the parameters of the model (workers' risk aversion, the unemployment benefit, and the number of firms). This discontinuity arises because of the divergence in interest between the employers' association and the individual firm since the employers' association internalizes the effect of increased employment on joint profits while the individual firm doesn't. The employers' association experiences a stronger marginal effect of employment on profits than the individual firm and it is consequently more reluctant than the firm to insure workers against income uncertainty by means of employment. If the parameters of the model are such that no excess labor is hired in equilibrium in neither the decentralized setting nor in the centralized setting, then wages are higher and employment is lower in the centralized setting than in the decentralized setting. On the other hand, if labor is excessively hired, output is unaffected by an increase in employment and the marginal effect of employment on profits is the same for the firm and the employers' association. Hence, if the parameters are such that excess labor is hired in equilibrium in both the decentralized and centralized settings, then wages and employment are the same.

Dowrick (1989) analyses the interaction of union-firm bargaining and oligopolistic price-setting when collusion in the product market is captured by conjectural variation. He finds, among other things, that the structural conditions in the product market which lead to higher profit margins also lead to higher wages. Also, when the wage bargaining is efficient, increased collusion in the product market leads to higher wages. Studying similar models, Davidson (1988) and Dowrick (1993) investigates the relation between bargaining structure and wages in unionized oligopolistic industries. Davidson main finding is that wages are higher when there is a central union and this benefits the firm-specific unions and reduces the profit of the firms. Dowrick (1993) shows that moving from bargaining at the national level to firm-level bargaining doesn't necessarily moderate wage pressure. Aggregate wage pressure and wage inequalities increase when the bargaining is between craft- and industry organizations.[2]

[2] Using the same type of models, Dowrick and Spencer (1994) study unions' attitudes toward

Bentolila *et al.* (1996) shows that wages are strategic complements from the unions' point of view and strategic substitutes from the firm owners' point of view. Comparative-statics analysis of how wages respond to changes in the labor- and product markets thus depend on the relative bargaining strength of unions and firm owners at the bargaining table. Haskel and Sanchis (1998) study theoretically firms' technical and allocative efficiency when firms and unions bargain over wages and effort when every firm is a monopolistic competitor. Their aim is to provide a theory that explains the inefficiencies the empirical literature seeks to estimate. The main results are closed form solutions of the two inefficiencies relating them to properties of the production function, union preferences and the firm's market power.

The main difference of our study and the studies above is the posed question. The focus is not on wages but on the connection between employment and firms' use of labor and the structures of the labor- and product markets. It differs from all the studies except Dowrick (1989) in that the bargaining is efficient and it differs from Dowrick (1989) in that firms have free disposal of labor and compete is in quantities. The main difference to Haskel and Sanchis (1998) is that the bargaining and product market are modelled differently and that also the labor-market structure is incorporated in the analysis.[3, 4]

The product-market stage and the decentralized labor market are presented in Section 2. Sections 3 and 4 contain the cases of one–sided centralized bargaining and centralized bargaining, respectively. The numerical example is given in Section 5 and Section 6 contains the discussion. All proofs are given in the Appendix.

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labor-saving innovations and Santoni (1996) studies strategic effects of international trade on domestic wage setting.

[3] Kahn and Reagan (1993) introduce working rules in their model of implicit contracts. They find that this may cause labor hoarding and inefficient use of the hired labor force. This finding has some empirical support. Especially, increases in the level of competition in the product market tend to increase productivity via less restrictive working rules and lower wages, see Nickell *et al.* (1994) and Nickell and Nicolitsas (1995).

[4] Dixit (1980) studies entry deterrence and investment in capacity. One main result is that excess capacity is never installed when capacity can be bought in an unlimited amount at a fixed price. In the model presented below, one may interpret employment as capacity. The reason why excess labor may be hired is that labor is not available in an unlimited quantity at some fixed wage. Instead, wage and employment are bargained over and consequently Dixit's result does not carry over.

2. The Model

The model consists of two stages, the labor-market stage and the product-market stage. Firms and workers are rational and thus forward looking at the labor-market stage. For this reason the properties of the product market is essential when modelling the bargaining. The product market is therefore presented before we proceed to the labor-market stage and the analysis.

2.1. The Product Market

Consider a product market in which there are $m \geq 1$ identical firms using only one input, labor. Let $\mathbf{n} = (n_1, ..., n_m)$ and $\mathbf{w} = (w_1, ..., w_m)$ denote the vectors of employment and wages of the firms. The wage in firm i is $w_i \geq 0$ and for reasons explained below the employment is $n_i \in [0, 1/m]$. The firms compete in quantities and they make their production decisions simultaneously. The inverse demand function is $P(\mathbf{q}) = 1 - \sum_{i=1}^{m} q_i$ where $\mathbf{q} = (q_1, ..., q_m)$ is the output vector. The firms have identical CRS technologies and every firm is assumed to have *free disposal* of its labor force. Hence, $q_i \leq n_i$. The profit of firm i is $\pi_i(\mathbf{q}, n_i, w_i) = q_i P(\mathbf{q}) - n_i w_i$ where the cost $w_i n_i$ is fixed. The unique profit-maximizing output of firm i for a given level of employment n_i is given by

$$q^o(n_i, \mathbf{q}_{-i}) = \min\left\{n_i, \frac{1 - \sum_{j \neq i} q_j}{2}\right\}$$

where $\mathbf{q}_{-i} = (q_1, ..., q_{i-1}, q_{i+1}, ..., q_m)$. Given $\mathbf{n}$, the system $q_i = q^o(n_i, \mathbf{q}_{-i})$, $i = 1, 2, ..., m$ has a unique solution. Let $\mathbf{q}^e(\mathbf{n}) = (q_1^e(\mathbf{n}), ..., q_m^e(\mathbf{n}))$ be this solution.

The difference between this simple product-market model and a standard Cournot model is the firms' capacity constraints. Labor is here viewed as irrevocably hired before the making of the production decision and this induces a (capacity) constraint on the individual firm's maximization problem, $q_i \leq n_i$. Because employment is irrevocable, the marginal cost of production can be viewed as zero up to the capacity limit and infinite for quantities above the capacity limit. In the symmetric case, if the capacity constraints are not binding then the firms produce as in the corresponding Cournot model with zero marginal cost and if the constraints are binding then the firms produce at their capacity limits. A simple example will illustrate this. Let $n_i = n$ for all i. If $n \geq 1/(m+1)$ then the product-market equilibrium is as in the standard Cournot case, $q_i^e(\mathbf{n}) = 1/(m+1)$, and if $n < 1/(m+1)$ then $q_i^e(\mathbf{n}) = n$.

Definition 1. *Firm i hires excess labor if $n_i > q_i$.*

Thus, if the capacity constraint doesn't bind we say that firm i hires excess labor. In the following we also say that the product market has *structure* m if there are m firms.

2.2. The Decentralized Labor Market

In the labor market, workers are identical, they have no mobility, and they are organized in m equally large firm-specific unions. The size of the labor market is normalized to 1 and the size of every union is $1/m$. At employment $n_i \in [0, 1/m]$, every worker in union i has equal probability of being employed, mn_i. The utility of income to a worker is given by the von Neumann - Morgenstern utility function $u(w_i) = 1 - e^{-\theta w_i}$ where $\theta > 0$ determines the marginal utility of income. Moreover, θ is equal to the Arrow-Pratt measure of absolute risk aversion which here is constant. With probability $1 - mn_i$ the worker will be unemployed and receive an unemployment benefit $b \in [0, 1)$ paid by the government. The benefit may be interpreted as also capturing the utility from leisure in case of unemployment.

Let (n_i, w_i) be the labor contract specifying employment and wage in firm i. If the firm and the union don't reach an agreement, then every worker will be unemployed and receive the unemployment benefit. The expected utility gain for a worker in union i from the labor contract is

$$v(n_i, w_i) = mn_i(u(w_i) - u(b)).$$

In the decentralized labor market, the labor contracts are determined in m pair-wise and simultaneous negotiations where each union bargains with "its" firm. The objective of union i is to maximize the expected utility of a representative member and the objective of the firm is to maximize expected profits. Prior to the bargaining all firms and unions hold the same belief over the future vector of employment, $\mathbf{n}^e = (n_1^e, ..., n_m^e) \in [0, 1/m]^m$, and this is common knowledge. Then, it is reasonable to assume every firm and union also to believe $\mathbf{q}^e(\mathbf{n}^e)$ to be the vector of future output decisions. Every labor contract (n_i, w_i) arrived to in the wage bargaining is assumed to be private information to the firm and union in question, i.e., after the bargaining firm and union i still believe the other firms to employ $\mathbf{n}_{-i}^e = (n_1^e, ..., n_{i-1}^e, n_{i+1}^e, ..., n_m^e)$ and know them to believe that firm i has employed n_i^e workers. Hence, firm and union i expect the other firms to produce $\mathbf{q}_{-i}^e(\mathbf{n}^e)$. Given this expectation, the best firm i can do is to produce $q^o(n_i, \mathbf{q}_{-i}^e(\mathbf{n}^e)$ and its expected profit is

$$\pi_i^e(\mathbf{n}^e, n_i, w_i) = \pi_i(\mathbf{q}_{-i}^e(\mathbf{n}^e), q^o(n_i, \mathbf{q}_{-i}^e(\mathbf{n}^e)), n_i, w_i).$$

The set of feasible labor contracts between firm and union i is

$$B_i^D(\mathbf{n}^e) = \left\{(n_i, w_i) \in [0, 1/m] \times [0, 1] \mid v(n_i, w_i), \pi_i^e(\mathbf{n}^e, n_i, w_i) \geq 0\right\}.$$

The Nash bargaining solution is

$$(n_i^D, w_i^D) = \arg \max_{(n_i,w_i)\in B_i^D(\mathbf{n}^e)} (v(n_i, w_i) - \underline{v})(\pi_i^e(\mathbf{n}^e, n_i, w_i) - \underline{\pi}) \tag{2.1}$$

where $\underline{v} = \underline{\pi} = 0$ because if there is no agreement then there is no utility gain to the union and the firm makes zero profit since it has no revenue and no expenses.[5] The Nash bargaining solution implicitly defines the employment and wage as continuous functions of the parameters of the model, $\eta_i^D(b, \theta, \mathbf{n}^e)$ and $\omega_i^D(b, \theta, \mathbf{n}^e)$ respectively. The wage is decreasing in θ and increasing in b. The workers' gain from trading wage for employment increases with their risk aversion, θ. Higher employment requires lower wage by the first-order conditions of the Nash bargaining solution. Increases in b decreases the workers' gain from trading wage for employment and increases therefore the wage. Employment is non-increasing in w and therefore non-decreasing in θ and non-increasing in b. The "non"-parts of the two "employment" statements are due to the restriction $0 \leq n \leq 1/m$ which can be interpreted as the union maximizing the expected utility of its current members. Employment higher than $1/m$ lowers the wage without increasing currents members' probability of being employed and this is not in the interest of the union. A vector of labor contracts

$$(\mathbf{n}^{D*}, \mathbf{w}^{D*}) = (n_1^{D*}, ..., n_m^{D*}, w_1^{D*}, ..., w_m^{D*})$$

is a labor-market equilibrium if $(n_i^{D*}, w_i^{D*}) = (\eta_i^D(b, \theta, \mathbf{n}^e), \omega_i^D(b, \theta, \mathbf{n}^e))$, $i = 1, ..., m$, and expectations are rational, $\mathbf{n}^e = \mathbf{n}^{D*}$. Due to the symmetry of the m bargaining problems and the continuity of η_i^D and ω_i^D there exists a symmetric equilibrium, $(n_i^{D*}, w_i^{D*}) = (n^{D*}, w^{D*})$ for all i. This is also the unique equilibrium.

Proposition 1. *There exists a symmetric labor-market equilibrium* $(\mathbf{n}^{D*}, \mathbf{w}^{D*})$ *to the decentralized labor market and, moreover, this equilibrium is unique.*

Proof. See the Appendix.

From the symmetry it follows that all firms make the same output decision and the same profit. The expected utility of an arbitrary worker is consequently the same in all unions. Using the first-order condition of the Nash bargaining solution with respect to employment and the symmetry of the equilibrium gives for every product-market structure m a critical wage such that excess labor is hired in equilibrium if and only if the equilibrium wage is lower than this critical wage.

[5] Throughout the study we use the break-down interpretation of the Nash bargaining solution, see Binmore *et al.* (1986).

Lemma 1. *Let the labor market be decentralized. Then labor is excessively hired in equilibrium if and only if*

$$w^{D*} < \overline{w}^D(m) = \frac{1}{2(m+1)}. \tag{2.2}$$

Proof. See the Appendix.

The equilibrium wage w^{D*} is higher than the unemployment benefit for all degrees of workers' risk aversion. This makes $\overline{w}^D(m) > b$ a necessary condition for excess labor in equilibrium at product-market structure m. The critical wage level $\overline{w}^D$ is decreasing in the number of firms and an increased product-market competition in the sense of an increased number of firms induces a decrease in the critical wage level $\overline{w}^D$. This decrease has two effects on the set of values of b and θ at which excess labor is hired in equilibrium. First, the highest value of b such that $\overline{w}^D(m) > b$ goes down. Secondly, the lowest value of θ such that $\overline{w}^D(m) > w^{D*}$ for a given b goes up. An increase in the number of firms thus reduces the range of parameter values at which excess labor is hired and in this sense, increased competition in the product market can be said to make hiring of excess labor in equilibrium less likely.

The extent to which excess labor is hired doesn't only depend on workers' risk aversion and the unemployment benefit. The product-market structure is also of importance. Suppose that θ and b are such that excess labor is hired at m and m' where $m' > m$. Then, the individual firm hires less excess labor under the more competitive product-market structure m' than under the less competitive structure m. There is also less aggregate excess labor under m' than under m. Moreover, given the unemployment benefit level excess labor cannot be hired under every product-market structure. The critical wage $\overline{w}^D$ is decreasing in m and if $b > 0$ then there exists a product-market structure $\overline{m}(b)$ at which firms may hire excess labor in equilibrium but not if one more firm is added, i.e., a product-market structure such that $\overline{w}^D(\overline{m}(b)) > b$ and $\overline{w}^D(\overline{m}(b)+1) < b$. By use of Equation 2.2, the critical product-market structure $\overline{m}(b)$ is derived. The two results are formalized in Proposition 2.

Proposition 2. *(i) Let $m' > m$ and let b and θ be such that excess labor is hired in the two corresponding labor-market equilibria, $(\mathbf{n}^{D*}, \mathbf{w}^{D*})$ and $(\mathbf{n}^{D*\prime}, \mathbf{w}^{D*\prime})$. Then, $n^{D*} - q_i^e(\mathbf{n}^{d*}) > n^{D*\prime} - q_i^e(\mathbf{n}^{d*\prime})$ for all i and $\sum_{i=1}^{m}\left(n^{D*} - q_i^e(\mathbf{n}^{d*})\right) \geq \sum_{i=1}^{m'}\left(n^{D*\prime} - q_i^e(\mathbf{n}^{d*\prime})\right)$.*
(ii) The critical product-market structure $\overline{m}(b)$ is given by

$$\overline{m}(b) = \begin{cases} +\infty & \text{if } b = 0 \\ \max\left\{x \geq 0 \mid x < \frac{1-2b}{2b} \text{ is an intiger}\right\} & \text{if } b > 0. \end{cases}$$

Proof. See the Appendix.

Thus, if the unemployment benefit is small enough then labor may be excessively hired even if the number of firms is large. For example, let $b = 0.02$, then the largest number of firms consistent with excess labor is 24. The key to Proposition 2(ii) is that workers may be infinitely risk averse. The equilibrium wage w^{*D} is decreasing in θ and approaching b as θ goes to infinity. Hence, if $b < \overline{w}^D(m)$ then there exists a well-defined value of θ such that the equilibrium wage is lower than the critical wage only if θ exceeds this value. We then say that the product-market structure is consistent with excess labor at unemployment benefit level b and $\overline{m}(b)$ is the most competitive product-market structure that is consistent with excess labor at b.

3. One-Sided Centralization

In the case of one-sided centralization, the firm-specific unions join together into a central union that simultaneously and separately bargains with the m firms. As in the decentralized setting it is common knowledge that the union and the firms all hold the belief $\mathbf{n}^e$. Let $\mathbf{w}^e = (w_1^e, ..., w_m^e)$ be the central union's belief over future wages. The union's objective is to maximize the sum of expected utilities of its members

$$V(\mathbf{n}, \mathbf{w}) = \sum_{i=1}^{m} v(n_i, w_i).$$

Workers have no mobility and every labor contract is assumed to be private information to the union and the firm in question. A change in the centralized union's contract with firm i does therefore not induce a change in any of the $m-1$ other bargaining problems. The union consequently treats the m negotiations as separate and its objective coincides with the objective of the firm-specific union in every bargaining. The utility gain of the union from an agreement with firm i is

$$V(\mathbf{n}_{-i}^e, n_i, \mathbf{w}_{-i}^e, w_i) - V(\mathbf{n}_{-i}^e, \frac{1}{m}, \mathbf{w}_{-i}^e, b) = v(n_i, w_i)$$

where $\mathbf{w}_{-i}^e = (w_1^e, ..., w_{i-1}^e, w_{i+1}^e, ..., w_m^e)$. Hence, the set of feasible labor contracts between firm i and the central union is

$$B_i^S(\mathbf{n}^e) = \{(n_i, w_i) \in [0, 1/m] \times [0, 1] \mid v(n_i, w_i), \pi_i^e(\mathbf{n}^e, n_i, w_i) \geq 0\} = B_i^D(\mathbf{n}^e).$$

Because the sets $B_i^S(\mathbf{n}^e)$ and $B_i^D(\mathbf{n}^e)$ are identical, given $\mathbf{n}^e$, every bargaining outcome in the one-sided centralized setting is identical to the corresponding outcome in the decentralized setting. Let $(\mathbf{n}^{S*}, \mathbf{w}^{S*})$ denote the equilibrium to the one-sided centralized labor market.

Proposition 3. $(\mathbf{n}^{S*}, \mathbf{w}^{S*}) = (\mathbf{n}^{D*}, \mathbf{w}^{D*})$ *for all* θ, b, *and* m.

Proof. See the Appendix.

From Proposition 3 it follows that the results arrived to in Section 2.2 carry over to the case of one-sided centralization.

Corollary 2. *Propositions 1 and 2 and Lemma 1 apply to case of one-sided centralization.*

Proof. See the Appendix.

4. The Centralized Labor-Market

In the centralized labor market the central union negotiates over wages and employment with an employers' association formed by the m firms. The employers' association maximizes total profits conditional on subsequent non-collusion in the product market. We have required the bargaining outcome to be symmetric, i.e., the central labor contract is required to stipulate the same wage and employment in all firms, $\mathbf{n} = (n, ..., n)$ and $\mathbf{w} = (w, ..., w)$. The symmetry of the central labor contract is common knowledge and every firm thus knows the employment of its competitors. Therefore, $\mathbf{n}^e = \mathbf{n}$ and the set of feasible contracts is

$$B^C = \{(n, w) \in [0, 1/m] \times [0, 1] \mid v(n, w), \pi^e(\mathbf{n}, w) \geq 0\}$$

where $\pi^e(n, w) = \pi_i^e(\mathbf{n}, w)$ for all i. The Nash bargaining solution is

$$(n^{C*}, w^{C*}) = \arg \max_{(n,w) \in B^C} (v(n, w) - \underline{v})(\pi^e(\mathbf{n}, w) - \underline{\pi}). \tag{4.1}$$

The bargaining outcome (n^{C*}, w^{C*}) where $w^{C*} = \omega^C(b, \theta)$ and $n^{C*} = \eta^C(b, \theta, m)$ is automatically a labor-market equilibrium since expectations are rational by assumption. However, for some values of b and θ the functions ω^C and η^C are not continuous if $m \geq 2$. Intuitively, the discontinuities arises because the employers' association internalizes the effect of employment on total profits and the assumption of the individual firm's free disposal of hired labor.

In the product market the individual firm produces until either the marginal revenue is zero or the capacity restriction is binding. The employers' association, on the other hand, aims to maximize total profits and it rationally anticipates the product-market equilibrium that follows from the central labor contract. The joint profit is maximized when the firms act as a monopolist but the firms cannot collude in the product market. Suppose that n is such that total employment is between the monopoly output and the Cournot output at zero marginal cost, i.e. $n \in (1/2m, 1/(m+1))$. Then an increase in employment induces an increase in production which in turn lowers the (negative) marginal joint revenue from employment. But if $n > 1/(m+1)$ then the production is unaffected by small changes in employment because of firms' free disposal of labor and the joint revenue from employment is zero. In both cases the marginal joint cost of employment is $-wm$. Hence, the marginal effect of employment on joint profits is discontinuous at $n = 1/(m+1)$.

The central labor contract is Pareto efficient and the marginal rate of substitution between wage and employment is the same for the union as for the employers' association. As explained above, the marginal effect of employment on joint profits is discontinuous and this causes a discontinuous shift in the central labor contract at some parameter values. This problem doesn't arise in the decentralized- and one-sided centralized labor markets because the individual firm considers its environment as fixed when bargaining in a decentralized labor market and the marginal effect of employment on profits is continuous. The first-order conditions to Equations 2.1 and 4.1 gives Proposition 4.

Proposition 4. *Let m, θ and b be given. Then:*
(i) If $n^{C*}, n^{D*} \leq 1/(m+1)$, then $w^{C*} > w^{D*}$ and $n^{C*} < n^{D*}$.
(ii) If $n^{C*}, n^{D*} > 1/(m+1)$, then $w^{C*} = w^{D*}$ and $n^{C*} = n^{D*}$.

Proof. See the Appendix.

Proposition 4(i) states that if no excess labor is hired in any of the two settings, then wages are higher and employment is lower in the centralized setting than in the decentralized. The intuition behind this result is that the marginal effect of employment on profits faced by the employers' association is higher than the corresponding effect faced by the individual firm. The employers' association is therefore more reluctant than the individual firm to insure the workers against income uncertainty by means of employment. If, on the other hand, excess labor is hired in both settings then the marginal effect of employment is the same in the two settings. Hence the identical wage and employment.

5. An Example

Consider the case of a duopoly, $m = 2$, and let the unemployment benefit be $b = 0.05$. Suppose the wage bargaining is decentralized or one-sided centralized. Then the equilibrium wage must be less than $\overline{w}^D(2) = 1/6$ for excess labor to be hired. The wage w^{D*} as a function of θ is shown in figure 5.1.
For the chosen values of b and m excess labor is hired if $\theta > 5.79$. The highest number of firms that is consistent with excess labor is $\overline{m}(0.05) = 8$.

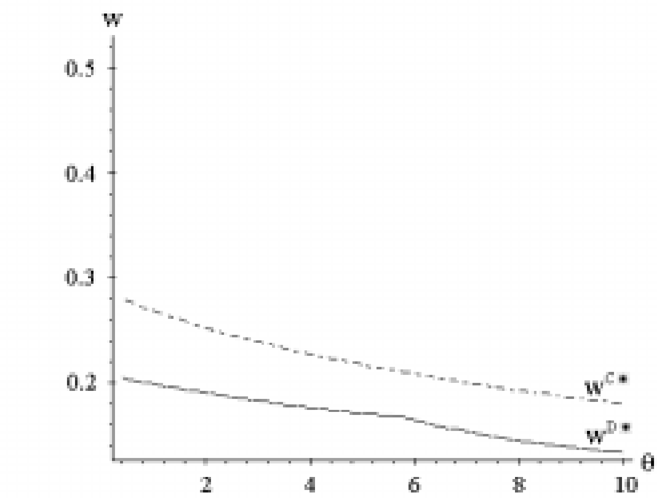

Figure 5.1: The wages w^{C*} (dashed) and w^{D*} as functions of θ for $b = 0.05$ and $m = 2$.

Consider the centralized labor market. The central labor contract specifies the same wage and employment in every firm by assumption and expectations are automatically rational, $\mathbf{n}^e = \mathbf{n}$. For the chosen parameter values, excess labor is not hired in the centralized setting for any θ. The wage w^{C*} as a function of θ is shown as the dashed line in Figure 5.1.

6. Discussion

One important assumption in the decentralized and one-sided centralized settings is that labor contracts are private information. This eliminates the strategic interaction among firms (and unions) at the labor-market stage. If labor contracts were not private information but announced before the making of the production decisions, then the labor contract between a firm and a union would no longer change continuously for all parameter values as the other firms marginally revise their contracts. Therefore it may not always exist a labor-market equilibrium. However, the qualitative results of the private-information setting carry over in those cases an equilibrium exists. The simplicity of the private-information framework is therefore an attractive property. Furthermore, hiring of excess labor is more easily obtained in the decentralized- and one-sided centralized settings if labor contracts are public information than if not. The reason is that the (negative) marginal effect of employment on the firm's profit may decrease when labor contracts are made public; increased employment in one firm makes that firm more aggressive and for some employment levels the best response of every other firm is to produce less. The expanding firm may consequently not have to bear the full costs of its action and this increases its willingness to employ. A union may consequently have an incentive to make labor contracts public information and so may the individual firm.

An other strong assumption is that of no labor mobility between firms. This is a simplifying assumption and allowing for labor mobility is not likely to eliminate the possibility of excess labor in equilibrium because of the symmetry of the model, but it may allow for multiple equilibria. In the production function there is no capital. If we allow for capital then excess labor may still exist if there is some minimal capital requirement or some non-substitutionalibility between labor and capital.

The perhaps most important aspect abstracted from in the model is the union's internal democratic decision process. Two possible conflicts are thereby ruled out; the conflict in interests between the leaders and the members and the conflict in interests between different groups of members. Such extensions may substantially change the results arrived to in this study. As an example, making the model dynamic and letting the union in time t represent workers employed in $t-1$ as in the "insider-outsider theory" (Lindbeck and Snower (1989)) is likely to eliminate employment of excess labor.

Appendix: Proofs

Proposition 1. This proof is carried out in two steps. First, the existence of a symmetric equilibrium is shown and secondly it is shown that this equilibrium is unique. Throughout the proof only $\mathbf{n}^D$ is considered and this is possible because the uniqueness of the labor contracts; w_i^D can thus be viewed as a function of n_i^D.

Step 1: In equilibrium $\mathbf{n}^e = \mathbf{n}$ and each labor contract $\eta_i^D(b,\theta,\mathbf{n})$ is a "best response" by firm-union pair i to $\mathbf{n}_{-i}$. Let $\mathfrak{N}(\mathbf{n}) = \times_{i=1}^m \eta_i^D(b,\theta,\mathbf{n})$. From the uniqueness and continuity of the Nash bargaining solution it follows that $\mathfrak{N}$ is a continuous function mapping $[0,1/m]^m$ into itself. Consider the compact and convex subset of $[0,1/m]^m$

$$D = \{(d,...,d) \in [0,1/m]^m \mid 0 \leq d \leq 1/m\}.$$

The correspondence $\mathfrak{N}$ maps D into D and hence has a fixed point $\mathbf{n}^D$ in D by Kakutani's fixed point theorem.

Step 2: Here it is first shown that if $\mathbf{n}^D, \mathbf{n}' \in [0,1/m]^m$ solves $\mathfrak{N}(\mathbf{n}) = \mathbf{n}$ and $\mathbf{n}^D$ is the symmetric equilibrium from step 1, then must $n_i', n_j' \neq n^D$ for some $i \neq j$. This is then used to show a contradiction.

Let $\boldsymbol{\varepsilon} = (\varepsilon_1, ..., \varepsilon_m) = \mathbf{n}' - \mathbf{n}^D$ and suppose $\mathbf{q}^e(\mathbf{n}^D) = \mathbf{q}^e(\mathbf{n}')$. Then the bargaining problem between firm and union i is the same in both equilibria. Hence, $\eta_i^D(b, \theta, \mathbf{n}') = \eta_i^D(b, \theta, \mathbf{n}^D)$ for all i and $\mathbf{n}' = \mathbf{n}^D$. It follows that if $\mathbf{n}' \neq \mathbf{n}^D$ then $q_i^e(\mathbf{n}^D) < q_i^e(\mathbf{n}')$ and $q_j^e(\mathbf{n}^D) > q_j^e(\mathbf{n}')$ for some $i \neq j$. From the relevant first-order conditions to the Nash bargaining solution (Equations 6.1 and 6.2) it follows that then is $\varepsilon_i > 0$ and $\varepsilon_j < 0$. Moreover, it also follows that $0 > dn_i/dn_j > -1$ along η_i when $n_i \in (0, 1/m)$ and $dq_j^e/dn_j \neq 0$. Then, the following must be true

$$\begin{array}{l} \varepsilon_i < -(\sum_{k \neq i,j} \varepsilon_k + \varepsilon_j) \\ \varepsilon_j > -(\sum_{k \neq i,j} \varepsilon_k + \varepsilon_i). \end{array}$$

The system can be rewritten

$$\begin{array}{l} \varepsilon_i < -(\sum_{k \neq i,j} \varepsilon_k + \varepsilon_j) \\ \varepsilon_i > -(\sum_{k \neq i,j} \varepsilon_k + \varepsilon_j). \end{array}$$

which is a contradiction. Hence, $\mathbf{n}^D$ is the unique equilibrium. ■

Lemma 1. If $n_i^D = q^o(n_i^D, \mathbf{q}_{-i}^e(\mathbf{n}^e))$ then (n_i^D, w_i^D) must satisfy the system of first-order conditions to the Nash bargaining solution

$$n_i^D - \frac{2(1 - w_i^D - \sum_{j \neq i} q_j^e(\mathbf{n}^e))}{3} = 0 \tag{6.1}$$

$$\frac{1 - w_i^D - \sum_{j \neq i} q_j^e(\mathbf{n}^e)}{3} - \frac{e^{-\theta(b - w_i^D)} - 1}{\theta} = 0 \tag{6.2}$$

and if $\frac{1}{m} > n_i^D > q^o(n_i^D, \mathbf{q}_{-i}^e(\mathbf{n}^e))$ then it must satisfy the system

$$n_i^D - \frac{(1 - \sum_{j \neq i} q_j^e(\mathbf{n}^e))^2}{8w} = 0 \tag{6.3}$$

$$w_i^D - \frac{e^{-\theta(b - w_i^D)} - 1}{\theta} = 0. \tag{6.4}$$

If $n = \frac{1}{m}$ then w_i^D must satisfy

$$\frac{(1 - \sum_{j \neq i} q_j^e(\mathbf{n}^e))^2}{4} - \frac{w}{m} - \frac{e^{-\theta(b - w_i^D)} - 1}{\theta} = 0. \tag{6.5}$$

When $w_i^D = (1 - \sum_{j \neq i} q_j)/4$ is $n_i^D = (1 - \sum_{j \neq i} q_j)/2$ which is the highest level of employment in firm i such that the firm doesn't hire excessive labor. At this point Equations 6.1 and 6.2 coincide with Equations 6.3 and 6.4. If

$w_i^D = m(1-\sum_{j\neq i} q_j^e(\mathbf{n}^e))^2/8$ then is $n_i^D = 1/m$ and Equations 6.3 and 6.4 coincide with Equation 6.5. Hence, the employment and the wage are continuous functions of the parameters of the model.

Using that the unique equilibrium is symmetric gives that Equations 6.1 and 6.2 coincides with Equations 6.3 and 6.4 when $n^{D*} = 1/(m+1)$ and $w^{D*} = (1-(m-1)/(m+1))/4 = 1/2(m+1)$. ■

Proposition 2. (i) This proof is carried out in two steps. From Equations 6.3 and 6.5 in the proof of Lemma 1 we have that $n^{D*} = 1/m$ iff $w^{D*} \leq \widetilde{w}(m) = \frac{m}{2(1+m)^2}$. Let $(\mathbf{n}^{D*}, \mathbf{w}^{D*})$ and $(\mathbf{n}^{D*\prime}, \mathbf{w}^{D*\prime})$ be the two labor-market equilibria associated with (θ, b, m) and (θ, b, m'), respectively. Let $n^{D*} \geq 1/(m+1)$ and let $n^{D*\prime} \geq 1/(m'+1)$.

Step 1: Here we show by contradiction that if $w^{D*\prime} \leq \widetilde{w}(m')$ then $w^{D*} \leq \widetilde{w}(m)$. Suppose the contrary, that $w^{D*} > \widetilde{w}(m)$. Then (w^{D*}, n^{D*}) solves the Equations 6.3 and 6.4. By assumption is $n^{D*\prime} = 1/m'$ and $w^{D*\prime}$ solves Equation 6.5. We will now show that $(\mathbf{n}^{D*\prime}, \mathbf{w}^{D*\prime})$ cannot be a labor-market equilibrium, i.e. $(w^{D*\prime}, n^{D*\prime})$ isn't the Nash bargaining solution of firm and union i. Let $\gamma = w^{D*\prime}n^{D*\prime}$ be total wages paid to the workers. Suppose the union freely could decide on n_i and w_i and distribute γ among its working members subject to $n_i \geq 1/(m'+1)$ which keeps the firm's profit constant at the current level. The union will set $w = \gamma/n$ and its maximization problem is

$$\max_{w\in[\gamma m', \gamma(m'+1)]} v(\gamma/w, w). \tag{6.6}$$

For the moment, consider the unrestricted maximization problem which first-order condition is given by Equation 6.4. w^{D*} is the solution to the unrestricted problem and $v(\gamma/w, w)$ is increasing in w for all $w < w^{D*}$. Equations 6.3 and 6.5 give $\gamma < w^{D*}/m'$ which is equivalent to $n < 1/m'$. Hence, if $\gamma \geq w^{D*}/(m'+1)$ then is w^{D*} the unique solution to the restricted maximization problem. If $\gamma < w^{D*}/(m'+1)$ then the corner solution $w = \gamma(m'+1)$ is the unique solution to the restricted maximization problem. Hence, $\left(w^{D*\prime}, n^{D*\prime}\right)$ don'ttt maximize the Nash product of firm and union i and $(\mathbf{n}^{D*\prime}, \mathbf{w}^{D*\prime})$ cannot be a labor-market equilibrium.

Step 2: Here we consider three different cases (the fourth is ruled out in step 1); (a) $w^{D*\prime} \leq \widetilde{w}(m')$ and $w^{D*} \leq \widetilde{w}(m)$, (b) $w^{D*\prime} > \widetilde{w}(m')$ and $w^{D*} \leq \widetilde{w}(m)$, and (c) $w^{D*\prime} > \widetilde{w}(m')$ and $w^{D*} > \widetilde{w}(m)$. In cases (a) and (b) Proposition 2(i) is trivially true since $n^{D*} = 1/m$, $n^{D*\prime} \leq 1/m'$, $q_i^e(\mathbf{n}^{D*}) = 1/(m+1)$, and $q_i^e(\mathbf{n}^{D*\prime}) = 1/(m'+1)$. In case (c) is $w^{D*} = w^{D*\prime}$, $n^{D*} < 1/m$ and $n^{D*\prime} < 1/m'$. By assumption is $m' > m$ and in equilibrium is

$$n^{D*} - q_i^e(\mathbf{n}^{D*}) \quad = \quad \frac{1}{m+1}\left(\frac{1}{2w^{D*}(m+1)} - 1\right) >$$

$$\frac{1}{m'+1}\left(\frac{1}{2w^{D*}(m'+1)}-1\right) = n^{D*\prime} - q_i^e(\mathbf{n}^{D*\prime})$$

and

$$\begin{aligned}\sum_{i=1}^{m}(n^{D*} - q_i^e(\mathbf{n}^{D*})) &= \frac{m}{m+1}\left(\frac{1}{2w^{D*}(m+1)}-1\right) > \\ \frac{m'}{m'+1}\left(\frac{1}{2w^{D*}(m'+1)}-1\right) &= \sum_{i=1}^{m'}(n^{D*\prime} - q_i^e(\mathbf{n}^{D*\prime})).\end{aligned}$$

(ii) First $\overline{m}(b)$ is derived and then it is shown that there exists a θ such that excess labor is hired at product-market structure $\overline{m}(b)$.
That $w_i^D > b$ for all $\theta > 0$ follows from the properties of the Nash bargaining solution. Excess labor at product-market structure m thus requires $b < \overline{w}^D(m) \Leftrightarrow m < (1-2b)/2b$. If $b > \overline{w}^D(1)$ then there exists no $m \geq 1$ such that $b < \overline{w}^D(m)$. Let $\overline{m}(b)$ be the largest integer m that satisfies $m < (1-2b)/2b$. If $b = 0$ then every integer satisfies the inequality and $\overline{m}(b) = +\infty$.
Let $b < \overline{w}^D(1)$ and let $m = \overline{m}(b)$. If there exists a θ such that $b < w^{D*} < \overline{w}^D(\overline{m}(b))$ then product-market structure $\overline{m}(b)$ is consistent with excess employment. Let $b < w^{D*} < \overline{w}^D(\overline{m}(b))$ and substitute w^{D*} into Equation 6.4. By the properties of the Equation 6.4 gives that a unique solution $\overline{\theta} > 0$ exists. Since $\overline{\theta}$ is such that $w^{D*} < \overline{w}^D(\overline{m}(b))$ there exists no solution to Equations 6.1 and 6.2 that maximizes the Nash product. Hence, $\overline{m}(b)$ is consistent with excess employment and by Lemma 1 so is every product-market structure $m < \overline{m}(b)$. ■

Proposition 3. Consider the expression $V - \underline{V}$. Simplifying gives

$$V(\mathbf{n}_{-i}^e, n, \mathbf{w}_{-i}^e, w) - V(\frac{1}{m}, \mathbf{n}_{-i}^e, b, \mathbf{w}_{-i}^e) = v(n, w) \tag{6.7}$$

which makes the bargaining problem of the firm and union identical to that of a decentralized labor market. Hence, $(n_i^S, w_i^S) = (n_i^D, w_i^D)$ for all θ, b, m, and i. ■

Corollary 1. Follows directly from the proof of Proposition 3. ■

Proposition 4. If $n^{C*} = q^o(n^{C*}, \mathbf{q}_{-i}^e(\mathbf{n}^{C*}))$ then (n^{C*}, w^{C*}) must satisfy the system of first-order conditions to the Nash bargaining solution

$$n^{C*} - \frac{2(1-w^{C*})}{3m} = 0 \tag{6.8}$$

$$\frac{1-w^{C*}}{3} - \frac{e^{-\theta(b-w^{C*})}-1}{\theta} = 0 \tag{6.9}$$

and if $\frac{1}{m} > n^{C*} > q^o(n^{C*}, \mathbf{q}^e_{-i}(\mathbf{n}^{C*}))$ then it must satisfy the system

$$n^{C*} - \frac{1}{2(m+1)^2 w^{C*}} = 0 \tag{6.10}$$

$$w^{C*} - \frac{e^{-\theta(b-w^{C*})}-1}{\theta} = 0. \tag{6.11}$$

When $n^{C*} = 1/m$ then w^{C*} must satisfy

$$\frac{m}{2(m+1)^2} - \frac{e^{-\theta(b-w^{C*})}-1}{\theta} = 0. \tag{6.12}$$

Suppose b, θ are such that $n_i^{D*}, n^{C*} \leq 1/(m+1)$. Compare Equation 6.9 with Equation 6.2 using that $(\mathbf{n}^{D*}, \mathbf{w}^{D*})$ is symmetric. From $\sum_{j\neq i} q_j^e > 0$ it follows that $w_i^{D*} < w^{C*}$. Using this inequality and Equations 6.8 and 6.1 gives $n_i^{D*} > n^{C*}$. Suppose b, θ are such that $n_i^{D*}, n^{C*} > 1/(m+1)$. The Equations 6.4 and 6.11 are identical and so are Equations 6.5 and 6.12 when using the symmetry of $(\mathbf{n}^{D*}, \mathbf{w}^{D*})$. Hence, $w_i^{D*} = w^{C*}$ and $n_i^{D*} = n^{C*}$. ■

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