Bundled Sales As Self-Selection Devise

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Abstract

This paper develops an adverse selection model of mixed bundling. By packaging its product with a competitively produced good unrelated in demand, a monopolist can induce self-selection of different types of consumers into buyers of the bundle and of the separate components. Private and social optimality conditions of bundling are derived. The effect on prices and welfare depends on the demand elasticities for the bundled and unbundled good. By overcoming information asymmetries, bundling may raise welfare if it leads to a sufficiently strong expansion of the market. It is possible that prices to all buyers rise; in this case welfare definitely falls if the bundle is introduced.

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1 Introduction

Commodity bundling is a pervasive vertical restraint in actual business practice, given that most goods are in fact at least partially separable bundle of attributes. Classic examples are travel packages, set dinners and season tickets. Less conspicuously, bundling occurs in club memberships, blanket servicing contracts, credit cards, student banking packages and software, although the rationale for bundling differs in these examples.

Can a firm with market power in the market for its own product gain by offering this product both by itself, and bundled with a fixed quantity of a competitive good? This is the case in the Japanese casualty insurance industry described in Wallner ([14]). The industry, operating with substantial and persistent profit margins, offers insurance policies both as a stand-alone product and as a bundle including a time deposit of some fixed amount. Regulation by the Ministry of Finance provides that the market for CD's as separate products is the exclusive domain of the competitive banking sector. As a result of significantly lower insurane claims from savings-type buyers, the companies realize substantial profits on the insurance portion of the bundle. In effect mixed bundling splits the pool of consumers, heterogeneous in their accident probabilities, into different risk classes and attracts the safer buyers into the bundle by implicitly offering lower insurance premia. Higher risk consumers are prevented from buying the bundle if their valuation for the bundled component is sufficiently low.

This essay endogenizes the amount of the competitive good offered with the monopoly good in a bundle. By focusing on the packaging decision, the role of bundling as self-selection device is emphasized. The optimal amount

of the competitive good in the bundle serves to draw the dividing line between the pools of separate and bundle buyers. A larger amount of the bundled good makes it more inconvenient for previous buyers of the monopoly good to take the bundle instead. This tends to preserve the pool of previous buyers and allows to extract a high profit margin via separate pricing. But it also deters potential new buyers, since a lower implicit price of the monopoly good in the bundle may not suffice to compensate for having to buy a larger than desired amount of the bundled good.

The prices of the bundled and unbundled good may be either higher or lower than the stand-alone monopoly level; in the former case welfare falls, while in the latter society benefits from bundling. The menu offered to buyers induces self-selection according to the intensity of their preferences. The monopolist can thereby partially overcome the information asymmetry of being unable to observe preference intensities directly. At the same time its monopoly power increases. Depending on which effect is stronger, society may gain or lose. The analysis carries implications for antitrust policy since it frames private and social optimality conditions of bundling in terms of precise conditions on the elasticities of demand for the bundle and the unbundled.

In contrast to the literature, the analysis is conducted in terms of the demand elasticities. The focus on the correlation of reservation prices among the population of buyers has limited previous research to stating results in terms of simple examples or simulations for a range of distribution functions and to graphical arguments.[1] While the elasticities in turn are derived from

[1]See, for instance, the papers by Stigler [12], Adams and Yellen [1] and Schmalensee [11].

the underlying primitive distribution of preference parameters, the focus on the local concept of elasticities allows for more sharpness and better understanding of the conditions of private and social optimality of bundling.

The paper is organized as follows. The next section reviews the relevant theoretical literature. Section 3 derives the demands for the monopolist's product sold separately and included in the bundle. The bundling decision is analyzed in section 4. Section 5 discusses welfare implications of bundling and concludes.

2 The Related Literature

The literature distinguishes between tying (the buyer decides in what proportion she consumes the two goods), as well as pure (only the bundle, but not the components individually are offered) and mixed bundling (both the bundle and the separate components are offered).[2]

It is well known that complementarity in demand between two goods can be exploited by a multi-product monopolist through bundling, making high-intensity users pay more through a positive profit margin on the bundled good.[3] This has become known as a metering device, allowing effective price discrimination among heterogeneous consumers if demand for the two goods is positively related.

[2] There is also an extensive literature on quantity-dependent pricing based on efficiency explanations, such as economies of scope in joint production (and marketing) of the goods, lower transaction and search costs for consumers, and efficient risk-bearing arrangements.

[3] See Telser [13] for a formal model and [2] for an application of this idea to the tying of punch cards to tabulating machines by IBM.

Subsequent work concentrates on incentives to bundle goods that are independent in demand. Stigler [12] demonstrates that a monopolist selling two such goods can gain by engaging in pure bundling, since the sum of the reservation prices for the two bundled components has a lower variance than the components individually. While Stigler suggests that a negative relationship in the distribution of the reservation prices is required for profitable bundling, Carbajo et al. [4] point out that the result applies to all distributions such that the reservation prices are not perfectly positively correlated. Further work by Adams and Yellen [1], McAfee et al. [6] and Schmalensee [11] examines the impact of distributional aspects of the reservation prices for the two components on the incentive to bundle. The upshot from the different models is that bundling can be profitable under any correlation between the values, suggesting that exclusive attention on correlation measures may be misleading.

Adams and Yellen [1] argue that with two monopoly products the mixed bundling strategy dominates pure bundling (it can always replicate the pure bundling outcome by setting the individual prices high enough), but it may be inferior to unbundled sales.[4] The bundle may induce buyers to acquire a good as part of the bundle which they would not want to buy at cost separately, reducing the ability to extract a higher margin on the other good in the bundle. On the other hand, with unbundled sales many consumers get priced out of the market in the attempt to extract surplus of the users

[4]This holds under the assumption that the bundle offers a discount relative to the individual components, since otherwise mixed bundling can trivially replicate, and hence weakly dominates, unbundled sales.

with high reservation prices. This tradeoff determines which pricing strategy is preferred.

Schmalensee [10] demonstrates that negatively correlated reservation prices can be a profitable motive for a single-product monopolist to bundle. In his model pure bundling is never more profitable than unbundled sales. Mixed bundling, however, by grouping buyers into different categories, makes price discrimination possible. The monopolist can extract more from buyers with low reservation prices for the bundled good through the separate sale of the monopoly product; these buyers might not have bought the bundle. Mixed bundling then allows to raise the price on the monopoly good sold alone, with the bundle capturing some of those buyers who get priced out of separate sales in the process.

The old antitrust question whether bundling can serve to extend monopoly power from one market to another has been sidelined since the forceful critiques mainly from the Chicago school (Posner [9], Bork [2]). Carbajo et al [4] and Whinston [15] revisit this 'leverage' debate. In their models, if the tied market is imperfectly competitive, bundling can indeed create market power by affecting the strategic behavior or entry/exit decision of rivals in that market. In the case of Cournot competition, bundling may commit the firm to a higher output than with unbundled sales, since the only way to realize the profit margin on the monopoly good is to sell a large number of bundles, i.e. get more aggressive in the tied market. Under Bertrand competition, bundling serves to differentiate the firm's product from that of the rival, which in turn relaxes competitiveness in the tied market and increases both firms' profits.

A recent paper by Mathewson/Winter [8] shows that bundling may actually enable a single-product monopolist to raise its price in a competitive market above marginal cost by tying it to the monopoly good. The monopolist withholds residual consumer surplus from buyers of its product by refusing to sell it unless they also buy the tied good at elevated prices. Doing this the monopolist can enhance its profitability by selecting Ramsey prices on both goods. Allowing for two-part pricing, tying is superior in a stochastic environment, if the demands are positively related. This paper thus combines elements of a leverage theory of bundling with a price discrimination rationale.

What are the lessons from this literature? The profitability of bundling under the various conditions derives either from price discrimination among heterogeneous consumers or from influence over the strategic conduct of a rival in the tied market. Conditions under which bundling is profitable are expressed in terms of the distribution of reservation prices, and typically supported by hand-selected examples. Few strong general results obtain, because many distributions of reservation prices are compatible with any given correlation measure, and for different particular distributions bundling may be profitable or not.

This paper argues that the joint distribution of reservation prices in the population of buyers is an incomplete statistic to tie profitability conditions of bundling to. It is the 'local' property of this distribution that alone matters. In the absence of an appropriate statistic of local correlation, the conditions should be framed in terms of the localized concepts of demand elasticities for the bundled and unbundled components.

Abstracting from all leverage arguments, the paper develops a model of bundling as a self-selection device. The endogenously determined amount of the bundled competitive good serves to prevent some buyers of the monopoly product from switching into the bundle. Mixed bundling is profitable if sufficiently more previous buyers of the stand-alone product relative to potential new buyers are thus precluded from the bundle.

3 The Demand for the Monopolist's Good

Let the preferences of a consumer described by (α, β) be represented by

$$U(x, y, \alpha, \beta) = \alpha x + \beta u(y) + e \text{ where } u(0) = 0, u'(y) > 0 \text{ and } u"(y) < 0,$$

and where $x \in \{0, 1\}$ is the indivisible level of the monopoly good, y is the divisible level of the competitive good, α and β are the individual-specific taste parameters associated with goods x and y respectively, and e is expenditure on other goods. The quasi-linear form is chosen to purge the analysis of all income-distributional effects. I assume that consumers are not financially constrained in buying either x or y, i.e. that the utility-maximizing choice contains a positive level of e. This requires the marginal utility of one dollar spent on y to fall below unity before the budget is exhausted, regardless of whether x is purchased or not.[5] The taste parameters are private knowledge of the consumer and are distributed according to some cumulative distribution function $F(\alpha, \beta)$ with a continuous probability density function $f(\alpha, \beta)$

[5] This applies to goods that do not make up the lion's share of the consumer's budget. An example is insurance, which is typically very small relative to the disposable income of the buyer.

on the unit square. Consumers cannot trade the good among themselves.

Let p_x denote the price of good x sold in unbundled form and let c_y denote the price of the competitive good sold by the competitive market at marginal cost. Let p_b denote the price of a bundle consisting of one unit of x and k units of the competitive good. Since a consumer has the option of buying the goods separately, a necessary condition for her to buy the bundle is that the price of the bundle is less than the sum of the prices of the goods sold separately, i.e.

$$p_b < p_x + kc_y. \tag{1}$$

Let $p_{\tilde{x}}$ be the imputed price of good x in bundled form. Then $p_{\tilde{x}} = p_b - kc_y$, which implies that (1) can be rewritten as $p_{\tilde{x}} < p_x$. In words, a necessary condition for a consumer to buy a bundle is that the imputed price of the good x in the bundle must be less than its price unbundled.

Now define $y(\beta, c_y)$ as the consumers most preferred level of good y when faced with the competitive price equal to marginal cost c_y, i.e.

$$y(\beta, c_y) = \arg\max_y \{\beta u(y) - c_y y\}.$$

Since β is the taste parameter associated with good y it will be the case that

$$\frac{\partial y(\beta, c_y)}{\partial \beta} > 0.$$

Let I denote the income of any given consumer. If she consumes zero units of good x and $y(\beta, c_y)$ units of y from the competitive market then their utility level is given by U_0, where

$$U_0 \equiv \beta u\{y(\beta, c_y)\} + I - c_y y(\beta, c_y). \tag{2}$$

If the consumer buys $y(\beta, c_y)$ units from the competitive market and one unit of x from the monopolist at a price of p_x her utility level is given by U_1, where

$$U_1 \equiv \alpha + \beta u\{y(\beta, c_y)\} + I - c_y y(\beta, c_y) - p_x. \tag{3}$$

If the consumer buys $y(\beta, c_y) - k > 0$ units of good y at price c_y from the competitive market and a bundle containing one unit of x and k units of y from the monopolist at a bundled price of p_b her utility is given by U_2, where

$$U_2 \equiv \alpha + \beta u\{y(\beta, c_y)\} + I - c_y\{y(\beta, c_y) - k\} - p_b. \tag{4}$$

If the consumer buys zery units of y from the competitive market and a bundle from the monopolist at a bundle price of p_b then her utility level is given by U_3, where

$$U_3 \equiv \alpha + \beta u(k) + I - p_b. \tag{5}$$

It will prove useful to work with the imputed price of good x in bundled form rather than the price of the bundle p_b. If we substitute $p_b = p_{\hat{x}} + kc_y$ into (4) and (5) we obtain

$$U_2 \equiv \alpha + \beta u\{y(\beta, c_y)\} + I - c_y\{y(\beta, c_y)\} - p_{\hat{x}} \tag{6}$$

and

$$U_3 \equiv \alpha + \beta u(k) + I - p_{\hat{x}} - kc_y. \tag{7}$$

Now define $\Delta CS_y(\beta, k, c_y)$ as the reduction in consumer surplus from good y as the consumption of good y increases from y_p (the most preferred level) to k (the amount contained in the bundle), given that the price of good y remains constant at c_y, i.e.

$$\Delta CS_y(\beta, k, c_y) \equiv [\beta u(k) - c_y k] - [\beta u\{y(\beta, c_y)\} - c_y y(\beta, c_y)] < 0. \tag{8}$$

If $y(\beta, c_y) < k$ and given that $y(\beta, c_y)$ is increasing in β then $\Delta CS_y(\beta, k, c_y)$ will also be increasing in β, i.e.

$$\text{if } y(\beta, c_y) < k \text{ then } \frac{\partial \Delta CS_y(\beta, k, c_y)}{\partial \beta} > 0. \tag{9}$$

In other words if y_p is less than the amount contained in the bundle then an increase in the consumer's preference for good y will close the gap between y_p and the quantity of y contained in the bundle. As a result the change in consumer surplus associated with the consumer having to increase her consumption from y_p to k at an imputed price of c_y per unit of y will be *less negative*.[6]

In order for the consumer to buy good x in unbundled form it must be the case that she receives a higher utility from doing so than from buying no x at all (i.e. $U_1 > U_0$) which implies (see (2) and (3))

$$\alpha > p_x, \tag{10}$$

i.e. that the reservation price of x must exceed the price of x in unbundled form.

Secondly a consumer buying good x separately must receive a higher utility from doing so than from buying the bundle. If $y(\beta, c_y) < k$ then this implies the requirement that $U_1 > U_3$ which in turn implies

$$p_x - p_{\tilde{x}} + \Delta CS_y(\beta, k, c_y) < 0 \text{ where } y(\beta, c_y) < k \tag{11}$$

[6] For example, if the buyer consumes either one or zero units of good y as in Schmalensee [10] then $y(\beta, c_y) < k$ implies $y(\beta, c_y) = 0$ and $k = 1$. If $u(1) = 1$ and given that $u(0) = 0$ then this implies that $\Delta CS(\beta, k, c_y) = \beta - c_y$ which is clearly increasing in β.

i.e. that the imputed cost saving from buying good x in bundled form, $p_x - p_{\hat{x}}$, does not compensate for the reduction in consumer surplus $\Delta CS_y(\beta, k, c_y)$ from buying more than y_p.

If $y(\beta, c_y) < k$ then the requirement that a consumer receive a higher utility from good x sold separately than from the bundle can be stated as $U_1 > U_2$ which implies $p_x < p_{\hat{x}}$. This in turn implies that the monopolist will never make any bundled sales (see the discussion preceding (1). As a result the conditions under which a consumer will buy the good separately given that the monopolist does make some bundled sales are given by (10) and (11). Hence the consumers who buy good x in unbundled form are those with a high α [7] and a low β.[8]

It will be useful to distinguish between those consumers whose most preferred level of good y is greater than the amount of y contained in the bundle (i.e. $(y_p > k)$) from those whose most preferred level of y is less than k (i.e. $(y_p < k)$).

If $y(\beta, c_y) > k$ and provided $p_b < p_x + kc_y$ (i.e. consuming the bundle is cheaper than buying the good separately) then a consumer will buy the bundle provided she gets greater utility from doing so than if they consume no x at all (i.e. $U_2 > U_0$) which implies

$$\alpha > p_{\hat{x}}, \tag{12}$$

i.e. the consumer's reservation price for good x exceeds the imputed price of good x in the bundle.

[7] This follows from (10).

[8] This follows from (9) and (11).

If $y(\beta, c_y) > k$ then the consumer will buy the bundle provided she receives a higher utility from doing so than from not buying good x at all (i.e. $U_3 > U_0$) which implies

$$\alpha - p_{\hat{x}} + \Delta CS(\beta, k, c_y) > 0, \tag{13}$$

i.e. the imputed consumer surplus from buying good x in bundled form, $\alpha - p_{\hat{x}}$, more than compensates for the reduction in consumer surplus from y of buying more than the preferred level y_p, $\Delta CS(\beta, k, c_y)$. Furthermore a consumer buying the bundle must receive more utility from doing so than from buying good x separately (i.e. $U_3 > U_1$), which implies

$$p_x - p_{\hat{x}} + \Delta CS(\beta, k, c_y) > 0, \tag{14}$$

i.e. the opposite inequality to the one given in (11). The inequalities given in (12), (13) and (14) describe the set of of parameters (α, β) and for which the consumer buys the bundle.

Define $\beta^*(p_x, p_{\hat{x}}, k, c_y)$ as the level of β which satisfies (14) with equality, i.e. which solves $\Delta CS(\beta^*, k, c_y) = p_{\hat{x}} - p_x$. Similarly define $\beta^{**}(k, c_y)$ as the level of β which solves (13) with equality when $\alpha = p_{\hat{x}}$, i.e. which solves $\Delta CS(\beta^*, k, c_y) = 0$.

Then the demand for good x in unbundled form $x(p_x, p_{\hat{x}}, k, c_y)$ can be calculated by integrating over all α between p_x and 1, and over all β between 0 and $\beta^*(p_x, p_{\hat{x}}, k, c_y)$:

$$x(p_x, p_{\hat{x}}, k, c_y) = \int_{p_x}^{1} \int_{0}^{\beta^*(p_x, p_{\hat{x}}, k, c_y)} f(\alpha, \beta)\, d\alpha d\beta. \tag{15}$$

The demand for good x in bundled form $\hat{x}(p_x, p_{\hat{x}}, k, c_y)$ can be calculated by integrating over two sub-areas. The first sub-area involves integrating

over all α between $p_{\hat{x}}$ and 1 and over all β between $\beta^{**}(k, c_y)$ and 1. The second sub-area involves integrating over all α between $p_{\hat{x}} - \Delta CS(\beta, k, c_y)$ and 1 and over all β between $\beta^{*}(p_x, p_{\hat{x}}, k, c_y)$ and $\beta^{**}(k, c_y)$. As a result we obtain that the demand for x in bundled form is given by

$$\hat{x}(p_x, p_{\hat{x}}, k, c_y) = \int_{p_{\hat{x}}}^{1} \int_{\beta^{**}(k,c_y)}^{1} f(\alpha, \beta)\, d\alpha d\beta + \int_{p_{\hat{x}} - \Delta CS(\beta,k,c_y)}^{1} \int_{\beta^{*}(p_x,p_{\hat{x}},k,c_y)}^{\beta^{**}(k,c_y)} f(\alpha, \beta)\, d\alpha d\beta. \tag{16}$$

4 The Bundling Decision

4.1 The Optimal Bundle

Mixed bundling in a trivial sense weakly dominates both pure bundling and separate sales. The former is established by noting that by setting the individual prices arbitrarily high the monopolist can replicate (trivially) any pure bundling outcome. The latter is true because if the price of the bundle is set arbitrarily high no consumer bundles are sold. Only the case where a meaningful bundle option is offered, i.e. the bundle is sold cheaper than its individual components, is considered as mixed bundling in this paper.

When does the monopolist gain by bundling, and how does the optimal bundle look like? The bundle to consider is $B(x, y) = (1, k)$, $k \in \Re^1_+$, and its price is p_b. Production of good x is costless for simplicity. The cost of producing the bundle c_b offers no economies of joint production, so that $c_b = kc_y$. The monopolist knows the distribution $F(\alpha, \beta)$, but cannot observe (or equivalently, cannot condition the price on) the combination (α, β) of any particular consumer. Since consumers cannot trade the goods among

themselves, the price of good x can differ for bundle buyers and separate buyers. Let the monopoly price of x be p_x^m and the corresponding demand x^m. Define

$$\tilde{k}\,(p_x, p_{\hat{x}}) \equiv \arg\max_k \pi(p_x, p_{\hat{x}}, k). \tag{17}$$

Suppressing the parametric argument c_y in the demand and profit function, the optimization problem of the monopolist can then be written as:

$$\max_{p_x, p_{\hat{x}}, k} \pi(p_x, p_{\hat{x}}, \tilde{k}) = p_x\, x(p_x, p_{\hat{x}}, \tilde{k}) + p_{\hat{x}} \widehat{x}(p_x, p_{\hat{x}}, \tilde{k}). \tag{18}$$

The first order conditions are:

$$x + p_x \frac{dx}{dp_x} + p_{\hat{x}} \frac{d\hat{x}}{dp_x} = 0 \tag{19}$$

$$\widehat{x} + p_{\hat{x}}\, \frac{d\hat{x}}{dp_{\hat{x}}} + p_x \frac{dx}{dp_{\hat{x}}} = 0. \tag{20}$$

Denote the solution to program (18) $p_x^*, p_{\hat{x}}^*$ and k^*; accordingly $x(p_x^*, p_{\hat{x}}^*, k^*) \equiv x^*$ and $\widehat{x}(p_x^*, p_{\hat{x}}^*, k^*) \equiv \hat{x}^*$. Since bundling is a weakly dominant strategy, $\pi(p_x^*, p_{\hat{x}}^*, k^*) \geq \pi(p_x^m, p_x^m, 0)$. The following subsection investigates when it does strictly better that unbundled sales, i.e. when $p^* x^* + p_{\hat{x}}^* \hat{x} > p_x^m x^m$.

4.2 The Marginal Incentive To Bundle

To understand the circumstances under which mixed bundling strictly raises profits, it will be useful to examine the marginal incentive to bundle, i.e. the incentive to add a bundled option to an existing separate sales offer at p_x^m. In this case, the monopolist's profits can be written as

$$\pi(p_x^m, p_{\hat{x}}, k) = p_{\hat{x}} \widehat{x}(p_x^m, p_{\hat{x}}, k) + p_x^m x(p_x^m, p_{\hat{x}}, k), \tag{21}$$

and the only choice variables are $p_{\hat{x}}$ and k. The function $\pi(p_x^m, p_{\hat{x}}, k)$ can be represented by a three-dimensional hyperplane whose shape is not uniquely determined by the correlation between α and β, but rather depends on the precise details of the distribution of consumers in the (α, β) space. Hence the correlation of α and β derived from the joint distribution $F(\alpha, \beta)$ is not a sufficient statistic for determining the incentive to bundle. In general the second order conditions are not satisfied and hence little can be said about where in the $(\widehat{p}_x, k)$-space profits reach a global optimum.

The first order conditions associated with maximizing program (21) are

$$\widehat{x} + p_{\hat{x}} \frac{d\widehat{x}}{dp_{\hat{x}}} + p_x \frac{dx}{dp_{\hat{x}}} = 0 \tag{22}$$

and

$$p_{\hat{x}} \frac{d\widehat{x}}{dk} + p_x \frac{dx}{dk} = 0. \tag{23}$$

Conditions (22) and (23) show that the choice of $\widehat{p}_x$ and k balances the marginal cost in terms of losing the profit margin from the lower sales of $\widehat{x}$ against the marginal benefit of a positive margin from the incremental sales of x when the choice variables are raised.

Note that an implication of the general failure of the second order conditions is that at $p_x = p_x^m$ and $k = 0$, $\frac{\partial \pi}{\partial p_{\hat{x}}} < 0$ and $\frac{\partial \pi}{\partial k} > 0$ are sufficient but not necessary for a profit incentive to introduce the bundle. This means that even if it is not profitable to introduce a 'marginal' bundle, (i.e. k close to zero and $\widehat{p}_x$ close to p_x^m), further increases in k and/or decreases in $\widehat{p}_x$ may eventually yield higher profits than separate pricing.

Program (21) can be turned into a univariate optimization problem by

replacing k with $\overline{k}$, where

$$\begin{aligned}\overline{k}(\widehat{p}_x) &= \arg\max_k \pi(p_x^m, p_{\hat{x}}, k) \\ &= \arg\max_k \left[p_{\hat{x}}\widehat{x}(p_x^m, p_{\hat{x}}, k) + p_x^m x(p_x^m, p_{\hat{x}}, k)\right]. \end{aligned} \tag{24}$$

Bundling is then profitable on the margin if

$$\widehat{x} + p_{\hat{x}}\frac{d\widehat{x}}{dp_{\hat{x}}} + p_x^m\frac{dx}{dp_{\hat{x}}} < 0, \tag{25}$$

which can be written as

$$\frac{\widehat{x}}{x}\left(1 + \epsilon^{\widehat{x},p_{\hat{x}}}\right) + \frac{p_x^m}{p_{\hat{x}}}\epsilon^{x,p_{\hat{x}}} < 0 \tag{26}$$

where $\epsilon^{\widehat{x},p_{\hat{x}}} < 0$ and $\epsilon^{x,p_{\hat{x}}} > 0$ are the elasticities of demand for the bundled and the independently sold X with respect to $\widehat{p}_x$ given the optimal bundled amount $\overline{k}$. Hence bundling is more likely to be profitable on the margin (i.e. at $\widehat{p}_x = p_x^m$) the more elastic the demand for bundles, the less elastic the demand for individual units of x, the higher the proportion of bundles sold relative to individual units, and the lower the monopoly price. The optimal $\overline{k}$ as a function of $\widehat{p}_x$ serves to pick the most favorable combination of demand elasticities for the bundled and unbundled good x.

When the bundle is introduced with a marginal discount from the monopoly price p_x^m, two effects happen. Some former buyers of the unbundled product now switch into the bundle. These are buyers of x who would have bought k units of y anyway and therefore can, at zero opportunity cost, take advantage of the lower $\widehat{p}_x$ by buying the bundle. This is a *switching effect*, reducing profits since $\widehat{p}_x < p_x^m$ (otherwise no bundles are sold). In addition, new buyers decide to buy x through the bundle, since it is available at $p_{\hat{x}} <$

p_x^m. This is a *market expansion effect*, raising profits through a positive profit margin on additional customers. Hence the bundle is profitable if the market expansion effect is large relative to the switching effect.

The two instruments $p_{\hat{x}}$ and k control the relative magnitudes of the switching and market expansion effects. A lower price $p_{\hat{x}}$ generates a market expansion effect by enticing consumers into the bundle who were in the no-bundle situation priced out of the market. At the same time, k is set optimally so as to prevent as many switches as possible, constrained by the need not to keep out too many new buyers in the process. The bundling requirement k is thus screening the population of consumers for those who have a high valuation for y and a low one for x, and grouping them into the bundle category.

Condition (26), while sufficient for a profit incentive for bundling, is not necessary. If $p_{\hat{x}}$ is lowered further, the elasticities of demand for both $\widehat{x}$ and x change and this may despite the initial losses lead to a situation where profits eventually increase sufficiently strongly for the optimal bundle to yield higher profits than unbundled sales alone.

Three cases can arise.

a) $p_x > p_{\hat{x}} > p_x^m$. In this case, a necessary condition for profitable bundling is from (19) that

$$x + p_x \frac{dx}{dp_x} + p_{\hat{x}} \frac{d\hat{x}}{dp_x} > 0, \text{ or} \tag{27}$$

$$1 + \varepsilon^{x,p_x} + \frac{\hat{x}}{x} \frac{dp_{\hat{x}}}{dp_x} \varepsilon^{\hat{x},p_{\hat{x}}} > 0, \tag{28}$$

and from (20) that

$$\widehat{x} + p_{\hat{x}} \frac{d\hat{x}}{dp_{\hat{x}}} + p_x \frac{dx}{dp_{\hat{x}}} > 0, \text{ or} \tag{29}$$

$$1 + \varepsilon^{\hat{x},p_{\hat{x}}} + \frac{x}{\hat{x}}\frac{dp_x}{dp_{\hat{x}}}\varepsilon^{x,p_x} > 0. \tag{30}$$

For bundling to be optimal, the inequalities in (28) and (30) must hold sufficiently strongly for some $p_x \in (p_x^m, p_x^*)$ and $p_{\hat{x}} \in (p_x^m, p_{\hat{x}}^*)$ that $\exists\, p_x, p_{\hat{x}} > p_x^m$ with $p_x x + p_{\hat{x}}\hat{x} > p_x^m x^m$. This requires both elasticities to be sufficiently low.

b) $p_x^m > p_x > p_{\hat{x}}$. The corresponding necessary conditions are

$$1 + \varepsilon^{x,p_x} + \frac{\hat{x}}{x}\frac{dp_{\hat{x}}}{dp_x}\varepsilon^{\hat{x},p_{\hat{x}}} < 0 \tag{31}$$

and

$$1 + \varepsilon^{\hat{x},p_{\hat{x}}} + \frac{x}{\hat{x}}\frac{dp_x}{dp_{\hat{x}}}\varepsilon^{x,p_x} < 0. \tag{32}$$

These conditions are sufficient if the elasticities are large enough, for some $p_x \in (p_x^*, p_x^m)$ and $p_{\hat{x}} \in (p_{\hat{x}}^*, p_x^m)$that $\exists\, p_x, p_{\hat{x}} \in (0, p_x^m)$ with $p_x x + p_{\hat{x}}\hat{x} > p_x^m x^m$.

c) $p_x > p_x^m > p_{\hat{x}}$. Now the necessary conditions are

$$1 + \varepsilon^{x,p_x} + \frac{\hat{x}}{x}\frac{dp_{\hat{x}}}{dp_x}\varepsilon^{\hat{x},p_{\hat{x}}} > 0 \tag{33}$$

and

$$1 + \varepsilon^{\hat{x},p_{\hat{x}}} + \frac{x}{\hat{x}}\frac{dp_x}{dp_{\hat{x}}}\varepsilon^{x,p_x} < 0, \tag{34}$$

sufficiently strongly for some $p_x \in (p_x^m, p_x^*)$ and $p_{\hat{x}} \in (p_{\hat{x}}^*, p_x^m)$, so that $\exists\, p_{\hat{x}} \in (0, p_x^m)$ and $p_x > p_x^m$ with $p_x x + p_{\hat{x}}\hat{x} > p_x^m x^m$. The elasticities must in this case be neither too large nor too small.

The following proposition summarizes the results of this section.

Proposition 1 *Mixed bundling is strictly more profitable than unbundled sales of good x alone (i.e. $\exists\, p_x, p_{\hat{x}} > 0$ s.t. $p_x x + p_{\hat{x}}\hat{x} > p_x^m x^m$) if*

i) equation (26) holds, or

ii) otherwise, if (28) and (30) (case a), or (??) and (32) (case b), or (33) and (34) (case c) hold as described above.

Proof. Follows from the derivation of (26) as well as as the discussion of cases a) through c). ■

It is not immediately obvious that all cases are possible. After all, it is a well-known result that a multi-product monopolist selling substitute goods raises the prices of both goods above their monopoly levels to create demand for the respective substitute good. To see that a) is possible, note that if k is sufficiently high to prevent most high-α buyers from switching into the bundle then clearly it is optimal to raise $p_x > p_x^m$ and capture those with lower α by offering the bundle at a price marginally above p_x^m. If it is the case that many previous buyers of x with lower α also have a high β, then this must raise profits. The difference to the multi-product monopolist case is that the additional instrument k can be used to keep the groups of buyers of the bundled and unbundled good separate from each other. By selecting the most favorable combination of elasticities the self-selection scheme allows to effectively raise the prices charged to all former buyers.

It is also possible for both prices to fall below the monopoly level (case b). Suppose that all former buyers of x have such a low value of β that they will not switch into a bundle offered at $p_{\tilde{x}} < p_x^m$. Setting p_x just marginally under p_x^m preserves the monopoly level of profits approximately, while the introduction of the bundle draws some new buyers into the market and hence has a positive first order effect on profits. Again it is k that makes the difference by screening out those buyers that have high α and low β. In the

unbundled sales situation the price could not be lowered without *all* buyers benefiting from it, and hence lowering profits.

The different cases are easily understood. For both prices to rise after bundling, the elasticities must not be too large since otherwise demand would fall sufficiently to render this unprofitable. Since a rise in the separate price p_x creates additional demand for the bundle, $p_{\tilde{x}}$ tends to rise as well. The opposite is the case if $p_{\tilde{x}}$ is lowered and this tends to steal customers from the separate sales of x. Case c) can only happen if the elasticities are neither too large (else both prices would have to drop to maximize profits) nor too small (else it would be optimal to exploit the unresponsiveness of demand by raising both prices).

The above conditions for the private profitability of bundling cannot be phrased with equal precision in terms of the correlation of reservation prices for x and y. While it is true (and has been repeatedly noted before) that the profitability in this model tends to stem from a general negative correlation between α and β, counterexamples can be constructed for all but the extreme case of a perfect positive correlation.[9] We have the following weak result concerning the relationship between the correlation of α and β and the private profit incentive to offer a bundle.

[9]Suppose all but one (of many) buyers have perfectly positively correlated values of α and β. If this one has a low β but a very high α, whereas all other buyers have both high α and β then bundling is profitable since it allows to sell to all previous buyers at the same price and to raise the price for the individual buying x alone, yielding strictly higher profits. Increasing the number of consumers with perfectly correlated values brings the correlation arbitrarily close to 1 but bundling is still profitable as there is one odd buyer in the pool.

Corollary 2 *Bundling cannot be profitable if α and β are perfectly positively correlated.*

This result has been previously noted in Schmalensee [10].

5 Welfare Effects

Commodity bundling and tie-in sales have traditionally been viewed with considerable suspicion by antitrust institutions in the United States. Both the Sherman Act (1890) and the Clayton Act (1914) contain provisions which have been used to strike down bundling or tying arrangements as anticompetitive. The courts typically argue that such vertical restraints serve to restrain competition in the market for the bundle components, and to transfer consumer surplus to sellers. The new Chicago School has in vain waged a campaign highlighting the efficiency rationales for such contractual provisions, and it is generally agreed upon by now that conditioning the sales of one good on quantities bought of another may indeed have anti-competitive consequences.

What are the welfare implications of the present model of bundling as a self-selection device? The following proposition distinguishes between the outcome of cases a), b) and c) from the previous section.

Proposition 3 *If bundling is privatly profitable, social welfare falls in case a) and rises in case b). Social welfare in case c) may be higher or lower than in the unbundled situation.*

Proof. The proposition follows from noting that the price to all buyers of good x rises in a) and falls in b). In c) some new buyers are attracted by

the cheap bundle price $p_{\hat{x}}$ while some former buyers of good x are priced out of the market by the higher p_x. Furthermore, some buyers of the bundle overconsume good y in order to get x cheaper. The welfare consequences depend on the relative magnitudes of these effects which are in turn determined by the details of the distribution of α and β among consumers. By continuity from cases a) and b) social welfare can be higher (if p_x is just marginally higher than p_x^m) or lower (if $p_{\hat{x}}$ is just marginally lower than p_x^m), respectively. ■

Bundling changes the elasticities that the seller faces in the market by inducing customers to self-select into groups buying the bundle and the unbundled x. This change in elasticities, despite rendering bundling privatly profitable, may benefit consumers as well (case b). Bundling here serves to overcome the information imperfection that had prevented the seller from separating the groups. Social surplus may rise if the lessening of information imperfection outweighs the increase in monopoly power that the seller experiences as a result of being able to group buyers according to their reservation prices. In the example of Japanese casualty insurance, as documented in Wallner ([14]) the introduction of savings-type products resulted in booming sales of policies to apparently new customers. While the individual policy sales at unchanged premium rates stagnated, the bundle posted dramatic growth rates over several years, suggesting a large welfare-enhancing market expansion effect.

The punchline for antitrust purposes is that an evaluation of the relevant elasticities can determine whether society benefits or suffers from bundling. In case c) only an examination of the magnitudes of the switching and mar-

ket expansion effects, determined by the demand elasticities of the bundled and unbundled good x, can shed light on the net effect on social welfare. Of course the issue of the distribution of surplus between the firm and consumers remains even if society on the whole gains. In the extreme, allowing a monopolist to engage in perfect price discrimination eliminates the deadweight loss completely, but it does so at the price of leaving the consumers empty-handed and the seller with all gains from trade.

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