Catching up with the Keynesians*

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Abstract

This paper examines the role for tax policies in productivity-shock driven economies with "catching-up-with-the-Joneses" utility functions. The optimal tax policy is shown to affect the economy counter-cyclically via procyclical taxes, i.e., "cooling down" the economy with higher taxes when it is "overheating" in booms and "stimulating" the economy with lower taxes in recessions to keep consumption up. Thus, models with catching-up-with-the-Joneses utility functions call for traditional Keynesian demand management policies. Parameter values from Campbell and Cochrane (1995) are also used to illustrate that the necessary labor taxes can be very high, in the order of 50 percent. However, Campbell and Cochrane's nonlinear version of the benchmark level in the catching-up-with-the-Joneses preferences has the implication that consumption bunching can be welfare enhancing.

1 Introduction

Envy is one important motive of human behavior. In macroeconomics, theories built on envy have been used in trying to explain the equity premium puzzle as described by Mehra and Prescott (1985). Abel (1990, 1995) and Campbell and Cochrane (1995) postulate utility functions exhibiting a desire to "catch up with the Joneses", i.e., if others consume more today, you yourself will experience a higher marginal utility from an additional unit of consumption in the future.[1] In some ways, the idea of "catching up with the Joneses" is a variation of the theme of "habit formation", see Constantinides (1990). The key difference is that "catching up with the Joneses" postulates a consumption externality since agents who increase their consumption do not take into account their effect on the aggregate desire by other agents to "catch up". Thus, this externality allows room for beneficial government intervention. The optimal tax policy would induce agents in the competitive equilibrium to behave in a first-best manner, which is given by the solution to a social planner's problem with habit formation.

While "catching up with the Joneses" has been the focus of quite some research in the asset pricing literature, its implications with respect to policy making have rarely been explored. The purpose of this paper is to do exactly that. In particular, we examine economies driven by productivity shocks where agents care about consumption as well as leisure, and there is a "catching-up" term in the consumption part of the utility function. For simplicity, the model abstracts from capital formation.[2] In this framework, we examine the role for taxing labor income. The optimal tax policy turns

[1] Galí (1994) explores an alternative assumption where agents' preferences depend on current instead of lagged per capita consumption ("keeping up with the Joneses" as compared to "catching up with the Joneses").

[2] As noted by Lettau and Uhlig (1995), the inclusion of capital formation in models based on catching-up-with-the-Joneses utility functions have the implication that consumption becomes excessively smooth. For a similar observation and a possible remedy in models with habit formation, see Boldrin, Christiano and Fisher (1995).

out to affect the economy countercyclically via procyclical taxes, i.e., "cooling" down the economy with higher taxes when it is "overheating" due to a positive productivity shock. The explanation is that agents would otherwise end up consuming too much in boom times since they are not taking into account the "addiction effect" of a higher consumption level. In recessions, the effect goes the other way around and taxes should be lowered to "stimulate" the economy by bolstering consumption. Thus, models with catching-up-with-the-Joneses utility functions call for traditional Keynesian demand management policies. We also use parameter values from Campbell and Cochrane (1995) to illustrate that the necessary labor taxes can be very high, in the order of 50 percent. However, Campbell and Cochrane's nonlinear version of the benchmark level in the catching-up-with-the-Joneses preferences has the implication that consumption bunching can be welfare enhancing. As an example, we show how welfare can be improved upon in their framework by inducing business cycles in an otherwise stationary environment.

The paper is organized as follows. In section 2, we examine a simple one-shot model as well as an infinite horizon version, where agents care about "keeping up with the Joneses". The assumption being that contemporaneous average consumption across all agents enters the utility function. In that case, it turns out that there is a constant tax rate on labor, which delivers the first best outcome independent of the productivity shock. In section 3, we allow the agents' benchmark level to be a geometric average of *past* per-capita consumption, i.e., specifying a utility function which exhibits "catching up with the Joneses". This framework has Keynesian-style countercyclical policy implications. In section 4, we examine the utility function used by Campbell and Cochrane (1995), adding another layer of complexity. Section 5 concludes.

2 Keeping up with the Joneses

We imagine an economy with many consumers, each with the same utility function

$$\frac{(c - \alpha C)^{1-\gamma} - 1}{1 - \gamma} - An\,,$$

where $c \geq 0$ is the individual's consumption, $C \geq 0$ is average consumption across all agents and $n \geq 0$ is labor supplied by the individual. The parameters $\alpha \in [0, 1)$, $\gamma \geq 0$ and $A > 0$ determine the relative importance of average consumption, the curvature of the consumption term and the relative importance of leisure. This utility function captures the notion of "keeping-up-with-the-Joneses", i.e., average consumption decreases an individual's level of utility and increases his marginal utility of an additional unit of consumption. This specification is different from the formulations in Abel (1990, 1995), who uses ratios rather than differences to aggregate consumption, but is in line with the catching-up formulation in Campbell and Cochrane (1995). No "keeping-up" is imposed on the leisure part of the utility function. In other words, we assume that agents are competing in, say, having the biggest car or the biggest house rather than having the most amount of leisure. The utility in leisure is also assumed to be linear. This assumption is partly done for convenience, but can also be motivated by indivisibilities in the labor market and is an often used assumption in the real-business cycle literature, see e.g. Hansen (1985) and the explanations therein. We imagine that the production function takes the form

$$c = \theta n\,,$$

where θ is a productivity parameter. Thus, there is no capital, and output is simply linear in labor.

The government levies a flat tax τ on all labor income and the tax revenues are then handed back to the agents in a lump-sum fashion. Let v be the lump-sum transfer to each agent. Since all agents are identical, the government's budget constraint can be written as

$$\tau \theta n = v\,.$$

A competitive equilibrium is calculated by having an agent maximize the utility function above with respect to c and n subject to his budget constraint,

$$c = (1-\tau)\theta n + v\,.$$

A consumer's optimal consumption is then found to be

$$c = \alpha C + \left(\frac{\theta}{A}(1-\tau)\right)^{\frac{1}{\gamma}}, \tag{1}$$

where average consumption C is taken as given by the individual agent. However, in an equilibrium it must be true that $c = C$, so the equilibrium consumption level is

$$c = C = \frac{1}{1-\alpha}\left(\frac{\theta}{A}(1-\tau)\right)^{\frac{1}{\gamma}}. \tag{2}$$

The government's optimal choice of τ can be deduced from the solution to the social planner's problem. The social planner would take the externality into account by setting $c = C$ in the utility function above, and then maximize with respect to consumption and labor subject to the technology constraint. The first-best outcome is then given by

$$C^* = \frac{1}{1-\alpha}\left(\frac{\theta}{A}(1-\alpha)\right)^{\frac{1}{\gamma}}.$$

Comparing the social planner's solution to the competitive equilibrium, we find:

Proposition 1 ("Keeping up with Joneses")
The first-best consumption allocation can be achieved with a tax rate

$$\tau = \alpha\,.$$

This result is quite intuitive. A fraction α of any increase in the representative agent's consumption does not contribute to his utility since it is offset through the consumption externality. It is therefore socially optimal to tax away a fraction α of any labor income so that the agent faces the correct utility tradeoff between leisure and consumption. It can also be noted that the optimal tax is independent of the productivity parameter θ. While the tax can potentially be high depending on the value of α, it does not react to current economic conditions. In particular, we do not get any Keynesian effects in the sense of setting taxes procyclically.

Given the solution above, one can easily examine a dynamic model, in which there are periods denoted by $t = 0, 1, 2, \ldots$ and agents have the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(c_t - \alpha C_t)^{1-\gamma} - 1}{1-\gamma} - A n_t \right),$$

where E_0 is the expectation operator conditioned upon information at time 0 and $\beta \in (0, 1)$ is a discount factor. The production function is the same as before, and so are the budget constraints of the government and the agents. There is now also some stochastic process driving productivity θ_t. Computing the competitive equilibrium and the social planner's solution amounts to the same calculations as above, since this dynamic model simply breaks into a sequence of one-shot models. The first-best solution is again achieved at $\tau = \alpha$, i.e., there are no cyclical consequences for the tax rate.

The parameter α governing the optimal tax rate also ties in with the value for the relative risk aversion η for gambles with respect to consumption, given by $\eta = \gamma/(1-\alpha)$ from the perspective of the individual agent, but given by $\eta_{SP} = \gamma$ from the perspective of the social planner, taking into account $c_t = C_t$. The social planner would thus be willing to forego a premium as a fraction of mean consumption approximately equal to $\gamma\sigma^2/2$ to avoid mean-zero random fluctuations in aggregate consumption with a standard deviation of $100 * \sigma$ percent. This is also the premium an individual agent would pay if that would avoid simultaneously fluctuations in his individual consumption as well as aggregate consumption. In the decentralized economy, however, the individual agent takes C_t as given. To avoid mean-zero random fluctuations in c_t with a standard deviation of $100 * \sigma$ percent, which are uncorrelated with fluctuations in C_t, he would be willing to pay a premium as a fraction of his mean consumption approximately equal to $\gamma\sigma^2/(2(1-\alpha))$. More relevant perhaps is the premium the agent would pay to avoid the fluctuations in his individual consumption, assuming them to be perfectly correlated and of equal size to the fluctuations in aggregate average consumption; that premium ρ is given by

$$\rho \;=\; \left(1 - \frac{\alpha}{1-\alpha}\right)\frac{\gamma\sigma^2}{2},$$

as can be seen from a second-order Taylor approximation.[3] Interestingly, that premium vanishes at $\alpha = 0.5$ and becomes negative for $\alpha > 0.5$; there, the marginal utility of an agent fluctuates less, when the same uncertainty affects both individual as well as aggregate consumption compared to the case when uncertainty only affects aggregate consumption. Similar calculations and remarks apply in the following sections, where we shall replace C with a benchmark level X, calculated from past aggregate consumption.

The premia calculated above require the calculation of σ^2, which could potentially be influenced by the taxation experiments considered here.[4] In-

[3] For a general utility function $u(c, C)$, and mean zero random variables ν, ξ, we solve for a risk premium ρ by setting

$$\begin{aligned} E_0[u(\bar{c}(1+\nu), \bar{C}(1+\xi))] \approx\; & u(\bar{c}, \bar{C}) + u_{11}(\bar{c}, \bar{C})\bar{c}^2\sigma_\nu^2/2 \\ & + u_{12}(\bar{c}, \bar{C})\bar{c}\bar{C}\mathrm{Cov}(\nu, \xi) + u_{22}(\bar{c}, \bar{C})\bar{C}^2\sigma_\xi^2/2 \end{aligned}$$

equal to

$$E_0[u(\bar{c}(1-\rho), \bar{C}(1+\xi))] \approx u(\bar{c}, \bar{C}) - u_1(\bar{c}, \bar{C})\bar{c}\rho + u_{22}(\bar{c}, \bar{C})\bar{C}^2\sigma_\xi^2/2\,.$$

[4] Additionally, one generally needs to consider the premium for the reduction in the variance of leisure. However, with our utility specification, the agent is risk neutral with

specting equation (2), we see that this is not so. When computing the variance σ^2 of the proportional changes in aggregate consumption, the multiplicative term involving τ drops out. A second-order Taylor approximation yields that σ^2 is equal to $1/\gamma^2$ times the variance of the proportional changes in θ.

Finally, the tax analysis presented here is closely related to the literature on redistributive taxation when individual welfare depends on relative income. Given a social welfare function, Boskin and Sheshinski (1978) analyze how the standard results of optimal tax theory are altered when individuals care about relative income, and they demonstrate that the scope for redistribution becomes much larger. Persson (1995) extends their argument by showing that high taxation can even constitute a Pareto improvement as long as individuals' pre-tax incomes are not too different. In fact, his discussion of the special case of identical individuals corresponds directly to our treatment of "keeping up with the Joneses".

3 Catching up with the Joneses

3.1 The model

We now assume that the utility function does not depend on current average consumption as assumed above, but rather on some measure X_t of *past* average consumption,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(c_t - X_t)^{1-\gamma} - 1}{1-\gamma} - An_t \right). \tag{3}$$

In particular, we let the benchmark level X_t be a geometric average of past per-capita consumption levels,

$$X_t = (1-\phi)\alpha C_{t-1} + \phi X_{t-1}\,, \tag{4}$$

respect to gambles in leisure.

with $0 \leq \phi < 1$ and $0 \leq \alpha < 1$. Otherwise, the production technology and the budget constraints of the consumers and the government are the same as before. In addition, we now need to be more careful about the productivity process. We postulate the following stochastic process,

$$\frac{1}{\theta_t} = \left(\frac{1-\psi}{\bar{\theta}} + \frac{\psi}{\theta_{t-1}} \right) (1 + \epsilon_t) , \tag{5}$$

where $\psi \in [0, 1)$ and ϵ_t is i.i.d, has mean zero and is bounded below by $\epsilon_t > -1$.[5]

For the competitive equilibrium in this model, one finds analogously to (1) that the agent will set consumption equal to

$$c_t = X_t + \left(\frac{\theta_t}{A} (1 - \tau_t) \right)^{\frac{1}{\gamma}} . \tag{6}$$

Thus, given a first-best path for consumption $c_t^* = C_t^*$, one can achieve this outcome with a sequence of taxes τ_t satisfying

$$\tau_t = 1 - \frac{A}{\theta_t} (C_t^* - X_t)^{\gamma} . \tag{7}$$

To characterize the optimal tax policy, we now turn to the social planner's problem.

3.2 Solving the social planner's problem

The social planner maximizes the utility function (3) subject to the production technology and the constraint (4), taking as given the process for θ_t

[5]The stochastic process (5) is approximately the same as postulating an AR(1) process for the logarithm of θ_t,

$$\log(\theta_t) = (1 - \psi) \log(\bar{\theta}) + \psi \log(\theta_{t-1}) + \epsilon_t .$$

Thus, our exact analytical results below pertaining to the stochastic process (5) can also be interpreted as approximations to the corresponding formulas valid for the more commonly used AR(1) process for the logarithm of θ_t.

and the initial conditions X_0 and θ_0. Since this maximization problem is a concave one, we can analyze it by using first-order conditions. Let μ_t be the Lagrange multiplier for the constraint (4). The two first-order conditions with respect to C_t and X_{t+1} can then be written as

$$\begin{aligned} (C_t - X_t)^{-\gamma} &= \frac{A}{\theta_t} + \alpha(1-\phi)\mu_t\,, & (8)\\ \mu_t &= \beta E_t\left[(C_{t+1} - X_{t+1})^{-\gamma}\right] + \beta\phi E_t\left[\mu_{t+1}\right]. & (9)\end{aligned}$$

The first equation contains the additional third term $\alpha(1-\phi)\mu_t$ as compared to the corresponding equation of the private agent's optimization problem. Here, the social planner takes into account the "bad" effect on future utility of additional aggregate consumption today, since it raises the benchmark level X_{t+1} tomorrow and beyond. In particular, a fraction $\alpha(1-\phi)$ of an increase in today's per-capita consumption spills over to X_{t+1}, and the shadow value of a higher X_{t+1} is given by μ_t. Equation (9) shows in turn how the shadow value μ_t is the sum of the expected effect on tomorrow's discounted marginal utility of consumption and its impact on still future periods. The latter effect is captured by the discounted expected value of μ_{t+1} multiplied by ϕ, where ϕ is the fraction of the benchmark level that carries over between two consecutive periods.

Using the two first-order conditions (8) and (9) as well as the constraint (4), the optimal steady-state[6] consumption level can be calculated to be

$$\bar{C}^* = \frac{1}{1-\alpha}\left(\frac{\bar{\theta}}{A}\left(1 - \frac{\alpha\beta(1-\phi)}{1-\beta\phi}\right)\right)^{\frac{1}{\gamma}}.$$

Comparing this expression to the agent's consumption rule in equation (6) and noting that $\bar{X} = \alpha\bar{C}$, we see that the first-best steady-state allocation is supported by a tax of

$$\bar{\tau} = \frac{\alpha\beta(1-\phi)}{1-\beta\phi}.$$

[6]The term 'steady state' is used in this paper to denote a deterministic steady state in which the productivity shock is always equal to $\bar{\theta}$.

For example, if the benchmark level is simply α times the level of yesterday's per-capita consumption ($\phi = 0$), we get $\bar{\tau} = \alpha\beta$. This formula is rather intuitive compared to the simple model above of "keeping up with the Joneses", where we got $\tau = \alpha$. Since the consumption externality now enters the utility function with a one-period lag, the adverse future effect of being "addicted" to today's consumption is discounted by β so the optimal steady-state tax rate is also scaled down by β.

In order to characterize the optimal consumption and taxation outside of a steady state, we can actually solve the dynamic equations in closed form. The substitution of equation (8) into (9) yields a first-order difference equation in the shadow value μ_t, which can be solved forward in the usual manner,

$$\mu_t = \beta A\, E_t \left[\sum_{j=0}^{\infty} \delta^j \frac{1}{\theta_{t+1+j}} \right], \tag{10}$$

where

$$\delta = \beta(\phi + \alpha(1-\phi)) < 1\,.$$

With the law of motion for θ_t in (5), one can then calculate μ_t to be

$$\mu_t = \frac{\beta A}{(1-\delta)\bar{\theta}} + \frac{\beta A \psi}{1-\delta\psi} \left(\frac{1}{\theta_t} - \frac{1}{\bar{\theta}} \right). \tag{11}$$

After substituting this expression into the first-order condition (8), the optimal consumption level is found to be

$$C_t^* = X_t + \left(\frac{A}{\bar{\theta}} \frac{1-\beta\phi}{1-\delta} + A \left(\frac{1}{\theta_t} - \frac{1}{\bar{\theta}} \right) \frac{1-\beta\phi\psi}{1-\delta\psi} \right)^{-\frac{1}{\gamma}}. \tag{12}$$

The tax necessary to support this optimal consumption allocation is then given by equation (7).

Rather than calculating the tax rate τ_t, it is more appealing to calculate the ratio of taxes to after-tax income. Using equations (7) and (8), we get

$$\frac{\tau_t}{1-\tau_t} = \frac{\alpha(1-\phi)}{A}\, \theta_t\, \mu_t\,. \tag{13}$$

With the productivity process in (5), μ_t is given by (11) and the tax ratio can then be rewritten as in the following proposition.

Proposition 2 ("Catching up with Joneses")
The tax rate τ_t supporting the first-best consumption allocation can be solved from

$$\frac{\tau_t}{1-\tau_t} = \frac{\alpha\beta(1-\phi)}{1-\delta\psi}\left(\psi + \frac{1-\psi}{1-\delta}\frac{\theta_t}{\bar{\theta}}\right), \tag{14}$$

with a steady-state value of

$$\bar{\tau} = \frac{\alpha\beta(1-\phi)}{1-\beta\phi}. \tag{15}$$

3.3 Tax policy implications

The implications for an optimal tax policy are seen to depend critically on the timing of the consumption externality. In the case of "keeping up with the Joneses" in Proposition 1, the optimal tax rate does not depend on the productivity shock. Since only contemporaneous average consumption affects the individuals' welfare, the social planner can correct the consumption level period-by-period without any intertemporal considerations. In each period, the social planner establishes the right tradeoff between consumption and leisure for individuals by taxing away a fraction α of any labor income. In contrast, "catching up with the Joneses" means that individuals care about past average consumption levels that are functions of past productivity shocks while current consumption opportunities depend on today's productivity shock. The social planner is now not only concerned about the tradeoff between consumption and leisure in any given period but also the effects of today's consumption on future utilities. Thus, the interdependence between the past, present and future gives rise to optimal time-varying tax rates that depend on the realizations of the productivity shock.

It is fruitful to compare this observation to the calculation of term premia in models with "keeping up" and "catching up" preferences. Abel (1995) defines the term premium to be the excess of the expected one-period rate of return on a n-period asset over the expected one-period rate of return on a one-period asset, when comparing assets for which the log dividends have the same constant proportionality to log consumption. For a slightly different specification of preferences[7] and a Markov regime-switching process for consumption growth, Abel (1995) shows that "keeping up" preferences imply that all term premia are identical to zero, while more complicated term premium structures may arise with "catching up" preferences. In his analysis, the stochastic properties of the consumption growth until the next period are enough to calculate the returns on these assets in the "keeping up" case, regardless of their remaining maturity, while the "catching up" case introduces additional interdependencies across periods. Analogously, the social planner here only needs to offset the current consumption externality in the "keeping up" case, while he needs to worry about the interdependencies across periods in the "catching up" case.

Corollary 1 ("Catching up with Joneses")
The optimal tax policy affects the economy countercyclically via procyclical taxes.

The corollary follows directly from equation (14), the tax ratio (and thus the tax rate itself) varies positively with productivity θ_t.[8] Thus, we get Keynesian-style policy recommendations. A government that maximizes welfare should "cool down" the economy during booms via higher taxes because agents would otherwise consume too much as compared to the first-best solution. Likewise, the government should "stimulate" the economy during

[7]Abel (1995) uses $u(c_t, X_t) = (c_t/X_t)^{1-\gamma}/(1-\gamma)$, where $X_t = C_t^{\zeta_0} C_{t-1}^{\zeta_1}$, i.e. he assumes that the representative consumer's utility depends on the ratio of c_t to X_t rather than on the difference between c_t and X_t.

[8]This result holds for a much larger class of stochastic processes than given by equation (5). According to equations (10) and (13), the optimal tax rate goes up with θ_t as long as $E_t\left[\sum_{j=0}^{\infty} \delta^j \theta_{t+1+j}^{-1}\right]$ decreases less than proportionally with the inverse of θ_t.

recessions by lowering taxes and thereby bolstering consumption. Of course, these optimal fiscal policies are here driven by a rather unorthodox argument. Taxation is needed to offset the externalities associated with private consumption decisions. One individual's consumption affects the welfare of others through agents' desire to "catch up with the Joneses".

To shed light on how different parameters affect the cyclical variations of optimal taxation, let ω_t be the relative deviation of the tax ratio $\tau_t/(1-\tau_t)$ from its steady-state value. That is, ω_t tells us, how the ratio of taxes to after-tax income responds to productivity shocks relative to its steady-state value. From equation (14), we can calculate

$$\omega_t \equiv \frac{\tau_t}{1-\tau_t}\left(\frac{\bar{\tau}}{1-\bar{\tau}}\right)^{-1} - 1 = \frac{1-\psi}{1-\delta\psi}\,\frac{\theta_t-\bar{\theta}}{\bar{\theta}}\,. \tag{16}$$

Doing comparative statics on this expression, we see that the size of the cyclical tax effect in absolute terms varies negatively with ψ and positively with α, β and ϕ. The intuition for this is straightforward by considering the tax response to a positive productivity shock. A higher ψ, i.e., a more persistent productivity shock, means that future production and consumption opportunities are also expected to be better than average. The anticipation of the economy being able to sustain a higher consumption level for a prolonged period of time mitigates the adverse effects of making people "addicted" to higher consumption today. It is therefore socially optimal to take more advantage of a persistent productivity shock, so the optimal tax hike is lower with a higher ψ. In contrast, preferences with a higher weight on yesterday's consumption (a higher α), a higher degree of persistence in the benchmark level (a higher ϕ), or a higher emphasis on the future (a higher β) give rise to a larger cyclical tax effect. The reason is, of course, that the consumption externality is more important for such preferences and the government must consequently be more resolute in moderating agents' consumption behavior.

As a point of reference, the largest tax effect as defined by (16) is attained for transient one-period productivity shocks ($\psi = 0$). The percentage deviation of the tax ratio from its steady-state value responds then one-for-one to the percentage change in the productivity from its steady state. However, besides noting that the cyclical tax effect can be large relative to the magnitude of the productivity shock, it is also important to keep in mind that most aggregate economic shocks are usually relatively small so the cyclical tax changes considered here are really examples of extreme "fine tuning" of taxes.

Finally, Figure 1 illustrates the consumption dynamics in response to a productivity shock. After a one-percent initial shock to θ_t at time $t = 0$, the hump-shaped dashed line traces out the response of consumption from the steady state when taxes are adjusted optimally and the solid line displays the consumption response when the tax rate is not changed but kept constant at its steady-state value. As a parameterization, we used $\psi = 0.9$, $\beta = 0.97$, $\alpha = 0.8$, $\phi = 0$ and varied $\gamma \in \{0.5, 1.5\}$. Not surprisingly, the consumption response becomes muted with a higher γ, since a more rapidly diminishing marginal utility of consumption reduces the attractiveness of increasing consumption. It is interesting to note that for both values of γ in Figure 1 the deviation of consumption from steady state is reduced by around 25 % under optimal tax adjustment as compared to keeping the tax rate constant at its steady-state value. The figure also contain the change in the tax ratio ω_t needed to accomplish this "cooling down" of the economy.

3.4 Welfare gains

It is interesting to examine the welfare gains due to taxation. We therefore compute welfare levels for three stochastic economies; laissez faire without taxation (LF), the social-planner outcome with optimal taxation (SP_{τ^*}), and an economy where the tax is kept constant at its steady-state value ($\mathrm{SP}_{\bar{\tau}}$). The calculations are based on 10,000 randomly generated sequences of the productivity shock θ, each one of length 1100 periods. Using steady states as initial conditions, we compute the economic outcomes associated with the three different economies. The welfare level for each economy is then obtained

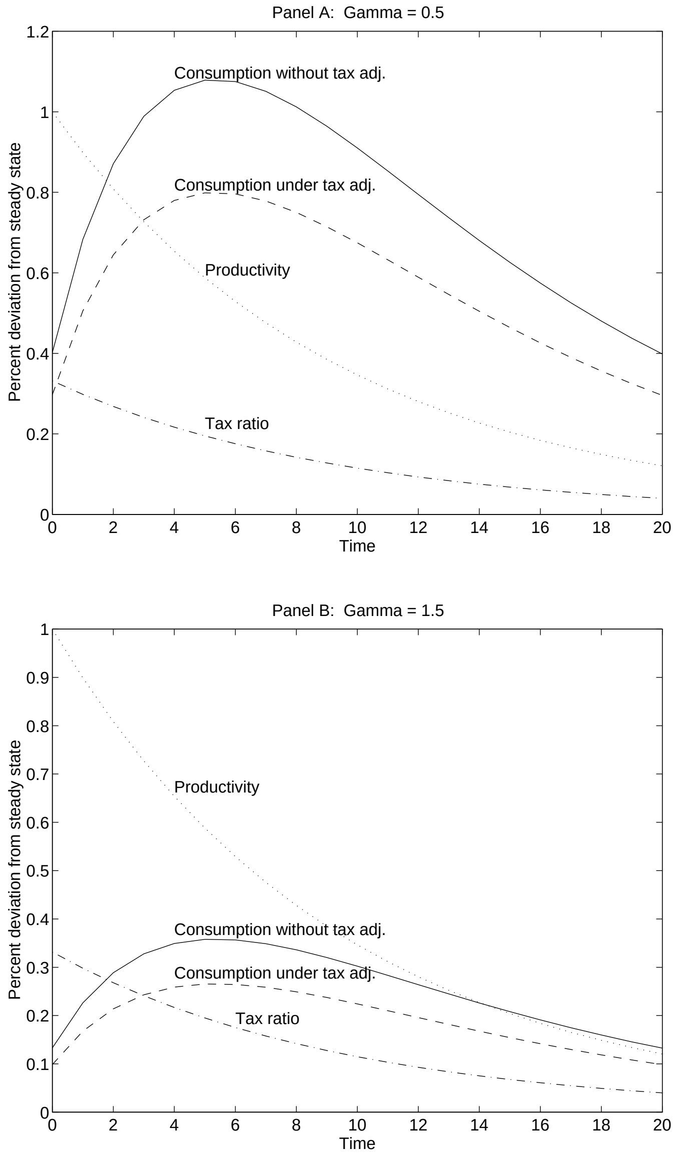

Figure 1: Consumption dynamics in response to a one-percent productivity shock from the steady state. The dash-dotted line depicts the optimal response in the tax ratio $\tau_t/(1-\tau_t)$. The parameters are $\gamma \in \{0.5, 1.5\}$ (panel A and panel B, respectively), $\psi = 0.9$, $\alpha = 0.8$, $\phi = 0$, and $\beta = 0.97$.

by discarding the first 100 periods in each sequence, and averaging over all 10,000 runs. For purposes of comparison, we also compute welfare levels for two nonstochastic economies where θ is constant and equal to its mean value; a laissez-faire outcome ($\overline{\text{LF}}$) and the social-planner solution ($\overline{\text{SP}}$).[9]

The welfare comparisons between the three stochastic economies use $\overline{\text{SP}}$ as a reference. In particular, we compute the fractional reduction $(-\triangle)$ in a single individual's consumption in economy $\overline{\text{SP}}$ which will make her as well off as in the alternative stochastic economy,

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{\left((1-\triangle)c_t^{\overline{\text{SP}}} - X_t^{\overline{\text{SP}}}\right)^{1-\gamma} - 1}{1-\gamma} - An_t^{\overline{\text{SP}}} \right)$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(c_t^j - X_t^j\right)^{1-\gamma} - 1}{1-\gamma} - An_t^j \right), \tag{17}$$

where the superscript on c_t, X_t and n_t denotes to which economy the values refer, and $j \in \{\text{SP}_{\tau^*},\ \text{SP}_{\bar{\tau}},\ \text{LF}\}$. In the simulations, we have chosen ϵ_t to be uniformly distributed with a standard deviation of $\sigma_\epsilon \in \{0.01, 0.04\}$. The other parameter values are $\psi = 0.9$, $\beta = 0.97$, $\phi = 0$, $\gamma \in \{1.5, 5\}$ and $\alpha \in \{0.2, 0.5, 0.8\}$.

The first two columns with results in table 1 confirm our previous observation that optimal cyclical tax changes constitute extreme "fine tuning" of taxes. There is hardly any loss in welfare when replacing the optimal time-varying tax with its steady-state value. However, there are relatively large welfare gains associated with the overall scheme of using taxation to overcome the externality arising from "catching up with Joneses", as shown in the third column of results. The welfare gains increase with the importance of the externality in the preferences, as parameterized by α.[10] When $\alpha = 0.8$ and $\gamma = 1.5$, an individual's consumption would have to be reduced

[9] Note that $\bar{\theta}$ is not the mean of the stochastic process in (5), instead we use the average value of θ computed over all the simulations.

[10] The simulated welfare results were rather insensitive to different values of the per-

σ_ϵ	γ	α	$\overline{\text{SP}} \to \text{SP}_{\tau^*}$	$\overline{\text{SP}} \to \text{SP}_{\bar{\tau}}$	$\overline{\text{SP}} \to \text{LF}$	$\overline{\text{LF}} \to \text{LF}$
0.01	1.5	0.2	0.030	0.030	1.377	0.032
		0.5	0.013	0.014	7.782	0.019
		0.8	0.009	0.009	11.844	0.024
	5	0.2	0.045	0.045	0.423	0.053
		0.5	0.026	0.026	1.976	0.046
		0.8	0.011	0.011	2.891	0.041
0.04	1.5	0.2	0.457	0.457	1.797	0.496
		0.5	0.336	0.338	8.058	0.461
		0.8	0.145	0.152	11.907	0.418
	5	0.2	0.635	0.636	1.002	0.755
		0.5	0.398	0.398	2.293	0.692
		0.8	0.173	0.174	2.980	0.637

Table 1: Welfare loss when switching from economy i to j ($i \to j$), measured by the percentage reduction in a single individual's consumption in economy i which will equalize her expected utilities across the two economies. $\overline{\text{LF}}$ and $\overline{\text{SP}}$ denote laissez faire and the social-planner solution, respectively, when productivity is equal to its mean value. The stochastic outcomes are represented by LF, SP_{τ^*} and $\text{SP}_{\bar{\tau}}$, where the latter two distinguish between the case when taxes are adjusted optimally (τ^*) and when the tax rate is kept constant at its steady-state value ($\bar{\tau}$). The parameters are $\beta = 0.97$, $\phi = 0$, $\psi = 0.9$, and ϵ_t is uniformly distributed with standard deviation σ_ϵ.

by roughly 12% in the $\overline{\text{SP}}$ economy to give rise to a level of welfare equal to the expected utility under laissez faire, and the required reduction remains fairly substantial at around 3% if γ is raised to 5. It can also be noted that the tax levels needed to offset the consumption externality for the specifications in table 1 are fairly high. According to equation (15), the steady-state tax rate is 19.4%, 48.5% and 77.6% for α equal to 0.2, 0.5 and 0.8, respectively. We will return to the question of high taxation in the next section when looking at Campbell and Cochrane's (1995) parameterization of their model.

Finally, the last column in table 1 reports on the welfare loss due to uncertainty in the laissez-faire economy. Specifically, we apply the formula in (17) after replacing the superscript $\overline{\text{SP}}$ by $\overline{\text{LF}}$ and setting j = LF. A comparison between the first and last column of the table indicates that the productivity shock gives rise to a higher risk premium in the laissez-faire economy relative to the social-planner outcome. Recall that the risk premia were invariant to the tax rate in the earlier case of "keeping up with the Joneses".

4 The Campbell-Cochrane utility function

We now turn to the utility function proposed by Campbell and Cochrane (1995) extended with a linear disutility term for labor. These preferences are then also given by our expression (3), but the benchmark level X_t is now a nonlinear function of current and past per-capita consumption as shown below.[11] A useful concept when studying this model is the "surplus con-

sistence parameter ϕ for the benchmark level X_t. Our results for $\phi \in \{0.3, 0.6, 0.9\}$ are therefore not reported here.

[11]Our notation differs from Campbell and Cochrane (1995) in order to stay consistent with the notation above. In particular, we use hats rather than small letters to denote logs, while small letters still denote the individual's choice variables. We use β instead of δ for the discount factor and we abstract from growth, i.e., their parameter g is here set

sumption ratio" defined for an individual as

$$s_t = \frac{c_t - X_t}{c_t},$$

and the upper case letter S_t will be used to denote the economy-wide value of s_t. In an equilibrium, S_t will of course be equal to s_t since all agents are identical.

Campbell and Cochrane postulate an implicit law of motion for X_t by writing a law of motion for S_t. Let $\hat{S}_t \equiv \log S_t$ be the logarithm of the economy-wide surplus consumption ratio, and likewise $\hat{C}_t \equiv \log C_t$ is the logarithm of average consumption across all agents. Abstracting from economic growth, it is then assumed that S_t evolves according to

$$\hat{S}_t = (1-\phi)\log \bar{S} + \phi \hat{S}_{t-1} + \lambda(\hat{S}_{t-1})\left(\hat{C}_t - \hat{C}_{t-1}\right), \tag{18}$$

where $\bar{S}$ is the steady-state value of S, and the function $\lambda(\hat{S})$ is given by

$$\lambda(\hat{S}) = \begin{cases} \bar{S}^{-1}\sqrt{1-2(\hat{S}-\log \bar{S})} - 1, & \hat{S} \leq \hat{S}_{max} \\ 0, & \hat{S} \geq \hat{S}_{max} \end{cases} \tag{19}$$

with $\hat{S}_{max} = \log \bar{S} + \left(1 - \bar{S}^2\right)/2$.[12] Given equation (18), one can back out the implied law of motion for X_t. Near the steady state, a log-linear approximation shows that the log of X_t is a moving average of past consumption in logs and it does not depend on contemporaneous consumption,

$$\hat{X}_t = \log(1-\bar{S}) + (1-\phi)\sum_{j=0}^{\infty} \phi^j \hat{C}_{t-j-1}.$$

For our purposes, the steady-value $\bar{S}$ can be thought of as a parameter in this model. Compare the model here to the model in section 3 where one

equal to zero.

[12]The purpose of Campbell and Cochrane's rather complicated preference specification is to assure that $c_t - X_t \geq 0$, and that the risk-free rate is constant when $\hat{C}_t$ is a random walk with drift.

would calculate $\bar{S} = (\bar{C} - \bar{X})/\bar{C} = (\bar{C} - \alpha\bar{C})/\bar{C} = 1 - \alpha$. Thus, by picking a value of $\bar{S}$, we are effectively choosing a particular preference specification, corresponding to the choice of $\alpha = 1 - \bar{S}$ before.

Taking X_t as given, the agent maximizes utility subject to the usual budget constraint. Analogously to the previous sections, the agent's optimal consumption is found to be

$$c_t = X_t + \left(\frac{\theta_t}{A}(1 - \tau_t)\right)^{\frac{1}{\gamma}},$$

which can also usefully be written as

$$(c_t s_t)^{-\gamma} = \frac{A}{(1 - \tau_t)\theta_t}. \tag{20}$$

Instead of solving the social planner's problem, we now turn to a more modest question. In a steady state, we ask what tax rate is needed to support the best possible constant consumption level. (The word 'constant' will soon be shown to be restrictive in terms of maximizing welfare.)

Proposition 3 (Campbell-Cochrane)
In a steady state with $\gamma > 1$, there exists a unique consumption level that cannot be improved upon through a once-and-for-all change to another consumption level. The steady-state tax rate supporting this best possible constant consumption level is given by

$$\bar{\tau} = \beta\frac{1 - \phi}{1 - \beta\phi}(1 - \bar{S}).$$

The derivation of this proposition is deferred to the appendix, and here we only note that the tax rate is the same as the steady-state tax rate for the usual linear version of the benchmark level in Proposition 2, using the analogy between $1 - \bar{S}$ and α.

η	$\bar{S}$	$\bar{\tau}$
2.372	1	0
3	0.791	0.109
5	0.474	0.273
10	0.237	0.396
20	0.119	0.458
50	0.047	0.495
∞	0	0.520

Table 2: This table varies the relative risk aversion η, calculating the implied value for $\bar{S}$ and thus the tax rate $\bar{\tau}$ needed to support the best possible constant consumption level. The other parameters are kept at $\beta = 0.973$, $\phi = 0.97$ and $\gamma = 2.372$.

It is also interesting to take a look at the quantitative tax implication of Proposition 3. Campbell and Cochrane use the parameters $\beta = 0.973$, $\phi = 0.97$, $\gamma = 2.372$ and $\bar{S} = 0.049$. For these parameters, we obtain

$$\bar{\tau} = 0.494,$$

i.e., almost 50 percent of labor income should be taxed away in steady state in order to support the best possible constant consumption level. Taken seriously, this would indicate that current labor taxes are too low in the United States, but about right in, say, the Netherlands or Sweden.

The parameters chosen by Campbell and Cochrane imply a relative risk aversion at the steady state of $\gamma/\bar{S} = 48.4$, which is probably necessary to explain the equity premium observation, but is arguably rather high. In table 2, we have varied the relative risk aversion, calculating the implied value for $\bar{S}$, and thus the tax rate $\bar{\tau}$. Since $\bar{\tau}$ is proportional to $(1 - \gamma/\eta)$, the tax rate $\bar{\tau}$ increases quickly with η, asymptoting to a level slightly above 50 percent.

Finally, we have refrained from using the word 'first-best outcome' simply since we have not presented the optimal solution to the social planner's problem. The nonconcave character of this maximization problem makes it analytically intractable, so here we rather use an example to demonstrate that consumption bunching can improve upon a constant consumption allocation in a stationary environment. We explore the welfare consequences of a one-period increase in consumption starting from a steady state with the best possible constant consumption level $\bar{C}$, as described in Proposition 3. That is, the environment being considered is one where the productivity shock is always equal to $\bar{\theta}$ and the agents are initially consuming $\bar{C}$. The agents are then induced to increase their work effort in a single period and the extra output is also consumed in that same period. In all future periods, work effort, output and thus consumption are again at their initial steady-state levels.

The x-axis in Figure 2 shows the size of the one-period consumption deviation as a percentage of $\bar{C}$, and the y-axis depicts the welfare gain associated with that policy. The welfare gain is measured as the percentage increase in the constant consumption stream $\bar{C}$ that would make an individual as well off as under the proposed temporary increase in consumption, i.e., the welfare calculation is analogous to formula (17). It is clear from the figure that there are one-time consumption deviations that can increase welfare. The intuition is that a temporary consumption increase acts as an 'investment' in the surplus consumption ratio S because of the persistence parameter ϕ in equation (18). But let us first consider what happens in the first period when the consumption hike takes place. Equation (18) shows how the log of S increases by the logdeviation in consumption multiplied by the steady-state value of the λ-function. Since the λ-function is decreasing in the surplus consumption ratio, it follows that the negative impact on S is smaller in the next period when consumption reverts back to $\bar{C}$. In fact, there is no effect at all if the log of the surplus consumption ratio has reached $\hat{S}_{max}$ in equation (19) when the λ-function becomes zero. (This critical point shows up as a kink on the curve in panel B of Figure 2.) In consecutive periods, welfare is

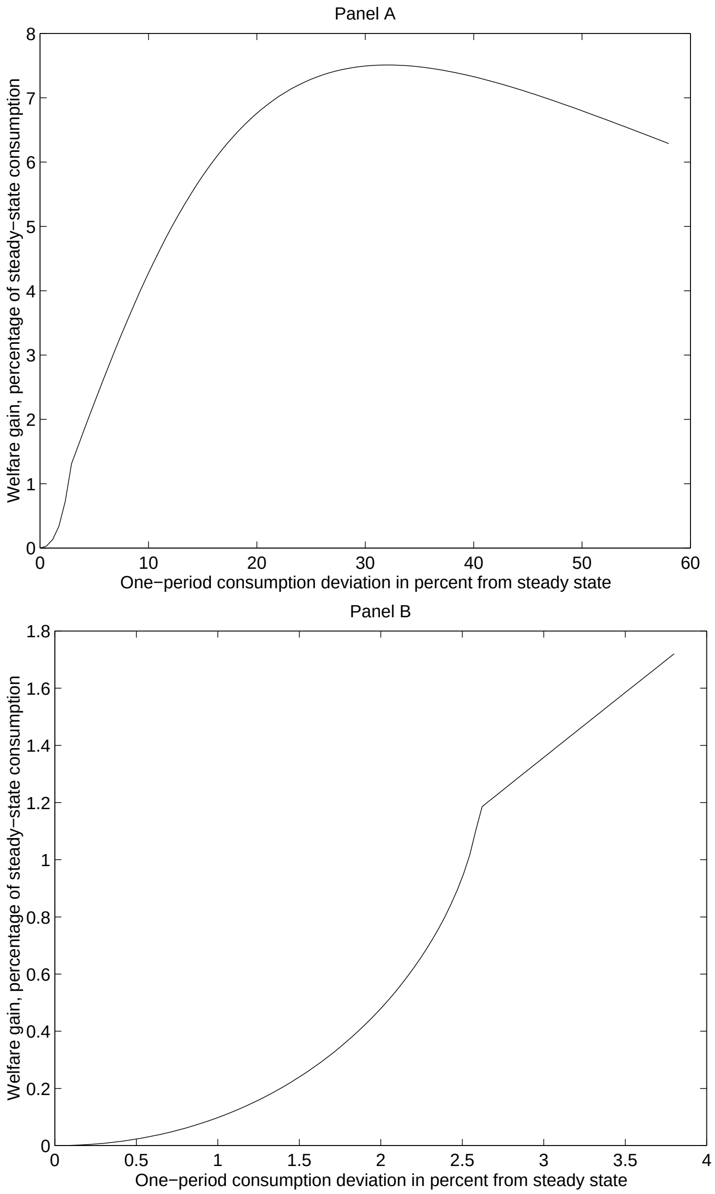

Figure 2: Welfare gain of a one-period consumption deviation from the best possible *constant* consumption level in Proposition 3. Panel B is a magnification of the left-hand portion of panel A. The parameters are $\beta = 0.973$, $\phi = 0.97$, $\bar{S} = 0.049$, and $\gamma = 2.372$.

positively affected by the slowly decaying surplus consumption ratio (while consumption is kept constant at $\bar{C}$). Concerning the parameterization in Figure 2, we have used Campbell and Cochrane's values mentioned above.

Figure 2 suggests that a first-best outcome for the Campbell-Cochrane utility function will involve consumption cycles even in an otherwise stationary environment. The social planner would like to exploit the law of motion for the surplus consumption ratio in order to increase the well-being of individuals. The existence of such welfare-improving cycles resembles the outcome in models with increasing-returns-to-scale production technologies. Murphy, Shleifer and Vishny (1989) discuss how an economy can take advantage of increasing returns through production bunching. It is efficient to produce at capacity some of the time and stay idle the remainder of the time. Similarly, Campbell and Cochrane's nonlinear version of the benchmark level imply that consumption bunching can be welfare enhancing.

5 Conclusions

The purpose of this paper has been to examine the role for tax policies in economies with "catching-up-with-the-Joneses" utility functions. These utility functions give rise to consumption externalities, but taxation can be used to get back to the first-best solution. The optimal tax policy turns out to affect the economy countercyclically via procyclical taxes. When the economy is "overheating" due to a positive productivity shock, a welfare-maximizing government should raise taxes to "cool down" the economy. Likewise, taxes should be cut in recessions to "stimulate" the economy by bolstering consumption. Thus, models with catching-up-with-the-Joneses utility functions call for traditional Keynesian demand management policies. We also used parameter values from Campbell and Cochrane (1995) to illustrate that the necessary labor taxes can be very high, in the order of 50 percent. However, Campbell and Cochrane's nonlinear version of the benchmark level in the catching-up-with-the-Joneses preferences has the implication that consumption bunching can be welfare enhancing. An example was used to illustrate how welfare can be improved upon in their framework by inducing business cycles in an otherwise stationary environment.

Appendix

Proof of Proposition 3

Let us start our argument from an arbitrary initial steady state with consumption $\bar{C}$ (and a surplus consumption ratio of $\bar{S}$). At time $t = 0$, we consider an alternative future consumption allocation of $C_t = C_0$ for all $t \geq 0$. According to equation (18), the associated sequence of surplus consumption ratios can be expressed in log form as

$$\begin{aligned}\hat{S}_0 &= \log \bar{S} + \bar{\lambda}(\hat{C}_0 - \log \bar{C}),\\ \hat{S}_t &= (1-\phi)\log \bar{S} + \phi\,\hat{S}_{t-1} = (1-\phi^t)\log \bar{S} + \phi^t\,\hat{S}_0, \quad \text{for } t \geq 1,\end{aligned}$$

where $\bar{\lambda} \equiv \lambda(\log \bar{S}) = \bar{S}^{-1} - 1$. That is, the sequence of surplus consumption ratios is given by

$$S_t = \bar{S}\left(\frac{C_0}{\bar{C}}\right)^{\phi^t\,\bar{\lambda}}, \qquad \text{for } t \geq 0.$$

The life-time utility associated with such an alternative consumption allocation is

$$\begin{aligned}&\sum_{t=0}^{\infty}\beta^t\left(\frac{(C_0\,S_t)^{1-\gamma}-1}{1-\gamma} - A\frac{C_0}{\theta}\right)\\ &= \sum_{t=0}^{\infty}\beta^t\left(\frac{\left(\frac{C_0}{\bar{C}}\right)^{(1-\gamma)(1+\phi^t\,\bar{\lambda})}(\bar{S}\bar{C})^{1-\gamma}-1}{1-\gamma} - A\frac{C_0}{\theta}\right).\end{aligned}$$

For $\gamma > 1$, this expression is concave in C_0 and the first-order condition with respect to C_0 is

$$\sum_{t=0}^{\infty}\beta^t\left((1+\phi^t\,\bar{\lambda})\left(\frac{C_0}{\bar{C}}\right)^{(1-\gamma)(1+\phi^t\,\bar{\lambda})-1}\bar{S}^{1-\gamma}\bar{C}^{-\gamma} - \frac{A}{\theta}\right) = 0. \qquad (21)$$

For any initial steady state $\bar{C}$, equation (21) can be used to solve for the best C_0 when we are constrained to only consider once-and-for-all changes in the consumption level. To find the unique constant consumption level that cannot be improved upon in this way, we solve for $\bar{C}$ in equation (21) such that $C_0 = \bar{C}$. The best possible constant consumption level is then found to be

$$\bar{C} = \left(\left(1 + \frac{1-\beta}{1-\beta\phi} \bar{\lambda} \right) \frac{\bar{\theta}}{A} \right)^{\frac{1}{\gamma}} \bar{S}^{\frac{1}{\gamma}-1}. \tag{22}$$

To support this consumption allocation in a competitive equilibrium, we first solve for the tax rate in the agent's first-order condition in equation (20),

$$\tau_t = 1 - \frac{A(c_t s_t)^\gamma}{\theta_t}$$

with a steady-state value of

$$\bar{\tau} = 1 - \frac{A(\bar{c}\bar{s})^\gamma}{\bar{\theta}}.$$

After substituting equation (22) and $\bar{\lambda} = \bar{S}^{-1} - 1$ into this expression, we arrive at the steady-state tax rate in Proposition 3.

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