Rational Bubbles and Fractional Alternatives

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Abstract

This paper suggests a new and more flexible framework for studying the existence of rational bubbles in stock prices. The present value model provides the robust no rational bubbles restriction of a stationary price-dividend ratio. The validity of this restriction has previously been investigated, but we extend the test procedure to allow for fractionally integrated alternatives. Thus, the price-dividend ratio may be a stationary process, where the mean-reversion is at a much slower (persistent) rate than that of stationary ARMA specifications. This pesistence may be hard to detect using traditional random walk tests. Indeed, when testing the no rational bubble restriction on US and Swedish data this distinction is important. For Sweden we conclude that the price-dividend ratio is ruled by a fractionally integrated process (no rational bubble), whereas it follows a unit root process for the US (a rational bubble). Using Dickey-Fuller type tests the unit root hypothesis cannot be rejected for any of the markets.

Key words: Present value models; Unit roots; Fractional Integration

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1. Introduction

During the last decade the existence or non-existence of rational bubbles in stock prices has been discussed extensively in the academic literature. The present value model states that the price of a stock is determined by the expected value of discounted future dividends. The term "bubbles" is a general expression for deviations from the fundamental price, that is in a rational expectations equilibrium the stock price equals the fundamental price and a possible "rational" bubble.

Several approaches have been suggested to empirically test for rational bubbles. In this paper we follow the line of work initiated by Campbell and Shiller (1987) who presents a (no rational bubbles) cointegration restriction between prices and dividends under the assumption of a constant discount factor. As an extension, Crain (1993) suggests a robust no rational bubbles restriction that allows for a stochastic discount factor. If the discount factor and dividend growth are stationary stochastic processes, then the fundamental price/dividend ratio is stationary and hence mean-reverting. Consequently the null hypothesis of rational bubbles may be investigated by testing for a unit root in the price/dividend ratio. Using US data and the Dickey-Fuller (1979) test, Crain concludes the possible existence of rational bubbles.

In the direction of Baillie (1996), this paper suggests a more flexible framework, to analyze the rational bubbles hypothesis by allowing for fractionally integrated alternatives. This flexibility permits fractionally integrated processes that are mean-revertings in the sense that the effect of a shock dies out at a slower hyperbolical rate compared to the geometric ARMA decay considered in earlier studies. We test for a fractional difference using the periodogram regression test of Geweke and Porter-Hudak (GPH, 1983).

The rational bubbles hypothesis is tested on annual price and dividend data from the US and Swedish stock market covering the years 1871 to 1997 and 1918 to 1996 respectively. The Dickey-Fuller test cannot reject the unit root null hypothesis for any of the markets. Furthermore, the GPH test concludes the possibility of a rational bubble for the US data, whereas, it rejects the unit root null hypothesis for Sweden and thus suggests a mean-reverting behavior. Therefore, we find no support for a rational bubble in the Swedish stock market.

The paper unfolds as follows: Chapter 2 presents the theory of rational bubbles and Chapter 3 the testing procedure. The fourth chapter contains the empirical investigation and Chapter 5 concludes the paper.

2. A Testable Restriction for Present Value Models

This section gives a brief introduction to the theory of stock prices and derives a testable restriction of no rational bubbles.

In general, the fundamental value of a stock P_t^* can be defined as the expected value of the sum of the discounted future dividends $D_{t+j}, j = 1, ..., J$, accordingly,

$$P_t^* = \lim_{J\to\infty} E\left[\sum_{j=1}^{J}\left(\prod_{k=1}^{J} m_{t+k}\right) D_{t+j}\,|\Omega_t\right], \tag{2.1}$$

where E is the expectations operator, m_{t+k} the (possibly stochastic) discount factor at time $t+k$ and Ω_t the information set available to the investor at time t. To assure that only discounted dividends contribute to the transversality condition, the following relation

$$\lim_{J\to\infty} E_t \prod_{i}^{J} m_{t+i} P_{t+J}^* = 0, \tag{2.2}$$

must hold. If stock prices are determined in a rational expectations equilibrium, the stock price P_t is the discounted value of the price plus the next period dividend, that is

$$P_t = E\left[m_{t+1}\left\{P_{t+1} + d_{t+1}\right\}|\Omega_t\right]. \tag{2.3}$$

Solving equation (2.3) recursively gives

$$P_t = P_t^* + B_t,$$

i.e., the price of the stock at time t is determined by the fundamental value P_t^* and, possibly, a rational bubble, $B_t = E\left[\prod_{i}^{J} m_{t+i} P_{t+J}\,|\Omega_t\right]$. For the bubble to be a viable outcome it is expected to continue to expand in the next period,

$$B_t = E\left[m_{t+1} B_{t+1}\,|\Omega_t\right]$$

In the light of the fact that the bubble must expand to survive, a decomposition of the market price into the fundamental price and a bubble is not hard to justify. However, as Crain (1993) points out, if the fundamental price P_t^* is non-stationary a robust decomposition is very hard indeed. Moreover, Crain suggests the following procedure to derive a robust testable restriction under the assumption of no rational bubbles. Divide the fundamental price P_t^* with the current dividend process gives

$$\frac{P_t^*}{D_t} = \lim_{J\to\infty}\sum_{j=1}^{J} E\left[\prod_{k=1}^{J} m_{t+k}\frac{D_{t+j}}{D_{t-1+j}}\,|\Omega_t\right] \tag{2.4}$$

Expression (2.4) then states a very robust restriction for the fundamental price dividend ratio, which depends on the process for the discounted dividend growth, $m_{t+k}D_{t+j}/D_{t-1+j}$. Note that the ratio P_t^*/D_t is a real-valued measurable function of the driving process. Therefore, it follows that if the discounted dividend growth rate is stationary, so is also P_t^*/D_t (see e.g. Durret (1996, p.336)). The restriction does not require a specified discount factor, but merely a stationary discount rate and a stationary dividend growth rate.

3. Testing

Crain (1993) tests the stationarity restriction (2.4) empirically by performing unit root tests on price-dividend ratios for aggregate US stock indices. This test can also been interpreted as a CI(1,1) cointegration test for prices and dividends with the cointegrating vector preset to (1,-1). Using the Dickey-Fuller (1979) test, Crain cannot reject the unit root null at the five percent level for any of the series analyzed. The failure to reject may be an effect of the Dickey-Fuller tests' quite low power against fractional alternatives, see Diebold and Rudebusch (1991). Alternatively, as Baillie (1996) indicates, is that a form of CI(1,1-d) fractional cointegration is appearant, that is the equilibrium error follows a fractionally integrated process, still with the cointegrating vector (1,-1). We investigate that more flexible alternative to the null, namely that the price-dividend ratio is fractionally integrated.

The idea of fractional integration extends the common ARMA methodology. Consider the ARMA specification

$$\phi(B)\, x_t = \theta(B)\, a_t, \tag{3.1}$$

where the members of the sequence $\{a_t\}$ are iid with finite variance. The process is stationary, and thus mean-reverting, if all roots of the autoregressive polynomial $\phi(B)$ is outside the unit circle. When a process is stationary in the levels, the process is said to be integrated of order zero (denoted $I(0)$), and if it is stationary in the first differences the process is $I(1)$. Instead of using the knife-sharp distinction between $I(0)$ and $I(1)$ we may consider fractional integration, that is allowing for non-integer differencing powers. A fractionally integrated ARMA, ARFIMA[1], process $\{x_t\}$ is generated by

$$\phi_p(B)(1-B)^d x_t = \theta_q(B)\, a_t, \tag{3.2}$$

where a_t again is iid, and d is allowed to assume any real value. Let all roots of ϕ_p be outside the unit-circle, if $d > 0$ the process is ruled by long memory. When

[1]The properties of the fractionally integrated ARMA model are presented by Granger and Joyeux (1980) and Hosking (1981).

$d < 1$, the process is mean-reverting and when $d < 0.5$, the process is covariance stationary. Compared to ARMA processes, shocks to an ARFIMA process die out slowly, i.e. they are persistent.

In this study we test for a fractional difference using the test of Geweke and Porter-Hudak (GPH, 1983). The GPH test is based on the following non-parametric periodogram regression equation:

$$\ln\left\{I_x\left(\omega_j\right)\right\} = \alpha - d\ln\left\{4\sin^2\left(\omega_j/2\right) + \eta_j\right\},\ j = 1,\ldots g(T), \tag{3.3}$$

where $I_x\left(\omega_j\right)$ is the periodogram across the harmonic frequencies $\omega_j = 2\pi j/T$. If the number of ordinates $g\left(T\right)$ is chosen properly, the ordinary least squares (OLS) estimator of d is consistent and the distribution of $\left(\hat{d}_{OLS} - d\right)/SE\left(\hat{d}_{OLS}\right)$ asymptotically normal. We use the known variance of η, that is $\pi^2/6$, to increase the efficiency of the test and $g\left(T\right) = T^v$, $v = 0.5$ and 0.9. $g\left(T\right) = T^{0.9}$ maximizes the power of the test, see Table 5.2 in Appendix.

4. Empirical Evidence

This section presents the results of price-dividend ratio unit-root tests on annual data for the US and Swedish stock markets. The US data, ranging from 1871 to 1997, are the well-known Standard and Poor 500 (S&P500) index. To approximate movements in the Swedish market we use an index constructed by Frennberg and Hansson (1992), which includes most of the stocks quoted on the Stockholm Stock Exchange and covers the period 1918 to 1996. The log of the equity price and dividend series are shown in Figures 5.1 and 5.2.

Table 5.1 presents the results of unit root tests against stationary autoregressive alternatives. According to the test statistics, the unit root hypothesis can not be rejected, even at the 10 percent level of significance, no matter of selected augmentation lag. Therefore, we cannot, using ADF tests, rule out the existence of a bubble in any of the stock indices.

For the US data, the fractional integration tests, see also Table 5.1, support the result of the Dickey-Fuller tests and hence we conclude the possibility of a rational bubble. However, for the Swedish series the unit-root null hypothesis is rejected at the five percent level. Moreover, also the hypothesis of $d = 0$ is rejected in favor of a fractional process. Subsequently, the GPH tests suggest that the Swedish price-dividends ratio is ruled by a mean-reverting fractionally integrated process. The hypothesis of a rational bubble, therefore, is not supported in the data.

5. Conclusions

This paper investigated the existence of rational bubbles in stock prices by testing for a unit root in the price-dividend ratio. If the expected present value model holds this ratio should, imposing minimal structure, be stationary and failure to reject the null of a random walk suggests the possible existence of a rational bubble. In contrast to previous studies that assume a stationary ARMA alternative to the unit root null, our test incorporates fractionally integrated processes. Hence, we allow for the equilibrium relation between stock prices and dividends to revert, to its long run mean, at a very slow rate.

Using long annual data series from the US and Swedish stock markets we cannot rule out the possible existence of a rational bubble using an ordinary unit root test. However using the more flexible framework proposed in the paper, we find the Swedish price-dividend ratio to be ruled by a fractionally integrated, mean reverting, process and thus conclude the non-existence of a rational bubble, whereas the unit root hypothesis holds still for the US.

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Table 5.1: ADF tests for unit roots.

	ADF $k=$							GPH, H_A	
	0	1	2	3	4	6	8	$d>0$	$d<1$
S&P500	**-2.77**	-2.67	-2.05	-2.21	-2.25	-1.99	-2.01	8.31	-0.03
Stockholm	-1.88	-2.57	-1.77	**-1.00**	-0.93	-0.66	-0.25	7.39	-2.23

For the ADF test, the table reports the t-statistic for $\rho = 0$ in the equation $\Delta \log(p/d)_t = \alpha + \beta t + \rho \log(p/d)_{t-1} + \sum_k \gamma_k \log(p/d)_{t-k} + \varepsilon_t$, k is the augmentation lag and the numbers in bold face are the lag selected by the AIC. GPH test; the table presents the t-statistic for d=0 vs d>0 and d=1 vs d<1. The periodogram regressions are based on $T^{.9}$ ordinates. The critical values are given in Tables 5.3 and 5.4 (Appendix).

Figure 5.1: S&P500 annual stock price and dividend series 1871-1997.

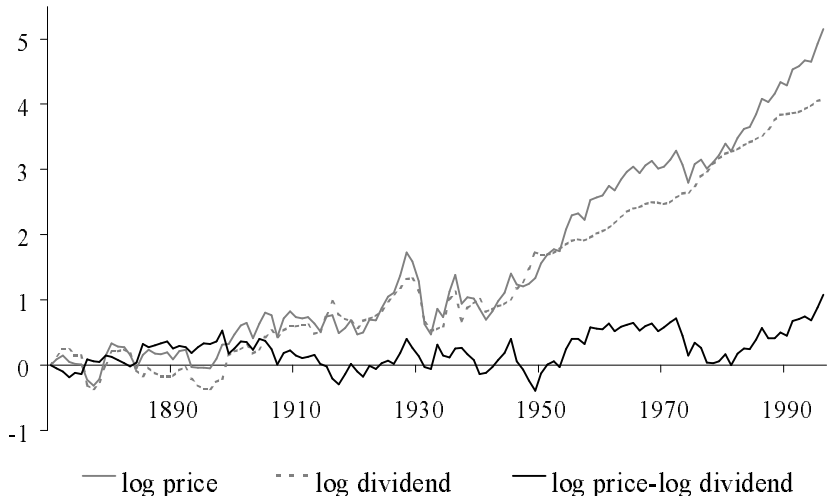

Figure 5.2: Swedish annual stock price and dividend series 1918-1996.

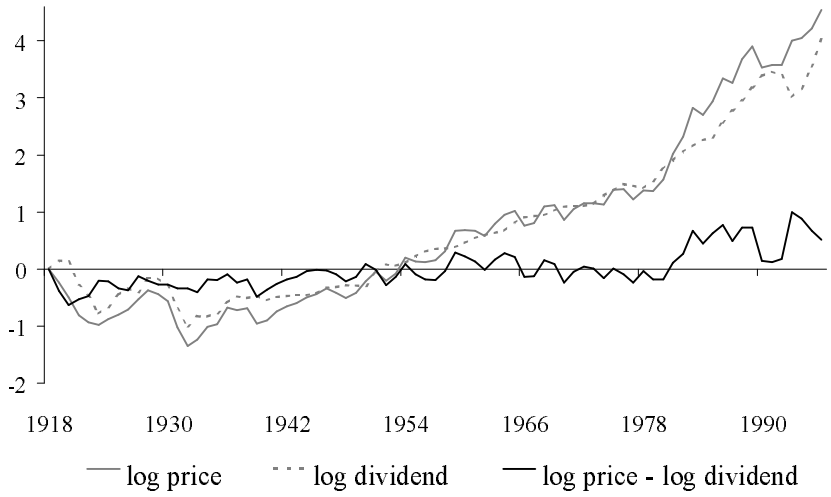

Table 5.2: Power of the GPH test.

	$T = 79$				$T = 127$			
	v				v			
d	.50	.65	.80	.90	.50	.65	.80	.90
.95	6.5	8.0	9.0	9.9	6.3	8.0	9.8	1.1
.85	8.8	14.8	22.1	26.2	11.3	19.4	32.2	38.0
.75	13.6	26.0	46.0	52.4	18.0	36.9	65.3	74.4
.65	19.7	41.5	71.5	77.6	27.9	60.1	90.5	94.8
.55	27.1	58.8	88.9	92.6	40.7	78.9	98.5	99.5
.45	34.5	73.7	96.9	98.0	51.2	90.7	99.9	100
.35	42.9	83.2	99.1	99.6	60.5	95.7	100	100
.25	49.4	89.5	99.7	99.9	67.3	97.7	100	100
.15	54.3	93.2	99.9	99.9	71.9	98.6	100	100
.05	56.6	94.3	100	100	72.6	98.8	100	100

Table 5.3: Critical values for the ADF and GPH tests, T=79.

	ADF						
level	0	1	2	3	4	6	8
1	-4.13	-4.24	-4.16	-4.20	-4.23	-4.27	-4.29
5	-3.52	-3.56	-3.53	-3.56	-3.58	-3.59	-3.63
10	-3.21	-3.23	-3.21	-3.25	-3.23	-3.26	-3.32
	GPH						
	d=0 vs d>0				d=1 vs d<1		
1	2.57				-2.68		
5	1.82				-1.79		
10	1.44				-1.33		

The critical values are obtained through 50,000 replicates.

Table 5.4: Critical values for the ADF and GPH tests, T=127.

	ADF						
level	0	1	2	3	4	6	8
1	-4.11	-4.11	-4.06	-4.17	-4.12	-4.13	-4.10
5	-3.48	-3.50	-3.50	-3.51	-3.50	-3.52	-3.54
10	-3.18	-3.21	-3.19	-3.19	-3.18	-3.20	-3.23
	GPH						
	d=0 vs d>0				d=1 vs d<1		
1	2.49				-2.65		
5	1.81				-1.80		
10	1.43				-1.36		

See note to Table 5.3.