Endogenous timing of investments yields modified Stackelberg outcomes*

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May, 1998.

Abstract: This paper deals with capacity constrained price competition in a duopoly model. The model resembles that in Kreps and Scheinkman (1983), but the timing of the investment/capacity choice is endogenous. In equilibrium, one of the firms will invest to become the Stackelberg leader, although the ratio between the leader's and the follower's capacities is smaller than in the standard Stackelberg outcome. Capacity is built too early, resulting in welfare losses. The leader and the follower will earn equal profits, except when capacity costs are small.

Keywords: Bertrand, Cournot, Stackelberg, strategic investment, excess capacity, games of timing, endogenous entry, rent equalization.

JEL classification: L13, D43

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* Work on this paper began while I was visiting the University of Wisconsin at Madison. I am grateful for their hospitality, and for valuable suggestions from Raymond Deneckere, Tore Ellingsen, Ante Farm, Larry Samuelson, Jörgen Weibull and participants at a seminar at Stockholm School of Economics. Financial support was provided by the Swedish Council for Forest and Agricultural Research.

1 Introduction

It is often claimed that firms in capital intensive industries invest too much too early. This appears to be true in the pulp and paper industry, where firms invest heavily in new machines (costing up to $400m a piece or $1.5 billion for a large integrated plant) when profits are high. Typically, the new equipment is completed after the business cycle's peak; the effect of all additions to capacity is to depress prices and protract the downturn. Only when market demand has grown enough to absorb the new capacity will prices and profits rise again. Similar boom-and-bust cycles are reported to characterise other markets, such as the commercial-property market and the shipping and airline industries.[1]

When demand is predicted to grow fast, firms in some industries appear to be too quick in installing new capacity, perhaps in an attempt to pre-empt rivals. Such behaviour has been reported for the Asian oil refinery and commodity chemical industries, as well as for the semiconductor industry in that region. All these industries are capital intensive: a new semiconductor fabrication plant costs around $1 billion, an oil refinery costs around $1.5 billion and a large petrochemical complex may be even more expensive.[2]

If the production technology employs inputs in fixed proportions, over-investment will result in excess capacity; with a variable proportions technology, there will be excess capital. In both cases, the purpose of investing excessively may be to deter entry (Spence, 1977). However, the entry-deterring effect of idle capacity hinges on a threat that may not be credible: that the excess capacity will be used once entry occurs (Dixit, 1980).[3] With variable proportions, each firm has an incentive to invest in more than the cost-minimising amount of capital, if this lowers marginal costs and if capital is sunk, since this is likely to improve performance in the

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[1] ***The Economist*** comments on the property market (February 20, 1993, p. 69, August 12, 1995, p.72) and the pulp and paper industry (November 11, 1995, p. 78).

[2] See the Economist on the semiconductor industry (January 20, 1996, pp. 75-76 and November 9, 1996, p. 111), oil refineries (October 12, 1996, pp. 73-74) and the chemical industry (March 13, 1993, pp. 25-30).

[3] Whether the threat to use the excess capacity is credible or not depends on the shape of the demand curve and on whether price or quantity competition is anticipated (Bulow, Geanakoplos and Klemperer, 1985). In models of incumbent-entrant interaction, where the incumbent firm invests to reduce its marginal cost of production, the standard results (see Gilbert, 1989) are the following. If the subsequent competition is of the Cournot type, the firm will over-invest. If the subsequent competition is of the Bertrand type, the firm will under-invest, unless it is more profitable to deter entry by over-investment.

ensuing oligopoly game (Dixit, 1980). Over-investments of these types may be one explanation for the tendency of oligopolistic markets to exhibit excess capacity.[4]

This article focus on a third rational for firms to over-invest: the race to be the first to accommodate an expected market opportunity by installing additional capacity. Firms may invest strategically, in order to occupy the ”Stackelberg-leader” position.

The boom-and-bust cycles observed in some of the industries discussed above are difficult to reconcile with profit maximising behaviour. Why would firms sink resources in assets that become productive in the trough of the business cycle - in particular since investment goods are likely to be more expensive during the boom periods? In other words: why is the timing of investments so bad? A naive, although unsatisfactory, explanation is that managers extrapolate recent trends. Another interpretation is that firms *have* to invest early in order to pre-empt rivals. At the time of investment, although the firm anticipates the coming downturn, it also anticipates the subsequent upturn. A firm that tries to delay the completion of an investment until the next upturn would be pre-empted by its rivals.

Most models of entry deterrence and other strategic oligopoly interactions impose an exogenous asymmetry between firms. One firm is assigned the incumbency position, and the other(s) enter(s). Only the incumbent firm has the option to make strategic investments. In contrast, the model described in this paper treats all firms symmetrically. In this respect, the model resembles that of Fudenberg and Tirole (1985), which is used to analyse pre-emption in the adoption of a new technology. (See also Riordan and Salant, 1994, and the references therein.)

Another closely related paper is Kreps and Scheinkman’s (1983). There, two firms simultaneously choose capacities in the first period. In the second period, the two firms engage in Bertrand competition. Even though the firms compete in prices, the unique equilibrium is the Cournot outcome. This important result reconciles the more plausible outcome of the Cournot model with the more appealing assumptions of the Bertrand model.[5]

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[4] The observed gap between output and reported capacity in many industries may also be the result of an optimal response to fluctuating demand, in which case the term ”over-investment” would be inappropriate.

[5] The results, however, must be modified if the Bertrand stage is repeated, and not just one-shot (Brock and Scheinkman, 1985), or if alternative assumptions are made concerning the rationing rules (Davidson and Deneckere, 1986). The implications of alternative rationing rules are discussed in Section 5. Herk (1993) shows that if consumers incur switching costs, Kreps and Scheinkman's results are more stable with respect to the rationing rules.

The two-period model has become the standard instrument for analysing strategic interaction. A much smaller number of studies analyse dynamic, multi-period rivalry between established firms that must invest in capacity in order to produce.[6] A few papers present models where a small number of symmetric firms (often two) compete in investments in such a way as to reduce profits to zero (e.g. Eaton and Lipsey, 1980, Gilbert and Harris, 1984, Maskin and Tirole, 1988).

None of the above models, however, appears to offer a credible explanation of premature, or excessive, investment in mature markets with both (all) firms active in both (all) periods.[7] The aims of the present paper are two-fold: to seek an explanation for the existence of premature and possibly excessive investments, in a duopoly market;[8] and to explore the consequences of endogenizing the timing of the investment decision in a model similar to that of Kreps and Scheinkman.[9]

A symmetric duopoly model is used to analyse strategic investments when demand is expected to increase. Two firms compete for Stackelberg leadership in a market where transactions take place in a single period. The firms are restricted to making a single investment each, but the investment can be made at any time prior to the transaction period. In equilibrium, one of the two firms accepts the Stackelberg follower position, but both firms will (ex ante) earn equal rents. The leader must invest prematurely in order to gain the leader position, but this entails a cost which exactly offsets the leader's gain from having a larger share of the market.

With symmetric firms and endogenous investment times, it is natural to assume that rent equalisation will occur. However, it is shown that for small capacity costs, this may not hold ex post. Intuitively, this can be understood as follows. Depending on the level of the cost of acquiring capacity, there are different classes of outcomes (corresponding to the equilibrium in the Stackelberg game, and to what is sometimes referred to as the "judo" equilibrium - see note 18). The earlier the leader invests, the higher the capacity cost. Therefore, at some point

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[6] See Fudenberg and Tirole, 1986, for an early review.

[7] Excess capacity or excessive investments may of course result in models of learning-by-doing and patent races.

[8] An early paper with a similar objective is Rao and Rutenberg, 1979. More recent papers are Mills (1988), Robson (1990), Holden and Riis (1994), Anderson and Engers (1994) and Pal (1996). None of these papers, however, uses the capacity constrained Bertrand-competition model applied in this paper.

[9] Kreps and Scheinkman's paper has spurred a number of papers that explore the consequences of modifying some of the assumptions of the original paper. Hviid (1990) assumes that firms set prices sequentially, in an endogenously determined order, and allows for demand uncertainty. Deneckere and Kovenock (1992) also study endogenous ordering in a price-setting game, with exogenous capacities. Deneckere and Kovenock (1994) extend Kreps and Scheinkman's model by allowing the firms to have different marginal costs of production; Allen, Deneckere, Faith and Kovenock (1994) extend the last-mentioned paper by also allowing for sequential entry in a three-period model; and Allen (1993) studies sequential entry in a three-period model with equal marginal costs.

in time, the leader's preferences between the different equilibria will change, resulting in a discrete shift in the size of the optimal investment. This means that there may be a time t^* before which both firms strictly prefer to follow, and after which they strictly prefer to lead. For rent equalisation to occur in this case, there must be a positive probability of simultaneous entry at time t^*. It follows that rent equalisation need not hold ex post. In effect, there is an investment war around time t^*. Holden and Riis (1994) analyse such investment wars. They obtain a discontinuity in the follower's pay-off by assuming that entry may be deterred. In contrast, the model of the present paper generates discontinuities even if this is impossible, which increases the applicability of the results.

2 The Model

Assume two competing firms in a market where a new good will suddenly be demanded at, and only at, a time normalised to *t*=0. Prior to that time, the firms choose capacities by making irreversible investments. Investments are assumed to be instantaneous, and there is no depreciation. Each firm can only invest once. This can be justified if there are increasing returns to scale and incremental investments are impossible. Time is continuous, and the firms' detection lag for observing an investment by the other firm is infinitesimal.[10]

For the sake of simplicity, the inverse aggregate demand function for the good produced by the firms is assumed to be linear and given by:

$$p = \max\left[a - x_1 - x_2, 0\right] \tag{1}$$

where *p* is the price, x_1 is firm 1's supply and x_2 is firm 2's supply. For the same reason, assume that the marginal cost of production is constant and equal between the two firms. Without loss of generality, it can be normalised to zero. The common rate of discount is *r*. The investment cost is given by $c(q_i) = cq_i$ where q_i is firm i's capacity measured in terms of maximal potential output and *c* is a (positive) constant, with $c < a$. The objective of firm *i* is to maximise profits:

$$\underset{p_i, q_i, t_i}{Max}\, \pi_i(p_i, p_j, q_i, q_j, t_i) = p_i x_i(p_i, p_j, q_i, q_j) - cq_i e^{-rt_i} \tag{2}$$

st $q_i \geq 0, t_i \leq 0, p_i \geq 0$

[10] I.e., if firm *i* invests at time *t*, firm *j, j≠i*, will be aware of that fact, and will be able to condition strategies upon firm *i*'s investment, at time *t+ε* for all ε>0.

where t_i is the time of investment, $x_i(\cdot)$ now denotes demand as a function of prices and capacities and *i,j*=1,2; *i*≠*j*. The solution concept is subgame-perfect equilibrium. Each firm's pure strategy set is a map from its set of information sets to its action set, where an information set consists of all previous actions chosen by the other firm - the history of the game. Thus a pure strategy specifies which action should be chosen in response to a certain information set, or history.

For technical reasons, I make the following assumption:

Assumption 1: Time t=0 consists of two atoms of time, during the second of which the market is realised. The first atom is the last instance when a firm can invest and participate in the market.

In Section 4, I make the simplifying assumption that an "auctioneer" sets the price at the market clearing level, $p = \max[a - q_1 - q_2, 0]$, at time 0. Then firm *i*'s action space is given by $T_i \times Q_i = (-\infty, 0] \times [0, \infty)$, where each action consists of a time of entry, $\mathrm{t}_\mathrm{i}^* \in (-\infty, 0]$, and a capacity choice $\mathrm{q}_\mathrm{i} \in [0, \infty)$. In Section 5, I assume instead that at time 0 the firms compete in prices, given their installed capacities. Firm *i* then has a second action set $P_i = [0, \infty)$, where each action consists of a price choice $p_i \in [0, \infty)$. The timing of the game is thus the following: First, one firm (firm 1, say) chooses a capacity. This capacity is observed by the other firm (firm 2), which subsequently chooses its capacity. Finally, the price is determined by the auctioneer (Section 4), or the two firms simultaneously choose prices (Section 5). The possibility that both firms invest simultaneously is not ruled out. In this latter case, there is no distinction between firms 1 and 2, and neither of the firms can condition its capacity choice on the other firm's capacity.

3 Preliminary intuition

In the game outlined above, the socially optimal time of investment is at t=0. Assume initially that both firms intend to invest at time zero, and that each firm realises that its own choice of capacity will influence the other firm's capacity. If one of the firms invests at time -ε, where ε is a small number, it can choose the Stackelberg leader quantity and achieve (virtually) the Stackelberg profit. Since investments are irreversible, the other firm's best reply is to choose the Stackelberg follower capacity (except for small capacity costs; see Section 5). If the other firm is rational, however, it will try to pre-empt its rival by installing the Stackelberg leader capacity at time -2ε. The argument can be repeated; therefore a subgame-perfect equilibrium in the present game cannot have both firms investing at times close to $t = 0$. The leader must invest sufficiently early so that it does not earn higher profits than the follower.

The leader's profit cannot fall below the Stackelberg follower profit level, since it will never be in the interest of the leader to choose a capacity in excess of the Stackelberg leader capacity (again, except for small capacity costs). Both firms will in most cases earn the same profit - i.e., in the subgame-perfect equilibrium, there will be rent-equalisation. The firm that invests first chooses such a capacity at such a date that the other firm is indifferent between choosing the same capacity at an infinitesimally earlier date, and acting as the follower. In general, the firms will earn a profit in excess of the Stackelberg follower profit. This is so, because the more premature the leader's investment is, the more costly each unit of capacity will be, compounded to present value. Thus the leader's capacity will fall short of the Stackelberg leader capacity, and the follower's capacity will exceed the Stackelberg follower capacity.

The solution will be characterised by excess capacity before time zero. Output will be higher than in the Cournot solution (but lower than in the traditional Stackelberg outcome), and the price and the profit levels will be lower. Welfare is improved, relative to the Cournot case, through increased output, but the early investment results in a welfare loss. The net effect is negative. Compared with the monopoly outcome, however, welfare is greater.

For small capacity costs, rents will not be equalised ex post. This type of equilibrium has the leader choosing a capacity in excess of the Stackelberg leader capacity, and the two firms will use mixed pricing strategies. Ex post rent equalisation will not necessarily occur, because there is a discontinuity in the follower's pay-off.

4 Imposed market clearing

In this section, it is assumed that the market clears with full capacity utilisation, i.e., that $p = \max[a - q_1 - q_2, 0]$ and that $x_i = q_i$ if $p > 0$.[11] It is easy to see that the following proposition holds:

Proposition 1. The second firm to invest will invest at *t*=0, unless both firms accidentally invest at the same time *t'*, *t'*<0.

[11] If the cost of capital is sufficiently high, the equilibrium price in the Bertrand model of Section 5 will be the market-clearing price. Note also that with imposed market clearing, we do not have to make assumptions on how consumers who cannot be satisfied by the firm offering the lowest price are rationed.

Proof. This follows immediately from the objective function of the firm, given by Eq. (2). ◊

Let q_1 be the capacity choice of firm 1 at time $t < 0$. Firm 2 will then invest at time 0 to maximise:

$$\pi_2 = (a - q_1 - q_2)q_2 - cq_2 \tag{3}$$

assuming that $q_1 < a - c$. From the first order condition:

$$q_2 = (a - c - q_1)/2 \tag{4}$$

Substituting Eqs. (1) and (4) into (2) and differentiating with respect to q_1 gives:

$$q_1 = \frac{a+c}{2} - ce^{-rt} \tag{5}$$

Again substituting into Eq. (4) gives:

$$q_2 = \frac{a + 2ce^{-rt} - 3c}{4} \tag{6}$$

Firm 1 must invest at such a date that its profit, when investing according to (5), yields the same profit as investing according to (6) at time 0 - otherwise, it will be pre-empted by firm 2. Equalising Eqs. (2) and (3), substituting Eqs. (5) and (6) for q_1 and q_2 and solving for *t*, we have:

$$\mathrm{t}^* = -\frac{1}{\mathrm{r}}\ln\left(\frac{(2-\sqrt{2})\mathrm{a} + (2+3\sqrt{2})\mathrm{c}}{2(\sqrt{2}+2)\mathrm{c}}\right) \tag{7}$$

Substituting Eq. (7) into Eqs. (5) and (6), we see that the leader's capacity will be $\sqrt{2}$ times larger than the follower's, and not twice as large, as predicted by the Stackelberg outcome in the linear case. Note that not only is the ratio of the firms' capacity choices independent of *r*, but also the absolute capacity levels and hence consumer surplus and welfare. We find that $q_1 = \sqrt{2}(a-c)/(2+\sqrt{2})$ and $q_2 = (a-c)/(2+\sqrt{2})$. Given that a firm is the first to invest at time *t*, it will invest according to Eq. (5); otherwise it will wait until *t*=0 and invest according to (6). Neither firm will invest before t^*; after that date each firm will wish to invest before the other firm.

Obviously, there exist two asymmetric subgame-perfect equilibria, with either of the firms investing at time t^*. However, in the symmetric equilibrium, both firms must use mixed strategies. Thus the investment process can be modelled as a timing game, or more specifically, as a pre-emption game. Following the analysis of Fudenberg and Tirole (1985), I define simple continuous time strategies for player i as a triplet of real-valued functions $(\mathrm{G_i}, \alpha_\mathrm{i}, \mathrm{q_i}) : (-\infty, 0] \times (-\infty, 0] \times (-\infty, 0] \rightarrow [0,1] \times [0,1] \times [0, \infty)$, where:[12]

(a) G_i is non-decreasing and right-continuous.

(b) $\alpha_i(t) > 0 \Rightarrow G_i(t) = 1$.

(c) α_i is right-differentiable.

(d) If $\alpha_i(t) = 0$ and $t = \inf(s \geq t | \alpha_i(s) > 0)$ then $\alpha_i(\cdot)$ has a positive right derivative at t.

(e) q_i is right-differentiable.

$G_i(t)$ is a cumulative distribution function, with $G_i(-\infty) = 0$, which gives the probability that firm *i* invests before time *t*, given that the other firm has not already invested. In continuous-time War of Attrition-models, a cumulative distribution function is sufficient to describe the strategies. In continuous-time pre-emption models another function, α_i, is required, which gives the probability, or intensity, with which player *i* moves in consecutive atoms of time following a time *t*.[13] The idea is that, although the probability of a move by either or both firms within an infinitesimal time after t^* is 1, the firms need not move simultaneously *exactly* at time t^*. In other words, the function α_i serves as a coordination device, enabling the firms to avoid the undesirable outcome of simultaneous investments. As before, q_i is the level of investment chosen by firm *i*.

Define $\{(\mathrm{G_i^t}, \alpha_\mathrm{i}^\mathrm{t}, \mathrm{q_i^t})\}$ as a closed-loop strategy, where $\{(\mathrm{G_i^t}, \alpha_\mathrm{i}^\mathrm{t}, \mathrm{q_i^t})\}$ specifies the behaviour of firm *i* from time *t* on for all possible histories up until time *t*. The closed-loop strategy consists of a collection of simple strategies, $(G_i^t, \alpha_i^t, q_i^t)$, one for each possible (including zero-probability) event and player.[14] Strategies $\{(\mathrm{G_i^t}, \alpha_\mathrm{i}^\mathrm{t}, \mathrm{q_i^t})\}$ constitute a subgame-perfect

[12] Fudenberg and Tirole do not allow for a continuous choice of capacity, only a dichotomous choice of entry and no entry; thus they only need a pair of functions.

[13] See Fudenberg and Tirole (1985, 1991) for an enlightening example with the "grab the dollar" game. If, *e.g.*, α equals 0.5 for both firms after time t', the probability of a simultaneous investment is 1/3, although the probability that at least one firm has invested before $t' + \varepsilon$ is 1.

[14] Two additional assumptions are necessary:

i) $G_i^t(t+v) = G_i^t(t+u) + (1 - G_i^t(t+u))G_i^{t+u}(t+v)$ for $t \leq u \leq v$

ii) $\alpha_i^t(t+v) = \alpha_i^{t+u}(t+v) = \alpha_i(t+v)$ for $t \leq u \leq v$

These ensure that the closed-loop strategies are intertemporally consistent. The pay-offs resulting from any pair of simple strategies are not formally defined, as this is complicated and not necessary for the results. For an explicit definition, see Fudenberg and Tirole (1985).

equilibrium if, at all times t and for all histories up until time t, it is a Nash equilibrium to play the strategies $(G_i^t, \alpha_i^t, q_i^t)$.

Proposition 2. The following symmetric strategies constitute a subgame-perfect equilibrium:

$$G^t(s) = \begin{cases} 0 \cdots\cdots s < t^* \\ 1 \cdots\cdots s \geq t^* \end{cases}$$
$$\alpha(s) = \begin{cases} 0 \cdots\cdots\cdots\cdots\cdots\cdots s < t^* \\ \dfrac{\pi^L(s) - \pi^F(s)}{\pi^L(s) - \pi^{LL}(s)} \cdots t^* \leq s < 0 \\ 1 \cdots\cdots\cdots\cdots\cdots\cdots s = 0 \\ 0 \cdots\cdots\cdots\cdots\cdots\cdots s > 0 \end{cases} \tag{8}$$
$$q_i(s) = \begin{cases} \dfrac{a+c}{2} - ce^{-rs} \cdots q_{j \neq i} = 0, s < 0 \\ 0 \cdots\cdots\cdots\cdots\cdots q_{j \neq i} > 0, s < 0 \\ \dfrac{a - q_j - c}{2} \cdots\cdots q_{j \neq i} > 0, s = 0 \\ \dfrac{a-c}{3} \cdots\cdots\cdots q_{j \neq i} = 0, s = 0 \end{cases}$$

where $G^t(s)$ and $\alpha(s)$ give the probability of investment according to $q_i(s)$. The functions π^L and π^F give the profits of the leader and the follower, if the leader invests at time $t = s$ and the follower at time $t = 0$. These are calculated by substituting Eqs. (5) and (6) into Eqs. (2) and (3) respectively. The function π^{LL}, calculated by substituting Eq. (5) for both q_1 and q_2 in Eq. (2), is the profit that accrues to each firm if they both invest at time $t = s$.

Proof:[15] It is obvious that no firm would want to deviate by investing at a time $t < t^*$, since it would then earn less than it would as a follower. Similarly, no firm would want to invest at a time t>0, since the market is realised at t=0, and all firms that have not invested before t=0 will invest at that time. It is clear that if only one firm has not invested prior to time 0, it can do no better than to choose the follower quantity given by Eq. (4). If neither firm has invested prior to time 0, we have in effect a two-period game, where quantities are (simultaneously) set in the first period, and in the second period, prices are set at the market clearing level. This is the standard Cournot model, and the unique NE is for both firms to choose $q = (a-c)/3$, i.e., the Cournot quantity.

[15] This proof is similar to that of Lemma 1 in Fudenberg and Tirole (1985).

For any subgame starting at t, $t^* \leq t < 0$, it is possible to show that the above strategies constitute an NE. If firm i chooses $G_i^t(s) = 0$ for all s<0, and then reverts to the equilibrium strategy, it earns $\pi^F(t^*)$. If $G_i^t(t) = \lambda$, $\alpha_i(t) = 1$, $0 < \lambda \leq 1$, firm i gets $\lambda\left[\alpha(t^*)\pi^{LL}(t^*) + (1-\alpha(t^*))\pi^L(t^*)\right] + (1-\lambda)\pi^F(t^*) = \pi^F(t^*)$.

If $G_i^t(t) = 1$, $\alpha_i(t) = a, 0 < a < 1$, firm i receives:

$$A + (1-a)((1-\alpha(t^*))A + (1-a)^2((a-\alpha(t^*))^2 A + \ldots = \frac{A}{a + \alpha(t^*) - a\alpha(t^*)} = \pi^F(t^*)$$

where $A = a\left[\alpha(t^*)\pi^{LL}(t^*) + ((1-\alpha(t^*))\pi^L(t^*)\right] + (1-a)\alpha(t^*)\pi^F(t^*)$.

If $G_i^t(t) = 1$, $\alpha_i(t) = 1$ and $q_i(t) = y$, $y \geq 0$, firm i gets $\alpha(t^*)\left[(a - q^L - y)y - cye^{-rt^*}\right] + (1-\alpha(t^*))\left[(a - y - (a - y - c)/2)y - cye^{-rt^*}\right]$, given that y is chosen so that the price is positive and where $q^L = \dfrac{a+c}{2} - ce^{-rt^*}$ is the leader quantity prescribed by Eq. (8). Since $\pi^L(t^*) = \pi^F(t^*)$, it follows that $\alpha(t^*) = 0$, and firm i obtains $(a - y - (a - y - c)/2)y - cye^{-rt^*}$. By Eq. (5), this expression is maximised when $y = q^L = \dfrac{a+c}{2} - ce^{-rt^*}$.

Thus no firm can increase its profit by deviating in any subgame from the strategies given by Eq. (8), which therefore constitute a subgame-perfect equilibrium. ◊

The idea is that, beginning at time t^*, the probability that each of the firms invests in a given atom of time rises slowly from zero. Since there are infinitely many atoms of time within a small finite period of time, ε, one of the firms will have invested before $\mathrm{t}^* + \varepsilon$, while the probability of both firms investing in the same atom of time is zero.

Obviously, letting either of the firms invest at t^* with probability one gives two additional equilibria. Social welfare can be calculated as consumer surplus plus profit. Although consumer surplus is higher than under Cournot competition, social welfare falls. The gain from increased competition is more than offset by the loss from the too early investment.[16]

[16] Welfare under alternative regimes are as follows: First-best: $\dfrac{(a-c)^2}{2}$; Cournot: $4(\dfrac{a-c}{3})^2$; modified Stackelberg: $(\dfrac{7}{2} + \sqrt{2})(\dfrac{a-c}{2+\sqrt{2}})^2$; monopoly: $\dfrac{3}{2}(\dfrac{a-c}{2})^2$. That consumer surplus is higher in the (modified) Stackelberg outcome than in the Cournot outcome follows immediately from the fact that the supplied quantity in the former case, $\dfrac{1+\sqrt{2}}{2+\sqrt{2}}(a-c)$, is greater than the Cournot quantity, $\dfrac{2}{3}(a-c)$.

Proposition 3. In all subgame-perfect equilibria, assuming market clearing, the two firms' profits will be equal.

Proof. Assume that firm 1 invests $q_1^{'}$ at time $t_1^{'}$ and obtains the profit $\pi_1^{'}$, while firm 2 invests $q_2^{'}$ at time $t_2^{'}$ and obtains $\pi_2^{'}$. Assume also that $\pi_1^{'} > \pi_2^{'}$ and $t_1^{'} \neq t_2^{'}$. From the analysis above, it is clear that in equilibrium either $t_1^{'} = 0$ or $t_2^{'} = 0$. Assume first that $t_1^{'} = 0$. Then firm 2 can increase its profit by investing later. For t_2 very close to 0, the Stackelberg leader profit can be achieved, i.e., $\pi_2 > \pi_1$. Therefore, this cannot be an equilibrium. Assume instead that $t_2^{'} = 0$. Since the leader's profit increases continuously with time, firm 2 can obtain almost $\pi_1^{'}$ by investing immediately before time $t_1^{'}$. Given that firm 2 is now the first to invest, firm 1 chooses to invest at time 0. Thus, firm 2 can obtain a higher profit than $\pi_2^{'}$ and this cannot be an equilibrium either.

Now assume that $t_1^{'} = t_2^{'} < 0$. Then each firm can obtain a higher profit by postponing investments until $t = 0$. If $t_1^{'} = t_2^{'} = 0$ we have a standard two-period Cournot game, which yields equal profits to the two firms.◊

5 Bertrand pricing

Under capacity constrained Bertrand competition, if $q_i \leq R(q_j)$ for *i,j*=1,2, and $i \neq j$, where *R(q)* is the optimal response function in Cournot competition, then the unique equilibrium is that in which both firms name the market-clearing price. (See Kreps and Scheinkman, 1983.) If the above inequality is violated for either (or both) of the firms, then there exists no pure strategy equilibrium. Instead, the firms must rely on mixed strategies. In this case, the strategies and the profits of the two firms depend on how the consumers are rationed. I will follow Kreps and Scheinkman, and most of the subsequent literature, in assuming that the high-valuation consumers buy first from the low-priced firm. This is known as efficient (or parallel) rationing.[17]

If the installed capacity slightly exceeds the Cournot capacity, the result may be Edgeworth cycles: starting at a high price, the firms will undercut each other until the price reaches the marginal cost. At this point, it will be in the interest of any firm to raise the price substantially, since the other firms cannot satisfy demand at a price equal to the marginal cost, and a new cycle will begin. The Nash equilibrium consists of mixed strategies (Kreps and Scheinkman,

[17] See Davidson and Deneckere (1986) and Deneckere and Kovenock (1992) for discussions of alternative rationing schemes. Davidson and Deneckere show, in a two-period model, that firms will compete more intensely and acquire more capacity with proportional rationing than with efficient rationing.

1983, Brock and Scheinkman, 1985). If n-1 of the n symmetric firms have sufficient capacity to satisfy demand, the only Nash equilibrium will be the Bertrand equilibrium, with price equal to the marginal cost. If the total capacity in the industry is less than or equal to the Cournot capacity, the single equilibrium price is the one that equalises demand with total capacity. In the intermediate range, a firm's expected profit will equal that which it would earn if all other firms were committed to produce to capacity.

With two asymmetric firms, if $q_1 \geq q_2$ and $q_1 > R(q_2)$, the expected revenue of firm 1 in the pricing stage, Γ_1, equals that which would be attainable if the smaller firm produced to capacity. Firm 2's revenue in the pricing stage, Γ_2, will be in the range $\left[q_2\Gamma_1 / q_1, \Gamma_1\right]$. (See Proposition 1 of Kreps and Scheinkman, 1983.)

Proposition 4. With a linear demand curve, and if $q_1 \geq q_2$ and $q_1 \leq \left[a + \sqrt{q_2(2a - q_2)}\right]/2$, then $\Gamma_2 = q_2\Gamma_1 / q_1$. If instead $q_1 > \left[a + \sqrt{q_2(2a - q_2)}\right]/2$ and $q_2 < a$, then $\Gamma_2 = q_2\left[a - \sqrt{q_2(2a - q_2)}\right]/2$, i.e., Γ_2 is independent of q_1.

Proof: See Kreps and Scheinkman (1983), Osborne and Pitchik (1986) or Davidson and Deneckere (1990). ◊

In the linear, zero marginal cost case, $R(q_2) = (a - q_2)/2$. Assume that $q_1 > R(q_2)$. Then:

$$\pi_1 = \Gamma_1 - \mathrm{c}e^{-\mathrm{rt}}\mathrm{q}_1 = \left(\frac{\mathrm{a} - \mathrm{q}_2}{2}\right)^2 - \mathrm{c}e^{-\mathrm{rt}}\mathrm{q}_1 \tag{9}$$

and:

$$\pi_2 = \begin{cases} \Gamma_2 - cq_2 = \dfrac{q_2}{q_1}\left(\dfrac{a - q_2}{2}\right)^2 - cq_2 & \text{if} \quad q_1 \leq \left[a + \sqrt{q_2(2a - q_2)}\right]/2 \\ \Gamma_2 - cq_2 = q_2\left[a - \sqrt{q_2(2a - q_2)}\right]/2 - cq_2 & \phantom{\text{if}} \quad q_1 > \left[a + \sqrt{q_2(2a - q_2)}\right]/2 \end{cases} \tag{10}$$

Let:

$$\mathrm{q}_2^{'} = \arg\max_{\mathrm{q}_2}\left[\frac{\mathrm{q}_2}{\mathrm{q}_1}\left(\frac{\mathrm{a} - \mathrm{q}_2}{2}\right)^2 - \mathrm{cq}_2\right] = \frac{2\mathrm{a} - \sqrt{\mathrm{a}^2 + 12\mathrm{cq}_1}}{3} \tag{11}$$

(such that $q_2 < a$). That is, $\mathrm{q}_2^{'}$ is firm 2's profit maximising response if q_1 is "small". Also, let:

$$q_2^{judo} = \arg\max_{q_2}\left[q_2\left(a - \sqrt{q_2(2a - q_2)}\right)/2 - cq_2\right] \tag{12}$$

(such that $q_2 < a$). Call q_2^{judo} firm 2's "judo capacity".[18] This is firm 2's optimal response if q_1 is "large". Then it will be true that:

$$q_2 = \begin{cases} q_2^{'} & \text{if} \quad \dfrac{q_2^{'}}{q_1}\left(\dfrac{a - q_2^{'}}{2}\right)^2 - cq_2^{'} > q_2^{judo}\left[a - \sqrt{q_2^{judo}(2a - q_2^{judo})}\right]/2 - cq_2^{judo} \\ q_2^{judo} & \quad\quad \dfrac{q_2^{'}}{q_1}\left(\dfrac{a - q_2^{'}}{2}\right)^2 - cq_2^{'} < q_2^{judo}\left[a - \sqrt{q_2^{judo}(2a - q_2^{judo})}\right]/2 - cq_2^{judo} \end{cases} \tag{13}$$

(as long as $q_1 > R(q_2)$). Let:

$$q_1^{judo} = \inf\left\{q_1 : \frac{q_2^{'}}{q_1}\left(\frac{a - q_2^{'}}{2}\right)^2 - cq_2^{'} \le q_2^{judo}\left[a - \sqrt{q_2^{judo}(2a - q_2^{judo})}\right]/2 - cq_2^{judo}\right\} \tag{14}$$

i.e., the smallest q_1 which makes firm 2 prefer q_2^{judo} over $q_2^{'}$. This is firm 1's judo capacity. Also, let:

$$q_1^{nrc} = \sup\left\{q_1 : q_1 \le R(q_2^{'}(q_1)) = (a - q_2^{'}(q_1))/2\right\} = (a + c)/3 \tag{15}$$

in which case $q_2 = q_2^{nrc} = (a - c - q_2^{nrc})/2 = (a - 2c)/3$. Call q_1^{nrc} and q_2^{nrc} firms 1 and 2's nrc (no randomisation constraint) capacities.[19] If $q_1 \le q_1^{nrc}$, q_2 is given by Eq. (4) and the price will be the market-clearing price. This is so, because q_1^{nrc} is the largest q_1 which makes firm 2 choose such a capacity q_2 that q_1 is not larger than firm 1's optimal response towards q_2 in Cournot competition. There exists an equilibrium in pure strategies only if $q_1 \le R(q_2(q_1))$ or if each firm alone can satisfy the whole market demand (the classic Bertrand outcome of price equal marginal cost). The former condition is satisfied if $q_1 \le q_1^{nrc}$.

Proposition 5. In equilibrium, $q_1 \le q_1^{nrc}$ (a "no randomisation constraint") or $q_1 \ge q_1^{judo}$ (the judo equilibrium). The latter relation holds with equality, unless $c = 0$. Furthermore, the two firms' profits will be equalised if $q_1 \le q_1^{nrc}$; if $q_1 \ge q_1^{judo}$, rent equalisation may or may not occur.

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[18] The term "judo" equilibrium is due to Gelman and Salop (1983), and refers to the fact that the follower chooses a small quantity, so as to be perceived as "harmless" in the ensuing pricing game.

[19] These are the quantities that would result if the two firms competed in quantities, and the marginal costs of firms 1 and 2 were 0 and c, respectively. Allen, Deneckere, Faith and Kovenock (1994) refer to this outcome as the "Dixit" equilibrium, after Dixit (1980).

Proof: Substituting Eq. (11) into Eq. (9) and differentiating, we find that:

$$\frac{\partial \pi_1}{\partial q_1} = c\left[\frac{1}{3}\left(1+\frac{a}{\sqrt{a^2+12cq_1}}\right)-e^{-rt}\right]<0 \tag{16}$$

for $c > 0$. That is, the profit of firm 1 is decreasing in its own capacity, given the reaction of firm 2, when $q_1^{nrc} < q_1 \leq q_1^{judo}$. Thus it will not be in the interest of firm 1 to choose a capacity in this range. The conclusion that rent equalisation will occur, if $q_1 \leq q_1^{nrc}$, follows from Proposition 3. Finally, it can be shown that $q_1 \geq q_1^{judo}$ can result in both rent-equalising equilibria and equilibria with asymmetric rents by examples (see below). ◊

Proposition 5 implies that either firm 1 chooses $q_1 \leq q_1^{nrc} = (a+c)/3$, in which case both firms will use pure price strategies (no randomisation), or firm 1 chooses a large capacity, so that $q_1 > \left[a+\sqrt{q_2(2a-q_2)}\right]/2$ will be satisfied (the judo equilibrium). If the leader expands capacity beyond q_1^{nrc}, this will reduce the capacity chosen by the follower, which will tend to increase the profit of the leader. However, this effect is not strong enough to offset the leader's increased investment cost. If the leader chooses a capacity above q_1^{nrc}, the equilibrium will be in mixed strategies, which means that some capacity may have to lie idle. When the leader's capacity reaches q_1^{judo}, however, the follower's strategy changes, which may result in a discrete increase in the leader's profit.

If $q_1 \leq q_1^{nrc}$ is optimal, then firm 1 chooses $q_1 = \min\left[q_1^{SL}, q_1^{nrc}\right]$, where $q_1^{SL} = \sqrt{2}(a-c)/(2+\sqrt{2})$ is the (modified) Stackelberg leader capacity derived in Section 4. Let $c' = (\sqrt{2}-1)a/(2\sqrt{2}+1) \approx 0.11a$ be the capacity cost for which $q_1^{SL} = q_1^{nrc}$. If $c > c'$, then $q_1^{SL} < q_1^{nrc}$.

Depending on the cost of capacity, there are three different constellations of leader and follower profits under the judo and nrc regimes. Figs. 1a-c illustrate the profits of the two firms in the three cases. The profits are shown as functions of the leader's entry time *t*, while, in each figure, the capacity cost *c* is held constant. Firm 1's profit will decrease the earlier the capacity is acquired. Firm 2 will always invest at time 0 - therefore its profit is independent of *t*. Just as in Section 4, t^* denotes the equilibrium time of investment. However, in that section a market-clearing assumption was imposed, while this section models the price competition explicitly. Only when conditions are such that one of the firms chooses q^{SL} will the value of t^* correspond to that of Eq. (7).

Figure 1a: The firms' profits, as a function of t, for a capacity cost c such that the judo quantities constitute the equilibrium. Disregard the numbers along the axis.

Define π_1^{judo} and π_2^{judo} as the firms' profit when the judo quantities are chosen, determined by the level of c and, for firm 1, the time of investment. Correspondingly π_1^{nrc} and π_2^{nrc} gives the profits when the nrc quantities are chosen. In Fig. 1a, to the left of t^*, the leader prefers the nrc strategy to the judo strategy. Also, for $t < t^*$, the follower profit, π_2^{nrc}, is greater than the best the leader can achieve, π_1^{nrc}. To the right of t^*, the leader's best option is to choose the judo strategy. As a consequence, the follower's profit falls discontinuously at t^*, from π_2^{nrc} to π_2^{judo}. It follows from Fig. 1a, that if π_1^{judo} intersects π_1^{nrc} between π_2^{nrc} and π_2^{judo}, then firm 1 will choose q_1^{judo} and there will not be rent equalisation.

Figure 1b: The firms' profits, as a function of t, for a capacity cost c such that the nrc quantities constitute the equilibrium. Disregard the numbers along the axis.

From Fig. 1b, if π_1^{judo} intersects π_1^{nrc} above π_2^{nrc}, then firm 1 will choose q_1^{nrc} and there will be rent equalisation. From Fig. 1c, if π_1^{judo} intersects π_1^{nrc} below π_2^{judo}, then firm 1 will again choose q_1^{judo}, but rents will now be equalised. Inspection of Figs. 1a-c leads to the conclusion that if the profit at the intersection of π_2^{nrc} and π_1^{nrc} is less than the profit at the intersection of π_1^{judo} and π_1^{nrc}, then firm 1 will choose q_1^{nrc}, and if the reverse relationship holds, it will choose q_1^{judo}.

Figure 1c: The firms' profits, as a function of *t*, for a capacity cost *c* such that the judo quantities constitute the equilibrium, but with rent equalisation. Disregard the numbers along the axis.

Consider Fig. 1a. Before t^*, both firms strictly prefer the nrc quantities over investing to become the judo leader. Furthermore, at times $t < t^*$, both firms prefer to be the follower rather than the leader. At t^*, the follower's profit falls discontinuously, so that after t^* both firms prefer to be the leader. This will result in an equilibrium in which both firms with positive probability invest to become the leader in the atoms of time immediately before and after t^*. Thus there is a positive probability that both firms invest simultaneously. Note also that rents will be equalised only ex ante, not ex post (except in the case of simultaneous investments).

In all three cases corresponding to Figs. 1a-c there are two asymmetric equilibria, with one of the two firms investing at (or very close to) time t^* and the other investing at $t = 0$. In the case corresponding to Fig. 1a, one of the firms (firm 1, say), chooses the nrc quantity at a time immediately before t^*; if firm 1 does not invest before t^* then the other firm (firm 2) chooses the judo quantity at time t^*; if firm 2 does not invest at time t^* then firm 1 chooses the judo quantity as soon as possible. Firm 1 cannot improve its profit by investing earlier, nor by investing at time t^* or after that time (given firm 2's behaviour). Firm 2 obtains a higher profit than firm 1, if firm 1 plays according to the equilibrium, but firm 1 cannot improve its profit by deviating.

In the case corresponding to Figs 1b-c, one of the firms invests at time t^* and the other invests at time 0. Both firms achieve the same profit, and the equilibrium is upheld by the follower's threat to invest at any time in the interval $(t^*,0]$ if neither of the firms has invested at an earlier date.

The asymmetric equilibria corresponding to Figs. 1b-c have the same structure as the equilibrium of Proposition 2, but with π^L and π^H replaced by π_1^{nrc} and π_2^{nrc} or π_1^{judo} and π_2^{judo} respectively, π^{LL} replaced by the appropriate measure of simultaneous-investment profit and q_i adjusted so that the leader firm chooses the nrc or judo quantities and the follower firm responds appropriately.

The symmetric equilibrium corresponding to Fig. 1a is more difficult to characterise. A game with a similar structure, but with discrete time, is analysed in Holden and Riis, 1994. As argued in the discussion of Fig. 1a, both firms want to invest as close to time t^* as possible; however, they do not want to invest *simultaneously* at time t^*. This suggests that in any symmetric equilibrium the probability must be concentrated on an infinitely small interval around t^*. At the same time, the probability of investing exactly at t^* must be less than 1 but strictly larger than 0, so that a positive risk of simultaneous investments motivates the firms not to invest with certainty at t^*. In the subgame-perfect equilibrium, the firms will randomise with strictly positive probabilities over a series of atoms of time around time t^*. After time t^*, the firms' strategies will yield an expected profit equal to the judo follower profit, π_2^{judo}.

At time t^*, the firms choose a probability p_{t^*} of investment so that the expected profit of the other firm is equal, whether it invests with probability 1 or 0 at time t^*, i.e.

$$p_{t^*}\pi^{LL} + (1-p_{t^*})\pi^L = p_{t^*}\pi^2_{nrc} + (1-p_{t^*})\pi^2_{judo} \qquad (17)$$

This equation can be solved to yield $p_{t^*} = (\pi^L - \pi^2_{judo})/(\pi^L + \pi^2_{nrc} - \pi^2_{judo} - \pi^{LL})$. The expected profit, given that no investment has been made before time t^*, is $(1-p_{t^*})\pi^L$. This procedure can be used to solve recursively for the probabilities of investing at all atoms of time prior to t^*. The expected profit will approach π^L and the probability of investment in a given atom of time will approach zero, as we take successive steps backwards in time.

Following Holden and Riis's analysis (case 2 in their Proposition 5), the probability mass of entry will be concentrated around t^*, there will be a positive probability of simultaneous entry, a positive probability of entry before t^* and a positive probability of an entry after t^*. The expected profit of the two firms will be equal to the leader profit at time t^*.

If $0 < c < c'$, then firm 1 will choose q_1^{judo} (the judo leader capacity), or q_1^{nrc} (the nrc leader capacity), depending on the value of *c*. With $q_1 = q_1^{nrc}$, there will be rent equalisation; with $q_1 = q_1^{judo}$ there may or may not be rent equalisation, depending on the value of *c*. For higher values of *c*, firm 1 will choose q_1^{judo} or q_1^{SL} (the Stackelberg leader capacity). Since q_2^{judo} and q_1^{judo} cannot be solved for analytically, we must resort to numerical methods to carry the analysis further. Recall that q_2^{judo} is given by Eq. (12), that firm 1's judo capacity is given by Eq. (14) and normalise *a* to 1.

From the analysis pertaining to Figs 1a-c, we need to know the position of three loci in order to know which equilibrium will be chosen. The locus ξ in the (c, π)-plane where $\pi_1^{nrc} = \pi_2^{nrc}$ holds can be solved analytically to be:

$$\xi = \left\{ \left(c, \left[\frac{1-2c}{3} \right]^2 \right), 0 \le c \le 1 \right\} \tag{18}$$

Similarly, the locus λ in the (c, π)-plane where $\pi_1^{SL} = \pi_2^{SF}$ holds is:

$$\lambda = \left\{ \left(c, \left[\frac{1-c}{2+\sqrt{2}} \right]^2 \right), 0 \le c \le 1 \right\} \tag{19}$$

When $c \le c'$ then the ξ locus will be relevant; when $c > c'$ then λ is relevant.

However, the locus μ where $\pi_1^{nrc} = \pi_1^{judo}$ (for $c \le c'$) or $\pi_1^{SL} = \pi_1^{judo}$ (for $c > c'$) holds must be obtained numerically. Fig. 2 plots the three loci. We see that μ intersects only with ξ and only once in the interval $[0, c']$. The loci μ and λ intersect when $c = c'$. If $\mu<\xi$, and $c \le c'$ then firm 1 will choose the judo capacity, and if $\xi<\mu$ and $c \le c'$ it will choose the nrc capacity. In the latter case, rents will be equalised; in the former, they may or may not be, as explained above. If λ<ξ, and $c > c'$ then firm 1 will chose the (modified) Stackelberg leader quantity.[20] The kinks in the μ locus corresponds to the cost at which the nrc-quantity becomes

[20] The implicit function theorem may be a potential avenue for analytical results, although the algebra appears to be prohibitively complex. Furthermore, it appears that the results of the numerical analysis are as general as can be, given the restrictive assumptions of, e.g., linear demand and constant marginal costs. The (Pascal) program for finding the numerical values is available upon request from the author.

less profitable than the modified Stackelberg leader quantity, c', and the cost at which the constraint that firm 2 does not chose a negative quantity becomes binding.

Figure 2: The loci μ, λ and ξ in the (c,π)-plane. For capacity costs to the left of the intersection between ξ and μ, firm 1 chooses the judo strategy; for higher capacity costs, it chooses the nrc strategy when ξ is above λ and the Stackelberg leader strategy when λ is above ξ.

The numerical calculations on which Fig. 2 is based show that for $c <\approx 0.02$ firm 1 will choose the judo capacity, and for $0.02 <\approx c < c'$ firm 1 will choose the nrc capacity. There will be no rent equalisation in the judo case (but there will be in the nrc case). The capacity choices of the two firms as functions of c are shown by the thick lines in Fig. 3. To summarise: for low capacity costs, the leader will choose the judo capacity; for intermediate capacity costs, it will choose the nrc capacity; and for high capacity costs, the leader will choose the modified Stackelberg leader capacity.

Figure 3: The firms' capacity choices as functions of the capacity cost are shown with thick lines. The segments correspond to (from the left) the judo quantities, the nrc quantities and the modified Stackelberg quantities.

If capacity is costless, there is no meaningful equilibrium time t^* - both firms would want to install the judo leader capacity infinitely early - and hence the present model cannot be applied.

5 Conclusions and extensions

The analysis above has shown, for the linear-demand, constant marginal cost case, that with endogenous investment times and irreversible investments, Kreps and Scheinkman's result no longer holds. For large investment costs ($c > c'$), a (modified) Stackelberg outcome will be the solution. The leader will choose a capacity $\sqrt{2}$ times as large as that of the follower, rather than the 2 to 1 ratio suggested by the standard Stackelberg solution. The excess rent of the leader will be totally dissipated by the competition between the firms, which forces the leader to invest earlier than the optimal time.

For an intermediate range of investment costs, the leader will invest in a capacity smaller than the (modified) Stackelberg solution of the last paragraph, but larger than the Cournot capacity. A "no-randomisation constraint" will bind the leader's choice of capacity: The leader's capacity choice will induce the follower to choose its capacity so that both firms will use pure pricing strategies in equilibrium. Both firms will earn equal profits. This outcome is also known as the Dixit equilibrium, and corresponds to the Cournot equilibrium where the leader has zero marginal cost, while the follower's marginal cost is c.

For small investment costs, the leader is better off choosing a much larger capacity, with the result that the equilibrium in the pricing stage will be mixed. This will induce the follower to choose a small capacity - the "judo" follower capacity. The follower will now earn a strictly smaller profit than the leader, although ex ante the firms' profits are equal. The equilibrium strategies will, with positive probability, result in the firms investing simultaneously.

An interesting extension of the model developed in this paper would be to allow for the possibility that two firms, already supplying an existing market, expect a sudden future upward shift in demand. Numerical calculations suggest that the larger of the two firms will be the first to invest in anticipation of the increased demand. The asymmetry between the firms enables the larger firm to maintain a profit level in excess of that of the smaller firm.

However, in order to maintain its position as the leader, the larger firm must invest far in advance of the demand increase. This is costly, in terms of capital costs for the excess capacity and a reduced profit flow due to a lower price resulting from the capacity expansion (the two firms are engaged in Bertrand competition). Therefore, the earlier the leader must invest, the smaller the capacity it will choose. Eventually, the ratio of the firms' sizes will converge to a number not much larger than 1.[21]

Another extension is to allow for the possibility that the marginal cost of production differs between the two firms. This idea is pursued by Allen, Deneckere, Faith and Kovenock (1994), in a three-period model with a rich set of possible outcomes, but where the sequencing and timing of the firms' investment is exogenous. Their model also includes a fixed entry cost, admitting the possibility of entry deterrence. With endogenous timing, it appears likely that the cost-advantaged firm would become the leader by investing just prior to the time when the cost-disadvantaged firm is indifferent between the leader and follower positions.

Finally, introducing a maximum horizon beyond which the firms cannot predict future market conditions, we would clearly expect to see some equilibria in which both firms invest simultaneously, as soon as a market opportunity comes within the horizon.

[21] With the numerical values used in footnote 15, if a increases to 16, the stable ratio of leader capacity to follower capacity is approximately 1.1.

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Figure 1a: The firms' profits, as a function of t^*, for a capacity cost c such that the judo quantities constitute the equilibrium. Disregard the numbers along the axis.

Figure 1b: The firms' profits, as a function of t^*, for a capacity cost c such that the nrc quantities constitute the equilibrium. Disregard the numbers along the axis.

Figure 1c: The firms' profits, as a function of t^*, for a capacity cost c such that the judo quantities constitute the equilibrium, but with rent equalization. Disregard the numbers along the axis.

Figure 2: The loci μ, λ and ξ in the (c, π)-plane. For capacity costs to the left of the intersection between ξ and μ, firm 1 chooses the judo strategy; for higher capacity costs, it chooses the nrc strategy when ξ is above λ and the Stackelberg leader strategy when λ is above ξ.

Figure 3: The firms' capacity choices as functions of the capacity cost cost are shown with thick lines. The segments correspond to (from the left) the judo quantities, the nrc quantities and the modified Stackelberg quantities.