Asymmetric shocks and policy responses.
A comparative analysis of the effects of a monetary union.

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Abstract

This paper analyses how optimal policy responses to productivity shocks change when the government loses the exchange rate as a policy tool after entering a monetary union. It is shown that over the business cycle (generated as cyclical changes in productivity), both deficit and inflation will be more volatile when there is no exchange rate to support the stabilization policies. The reason is that the exchange rate is quite an efficient weapon in addressing the impact on the price level from the different shocks. Losing it therefore makes prices less stable and triggers a more extensive response in the only policy tool left, the deficit.

With respect to unemployment, the outcome depends on the source of the asymmetric shock. If the government dislikes fluctuations in the unemployment rate, and supply shocks are mainly domestic in origin, a monetary union is preferable. If supply shocks are instead mainly foreign in origin, unemployment is less volatile under a flexible exchange rate regime.

Keywords: Asymmetric shocks, monetary union, stabilization policy, reaction function.

JEL Classification: E31, E52, E63.

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1 Introduction

The objective of this paper is to compare how the government's optimal policy response to a stylized business cycle differs between on the one hand a situation with a flexible, discretionary exchange rate, and on the other hand a monetary union. The policy tools available are the exchange rate and the deficit. The business cycle is roughly proxied by changes in labor productivity. In order to do this, we use the set up used by Agell et al in their paper "Fiscal policy when monetary policy is tied to the mast" [1], and extend the model to include a stochastic productivity level.

2 Background

With the imminent creation of a monetary union in Europe, EMU, the issue of how well the eleven future members fit in to a currency union is still unresolved. In a world of asymmetric shocks, a high degree of factor mobility is the main requirement, according to the original Mundell theory of optimum currency areas [21]. Without it, a shift in demand from one country's goods to the other's will lead to recession in the former and inflationary pressures in the latter. With separate currencies, this adverse effect can be remedied by an adjustment in the bilateral exchange rate. Even though the European economies are not especially specialized in production with respect to each other, asymmetric disturbances are far from extinct. Bayoumi and Eichengreen [5] claim that supply shocks are larger in magnitude and less correlated across regions in Europe than in the US. Furthermore, among the core of EU countries supply shocks are relatively small and highly correlated, whilst among the "EU periphery" supply shocks are bigger and more country specific. In a different study, Caporale [9] estimates that the percentage of output variance due to asymmetric shocks has a lower bound ranging between 30 and 70 percent among the EU countries.[1] The rather high degree of asymmetry within Europe is therefore seen by some economists as one of the greater challenges to the EMU project.

The above reasoning leads to the question of how much of a buffer the exchange rate is in addressing country specific disturbances. According to Canzoneri, Vinals and Valles [8],

[1] Her analysis, using a *principal components technique*, is based on the EU-12 group, ie the present 15 countries minus the lastest entrants, Austria, Finland and Sweden. In the case of Sweden, Hansson and Sjöholm [13] claim shocks to output are mainly asymmetric (*vis-a-vis* the EU). Looking at correlations, they find that with respect to supply shocks, the only correlations which are statistically significant are with Great Britain, Italy, and the Netherlands. With respect to demand shocks, the only statistically signifcant correlation is with Greece. In a different study, Assarsson and Olsson [4] conclude that Swedish supply and demand shocks are only weakly correlated with those of Germany, indicating that an adjustable exchange rate may be preferable. However, they also show that domestic supply shocks have not been the dominant shocks in the Swedish economy.

exchange rates among the countries in the EU do not move in response to macroeconomic imbalances, but rather respond to financial shocks. Therefore, for the countries they look at, giving up the exchange rate is not a very costly action with respect to stabilization issues.

Furthermore, the lack of a federal fiscal policy of any significant dimension in the EU reinforces this problem. Eichengreen [11] states that while state taxes paid to the federal government in America go down by 30 percent for each dollar decline in state income, the corresponding figure for a European country's payments to Brussels is a reduction of no more than one percent. Moreover, Sachs and Sala-i-Martin [23] conclude that a one dollar income drop in house hold income raises federal transfers in the US by between 6 and 10 cents. Thereby, 40 percent of the income drop is eliminated through the federal insurance system.[2] The two studies conclude that this aspect may pose a big problem for the EMU region.

Yet the situation may not be as bad as that. Firstly, Bayoumi and Masson [7] find that although the federal stabilization budget within the EU is nowhere near that of the US, national fiscal policies have been as effective as the American federal system in cushioning shocks. However, this cushioning effect may be restrained by the stability pact's requirement that deficits in the union stay below three percent of GDP.[3] One of the drawbacks of imposing fiscal restrictions under a monetary policy dedicated to EMU-wide price stability is that fiscal flexibility may be essential to make the monetary union functional. Fiscal policy can potentially ensure an adjustment to shocks since it can both affect the consequences of shocks and stabilize income. Not least if labor markets prevent speedy adjustments to shocks.[4] To further complicate things, restrictions regarding fiscal policies may in turn increase the pressure on the European Central Bank for a looser monetary stance (Masson [20]).

Secondly, Italiener and Vanheikelen [17] assert that a stabilization tax of only 0.2 percent of GDP would be sufficient to compensate for the lack of national stabilization policies by allowing transfers between countries in the event of asymmetric shocks. That is, a rather limited insurance scheme is necessary.[5]

In the Agell et al paper [1], the government can address a shock and stimulate the economy

[2] Sala-i-Martin and Sachs also estimate the tax reduction to 30 cents on each dollar.

[3] Artis and Winkler [3] argue on the other hand that the leeway in case of a temporary disturbance is quite large.

Yet the actual restrictions on fiscal policy may be even more restraining than the Maastricht treaty indicates. According to Masson [20], in order not to exceed the stability pact's limits the countries need to have an average deficit less or equal to one percent, since the standard deviation between actual and structural deficits has been two percent of GDP among the EMU participants.

[4] See for instance de Grauwe, 1992 [10], and Kenen, 1995 [18].

[5] This may be a too low estimate, though. See for instance Andersen , 1996 [2].

through two different channels; the exchange rate and the deficit. The government, which chooses its policy variables after unions have set nominal wages, cares about three things: unemployment, the deficit and the level of inflation. In the discretionary case, the outcome is a wage-depreciation cycle and a time inconsistency problem both with respect to monetary policy *and* fiscal policy.[6] However, the main focus of the Agell et al study is the effect on the fiscal policy from giving up the exchange rate. Their results indicate that a deterioration of the fiscal stance (increased deficits) is likely to follow once a monetary union is created. The reason is that as monetary policy is taken away from the hands of national policy makers, fiscal policy gets to bear more of the burden in achieving the government's goals. Hence restraining rules regarding fiscal policy might be appropriate. The Maastricht Treaty's convergence criteria and the Stability Pact's requirements may be seen against this background.

This paper uses the same model as Agell et al [1], but adds a productivity variable in the production function. Changes in domestic labor productivity affects the supply side of the economy, whilst changes in foreign productivity will affect the domestic economy on the demand side, via fluctuations in export demand. The analysis focuses on how the optimal stabilization policies regarding the deficit and the exchange rate change over the business cycle (represented by changes in labor productivity) in two different worlds. That is, one with a flexible, discretionary exchange rate regime, and one with a monetary union. The model is chosen because of some attractive features. It is simple in its set up, is based on optimizing agents, and has an "overambitious" employment target on behalf of the government, leading to combined deficit and inflationary biases.[7]

The results indicate that both the deficit and inflation will become more volatile in a monetary union. The former will fluctuate more mainly because it will be increasingly utilized as a shock absorber when there is no exchange rate to help buffer the disturbance, while movements in inflation will be dramatically larger in a monetary union due to the fact that the exchange rate is a very potent price stabilizer in this model.

[6] The wage-depreciation cycle occurs in many models, see for instance Horn and Persson [16] and Gylfason and Lindbeck [14]. The fiscal time inconsistency problem/outcome is less common, however.

[7] Although both consumers and firms are optimizing, their horizon is restricted to the present period. Furthermore, consumers optimize over consumption only, ie there is no disutility from labor in the utility function.

3 The model

The economy consists of four parts; an aggregate demand side, a production function, a wage equation and the government's loss function. We assume a Cobb Douglas style production function which only depends on labor.. We have two different productivity shocks, either to domestic production, A_t :

$$Y_{j,t} = L_{j,t}^{\gamma} A_t$$

or to foreign production, A_t^*. (We will denote foreign levels by a star superscript throughout the paper.) A shock to foreign output affects foreign income, and hence foreign demand for domestic exports. We think of the foreign shocks as being of the same source as the domestic ones, affecting the supply side. Furthermore, the shocks affect all production within the national borders identically. Hence A can be seen as a technology parameter or a cost parameter that is universal to the country in question. Also, calling it a shock is not really correct, since it is productivity factor. In this sense, it would be more correct to denote the change in productivity as the shock. However, we will refer to the level as the shock. The logarithmic values of the productivity shocks (denoted by lower case letters) follow an AR(1) process:

$$\begin{aligned} a_t^i &= \rho a_{t-1}^i + \varepsilon_t^i \qquad\qquad i = domestic \,\&\, foreign \;\; (*) \\ & \quad \varepsilon_t^i \sim N(0, \sigma_\varepsilon^2) \\ E(\, a_t^i) &= \rho a_{t-1}^i \end{aligned} \tag{3.1}$$

where $\rho \in (0,1)$, and both the persistence in productivity and the variance of innovations are the same in the domestic and in the foreign economy ($\rho = \rho^*$, $(\sigma_\varepsilon^2) = (\sigma_\varepsilon^2)^*$)[8]. Furthermore, we assume:

i) Identical consumers and identical firms.

ii) The world consists of two areas; a domestic economy which produces one single good, and the rest of the world that also produces a single, different good.

iii) No intertemporal aspects on behalf of consumers or unions, who maximize expected instantaneous utility each period.

In order to let the government react to contemporaneous productivity changes, the timing in each period in our economy is as follows:

[8] In order to keep the analysis simple and clear, we ignore propagation between our two different types of shocks. I e shocks are totally independent from one another. See appendix D for an analysis based on the empirical estimation of propagation of technology shocks.

i) first to move are the unions who unilaterally decide wages before shocks are realized.

ii) shocks are realized.

iii) second to move is the government, which can react to shocks by choosing its policy variables (exchange rate S and deficit as proportion of domestic output, D) appropriately, which in turn determines the period's price level.

iv) Finally, firms choose employment levels as a function of the real wage and realized shocks.

As a result of this ordering of events, it will turn out that the government can affect both the inflation and the employment level. Consequently, consumers and unions have an informational disadvantage.

3.1 Aggregate demand (AD)

Consumers get utility from consuming a combination of the two goods produced in the world, the domestic good X, and the foreign good M. Despite the income uncertainty, we do not allow for savings, but instead assume consumers spend all their income each period. Given this assumption, consumers face the following maximization problem each period:

$$\begin{aligned} \max U_i &= X_i^{\alpha} M_i^{1-\alpha} \\ s.t.\ P(1+D)(L_i^{\gamma} A_t) &= PX_i + SM_i \end{aligned}$$

where α is the share of domestic goods in total consumption. In this sense, α will be an indication of the openness of the country. The smaller is α, the more imports the domestic economy consumes. $L_i^{\gamma} A_t$ is gross income per identical consumer, measured in domestic goods. D is the budget deficit ($D > 0$) or surplus ($D < 0$) as proportion of gross income. P indicates the domestic price level, and S is nominal exchange rate (domestic currency/foreign currency). We choose to assume a foreign monetary policy set to keep the foreign price level constant.[9] Given this assumption, p^* can be normalized to unity, and dropped from the equations. Since there are no savings, there are no credit markets. Total real consumption is therefore equal to income plus any deficit or surplus. Consequently, the implicit assumption is that the government borrows from abroad to pay for a possible deficit. Furthermore, we implicitly assume that the world economy is sufficiently much larger than the domestic one so that a deficit D can be financed without affecting the world's demand for domestic exports. In this simple set up, we furthermore

[9] A fluctuating foreign price level results in the cancellation of the foreign shock in the expression for the domestic price level. The reason is that its indirect effect via the foreign price level (p^*) is fully offset by its direct effect (a^* in p^* cancels with the direct effect of a^*).

ignore possible borrowing limits for the government. That is, there is no intertemporal budget constraint. This is obviously an extreme assumption. However, empirical facts indicate that governments in industrialized countries such as the OECD group have quite extensive borrowing possibilities on international capital markets. Since our analysis focuses on how the use of the deficit as a shock absorber changes as the government loses the exchange rate as a policy tool, and the interesting results come when borrowing is unconstrained, the paper accepts this weakness of the model.[10]

Given this set up, changes in the deficit level affect aggregate demand, so Ricardian Equivalence does not hold. Aggregating over all identical consumers, solving for domestic demand in the budget constraint, and using this in the utility function, we eventually get demand as :

$$M = (1+D)(L^{\gamma}A)(1-\alpha)\frac{P}{S} \tag{3.2}$$

$$X = \alpha(1+D)(L^{\gamma}A) \tag{3.3}$$

(See appendix A for a more thorough exposition of the calculations behind section 3.) Domestic demand for foreign goods (M) is a function of the domestic economy's openness $(1-\alpha)$, relative prices and disposable income. So the higher is the domestic price level, and the stronger is the nominal exchange rate (low value of S), the stronger is import demand. By symmetry, world demand for domestic exports equal to:

$$X^{*} = (1-\alpha^{*})\frac{S}{P}\left((L^{\gamma})^{*}A_t^{*}\right) \tag{3.4}$$

where $(1-\alpha^{*})$ is the share of imports in foreign consumption. So the weaker the real exchange rate, the higher its value, and the higher is the outside world's demand for the domestic good. Adding up demand for the domestically produced good ((3.3) and (3.4)), and dropping the constant (it will not affect the comparative analysis which is the focus of the paper) we have total demand for the domestic output (good X) as:

$$\gamma l_t + a_t = s - p + \frac{\alpha}{1-\alpha}D + a_t^{*} \tag{3.5}$$

where lower letters indicate logarithmic values. The expression says the demand for domestic goods is a function of the real exchange rate $(s-p)$, the fiscal stance and foreign supply shocks (working via higher foreign demand). Domestic shocks will enter on the right hand side (RHS) in p, and also in the policy variables s and D.

[10] See appendix C for a presentation of how the model behaves when there is a limit to the government's aggregate debt level.)

3.2 Aggregate supply (AS) part 1: Production and employment.

There is a continuum of identical firms, normalized to unity. As a result, all firms will employ the same amount of labor.. Each firm takes the price level and the exchange rate as given, and faces a demand equal to $X_{j,t} + X^*_{j,t}$. We assume that the law of one price holds, since there are no impediments to trade and there is perfect competition in each of the two goods. Hence firms get the same price irrespective of the market in which they sell. That is, the price of the domestic good in the foreign market is $\frac{P}{S}$ (in foreign currency). The interest rate is set to zero (it does not matter for the analysis). Consequently, we have that profits for each firm j equals:

$$profit_{j,t} = P_t L^{\gamma}_{j,t} A_t - L_{j,t} W_{j,t}$$

Since firms act after shocks are realized and the price level and the real wage are known, there is no uncertainty at the time of firms employment decision. Profit maximization (with respect to firms' discretionary employment decision) gives us log employment in each firm as:

$$l_{j,t} = \widetilde{l} - \theta(w_{j,t} - p_t - a_t) \tag{3.6}$$

where $\theta \equiv \frac{1}{1-\gamma} > 1$ is elasticity of labor demand, and $\widetilde{l} \equiv \theta \ln \gamma$. This expression gives a relationship of how much the firm's employment level differs from a bench mark case, indicated by $\widetilde{l}$, (where $(w - p - a) = 0$, that is, the shock augmented real wage $\frac{W}{PA}$ is normalized to unity), due to effects from the real wage and domestic supply side shocks. A positive domestic shock raises productivity (by assumption) and raises firm employment. Thus aggregate unemployment in our normalized economy will be:

$$u_t \equiv \widetilde{l} - l_t = \theta(w_t - p_t - a_t) \tag{3.7}$$

Writing the production function in logs, we have:

$$y^{AS}_t = \widetilde{y} - (\theta - 1)(w_t - p_t) + \theta a_t \tag{3.8}$$

Hence, actual supply will deviate from our bench mark level, $\widetilde{y} = \gamma \widetilde{l}$, due to adjustments for the real wage and supply side shocks. Using (3.5) , (3.8) , and the equilibrium condition $y^{AS} = y^{AD}$, we can solve for the price level (ignoring the constant term):

$$p_t = \frac{1}{\theta} s_t + \frac{\alpha}{\theta(1-\alpha)} D_t + \gamma w_t + \frac{1}{\theta} a^*_t - a_t \tag{3.9}$$

Adding shocks has two direct effects on the price level. A foreign supply shock increases foreign productivity, raising foreign income, and hence foreign demand for our exports. This in turn raises the domestic price level.

$$a^* \uparrow \Longrightarrow \; y^* \uparrow \Longrightarrow \; X^* \uparrow \Longrightarrow \; p \uparrow$$
$$\Longrightarrow \; \frac{\partial p}{\partial a^*} > 0$$

A domestic positive productivity shock works the opposite way, as it makes production more efficient, and thereby cheaper, resulting in a lower domestic price level.

$$a \uparrow \Longrightarrow \; more\; efficient\; production \; \Longrightarrow \; p \downarrow$$
$$\Longrightarrow \; \frac{\partial p}{\partial a} < 0$$

However, there are also the indirect effects entering via the two policy variables s and D, as they will be functions of the shocks. The government can reduce the real wage, and thereby stimulate output and employment, by creating a price surprise in two different ways. It can either use an unexpected depreciation or an unexpected fiscal expansion. In the first case, both domestic and foreign expenditures switch to domestic goods through a real exchange rate depreciation. In the second case, there is a domestic aggregate demand increase at the same time as the real exchange rate appreciates (the price level is increased).

3.3 AS part 2: Wage setting

We assume unions have monopoly power with regard to setting the wage, but firms have "right to manage," as they decide unilaterally what amount of labor to hire once unions have set wages. Furthermore, we assume one union per firm and no labor mobility. Unions care about the real consumption wage and employment, and each union's utility function is a combination of these two. The union is small enough to ignore the effect its wage may have on the aggregate wage level. The set up implies unions set wage demands given expectations of policy parameters S and D. Real consumption wage is defined by:

$$\begin{aligned} real\ consumption\ wage &= W(1+D)/CPI \\ CPI &= \alpha P + (1-\alpha)S \end{aligned}$$

Recall that D is transfers as a proportion of Y, so in symmetric equilibrium it will spread evenly among all consumers, hence we can multiply the wage by the factor $(1 + D)$. Let superscript e

denote unions' expectation. Union expected utility (in logs) in firm j at time t is given by:

$$V_{j,t}^{e}=(w_{j,t}+D_{t}^{e}-cpi_{t}^{e})-E\left[\frac{\lambda}{2}(\widetilde{l}-l_{j,t})^{2}\right] \tag{3.10}$$

or in words, expected union utility is equal to the log of the real wage ($\ln(1+D)$ is approximated with D) minus a quadratic unemployment term. λ is the weight unions put on employment relative to the real wage. In periods with low unemployment, union cares mainly about the real wage. However, due to the quadratic term, as unemployment grows its weight automatically increases in the union's objective function. There are no intertemporal links, the union just maximizes expected $V_{j,t}$ in each period. We focus on the Nash equilibrium in the "game" between the union and the government. That is, the union knows the objective function of the government and maximizes its utility given its assessment of p^e and D^e, where the latter is consistent with the set up of the economy (rational expectations). Using equation (3.7) and differentiating (3.10) with respect to $w_{j,t}$ yields:

$$w_{j,t}=\frac{1}{\lambda\theta^{2}}+p_{t}^{e}+a_{t}^{e} \tag{3.11}$$

under the assumption of exogenous shocks and that the unions believe the wages they set affects neither the general price level nor the fiscal position chosen by the government ($\frac{\partial p_t^e}{\partial w_{jt}}=\frac{\partial D_t^e}{\partial w_{jt}}=0$). Symmetric equilibrium implies $w_j=w\ \forall j$. We see that the nominal wage is related 1:1 to the expected price level. Using the price level expression (3.9), we have:

$$p_{t}^{e}=\frac{1}{\theta}s_{t}^{e}+\frac{\alpha}{(1-\alpha)\theta}D_{t}^{e}+\gamma w_{t}+\frac{1}{\theta}\left(a_{t}^{*}\right)^{e}-a_{t}^{e}$$

Plugging this into our wage expression, we get the wage as:

$$w_{t}=\frac{1}{\lambda\theta}+s_{t}^{e}+\frac{\alpha}{(1-\alpha)}D_{t}^{e}+\left(a_{t}^{*}\right)^{e} \tag{3.12}$$

The union's rational expectations of s^e and D^e will follow from solving for them below. However, we can already see how the government can improve the unemployment situation. Plugging in the price expression into firms' employment decision, and then substituting in for w, we get unemployment as:

$$u_{t}=\frac{1}{\lambda\theta}-(s_{t}-s_{t}^{e})-\frac{\alpha}{(1-\alpha)}(D_{t}-D_{t}^{e})-(a_{t}^{*}-(a_{t}^{*})^{e}) \tag{3.13}$$

Once again we see that a surprise depreciation and a surprise increase in the deficit reduces unemployment. Furthermore, plugging the price expression (3.9) into the unemployment equation (3.7), we have:

$$u_{t}=w_{t}-s_{t}-\frac{\alpha}{(1-\alpha)}D_{t}-a_{t}^{*} \tag{3.14}$$

Assume for a moment that there is no government interaction in any period. Then s and D are zero (exchange rate in levels normalized to unity), and unemployment is related $1:1$ to the wage. Hence without a government, as wage goes, so goes unemployment. The implication is that changes in domestic productivity do not affect labor demand (when there is no government), and the reason is that a change in productivity, which ought to induce firms to adjust employment, is fully offset by a counter-effect through the price level. For example, an increase in productivity makes the firm want to hire more labor. However, the same increase lowers prices, thereby raising the real wage. If the price level is not affected by policy variables, these two effects cancel out. When we introduce a government, it will choose its policy variables in response to present shocks. Hence the unemployment level will be affected, since policy variables will add additional changes to the price level, thereby nullifying the previous 1:1 link between the rise in productivity and the rise in the real wage.

3.4 Government

The government is concerned with unemployment, deficit and inflation.[11] The government's loss function is therefore defined as:

$$R_t = \frac{1}{2}{u_t}^2 + \mu_D \frac{1}{2} D_t^2 + \mu_\pi \frac{1}{2} \pi_t^2 \tag{3.15}$$

where consumer price inflation is defined as:

$$\pi_t = \alpha(p_t - p_{t-1}) + (1-\alpha)(s_t - s_{t-1}) \tag{3.16a}$$

That is, inflation is caused by the effect from domestically produced consumption times its share of consumption (α), plus price changes on imported goods (caused only by changes in the exchange rate, as the foreign price level is normalized to unity), times its share $(1-\alpha)$. μ_D and μ_π indicate the government's preferences over the three terms in the loss function.

4 Solving the model

[11] As the model is set up here, the government wishes to have full employment. But as equation (3.13) indicates, the steady state natural level of unemployment is $\frac{1}{\lambda\theta}$. This "overambitious" employment target is what drives the deficit and inflationary biases. Had we set the employment target in the loss function to $u_t - \frac{1}{\lambda\theta}$, the steady state deficit is zero, as is the inflation rate.
For a presentation of how the model behaves when there is a limit to the government's aggregate debt level, see appendix C.

4.1 The flexible exchange rate case (government chooses both s and D).

In the case where the government has discretion over both the exchange rate and the deficit, it solves the maximization problem:

$$min_{s,D}\ H = \sum_{t=1}^{\infty} \beta^{t-1} R_t$$

subject to (3.15) and where β is the government's discount factor. Using (3.7), (3.16a), (3.9) and (3.12):

$$\begin{aligned}
u_t &= \theta(w_t - p_t - a_t) \\
\pi_t &= \alpha(p_t - p_{t-1}) + (1-\alpha)(s_t - s_{t-1}) \\
p_t &= \frac{1}{\theta}s_t + \frac{\alpha}{(1-\alpha)\theta}D_t + \gamma w_t + \frac{1}{\theta}a_t^* - a_t \\
w_t &= \frac{1}{\lambda\theta} + s_t^e + \frac{\alpha}{(1-\alpha)}D_t^e + \rho a_{t-1}^*
\end{aligned}$$

we can plug in for the wage level and the price level and get the terms in the loss function R to be a function of policy variables and their expectations, and lagged and present shocks. In order to solve the problem, we use the Bellman technique of dynamic programming. The value function $J(p,s)$ is set up with lagged price level and exchange rate as the state variables, and the minimization problem becomes:

$$J(p_{t-1}, s_{t-1}) = E_t \min_{D_t, s_t,} \left[R_t + \beta J_{t+1}(p_t, s_t)\right] \tag{4.17}$$

Since the loss function is linear quadratic, we know that the value function J must be quadratic. Hence we can write its general form as:[12]

$$\begin{aligned}
J(p_t, s_t) &= \nu_0 + \nu_1 p_t + \frac{1}{2}\nu_2 p_t^2 + \nu_{10} s_t + \frac{1}{2}\nu_{11} s_t^2 + \nu_3 a_{t+1}^* + \frac{1}{2}\nu_4 \left(a_{t+1}^*\right)^2 + \\
&\quad + \nu_5 a_{t+1} + \frac{1}{2}\nu_6 \left(a_{t+1}\right)^2 + \nu_{12} p_t s_t + \nu_7 p_t a_{t+1}^* + \nu_8 p_t a_{t+1} + \\
&\quad + \nu_{13} s_t a_{t+1}^* + \nu_{14} s_t a_{t+1} + \nu_9 a_{t+1}^* a_{t+1}
\end{aligned} \tag{4.18}$$

which gives us the derivatives as:

$$\begin{aligned}
\frac{\partial J_{t+1}}{\partial p_t} &= \nu_1 + \nu_2 p_t + \nu_{12} s_t + \nu_7 a_{t+1}^* + \nu_8 a_{t+1} \\
\frac{\partial J_{t+1}}{\partial s_t} &= \nu_{10} + \nu_{11} s_t + \nu_{12} p_t + \nu_{13} a_{t+1}^* + \nu_{14} a_{t+1}
\end{aligned}$$

[12] The timing subscripts follow from the fact that when the government acts, the present period's productivity levels are known.

The government minimizes (4.17) given the private sector's expectation of the price level, and given wages. This gives us the two first order conditions ($FOCs$):

$$\frac{\partial R_t}{\partial s_t} + E_t\beta\left[\frac{\partial J_{t+1}}{\partial p_t}\frac{\partial p_t}{\partial s_t} + \frac{\partial J_{t+1}}{\partial s_t}\right] = 0 \quad (4.19)$$

$$\frac{\partial R_t}{\partial D_t} + E_t\beta\frac{\partial J_{t+1}}{\partial p_t}\frac{\partial p_t}{\partial D_t} = 0 \quad (4.20)$$

From these equations we solve for s and D as functions of a constant, lagged price level and exchange rate, and lagged and present shocks to eventually get:

$$s_t^{Flex} = s\left(p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right) \quad (4.21)$$

$$D_t^{Flex} = D\left(p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right) \quad (4.22)$$

In order to solve for the nine unknown ν coefficients, we use a numerical method (method of undetermined coefficients), as an analytical solution would be prohibitively messy. See appendix **??** for a more thorough exposition.

4.2 The monetary union case

One of the main findings in the Agell et al paper [1] was that under commitment of the exchange rate, the deficit may be used to a greater extent to stimulate the economy than under a flexible exchange rate. Our question is how passive the government becomes with respect to stabilization policy when it loses the exchange rate as a tool. That is, will the monetary union economy be more volatile then the flexible exchange rate one?

Let the nominal exchange rate S be normalized to unity, and hence log $S_t = s_t = s_t^e = 0$ $\forall\ t$. All structural equations from above remain valid. To solve the government's problem and get an explicit value of D, we again set up the government's objective and constraints, but now dropping the exchange rate term in each equation:

$$min_{s,D}\ H = \sum_{t=1}^{\infty}\beta^{t-1}R_t$$

$$s.t.\ R_t = \frac{1}{2}{u_t}^2 + \frac{\mu_D}{2}D_t^2 + \frac{\mu_\pi}{2}\pi_t^2$$

where (3.7), (3.16a), (3.9) and (3.12)become:

$$u_t = \theta(w_t - p_t - a_t)$$

$$\pi_t = \alpha(p_t - p_{t-1})$$

$$
\begin{aligned}
p_t &= \frac{\alpha}{(1-\alpha)\theta} D_t + \gamma w_t + \frac{1}{\theta} a_t^* - a_t \\
w_t &= \frac{1}{\lambda\theta} + \frac{\alpha}{(1-\alpha)} D_t^e + \rho a_{t-1}^*
\end{aligned}
$$

We follow the same solution algorithm as in the flexible exchange rate case. Now we only have one first order condition, however:

$$
\frac{\partial R_t}{\partial D_t} + E_t \beta \frac{\partial J_{t+1}}{\partial p_t} \frac{\partial p_t}{\partial D_t} = 0 \tag{4.23}
$$

which eventually results in a closed form solution for the deficit, D, as a function of lagged price level, and lagged and present shocks.

$$
D_t^{MU} = D\left(p_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right) \tag{4.24}
$$

5 Application and analysis

We use the following values on our parameters:[13]

$$
\begin{aligned}
\alpha &= 0.7 \\
\gamma &= 2/3 \Rightarrow \theta = 3 \\
\lambda &= 5.7 \\
\rho &= 0.75 \\
\beta &= 0.96 \\
\mu_D &= 2.4 \\
\mu_\pi &= 2.4
\end{aligned}
$$

That is, the import share $(1-\alpha)$ is 30 percent, labor's share of output is 2/3, the unions place almost three times as high weight on the unemployment factor as it does on the real wage $(\lambda/2 = 2.85)$, and the government puts equal weight on the deficit term and on the inflation term in the loss function, both of which carry more than double the weight of the unemployment term. Obviously, the choice of parameter values is critical to the results we obtain. That is, with unemployment given a less than half of the weight of the other two variables in the loss function, our model is prone to yield relatively high unemployment. Yet given the last two decades of

[13] The model is parameterized so as to match Swedish data 1989-98, assuming this to be a period of flexible Swedish exchange rate. I e $\overline{u}_{1989-98} = 5.8$ %, $\overline{B}_{1989-98} = 3.2$ % and $\overline{\pi}_{1989-98} = 4.7$ %. The share of trade to GDP is from the annual Swedish national accounts, Central Bureau of Statistics (SCB). Data on technology shocks come from Zimmerman [25]. Inflation, deficit and unemployment figures are from OECD Economic Outlook, nr 63, June 1998.

EU's experience in this respect, one could argue that this is not an unreasonable assumption. The bottom line is that the choice of $\mu_D = \mu_\pi = 2.4$ is open to criticism, however.[14]

With these parameter values, equations (4.21) and (4.22) become:

$$\begin{aligned} s_t^{Flex} &= c_1 + c_2\left(\alpha p_{t-1} + (1-\alpha)s_{t-1}\right) + c_3 a_t^* + c_4 a_{t-1}^* + c_5 a_t + c_6 a_{t-1} \\ s_t^{Flex} &= -0.03 + 0.70 p_{t-1} + 0.30 s_{t-1} - 0.60 a_t^* - 0.08 a_{t-1}^* + 0.94 a_t - 0.18 a_{t-1} \end{aligned} \quad (5.1)$$

$$\begin{aligned} D_t^{Flex} &= c_{11} + c_{12}\left(\alpha p_{t-1} + (1-\alpha)s_{t-1}\right) + c_{13} a_t^* + c_{14} a_{t-1}^* + c_{15} a_t + c_{16} a_{t-1} \\ D_t^{Flex} &= 0.03 - 0.10 a_t^* + 0.07 a_{t-1}^* - 0.23 a_t + 0.17 a_{t-1} \end{aligned} \quad (5.2)$$

with $c_2 = 1$ and $c_{12} = 0$. This is a result of the fact that all the ν's in the Bellman equation come out as zero. Inflation will be constant and independent of lagged endogenous variables in the model. In order to see why, we focus on the case when there are no shocks. Then expectations of the policy variables are the same as their outcome, that is, $s = s^e$ and $D = D^e$. This in turn implies that unemployment is constant at $u = \frac{1}{\lambda\theta}$. Under these circumstances, the price level reduces to:

$$p_t = \frac{\gamma}{\lambda\theta} + s_t + \frac{\alpha}{(1-\alpha)} D_t$$

after plugging in for the wage (and assuming no shocks). Now assume the government stands in a hypothetically end period T. Then, the FOC with respect to s reduces to:

$$\frac{\partial R_T}{\partial s_T} = -u_T + \mu_\pi \left(\frac{\alpha}{\theta} + 1 - \alpha\right) \pi_T = 0$$

and therefore we have inflation as:

$$\pi_T = \frac{1}{\lambda\theta\mu_\pi\left(\frac{\alpha}{\theta} + 1 - \alpha\right)} \quad (5.3)$$

That is, inflation depends only on parameters and is therefore constant. Given that this is the solution in the last period, and that it is independent of any action taken in period $T-1$, it must be the case that this will be the outcome in all earlier periods, too. Furthermore, given that both unemployment and inflation are constant in a shock free world, the FOC with respect to D in the last period will be:

$$\frac{\partial R_T}{\partial D_T} \quad = \quad -\frac{\alpha}{(1-\alpha)} u_T + \mu_D D_T + \mu_\pi \frac{\alpha^2}{(1-\alpha)\theta} \pi_T = 0 \quad =>$$

[14] Agell et al [1] use the following parameter values in their base case: $\beta = 0.9$, $\alpha = 0.75$, $\theta = 1.4$, $\mu_D = \mu_\pi = 1$ and $\lambda = 15$. See appendix F for a sensitivity analysis of how the coefficients in front of the shocks in the government's reaction functions depend on the parameter values chosen.

$$
\begin{aligned}
D_T &= \frac{1}{\mu_D}\left[\frac{\alpha}{(1-\alpha)\lambda\theta} - \mu_\pi \frac{\alpha^2}{(1-\alpha)\theta}\frac{1}{\lambda\theta\mu_\pi\left(\frac{\alpha}{\theta}+1-\alpha\right)}\right] \\
&= \frac{\alpha}{\lambda\theta\mu_D\left(\frac{\alpha}{\theta}+1-\alpha\right)}
\end{aligned} \tag{5.4}
$$

That is, equal to a constant and independent of any earlier decisions, too. Hence under a flexible exchange rate, the problem facing the government is static rather than dynamic.[15]

In the case of a monetary union, the $\nu's$ are non-zero and the government's reaction function follows:

$$
D_t^{MU} = 0.03 + 0.17p_{t-1} - 0.27a_t^* + 0.08a_{t-1}^* + 0.10a_t + 0.05a_{t-1} \tag{5.5}
$$

6 Analysis

At first sight there seem to be some peculiarities in the results. A drop in domestic labor productivity raises prices and unemployment according to equations (3.9) and (3.7) and is in general a bad thing for the economy in the eye of the government. However, equation (5.1) tells us that if the economy is exposed to such a shock, the government will choose to appreciate the currency, thereby contracting the economy further. In the monetary union case, the same reasoning makes equation (5.5) equation look un-intuitive regarding the sign of the domestic productivity shock's coefficient, too. Yet after a closer look it all makes sense.

6.1 Domestic productivity shock

Assume we start off in a situation with employment at its benchmark, full employment level ($\widetilde{l}$, not to be confused with the natural level, $\widetilde{l} - \frac{1}{\lambda\theta}$) which implies zero unemployment, zero deficit and zero inflation. In this case, the value of the loss function is also zero.[16]

6.1.1 Flexible exchange rate case

A drop in domestic labor productivity will raise the price level. Hence the government uses the exchange rate to counter-act the upward movement in the general price level. The exchange

[15] The two expressions for inflation and deficit above are identical to Agell et al's [1] solutions in the flexible exchange rate case (their equations (22) and (23) on page 1423).

[16] Recall that full employment is given by the expression $\widetilde{l} \equiv \theta \ln \gamma$, implying $\theta\,(w - p - a_t) = 0$. That is, the shock augmented real wage $\frac{W}{PA}$ is normalized to unity. Since this level has no practical meaning, we can for the sake of clarity when reasoning about what the equations say, assume that employment can be even higher, thereby generating negative unemployment.

rate is a potent tool for this objective, as it affects inflation both indirectly via the price level p, and directly. Furthermore, it carries no cost in itself (as opposed to the deficit).

$$a \downarrow \Rightarrow p \uparrow \Rightarrow s \downarrow \textit{ to support price stability}$$

However, the use of the exchange rate to mitigate the inflationary pressure causes negative effects on the other terms in the loss function. Given a starting point where all terms in the loss function are zero, and shocks last period were zero, the unions' expectations of the exchange rate will be above what the government sets it to in the absence of any productivity shocks. Hence equation (3.13):

$$u_t = \frac{1}{\lambda\theta} - (s_t - s_t^e) - \frac{\alpha}{(1-\alpha)}(D_t - D_t^e) - (a_t^* - (a_t^*)^e)$$

tells us that unemployment rises (if we for the moment disregard any movement in the deficit). This is the off-setting effect from further usage of the exchange rate to handle the inflation, caused by the fact that the terms in the loss function are quadratic. It results in a tendency to "even out" the deviations in the three variables. Therefore, the exchange rate is not used so extensively as to totally cancel the inflation triggered by the domestic shock, and the price level is still above its initial value, but below what it would have been without an exchange rate response. Thus, a drop in domestic productivity will drive unemployment up above its optimal level given by the benchmark real wage, causing in turn a deficit response to counteract this negative unemployment effect. Therefore, the domestic shock has a negative coefficient in equation (5.2), which works to decrease unemployment in (3.13) .

$$a \downarrow \Rightarrow p \uparrow \Rightarrow s \downarrow \textit{ to support price stability} \Rightarrow u \uparrow \Rightarrow D \uparrow$$

So given our starting position and a flexible exchange rate regime, the final result from an unexpected deterioration in domestic productivity will be some inflation, unemployment above its optimal level and a budget deficit. A rise in domestic productivity will cause the reverse scenario, being equally bad for the government since the terms in the loss function enter quadratically. [17]

6.1.2 Monetary union case

In a monetary union, there is by definition no exchange rate to counter-act the inflationary pressures from a negative domestic productivity shock. Instead, the government is confined to

[17] Hence unemployment below the bench mark level is no better than unemployment above it, and budget surpluses and deflation no better than deficits and inflation.

fiscal policy, running a budget surplus to decrease the price level (by a factor of $\frac{\alpha}{(1-\alpha)\theta}$). As the government uses the negative deficit to react to the shock, the price level loses its "neutrality" on employment along the lines stated above in section (3.3) , that is, that employment is unaffected by the domestic shock effect since the price level effect cancels with the wage effect.

$$a \downarrow \Rightarrow p \uparrow \Rightarrow D \downarrow \textit{ to support price stability} \Rightarrow u \uparrow$$

Assuming a starting point with zero inflation, zero deficit and benchmark unemployment, the induced surplus increases unemployment. Just as under a flexible exchange rate, the result is a situation where the optimal response by the government eases the first effect from the inflation by causing costly changes in unemployment and deficit, respectively. The extent to which the deficit is used is governed by the "internal balance" given by the FOC (4.24).

6.2 Foreign productivity shock

6.2.1 Flexible exchange rate case

A decrease in foreign productivity lowers the price level according to (3.9). The mechanism is that foreign income drops, and with it foreign demand for domestic exports (equation (3.5)), resulting in lower prices. Assuming the same starting position and following the same reasoning as above, the government uses the exchange rate to dampen the deflationary pressure by making imports more expensive via a depreciation of the exchange rate.

$$a^* \downarrow \Rightarrow \; y^* \; \downarrow \Rightarrow \; X^* \downarrow \Rightarrow \; p \downarrow \Rightarrow \; s \uparrow \; \textit{ to support price stability}$$

However, $|\frac{\partial s}{\partial a^*}| < 1$ from equation (5.1), that is, the drop in the level of the foreign productivity is larger than the depreciation of the exchange rate. With no changes in the deficit, this would result in a rise in unemployment, according to equation (3.13) , that is $a^* \downarrow \Rightarrow \; u \uparrow$.

Just as in the case of a domestic shock, the inflation term and the unemployment term in the loss function work against each other. On the one hand, the unemployment equation suggests that the government should counteract the foreign productivity decrease one for one. On the other hand, in the inflation equation (3.16a) a change in s has a greater impact than a change in a_t^*, since a_t^* only affects inflation via p, so the aggregate effect on inflation is $\frac{\alpha}{\theta}$. Yet the exchange rate has its direct effect $(1-\alpha)$, *and* its indirect effect via the price level, which is identical to the foreign shock's impact.

$$\frac{\partial \pi}{\partial a^*} = \frac{\alpha}{\theta} \quad < \quad \frac{\partial \pi}{\partial s} = \frac{\alpha}{\theta} + (1-\alpha)$$

The result is therefore an "overreaction" of the exchange rate regarding the inflation term, reversing the initial deflation to inflation. The explanation is that the depreciation does too little with respect to unemployment, and too much with respect to the inflation. That is:

$$|\frac{\partial u}{\partial a^*}| = |\frac{\partial u}{\partial s}| \quad but \quad |\frac{\partial \pi}{\partial a^*}| < |\frac{\partial \pi}{\partial s}|$$

Hence the use of the deficit, which also rises in response to a drop in foreign productivity, thereby mitigating the loss created in unemployment according to equation (3.13).

6.2.2 Monetary union

When foreign productivity decreases in the monetary union case, the price level drops and the government runs a deficit to both reduce the deflationary pressures and to mitigate the rise in unemployment. The intuition is the same as in the flexible exchange rate case, but since there is no exchange rate to support the task, the deficit is used more extensively.

$$a^* \downarrow \Rightarrow p \downarrow \Rightarrow D \uparrow \ \textit{to support price stability}$$

This is clearly shown by the reaction equations (5.2) and (5.5), where the coefficient on the deficit almost triples in absolute value in the case of a monetary union.

$$|\frac{\partial D^{Flex}}{\partial a^*}| \;=\; 0.10 \quad > \quad |\frac{\partial D^{MU}}{\partial a^*}| \;=\; 0.29$$

Hence one of the main results from Agell et al's paper [1] carries over to the case with stochastic productivity shocks. Namely that giving up the exchange rate as a stabilization tool results in a substantial increase in the usage of the deficit.

7 Steady state

In the absence of any productivity shocks, the steady state under a flexible exchange rate implies an unemployment rate equal to $\frac{1}{\lambda\theta} = 5.7$, a deficit at 3.2 percent, and a 4.8 percent inflation rate.[18] By comparison, the monetary union steady state is characterized by zero inflation and a deficit of 5.1 percent.

8 Business cycle analysis

In order to see how our model behaves when exposed to a series of productivity shocks, a fictionary business cycle is created by letting productivity vary +/- 10 percent over an eight

[18] This is the natural level of unemployment, i e when $s = s^e$ and $D = D^e$.

interval period. The cycle is symmetric and generated as a sine curve. Aggregate utility is unaffected by the business cycle since the latter is symmetric. Therefore, the only welfare statement the analysis offers is one regarding the government's aggregated loss.

8.1 Domestic productivity cycle

8.1.1 Flexible exchange rate case

With $\mu_D = \mu_\pi = 2.4$, the average deficit over the cycle is 3.2 percent. As the equations $(3.7), (3.9), (5.2)$ and (5.1) indicate, unemployment, inflation and the deficit are all countercyclical with respect to domestic productivity (although the peak of the productivity and the trough of the three variables differ by one period, resulting in a correlation of -0.66).[19] Both unemployment and the inflation rate oscillate more than the deficit. As a point of reference, unemployment would fluctuate between 2 and 15 percent over the cycle if there was no deficit to mitigate the adverse effect the exchange rate has on it.

Table 8.1: **Correlation matrix.**
Flexible exchange rate. Domestic productivity cycle.

	a	**u**	**D**	**π**
a	1			
u	-0.66	1		
D	-0.66	1	1	
π	-0.66	1	1	1

8.1.2 Monetary union case

Under a monetary union, unemployment is countercyclical as in the flexible exchange rate case. Yet the deficit is now procyclical. The reason is the same as stated above, that the inflationary pressure caused by a negative domestic shock is mitigated by decreasing the deficit. Its average level is now 5.1 percent, 60 percent higher than the flexible exchange rate outcome. Inflation has a slight bias towards the positive side, averaging 0.1 percent over the cycle. Hence on the

[19] That the correlation is less than unity stems from the fact that as the productivity parameter reaches its peak, each period's innovation is reduced. Hence the impact on the price level becomes smaller and smaller, which triggers less of a response in the policy variables s and D. The outcome is that our three loss function variables have their "turning points" one period ahead of the productivity cycle. An easy example can illustrate the point: when domestic productivity drops in a monetary union, the price level increases and unemployment with it. The price instability triggers a decrease in the deficit, which in turn leads to a rise in unemployment. As the cycle approaches its trough, the innovation is smaller, and hence inflation is lower, i e less of a problem. Hence the government's focus is shifted towards the unemployment situation. The result is an increase in the deficit (and consequently a reduction in unemployment) one period ahead of the turn in productivity.

one hand inflation is much more volatile than it is under a flexible exchange rate regime. On the other hand unemployment is dramatically more stable. The reason is that the exchange rate is used relatively efficiently as a price stabilizer under a flexible exchange rate. This, however, causes adverse effects on the unemployment as we saw above, resulting in the use of the deficit to mitigate the latter effect. Yet despite the deficit's effort, the large response in the exchange rate makes unemployment almost twice as volatile under a flexible exchange rate regime.[20]

With respect to the government's aggregated loss, giving up the exchange rate has a marginal negative effect. The gain from price stability (or rather, inflation stability) under a flexible exchange rate outweighs the loss caused by higher average inflation.

Table 8.2: **Correlation matrix. Monetary union. Domestic productivity cycle.**

	a	**u**	**D**	π
a	1			
u	-0.66	1		
D	0.30	-0.91	1	
π	-0.50	0.98	-0.98	1

Table 8.3: **Standard deviations in response to a domestic productivity cycle.**

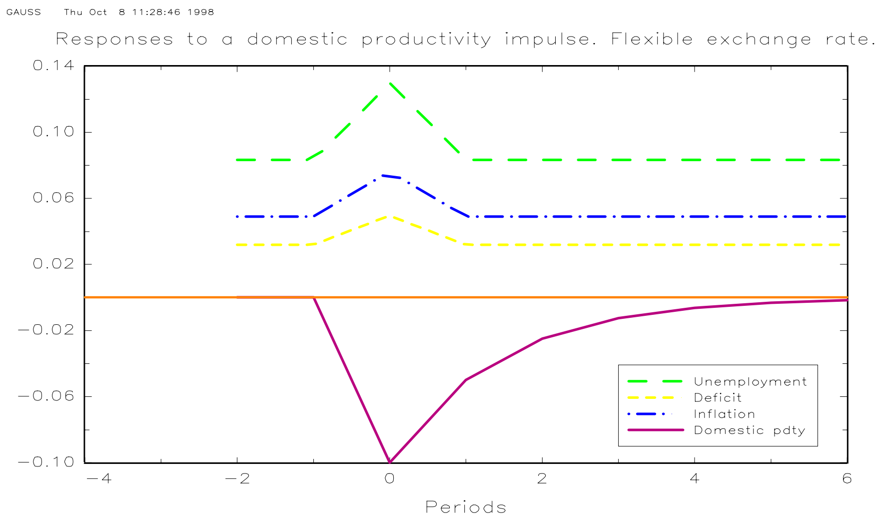
	Flexible exchange rate	**Monetary Union**
u	2.1 (5.8)	1.2 (5.8)
D	1.1 (3.2)	0.7 (5.1)
π	1.6 (4.6)	3.7 (0.1)
$1000 * \sum R$	54	58

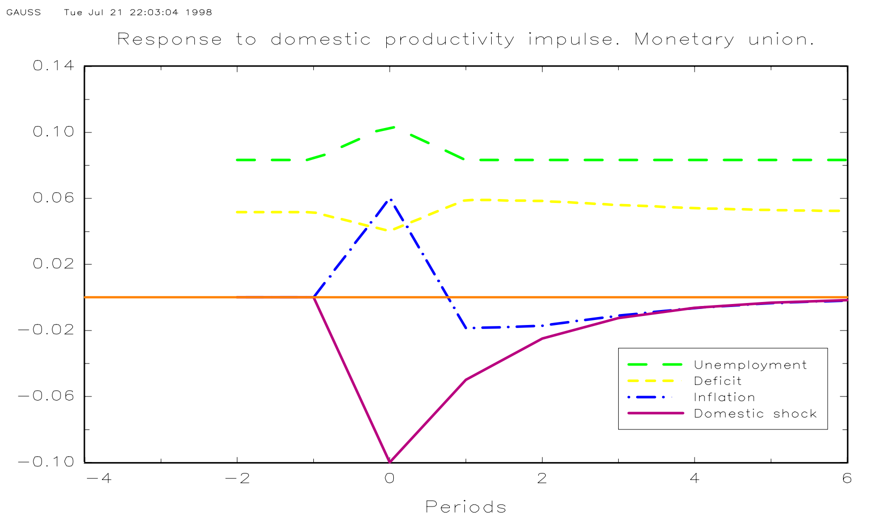
Notes: Except for the loss value, numbers are in percentage points. Average levels over the business cycle within parentheses.

8.1.3 Impulse response

If we, instead of letting productivity gradually decrease by 10 percent, let it drop at once to its trough, and then assume no more innovations in productivity, we get an impulse response analysis. Due to the model's set up, the results are rather trivial under a flexible exchange rate regime in the sense that the impulse will affect the economy only one period. After that, expectations are equal to actual levels, and therefore wages and unemployment are back at steady state levels, as are deficit and inflation.

[20] Note: This is a clear example of how the specfic results hinge upon the relative weights μ_D and μ_π in the government's loss function. See appendix E.1 for a similar analysis but with all weights equal to unity in the loss function, and appendix F for an analysis with the parameter values used by Agell et al [1].





In a monetary union, the response in unemployment is the same as under a flexible exchange rate. That is, any impact lasts only one period. After that, the unions' expectations are correct, and unemployment returns to its steady state value $\frac{1}{\lambda\theta}$. Regarding the deficit and the inflation rate, the dynamics are more pronounced. The deficit first drops in response to a negative domestic impulse, then rises above its steady state level after which it gradually returns to the steady state level.

The reason is the deficit is now affected by the state variable, p_{t-1}. Although the unions' expectations are correct in the second period, and consequently unemployment is at its natural level, the negative "decay" in productivity back up to its steady state value means that there is a positive jump in domestic productivity in each subsequent period, imposing a downward pressure on the price level in every period. Since the government is concerned with *changes* in the price level, deflation is as bad as inflation, and as a result the deficit is now used to increase the price level. This process goes on until domestic productivity has settled down at its long run level, resulting in an inflation rate approaching zero from below and a deficit approaching its steady state level from above.

8.2 Foreign productivity cycle

8.2.1 Flexible exchange rate case

Under a flexible exchange rate, all three variables are countercyclical and they fluctuate less than when exposed to a domestic cycle. Their average levels are unchanged, however. Since foreign productivity has less of an impact on the price level, the price instability is less of a problem for the government and the standard deviation of inflation drops from 1.6 to 0.7 percentage points. The correlation matrix is identical to the domestic shock case (table 8.1).

8.2.2 Monetary union case

Under a monetary union, inflation is procyclical whilst the deficit and unemployment are countercyclical. When the foreign productivity level is below its expectation, the deficit works in the same direction in its effort to mitigate on the one hand the price effect, and on the other the unemployment effect, resulting in a countercyclical behavior. The difference from the domestic shock is that unemployment is more volatile in a monetary union than under a flexible exchange rate. This steams from the shock's effect on the price level, which in turn affects the unemployment via the real wage. As opposed to the flexible exchange rate case, there is now no exchange rate to mitigate the shock's effect. And since the deficit carries a direct cost in itself in the loss function (which in our analysis here is more than two times as high as the relative cost of unemployment), the government's optimal response is to let the unemployment vary much more than the deficit. The result is a standard deviation of unemployment of 1.9 percentage points, leading to a slightly higher loss than under flexible exchange rate. The increased loss is partly reduced by the fact that price instability is less of a problem under a foreign business cycle than under a domestic one, and therefore giving up the exchange rate is less costly.

Table 8.4: **Correlation matrix. Monetary union. Foreign productivity cycle.**

	a	**u**	**D**	π
a	1			
u	-0.66	1		
D	-0.82	0.96	1	
π	0.95	-0.75	-0.90	1

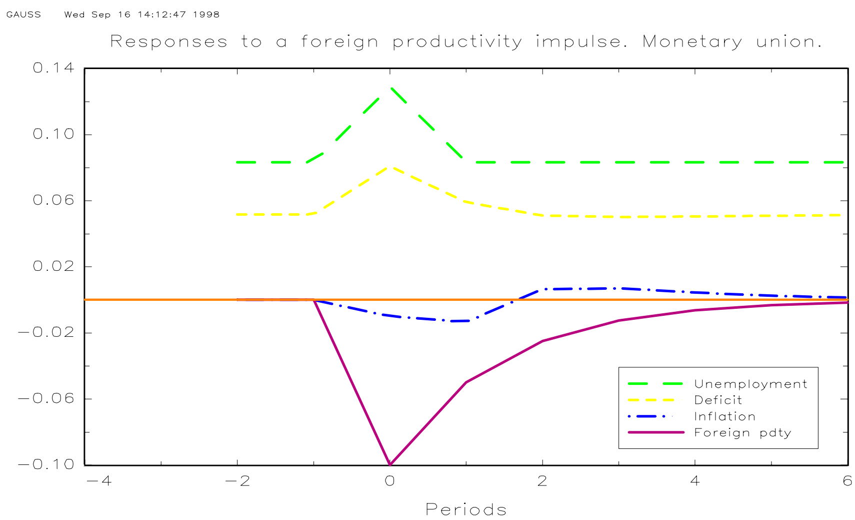
Table 8.5: **Standard deviations in response to a foreign productivity cycle.**

	Flexible exchange rate	**Monetary Union**
u	0.9 (5.8)	1.9 (5.8)
D	0.5 (3.2)	1.8 (5.3)
π	0.7 (4.6)	1.9 (−0.4)
$1000 * \sum R$	50	54

Notes: Except for the loss value, numbers are in percentage points. Average levels over the business cycle within parentheses.

8.2.3 Impulse response

When foreign productivity drops by 10 percent at once, our flexible exchange rate economy reacts similarly to when it was hit by a domestic impulse (see graph 8.1.3.1). Any effects last only one period. Since the impact on the economy of a foreign impulse is smaller than a domestic one (mainly the effect on the price level), so are the magnitude of the responses.



In a monetary union, the deficit and the inflation rate again convey a more dynamic pattern. Just as in the case of a negative impulse in domestic productivity, there is a gradual return to steady state levels. The deficit rises the first period to address both the deflationary pressure from the shock, and the unemployment the shock triggers (equations (3.9) and (3.14)), then drops below its steady state level, after which there is a gradual increase back to equilibrium. The reason is that from the second period on, unemployment is back at its steady state level since the unions' expectation of the productivity level is equal to actual levels. Yet the "decay" from below in foreign productivity causes a price increase in the second period, which triggers a deficit below its long run level. Hence inflation and deficit once again mirror each other's behavior. The above reasoning thereby explains why the deficit moves in opposite directions depending on whether the shock is domestic or foreign, whilst unemployment deteriorates in both cases (given that productivity drops).

8.3 Synchronized domestic and foreign business cycles

What happens if our model economy is hit by a global productivity shock, that is, domestic and foreign productivity cycles overlap? Will the effects on the domestic economy be larger or smaller than when it is exposed to either of them separately? In Mundell's original paper [21] the shock is a slump in demand. When this affects two the countries symmetrically, the desired policy response is the same in the two countries, and hence a monetary union is not a problem. Furthermore, under symmetric shocks to an open economy, a coordinated fiscal policy is likely to be advantageous due to spill-over effects.[21] In our model, overlapping technology shocks cause fluctuations in the domestic economy that are generally larger than any one of the single shocks do.

8.3.1 Flexible exchange rate case

Under a flexible exchange rate, a global productivity cycle reinforces the results above, in the sense that the time series patterns of our three key variables are more volatile. For example, unemployment now has a standard deviation of 3 percentage points. The correlation matrix is identical to the domestic shock case, table (8.1), indicating the dominance of the domestic shock compared to the foreign shock regarding their effect on the economy.

8.3.2 Monetary union case

In a monetary union, the pattern is less uniform. Inflation lags the business cycle by two periods, indicating that the domestic shock dominates the price behavior since inflation is positive when the domestic shock decreases, and negative when it increases. The deficit remains nicely

[21] For instance, see Masson [20].

countercyclical, increasing as soon as productivity starts to drop. As above, the countercyclical deficit is a result of the need to offset the price effect.

Table 8.6: **Correlation matrix.**
Monetary union. Overlapping productivity cycles.

	a	**a***	**u**	**D**	**π**
a	1				
a*	1	1			
u	-0.66	-0.66	1		
D	0.97	-0.97	-0.82	1	
π	-0.02	-0.02	0.75	0.24	1

Table 8.7: **Standard deviations in response to overlapping productivity cycles.**

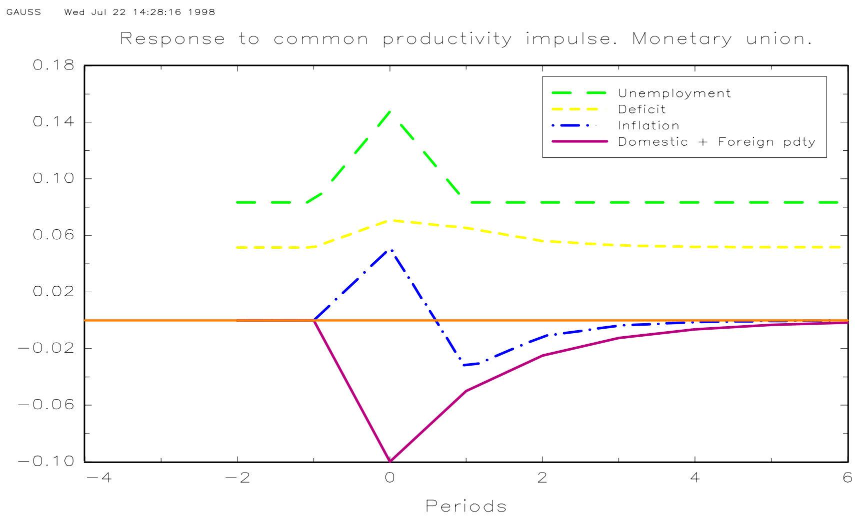
	Flexible exchange rate	**Monetary Union**
u	3.0 (5.8)	3.1 (5.8)
D	1.6 (3.2)	1.3 (5.3)
π	2.3 (4.6)	2.9 (−0.3)
$1000 * \sum R$	60	59

Notes: Except for the loss value, numbers are in percentage points. Average levels over the business cycle within parentheses.

From the government's point of view, overlapping shocks make things a little bit worse. The aggregated loss increases to 0.060 under a flexible exchange rate, and 0.059 under a monetary union regime. The latter regime is marginally preferable because of the lower average inflation rate under a monetary union.

8.3.3 Impulse response

In line with the results above, the impulse response results are reinforced when the two productivity cycles overlap under a flexibility exchange rate regime. Apart from that, they are identical to the impulse response graph in section 8.1.3. And they again die out after one period. Giving up the exchange rate yields more dynamic results, with a gradual return in deficit and inflation. However, with a common productivity cycle the deficit jumps only once, after which it returns to the equilibrium level smoothly. (In the cases of a domestic impulse and a foreign impulse above, the deficit first jumped down, then up above its long run level, and *then* started its gradual return.)



8.4 Counteracting business cycles

What happens when the domestic economy is totally out of sync with the rest of the world? That is, when domestic productivity increases as foreign productivity drops. How will this affect the choice of policy variables, and what is the impact on amplitude of the business cycle fluctuations from such a scenario?[22]

8.4.1 Flexible exchange rate case

Letting domestic productivity be half a cycle behind/ahead of the foreign cycle, all three variables are countercyclical under a flexible exchange rate regime. Again, the domestic shock's greater impact on the price level is the reason for the price level's behavior.[23] The exchange rate, which is utilized to dampen the disruptive price effect and therefore is procyclical with respect to the domestic shock, affects the unemployment in an opposite direction to its own movement. This effect is sufficient to dominate over the foreign shock's impact on unemployment, rendering the latter countercyclical with respect to the domestic shock, and procyclical with respect to the foreign shock. Consequently, the variables' correlations with the shocks are identical to table

[22] For countries in a monetary union, this may be of decreasing interest (Frankel and Rose, 1996 [13]). The reason is that economic integration with the rest of the union will most likely increase, thereby making the business cycle more correlated. Hence it is plausible that the degree of symmetry of shocks is endogenous. This will likely be the case regarding demand shocks. Supply shocks may or may not become more correlated, depending on whether integration leads to further specialization (as argued by for instance Bayoumi and Eichengreen [6], Eichengreen [12], and Krugman [19]), or to more diversification. However, despite relative integration among the Scandinavian countries, Hansson and Sjöholm [15] estimate that Sweden has negatively correlated supply shocks with both Norway and Finland (although statistically *in*significant).

[23] Recall that the partial derivatives are $\frac{\partial p}{\partial a} = -1$ and $\frac{\partial p}{\partial a^*} = \frac{1}{\theta}$, respectively.

(8.1), but now have a positive sign with respect to the foreign shock (but the same absolute value.)

8.4.2 Monetary union case

In the monetary union case, the deficit and the unemployment is procyclical with respect to the domestic cycle, with inflation being countercyclical. As above, the domestic shock's impact dominates the price effect. As domestic productivity starts to fall, the price level starts to increase. But it is a gradual increase, since the government uses the deficit to mitigate the price increase. However, the deficit and the foreign shock is not enough to cancel the impact of the domestic shock, and hence inflation is countercyclical with respect to the domestic cycle. Unemployment is mainly affected by the foreign shock, and is therefore countercyclical to the foreign shock, according to equation (3.13).

When the domestic economy is out of sync with the world economy, the loss of the exchange rate makes a substantial difference. The deficit fluctuates four times as much as under a flexible exchange rate, and inflation more than five times as much. As a result, the aggregated loss exceeds the loss under a flexible regime by 40 percent.

Table 8.9: **Correlation matrix. Monetary union. Counteracting productivity cycles.**

	a	**a***	**u**	**D**	π
a	1				
a*	-1	1			
u	0.66	-0.66	1		
D	0.71	-0.71	0.99	1	
π	-0.71	0.71	-0.99	-1.00	1

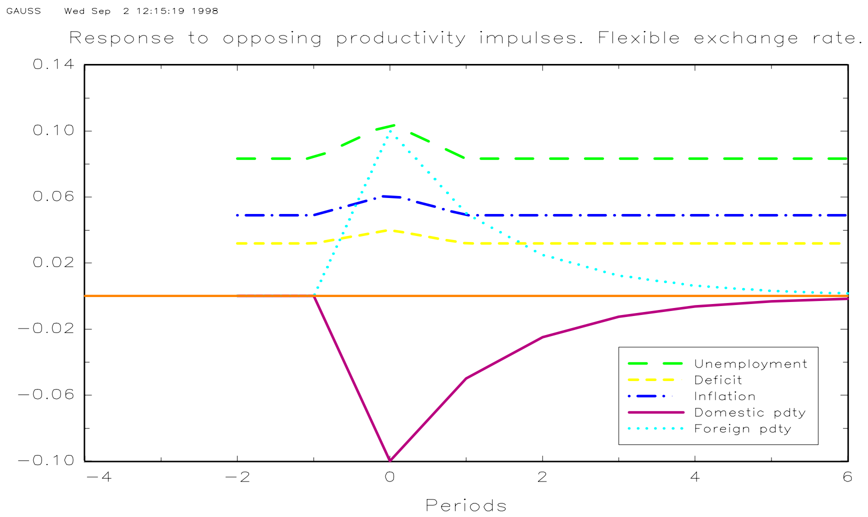
Table 8.10: **Std deviations in response to counteracting productivity cycle.**

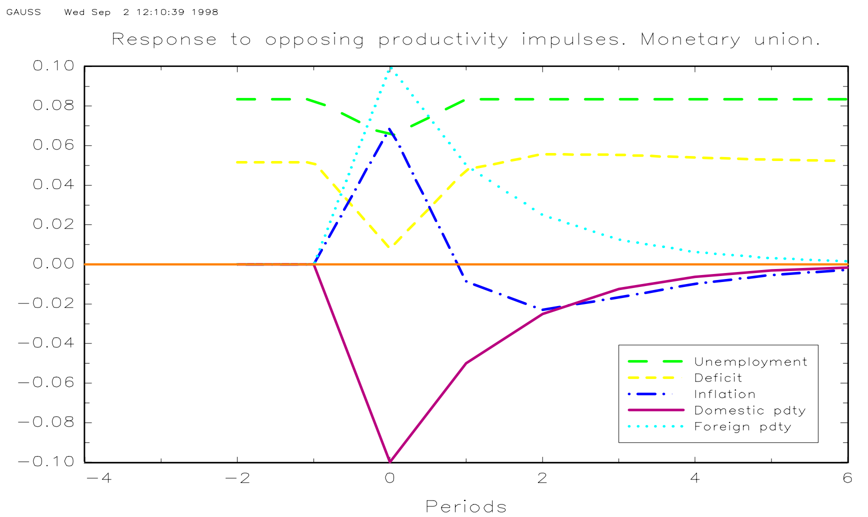
	Flexible exchange rate	**Monetary Union**
u	1.2 (5.8)	0.7 (5.8)
D	0.6 (3.2)	2.4 (5.0)
π	0.9 (4.6)	5.0 (0.5)
$1000 * \sum R$	51	72

Notes: Except for the loss value, numbers are in percentage points. Average levels over the business cycle within parentheses.

8.4.3 Impulse response

Under a flexible exchange rate, opposing impulses cause a similar reaction to a pure domestic impulse. The contribution of the opposing foreign impulse is to dampen the responses. In an monetary union, the deficit determines the dynamics of the price level after the first period, and as the deficit starts its decline back to the equilibrium level, so does the price level, creating a negative inflation that gradually approaches zero from below.





9 Conclusion

With respect to unemployment, the comparative outcome depends on the source of the asymmetric shock. A domestic productivity cycle has a relatively large impact in the price level, which in turn triggers an exchange rate response in order to stabilize the price level that via its surprise effect $(s - s^e)$ in equation (3.13) renders unemployment remarkably more volatile than it is in a monetary union. Under a foreign productivity cycle, the opposite holds true. That is, there is no "trade off" between addressing the unemployment problem or the price stability problem. A drop in foreign productivity will induce an increase in both the exchange rate (depreciation) and in the deficit, both of which mitigate the rise unemployment *and* the deflationary pressure caused by the foreign shock. However, this response is less effective when there is no exchange rate, and consequently unemployment fluctuates more under a monetary union. That is:

$$\begin{aligned} Var\left(u^{Flex}\right) &> Var\left(u^{MU}\right) |_{Domestic\ shock} \\ Var\left(u^{Flex}\right) &< Var\left(u^{MU}\right) |_{Foreign\ shock} \end{aligned}$$

Apart from that, the main conclusion from shifting from a flexible, discretionary exchange rate regime to a monetary union is the following. On the one hand, the level of the deficit is both higher and more volatile, as the government now only has the deficit to enforce its stabilization policies with. Inflation on the other hand is lower, but substantially more volatile since the potent price stabilizing tool, the exchange rate, is not available.

Regarding the government's loss, the high volatility of the inflation rate under a monetary union makes a flexible exchange rate a preferable regime. This is especially true when the two cycles counter-act. Then the aggregated loss is substantially larger when the government has no exchange rate to work with. The reason is that the loss under a monetary union is aggravated by the fact that the inflation is sufficiently volatile to seriously undermine the advantage of zero average inflation, the trade mark of the monetary union case. In addition to this, the loss function's value is worsened by the most fluctuating deficit among all the cases analyzed. When the cycles overlap, there is little difference regarding the loss value.

References

[1] Agell, Jonas, Calmfors, Lars, and Jonsson, Gunnar, 1996, Fiscal policy when monetary policy is tied to the mast, European Economic Review 40 1413-1440.

[2] Andersen, Torben M, 1996, Fiscal policy in the EMU and outside, Background report for the Swedish government Commission on EMU, SOU 1996:158

[3] Artis, Michael J, and Winkler, Bernhard, 1997, The Stability Pact: Safeguarding the Credibility of the European Central Bank, CEPR Discussion Paper No 1688.

[4] Assarsson, Bengt, and Olsson, Clas, 1993, Makroekonomiska chocker och ekonomisk struktur: En jämförelse mellan Sverige och EG-länderna, appendix 2 till EG-konsekvensutredningen, samhällsekonomi, SOU 1994:100.

[5] Bayoumi, Tamim, and Eichengreen, Barry, 1992, Shocking aspects of European monetary unification, NBER working paper No 3949.

[6] Bayoumi, Tamim, and Eichengreen, Barry, 1994, One money or many? Analyzing the prospects for monetary unification in various parts of the world, Princeton Studies in International Finance No 76, Sept, Princeton, USA.

[7] Bayoumi Tamim, and Masson Paul R, 1995, Fiscal flows in the United States and Canada: Lessons for monetary union in Europe, European Economic Review, 39, p 253-274.

[8] Canzoneri, Valles and Vinals (1996), Do exchange rates move to address international macroeconomic imbalances, CEPR discussion paper 1498.

[9] Caporale, Guglielmo Maria, 1993, Is Europe an optimum currency area? Symmetric versus asymmetric shocks in the EC, National Institute Economic Review, No 143.

[10] de Grauwe, Paul, 1992, The Economics of Monetary Integration, Oxford University Press.

[11] Eichengreen, Barry, 1992, One money for Europe? Lessons from the US currency union, Economic Policy 10.

[12] Eichengreen, Barry, 1992, Should the Maastricht treaty be saved, Princeton Studies in International Finance, No 74, Dec, Princeton, USA.

[13] Frankel, Jeffrey A, and Rose, Andrew K, 1996, Economic structure and the decision to adopt a common currency, Background report for the Swedish government Commission on EMU, SOU 1996:158.

[14] Gylfason, Lindbeck, Assar, 1994, The interaction of monetary policy and wages, Public choice 79, 33-46.

[15] Hansson, Jesper, and Sjöholm, Fredrik, 1996, Asymmetriska chocker - ett hinder för svenskt medlemskap i EMU?, Ekonomisk Debatt No3, p 193-203.

[16] Horn H, Persson , 1988, Exchange rate policy, wage formation and credibility, European Economic review 32, 1621-1623.

[17] Italiener, A, and Vanheukelen, M, 1993, Proposals for Community Stabilization Mechanisms: Some historical applications in the Economics of Community Public Finance, European Economy, Reports and Studies No5, Brussels.

[18] Kenen, P B, 1995, Economic and Monetary Union in Europe, Cambridge University Press.

[19] Krugman, Paul, 1993, Lessons of Massachusetts for EMU, in Giavazzi, F, and Torres, F (eds) The Transition to Economic and Monetary Union in Europe, Cambridge University Press.

[20] Masson, Paul R, 1996, Fiscal dimensions of EMU, The Economic Journal, 106, (July), p 996-1004.

[21] Mundell, Robert, 1961, A theory of optimum currency areas, American Economic Review.

[22] OECD Economic Outlook, June 1998, nr 63.

[23] Sala-i-Martin, Xavier, and Sachs, Geoffrey, 1992, Fiscal federalism and optimum currency areas: Evidence for Europe from the United States, in Canzoneri, Grilli and Masson, eds, Establishing a Central Bank: Issues in Europe and lessons from the US, Cambridge University Press, p 195-219.

[24] Svensson, Lars E O, 1997, Optimal inflation targets, "conservative" central banks, and linear inflation contracts, American Economic Review 87, No 1, p 98-114.

[25] Zimmermann, Christian, 1994, Technology innovations and the volatility of output: An international perspective, Working Paper, University of Quebec at Montreal.

A Calculus 1

A.1 Aggregate demand

Aggregation over all identical consumers yields:

$$P(1+D)(L^{\gamma}A) = PX + SM$$

Solving for domestic demand in the budget constraint, we get:

$$X = (1+D)(L^{\gamma}A) - \frac{S}{P}M$$

Using this in the utility function yields:

$$\max U = \left[(1+D)(L^{\gamma}A) - \frac{S}{P}M\right]^{\alpha} M^{1-\alpha}$$

Differentiating this with respect to M:

$$\begin{aligned} M &= (1+D)(L^{\gamma}A)(1-\alpha)\frac{P}{S} \\ X &= \alpha(1+D)(L^{\gamma}A) \end{aligned}$$

By symmetry, world demand for domestic exports equal to:

$$X^{*} = (1-\alpha^{*})\frac{S}{P}((L^{\gamma})^{*}\, A^{*})$$

Adding up demand for the domestically produced good, we have:

$$L^{\gamma}A = X + X^{*} = \alpha(1+D)(L^{\gamma}A) + (1-\alpha^{*})\frac{S}{P}(L^{\gamma *}A^{*})$$

Collecting terms, taking logs (letting lower case letters denote log values):

$$\gamma y + a_t = -\ln\left[1-(1+D)\alpha\right] + s_t - p_t + \ln\left[(1-\alpha^{*})\left(L^{\gamma}\right)^{*}\right] + a^{*}$$

Using a Taylor expansion of the first term of the RHS $\left[f(x) = f(x_0) + f'(x_0)(x-x_0)\right]$ and the fact that the derivative is to be evaluated at x_0, hence $D = 0$ in its evaluation, yields:

$$\ln\left[1-(1+D)\alpha\right] = \ln(1-\alpha) - \frac{\alpha}{1-\alpha}D$$

where $-\alpha$ is the inner derivative. Using this, we get:

$$\gamma y + a = -\ln(1-\alpha) + \frac{\alpha}{1-\alpha}D + s - p + \ln\left[(1-\alpha^{*})\left(L^{\gamma}\right)^{*}\right] + a^{*}$$

Letting $\phi \equiv \ln\left[(1-\alpha^{*})\left(L_t^{\gamma}\right)^{*}\right] - \ln(1-\alpha)$, we have total demand for the domestic output (good X) as:

$$\gamma y + a = \phi + s - p + \frac{\alpha}{1-\alpha}D + a^{*}$$

A.2 Aggregate supply (AS) part 1: Production and employment.

Profit maximization in firm j yields:

$$\frac{\partial(profit)_j}{\partial L_j} = \gamma P L_j^{\gamma-1} A - W_j = 0$$

$$\Rightarrow L_j^{\gamma-1} = \frac{W_j}{\gamma P A}$$

Hence log employment in each firm is:

$$\begin{aligned} l_j &= -\frac{1}{1-\gamma}(w_j - \ln\gamma - p - a) \\ &= -\theta w_j + \theta \ln\gamma + \theta p + \theta a \\ &= \bar{l} - \theta(w_j - p - a) \end{aligned} \tag{A.1}$$

where $\theta \equiv \frac{1}{1-\gamma} > 1$ is elasticity of labor demand, and $\bar{l} \equiv \theta \ln\gamma$. By the symmetry of our normalized economy, $(A.1)$ will also be aggregate employment. Thus aggregate unemployment in our economy will be:

$$u_t \equiv \widetilde{l} - l_t = \theta(w_t - p_t - a_t)$$

Writing the production function in logs, we have:

$$\begin{aligned} y_t^{AS} &= \gamma y_t + a_t \\ &= \gamma(\widetilde{l} - (\widetilde{l} - l_t)) + a_t \\ &= \widetilde{y} - (\theta - 1)(w_t - p_t) + \theta a_t \end{aligned}$$

with $\gamma = \frac{\theta-1}{\theta}$.

A.3 AS part 2: Wage setting

Union expected utility in firm j at time t is given by:

$$V_{jt}^e = (w_{jt} + D_t^e - p_{ct}^e) - E\left[\frac{\lambda}{2}(\bar{l} - l_{j,t})^2\right]$$

Using equation (3.7) we get:

$$V_{jt}^e = w_{jt} - p_{ct}^e + D_t^e - E\left[\frac{\lambda}{2}[\theta(w_{jt} - p_t - a_t)]^2\right]$$

Differentiating with respect to $w_{j,t}$ yields:

$$w_{j,t} = \frac{1}{\lambda\theta^2} + p_t^e + E\left(a_t\right)$$

under the assumption that the unions do not believe their wage affects neither the general price level nor the fiscal position chosen by the government ($\frac{\partial p_t^e}{\partial w_{jt}} = \frac{\partial D_t^e}{\partial w_{jt}} = 0$), and the exogeneity of shocks. Symmetric equilibrium implies $w_j = w \ \forall j$. Using (3.9), we have:

$$p_t^e = \frac{1}{\theta}{s_t}^e + \frac{\alpha}{\theta(1-\alpha)} D_t^e + \gamma w_t + \frac{1}{\theta} E\left(a_t^*\right) - E\left(a_t\right)$$

Plugging this into our wage expression, and cleaning up, we get the wage as:

$$w_t = \frac{1}{\lambda\theta} + {s_t}^e + \frac{\alpha}{(1-\alpha)} D_t^e + E\left(a_t^*\right)$$

B Calculus 2

From the two first order conditions (FOCs) :

$$\begin{aligned}\frac{\partial R_t}{\partial s_t}+\beta E_t\left[\frac{\partial J_{t+1}}{\partial p_t}\frac{\partial p_t}{\partial s_t}+\frac{\partial J_{t+1}}{\partial s_t}\right] &= 0\\ \frac{\partial R_t}{\partial D_t}+\beta E_t\frac{\partial J_{t+1}}{\partial p_t}\frac{\partial p_t}{\partial D_t} &= 0\end{aligned}$$

we solve for s and D, substitute so that each's solution does not include the other policy variable, after which we take expectations. The expectations can be rearranged so that they are functions of lagged price level and the productivity shocks only. That is, step by step we have the solutions following the FOCs as:

$$\begin{aligned}s_t &= s\left(s_t^e, D_t, D_t^e, p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\\ D_t &= D\left(s_t^e, D_t, D_t^e, p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\end{aligned}$$

Further rearrangement gives us the policy variables as functions of their expected value, state variables (lagged p and s), and shocks:

$$\begin{aligned}s &= s\left(s_t^e, D_t^e, p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\\ D &= D\left(s_t^e, D_t^e, p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\end{aligned} \tag{B.1}$$

Taking expectations of the latter ($E(x)=x^e$) and rearranging, we have:

$$\begin{aligned}s^e &= s\left(D_t^e, p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\\ D^e &= D\left(s_t^e, p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\end{aligned}$$

Substitution gives us:

$$\begin{aligned}s^e &= s\left(p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\\ D^e &= D\left(p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\end{aligned}$$

Plugging in the last two equalities into $(B.1)$ gives us the optimal policy variables as functions of a constant, lagged price level and exchange rate, and lagged and present shocks.

$$\begin{aligned}s_t^{Flex} &= s\left(p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\\ D_t^{Flex} &= D\left(p_{t-1}, s_{t-1}, a_t^*, a_{t-1}^*, a_t, a_{t-1}\right)\end{aligned}$$

However, these expressions will include the nine ν's from the value function. In order to solve for the ν's, we recognize the fact that the derivative of the value function can be written in two different forms:

$$\begin{aligned}E_{t-1}\frac{\partial J_t}{\partial p_{t-1}} &= \nu_1+\nu_2 p_{t-1}+\nu_{12}s_{t-1}+\nu_7\rho a_{t-1}^*+\nu_8\rho a_{t-1}=\\ &= E_{t-1}\left[\frac{\partial R_t}{\partial p_{t-1}}+\beta\left(\frac{\partial J_{t+1}}{\partial p_t}\frac{\partial p_t}{\partial p_{t-1}}+\frac{\partial J_{t+1}}{\partial s_t}\frac{\partial s_t}{\partial p_{t-1}}\right)\right]\end{aligned}$$

where we use that:

$$E_{t-1}a_t^i=\rho a_{t-1}^i \qquad i = domestic\ and\ foreign$$

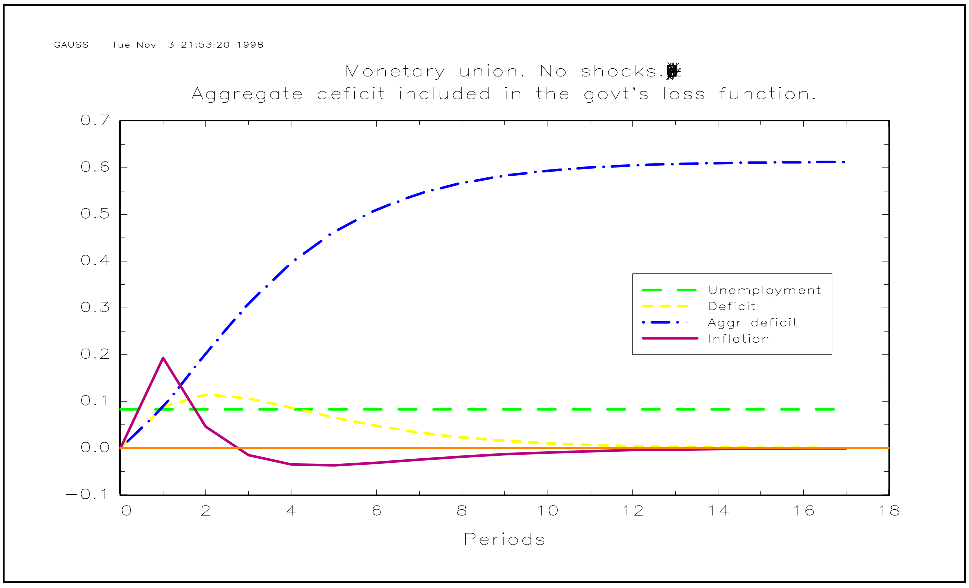
By calculating the second equality, we can then solve for the ν's by equating the coefficients on each of the constant and variables p_{t-1}, a_{t-1}^*, and a_{t-1} (method of undetermined coefficients). Doing the same thing to $E\frac{\partial J_t}{\partial s_{t-1}}$ gives us nine equalities, and nine coefficients to determine. The identification is done numerically.

C An aggregate deficit term in the loss function

In order to see how the model behaves when the government also cares about the aggregate level of debt, we assume the following loss function:

$$\begin{aligned} min\ H &= \sum_{t=1}^{\infty} \beta^{t-1} R_t \\ subject\ to\ R_t &= \frac{1}{2} {u_t}^2 + \frac{\mu_B}{2} B_t^2 + \frac{\mu_{AB}}{2} \left(\sum_{i=-\infty}^{t} B_i \right)^2 + \frac{\mu_\pi}{2} \pi_t^2 \end{aligned}$$

The aggregate deficit level (= debt level) will rise in a concave fashion until eventually the deficit is set to zero and the debt stabilizes.



D Technology shocks with propagation

A more realistic description of the technology shocks is as a two variable VAR(1).

$$\begin{aligned} \begin{bmatrix} a_t \\ a_t^* \end{bmatrix} &= \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} a_{t-1} \\ a_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix} \\ \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix} &= \tilde{}\quad N\left(0, \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_\varepsilon \sigma_{\varepsilon^*} \\ \sigma_{\varepsilon^*}\sigma_\varepsilon & \sigma_{\varepsilon^*}^2 \end{bmatrix}\right) \end{aligned}$$

Using Zimmerman's [25] calibration in the case of Sweden versus the rest of the world, we obtain:

$$\begin{aligned} \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} &= \begin{bmatrix} 0.75 & 0.18 \\ -0.02 & 1.01 \end{bmatrix} \\ \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_\varepsilon \sigma_{\varepsilon^*} \\ \sigma_{\varepsilon^*}\sigma_\varepsilon & \sigma_{\varepsilon^*}^2 \end{bmatrix} &= \begin{bmatrix} 1.82 & 0.20 \\ 0.20 & 0.30 \end{bmatrix} \end{aligned}$$

Including the propagation, the impulse responses have dramatically more persistence. The graphs below show four different cases: an initial domestic shock under our two regimes, and ditto for an initial foreign shock. Higher persistence in the shocks affects the responses in the variables only marginally, however. Unemployment, the deficit, and inflation rate behave much as they do on section (8) .

Under an initial domestic shock, the responses in the three variables in the loss function all behave as they did in section (8.1.3) , but with more persistence. Furthermore, domestic productivity grows above its steady state level before eventually settling down at its long run level in the flexible exchange rate case.

In the case of a foreign initial shock, persistence is very high. The shocks do not die out until after several hundred periods. Moreover, as the autocorrelation coefficients indicate, the foreign shock has a substantial impact on the domestic productivity level.

E Sensitivity analysis 1: averages and standard deviations

E.1 $\mu_D = \mu_\pi = 1$

To see what the weights in the loss function do for the results, we look at the case of a domestic shock. If we, instead of setting $\mu_D = \mu_\pi = 2.4$, let all three variables in the loss function carry the same weight (i e set the two $\mu' s$ to unity), the average levels of unemployment is unaffected but the volatility is reduced since the unemployment term now carries a relatively higher weight. (Recall table 8.3 with standard deviations of 2.4 and 1.4 percent.) However, in both the flexible exchange rate case and the monetary union case, the average level of the deficit increases dramatically, to 9.6 and 11.1 percent, respectively. Under a flexible exchange rate, the deficit is also approximately 50 percent more volatile than when $\mu_D = \mu_\pi = 2.4$. The same goes for the flexible exchange rate inflation. Furthermore, higher average levels bring up the aggregated loss.

Table E.1: **Std deviations in response to a domestic productivity cycle.** $\boldsymbol{\mu_D = \mu_\pi = 1}$

	Flexible exchange rate	Monetary Union
u	1.1 (8.3)	1.0 (8.3)
D	1.3 (9.6)	0.6 (11.1)
$\boldsymbol{\pi}$	1.9 (14.7)	3.3 (0.0)
$1000 * \sum R$	172	92

Notes: Except for the loss value, numbers are in percentage points. Average levels over the business cycle within parentheses.

F Sensitivity analysis 2: Using the Agell et al parameters

If we use the parameterization of Agell et al [1], average unemployment is substantially lower due to the higher value of λ (recall that the steady state level of unemployment is equal to $\frac{1}{\lambda\theta}$, which in our base case meant an unemployment of 5.8 percent). This in turn brings down the aggregated loss (which was 54 (flexible exchange rate) and 58, (monetary union) in section (8.1.2)). As a consequence of the lower relative weights on deficit and unemployment in the loss function, their average levels increase. In the flexible exchange rate case, volatility is also increased. (See table 8.3)

Table E.2: **Std deviations in response to a domestic productivity cycle.** $\boldsymbol{\alpha = 0.75,\ \theta = 1.4,\ \beta = 0.9\ ,\ \lambda = 15,\ \mu_D = \mu_\pi = 1}$

	Flexible exchange rate	Monetary Union
u	1.4 (4.8)	2.1 (4.8)
D	1.3 (4.5)	0.8 (6.5)
$\boldsymbol{\pi}$	1.8 (6.1)	3.0 (0.0)
$1000 * \sum R$	39	35

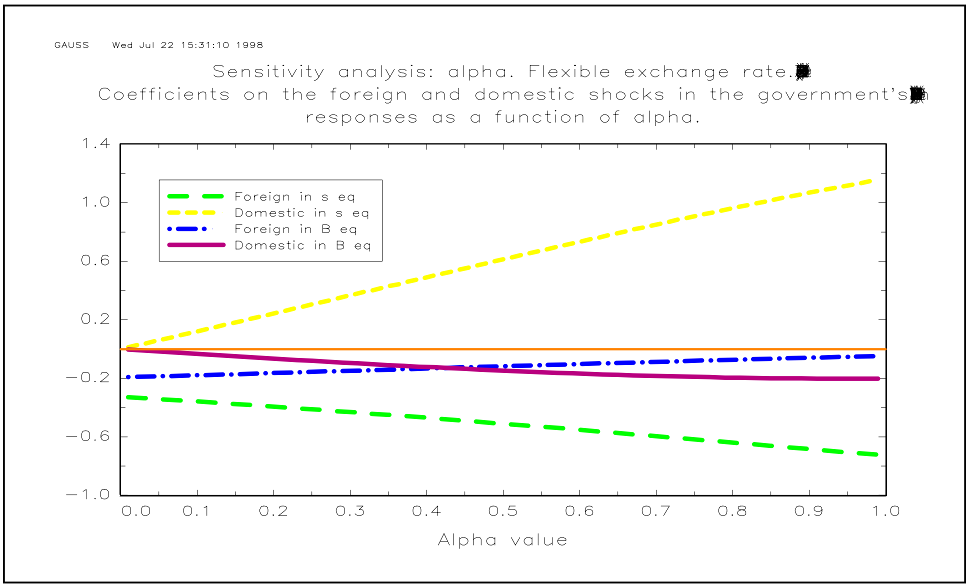
Notes: Except for the loss value, numbers are in percentage points. Average levels over the business cycle within parentheses.

G Sensitivity analysis 3: shock coefficients

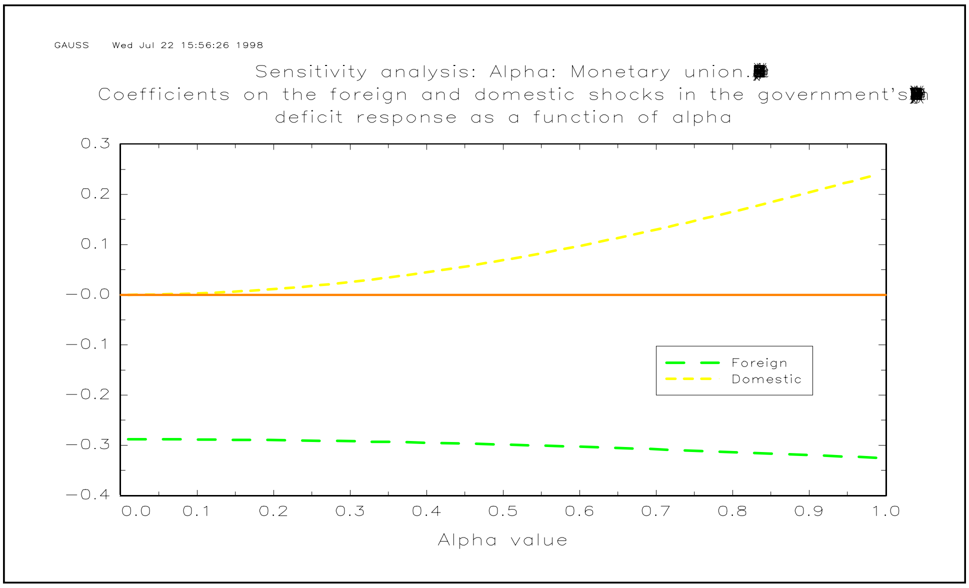
In each of the following sensitivity analyses, we let all coefficients keep their designated value from above, but for the parameter under scrutiny. We focus on the coefficients on the present shocks in the three reaction functions (5.1), (5.2) and (5.5). Some of the shock coefficients change substantially as our "basic" parameters change. Most important for our purposes, however, is that there is no change in the sign of the shock coefficients throughout the following analysis.

G.1 Alpha, α

The most conspicuous characteristic of the results from letting alpha, the share of domestic purchases in total consumption, vary from zero to unity is a continuous increase in the exchange rate response to a domestic productivity shock, combined with an increasingly negative coefficient on the same shock in the deficit equation. The reason is that the exchange rate gets less and less weight in the inflation expression $(3.16a)$. Therefore, the exchange rate, increasingly affecting the inflation rate only indirectly via the price level, is used with full force to handle the disturbance from the domestic shock.



As we move towards the other extreme (a country that consumes only imports and exports all its production), only the responses to a foreign shock differ from zero. The reason is that in this case, the inflation is determined solely by the exchange rate, which therefore must be used with more and more caution as a stabilizing tool. Hence a decline in magnitude of its response to a foreign productivity shock. Given this reasoning, the only way for the government to handle the employment effect from a foreign shock is to use the deficit more. Therefore, its response increases in magnitude (becomes more negative).

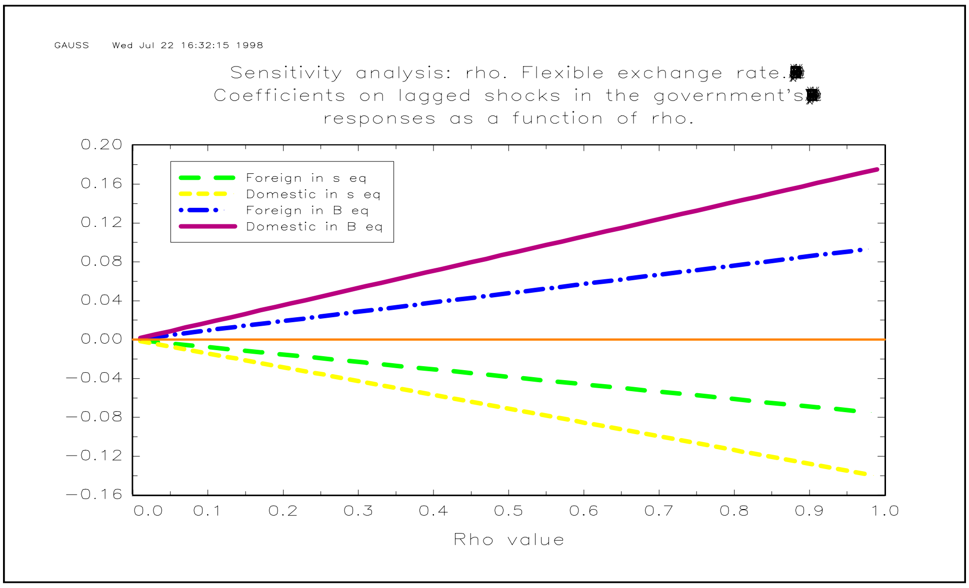


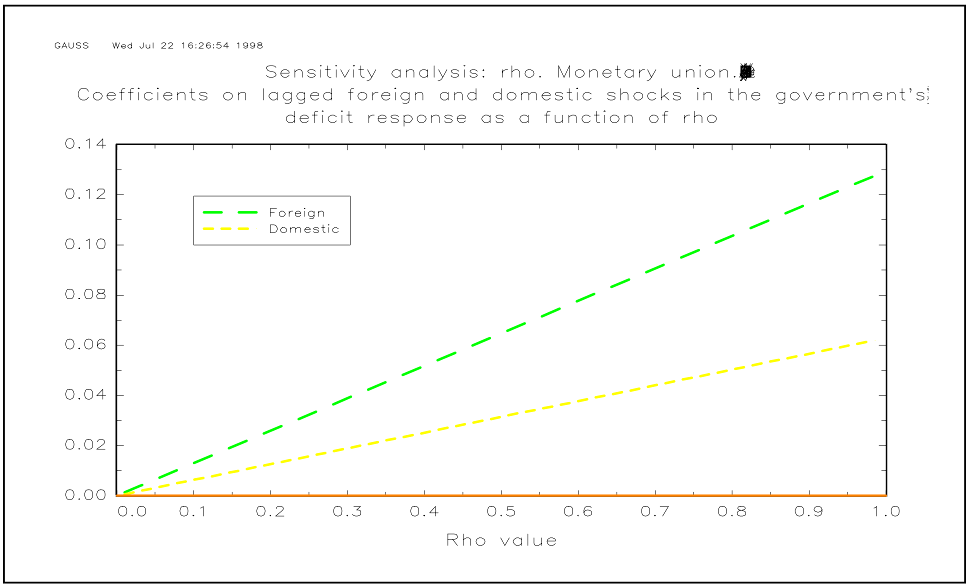
Under a monetary union, the reaction is the same. As the economy becomes more introvert, the exchange rate is used more aggressively to keep its potency.

G.2 Rho, ρ

Letting ρ (the persistence of the productivity shocks) vary between zero and one has no effect on the government's response to present shocks, irrespective of the choice of exchange rate regime.

Looking at the coefficients on the lagged shocks the deficit response function, they grow from zero to approximately 0.1 and 0.2 respectively, as ρ goes from zero to unity. Hence high persistence results in a more pronounced response in the deficit to lagged shocks, balancing the deficit response to present shocks. The result is that as persistence grows, the deficit's overall impact as a tuning tool is substantially reduced. High persistence makes the deficit a too costly instrument to use, and the government is better off being more passive in response to shocks.





In the monetary union case, the story is the same. The coefficients on lagged shocks increase as the persistence in the shocks rise. The only difference is that the coefficient on the lagged foreign shock increases more than the domestic one, as opposed to the discretionary case.

G.3 Beta, β and lambda, λ

β is the government's discount factor and lambda the unions' weight on unemployment relative to the real wage. The higher is β, the more the government cares about the future. And hence the more the government ought to want to see to it that the present shocks influence the future as little as possible. That is, it seems likely that the government would want to kill the shocks as much as possible. Just as a central bank may wish to do in models with output persistence (see for instance Svensson, [24]).

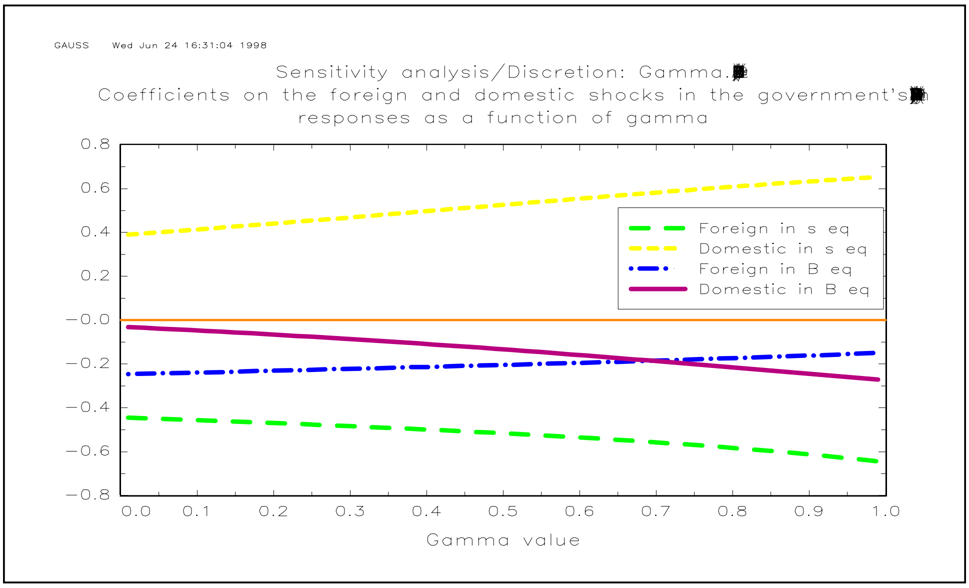
However, since the governments problem is non-dynamic under a flexible exchange rate, it turns out that changes in β do not affect the coefficients. The same goes for λ. The latter will only affect the average level of unemployment, and not any policy responses.

G.4 Gamma, γ

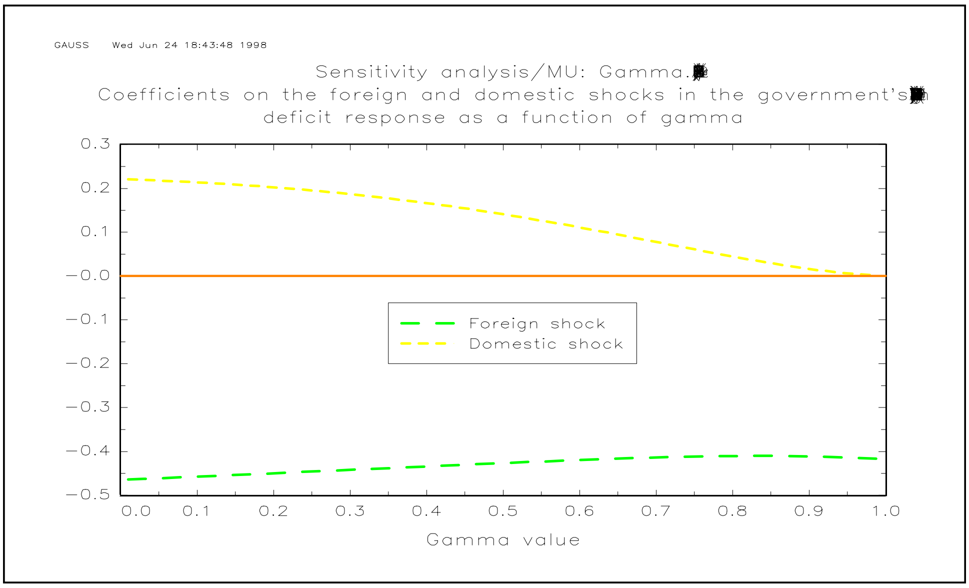
γ is labor's share in the Cobb-Douglas production function. It transforms into θ $\left(\theta = \frac{1}{1-\gamma}\right)$, the elasticity of labor demand (equation (3.6)). As γ grows from zero to unity, the exchange rate response to domestic productivity shocks and the deficit response to foreign shocks increase, whilst the other two decrease.

$$\gamma \quad \uparrow \quad => \frac{\partial s^{Flex}}{\partial a} \; \& \; \frac{\partial D^{Flex}}{\partial a^*} \; \uparrow$$
$$\gamma \quad \uparrow \quad => \frac{\partial s^{Flex}}{\partial a^*} \; \& \; \frac{\partial D^{Flex}}{\partial a} \; \downarrow$$

Furthermore, θ goes from unity towards infinity. This means that the coefficients on the exchange rate and the foreign shock in the price level goes from unity towards zero. Hence as γ grows, the impact of a foreign productivity shock decreases, and the exchange rate becomes increasingly impotent as a tool with which to affect the price level. Despite the fact that these two effects are weakened by the same amount, it appears that the coefficient on the foreign shock in the exchange rate's response becomes increasingly more negative as a reaction to its increasing ineffectiveness. As γ goes towards unity, the effect from the foreign shock on the loss function is focused on the unemployment effect. Yet the deficit's response to such a shock decreases as γ grows, instead of balancing the foreign innovation by decreasing $D_t - D_t^e$ further (a more negative coefficient on D).



With respect to a domestic shock, the exchange rate response grows as γ increases. As above, this is likely a function of the growing impotence of the exchange rate on the price level, and hence an effort to compensate this loss of power. The deficit, which counteracts the exchange rate (see the analysis above under section (6.1)), also becomes less potent as it too has a $\frac{1}{\theta}$ factor in front of it in the price equation. Hence its coefficient increases in absolute value. Over the whole range $\gamma \in (0,1)$ none of the coefficients change sign however, so the qualitative results in the applied response functions above are unchanged.



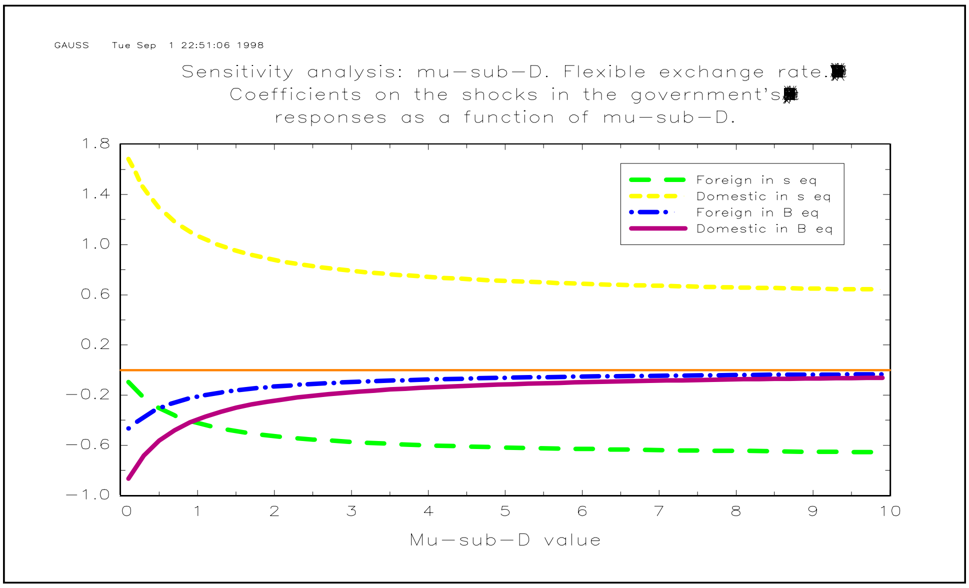
In a monetary union, the response to a foreign shock increases, and the response to a domestic shock decreases, just as they do in the discretionary case. Now however, the coefficient on the domestic shock drops from above. That is, it is positive throughout the interval.

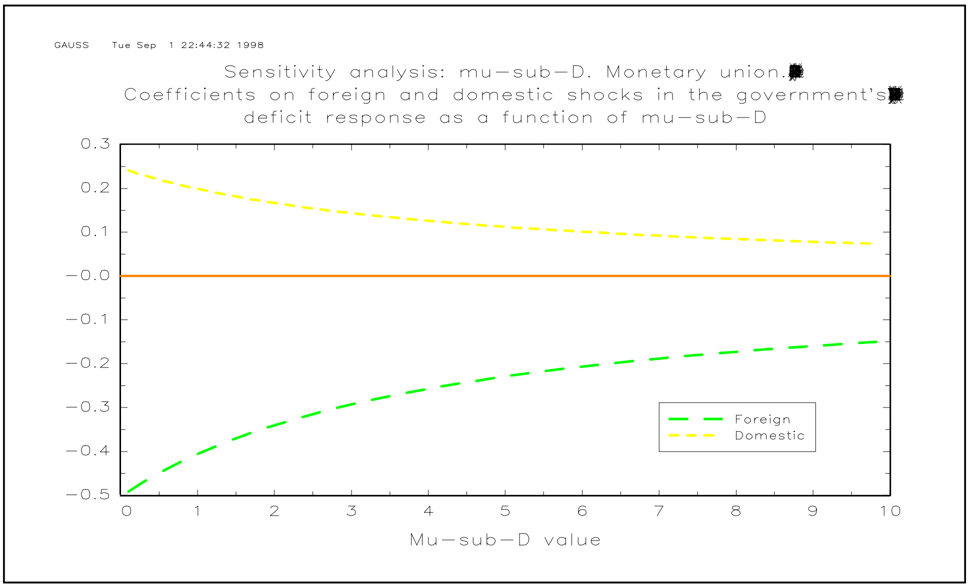
G.5 Mu-sub-D, μ_D

μ_D is the weight on the deficit in the loss function. As μ_D increases above zero, the shocks' coefficients in the deficit response function goes towards zero. This is reasonable, since it becomes imperative for the government to have a low deficit. As a result, the exchange rate now does all the work in trying to mitigate the effects of shocks under discretion.

In response to a foreign shock, the deficit moves in the same direction as the exchange rate. Hence as μ_D grows, the coefficient on the foreign shock in the exchange rate's response function becomes increasingly negative in order to compensate for the loss of help from the deficit, keeping down both the inflationary pressure from the shock, and its reduction of the unemployment.

In response to a domestic shock, the deficit counteracts the effects of the exchange rate (see section (6.1)). Therefore, as the deficit goes towards zero, the "disciplinary effect" from the deficit is reduced, inducing a less forceful response in the exchange rate. Hence the decline in its response to a domestic shock as μ_D increases.



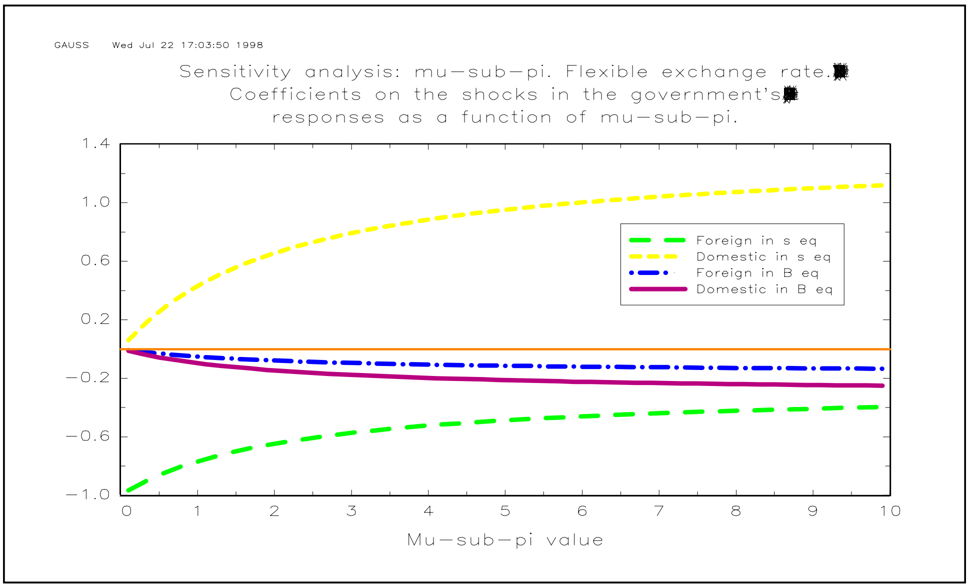


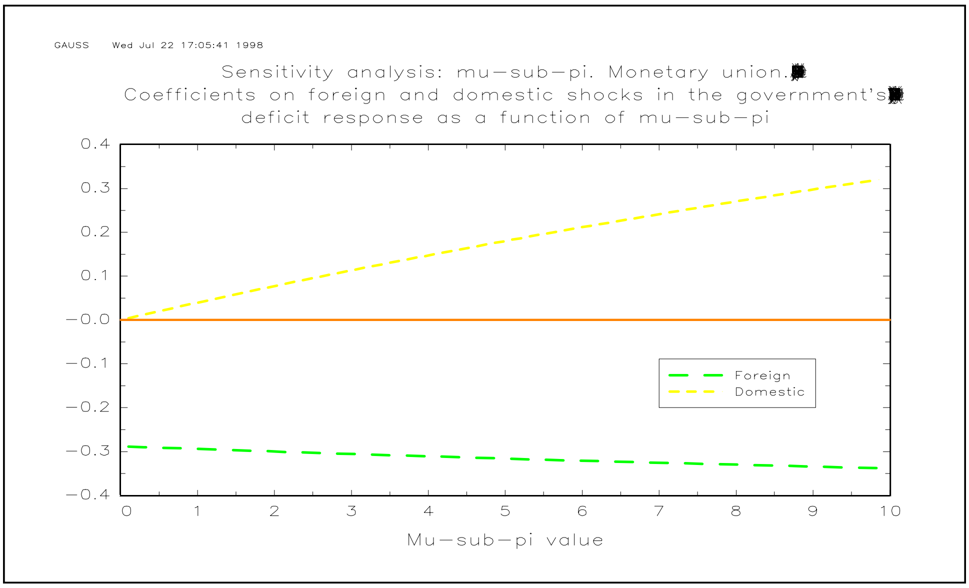
Just as in the discretionary case, the willingness to use the deficit decreases in a monetary union as μ_D increases and the government cares ever more about its deficit. However, the internal changes are reversed, in the sense that now the domestic shock's coefficient goes to zero rather than the foreign shock's coefficient. This can be due to the fact that the domestic shock affects the price level more, and when there is no exchange rate to address the domestic shock with, it is therefore relatively more important to use the deficit against a domestic shock.

G.6 Mu-sub-pi, μ_π

μ_π is the weight the government places on the inflation term in the loss function. As it increases above zero the exchange rate's response to a foreign shock decreases dramatically, and its response to a domestic shock increases equally dramatically. When inflation carries zero weight, the exchange rate only affects the unemployment term. In this case, it is used 1:1 to counteract the foreign shock's impact on unemployment (equation (3.13)), giving the foreign shock a coefficient of -1, and the deficit a zero coefficient. Domestic shocks, only affecting the price level, have no effect on the loss function.

As μ_π increases and keeping a stable price level starts to dominate the government's objectives, the deficit starts to take on more of the work to stabilize unemployment and the price level. Both equation (3.9) and (3.13) clearly show how the deficit can replace the exchange rate in response to a foreign shock, with the result that the direct effect on inflation from the exchange rate can be minimized. In response to a domestic shock the substitutability between the two policy tools is smaller, as the domestic shock has a much more forceful impact on the price level. Therefore, the exchange rate must also be taken into the arsenal to counter-act the deflationary pressure (given that the domestic shock is positive) according to equations (3.9) and (3.16a).





Under a monetary union, the foreign shock's coefficient is relatively stable, but the domestic shock's coefficient increases dramatically, from zero when μ_π is zero, to above 0.5. The reason is likely the same as in the monetary union case of μ_D. That is, that the domestic shock has a larger impact on the price level than the foreign shock, and hence as price stability becomes the key worry of the government, the deficit's response is increasingly focused on balancing the adverse effects on the price level, which is more sensitive to domestic shocks than to foreign shocks.