Smooth transitions in a UK consumption function

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Abstract

This paper reconsiders the equilibrium correction model of nondurable consumption in the UK by Davidson *et al.* (1978), denoted DHSY. The DHSY model fails outside the original observation period and several studies claim that this is due to neglected nonlinearities or time-varying parameters. This paper will take both features into account simultaneously by using the methodology of smooth transition regressions (STR). The study is performed both for the original sample and for an extended one. It turns out that nonlinearities are present in both samples when keeping the original model specification. The resulting consumption functions are characterised by time-varying parameters rather than nonlinear relationships between the explanatory variables. The estimated nonlinear models encompass, and dominate in variance, their linear equivalents for each sample.

Keywords. DHSY, dynamic model, econometric model building, encompassing, parameter constancy, smooth transition regression.

JEL Classification Codes: C52, C53, C22, E21.

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1. Introduction

During the last twenty years equilibrium correction models (ECM) have successfully been applied to describing macroeconomic relationships. The advantage of the methodology is that it makes it possible to model short-run as well as long-run dynamics of integrated economic variables in a single model without running into problems of spurious regressions. In order for this to be true the integrated variables have to be cointegrated, see Engle and Granger (1987). A pioneering work in equilibrium correction modelling is Sargan's (1964) study of wages and prices in the UK, but the best-known example is the ECM of nondurable consumption in the UK presented in a seminal paper by Davidson, Hendry, Srba and Yeo (1978), DHSY for short.

Later evaluations of the DHSY model have shown that it fails outside the original observation period and various explanations of the failure have been suggested. Harvey and Scott (1994) claim that changing seasonality is the reason for the failure whereas Carruth and Henley (1990) and Gausden and Brice (1995) suggest time-varying parameters and neglected nonlinearities. This paper will reconsider the above explanations by taking nonlinearity and time-varying parameters into account simultaneously. This is done by using the methodology of smooth transition regressions (STR), see Granger and Teräsvirta (1993). In that framework time-varying parameters are just another type of nonlinearity.

The analysis will be carried out for two different samples. The first sample is the one used in the DHSY study, 1958(2)-1970(4), and the second is an extended sample, 1958(2)-1992(2). It should be noted that the longer sample is not a simple extension of the original sample. The two data sets belong to different revisions of data and some variables have been redefined, see Hendry (1994). This will have consequences for the estimated consumption functions, and simple comparisons between the models will not be straightforward.

In the DHSY model the short-run dynamics are specified as annual changes (fourth differences). This might be too restrictive in the case of time-varying seasonality and this paper will also use an alternative specification of quarterly changes (first differences). Testing reveals a nonlinear relationship between the explanatory variables when the model is specified in fourth differences using the original sample period. But when it is defined for quarterly changes the model appears to be linear with

time-varying parameters instead. For the second sample both the model specified in fourth differences and the one in first differences have time-varying parameters. The resulting nonlinear models variance dominate and encompass their linear equivalents for each sample.

There are large differences between the four final nonlinear consumption functions presented in this paper. This emphasizes the sensitivity to data transformations and choice of data set when specifying a UK consumption model. Considering this, it is not surprising that there are a wide variety of explanations to the DHSY model failure.

The outline of the paper is as follows. In Section 2 the DHSY model and some of the criticism against it is presented. Section 3 views the data that is used in the analysis and in Section 4 there is a short introduction to STR models. Section 5 suggests a specification of the consumption behaviour during 1958(2)-1970(4) and in Section 6, the period 1958(2)-1992(2) is analysed. Section 7 concludes.

2. Some previous consumption studies

There are numerous descriptions of UK nondurable consumers' expenditures. The choice of methodology is highly varied, see e.g. the DHSY model mentioned in Section 1, HUS-models introduced by Hendry and von Ungern-Sternberg (1981), periodic autoregressive models (PAR) used by Osborn (1988) and "structural models" estimated by Harvey and Scott (1994). In this study the DHSY model will be reconsidered and some of the criticisms are reviewed in this section.

In DHSY the authors start by comparing previously published empirical models of UK nondurable consumption. They point out inconsistencies and provide possible explanations for the discrepancies between the models. Finally they estimate an ECM for 1958(2)-1970(4). The short-run dynamics are specified as annual changes since the authors find this to be a sensible choice for capturing the consumption behaviour in different quarters. The long-run dynamics are captured by the fourth lag of the equilibrium correction term. The resulting model encompasses the models discussed earlier and it performs well in the forecasting period, 1971(1)-1975(4). The DHSY

model has the form

$$\begin{aligned}\Delta_4 c_t &= \alpha_1 \Delta_4 y_t + \alpha_2 \Delta_1 \Delta_4 y_t + \alpha_3 (c-y)_{t-4} + \\ &\quad \alpha_4 \Delta_4 D_t^0 + \alpha_5 \Delta_4 p_t + \alpha_6 \Delta_1 \Delta_4 p_t + \varepsilon_t\end{aligned}$$

where c_t is consumption of goods and services, y_t is real personal disposable income, $\Delta_1 \Delta_4 y_t$ is the first difference of the annual change of income, $(c-y)_{t-4}$ is the equilibrium correction term, D_t^0 is a dummy variable[1], $\Delta_4 p_t$ is the level of inflation and $\Delta_1 \Delta_4 p_t$ the rate of change of inflation. Furthermore, Δ_j is the difference operator, $\Delta_j x_t = x_t - x_{t-j}$, and lower-case letters denote logarithms.

As mentioned in Section 1 the DHSY model fails outside the original sample, that is after 1976(2). According to Bollerslev and Hylleberg (1985) the DHSY model underpredicts consumers' expenditures over the period 1977(3)-1980(2); meanwhile Drobny and Hall (1989) find a distinct tendency for DHSY to overpredict consumption over the period 1980-1986. Carruth and Henley (1990) claim that the DHSY model underpredicts consumption during 1985-1987. Although it is a commonly accepted view that the DHSY model breaks down outside the original observation period, the reason for the failure is still open to debate. A few comments are listed below.

- Hendry (1994) stresses the importance of data revisions and points out that recent data vintages are not cointegrated with earlier vintages. Hence, if the DHSY were estimated using present data vintages a different model would be chosen. The different data vintages are shown and compared in Section 3.

- Hendry, Muellbauer and Murphy (1990) in turn find that consumption, income and inflation are cointegrated, 1957-1976. This should imply a unit root in $\Delta_4 p_t$ and $(c-y)_{t-4}$. The unit root tests in Appendix A show that $(c-y)_{t-4}$ has a unit root 1958(2)-1970(4) whereas $\Delta_4 p_t$ contains a unit root only when the estimation period is extended to 1976(2). For the period 1958(2)-1970(4) the unit-root hypothesis is rejected.

- Carruth and Henley (1990) check whether or not the existing consumption models, such as HUS and DHSY, are adequate for describing quarterly UK non-

[1] $D_t^0 = D_{t-1} - D_t$. D_t is a dummy variable which assumes unity 1968(ii) and zero elsewere. It is included to capture a change in consumption as a result of information of a purchase tax increase in the 1968 budget. For a discussion of the dummy variable, see Davidson *et al.* (1978).

durable consumption behaviour after the original sample periods. They estimate the DHSY model for 1969(2)-1984(4) and find that the (Chow) parameter constancy test fails. Since the purpose of their study is to evaluate existing models rather than to suggest a new model with time-varying parameters, they settle by concluding that a majority of the models they evaluate are inadequate for forecasting UK consumption.

- Harvey and Scott (1994) claim that seasonality varies over time and that this is the main reason for the failure of the DHSY model. In order to account for this effect the authors estimate a "structural model" both for the original data set (1958(2)-1970(4)) and for the extended one (1958(2)-1992(2)). The "structural model" is specified for quarterly changes instead of annual in the short-run dynamics and dummy variables are used to capture the seasonality. The authors find that the impact of seasonality changes over time in both samples and model this by introducing a stochastic seasonality component into the ECM specification. The "structural model" out-performs the DHSY model: there is no error autocorrelation, which is a severe problem in the extended DHSY model, and the standard deviation of the structural model is 74% of that of the DHSY for the sample 1958(2)-1992(2).

- Gausden and Brice (1995) examine the performance of the DHSY model in predicting aggregate consumers' expenditures in UK during 1956(2)-1984(4) for seasonally adjusted and seasonally unadjusted data. They perform their study for both consumption of nondurable goods and services and total expenditures. According to their results it is the impact of income growth that changes over time making a more flexible specification necessary and, hence, they allow for time-varying parameters. To begin with they compute sub-period estimates of the income growth coefficient and find that it is not constant over time. Using a "variable-coefficient" model improves the goodness-of-fit within the sample compared to a "fixed-coefficient" applied to seasonally adjusted data for both total and nondurable consumers' expenditures.

- Song (1995) re-estimates the DHSY and the HUS consumption functions using time-varying parameters in order to improve the forecasting performance of the models, using quarterly data over the period 1961-1991. The time-variation

is modelled using a Kalman filter where the parameters are assumed to be a function of their value in the last period as well as of lagged wealth and inflation. The model with time-varying parameters gives better forecasts than the original DHSY.

In four of the studies mentioned above neglected nonlinearities are suggested as the reason for the failure of the DHSY model. However, it is not obvious whether the problem lies in time-varying seasonality or nonlinear relationships between the variables. The analyses in Sections 5 and 6 will therefore consider both types of nonlinearities simultaneously. The main interest of this study lies in the dynamic properties of DHSY, the effect of the functional form on the fit and the conclusions from the model. The choice of variables is the same as in the DHSY specification. This means that other suggested equations for explaining consumer behaviour, such as those including, for example, wealth and interest rate variables, credit constraints and demographic factors[2] are ruled out. This does not imply that these variables are superfluous in describing nondurable consumption, but it is beyond the scope of this paper to address other consumption functions than the DHSY.

3. The Data

This section presents some stylized facts of the quarterly observations of logarithmic nondurable consumers' expenditures (c_t) and logarithmic real personal disposable income (y_t), for the United Kingdom. The analysis will be carried out for two different time periods. The first period is the one used in the DHSY article, and the corresponding time series will be referred to as the original data. This time series, 1958(2)-1976(2), is from Economic Trends (1976 Annual Supplement) and is quarterly unadjusted in £ million at 1970 prices. The second time period is 1958(2)-1992(2), and is quarterly unadjusted in £ million at 1955 prices. It will be referred to as the extended data set. The two series belong to different vintages of data revisions and will therefore be shown in separate graphs.

The logarithms of nondurable consumption (c_t) and disposable income (y_t) can be found in Figures 3.1 and 3.2. The salient features of the data are a distinct seasonal

[2] See for example Hendry and von Ungern-Sternberg (1981), Hendry, Muellbauer, Murphy (1990).

pattern and strong positive trends in both c_t and y_t. In 1990 the growth slows down because of a recession. When the series are transformed into annual changes (four-quarter differences) the seasonality disappears from sight, see Figures 3.3 and 3.4. Consumption is smoother than income until 1985 while it is the other way around during 1985(3)-1988(3). From 1988(4) onwards the stylized fact of consumption being smoother than income seems to be valid again.

In Figure 3.5 the equilibrium correction term, $(c-y)_t$, is shown for both samples during 1958(2)-1976(2). The difference between the data vintages is conspicuous, especially from 1970 and onwards. The equilibrium correction term of the original sample is greater in absolute value than that of the extended sample based on a later vintage of data. The time series of the two vintages are not cointegrated according to results presented in Hendry (1994), which implies that there is no linear long-run relationship between the values of the variables for the different vintages.

4. Nonlinear modelling

In this section, modelling with smooth transition regression (STR) models is discussed. When nonlinearities in the conditional mean cannot be excluded a priori, a flexible nonlinear model specification is necessary. A large number of parametric and nonparametric nonlinear models exist in the literature. In modelling macroeconomic relationships the limited amount of data which is usually available sets a practical limit to the number of alternative models and techniques available. Smooth transition regression (STR) models enable a rich parameterisation of the conditional mean once linearity has been rejected, and these models may be applied to rather short time series since there exists a modelling strategy that does not necessarily require very long time series, Teräsvirta (1994). STR models have been successfully applied in macroeconometric modelling, see for example Lütkepohl, Teräsvirta and Wolters (1999), Michael, Peel and Taylor (1997) and Teräsvirta and Eliasson (1998) who model asymmetries in money demand functions, Michael, Nobay and Peel (1997) who describe nonlinear adjustment in real exchange rates, van Dijk and Franses (1999) who estimate a nonlinear relationship for interest rates, Teräsvirta (1998) who describes house prices and Johansen (1999) who models wage equations.

4.1. Smooth transition regression models (STR)

The exposition below is a brief introduction to STR models. For a more comprehensive presentation, see Granger and Teräsvirta (1993) or Teräsvirta (1998).

Start by considering the following STR model,

$$y_t = x_t'\varphi + x_t'\theta F(s_t) + u_t. \qquad t = 1,..,T. \tag{4.1}$$

The model consists of a linear and a nonlinear component. The linear component is $x_t'\varphi$, where $x_t = (1, x_{1t}, .., x_{mt})' = (1, y_{t-1}, .., y_{t-p}, z_{1t}, .., z_{qt})'$ is a $((m+1)\times 1)$ vector of explanatory variables where $m = p + q$. Furthermore, $\varphi = (\varphi_0, .., \varphi_m)'$ and $\theta = (\theta_0, .., \theta_m)'$ are parameter vectors. The nonlinear component is specified as a linear combination of variables multiplied with a nonlinear transition function $F(s_t)$ which is continuous and bounded. The error sequence $\{u_t\}$ consists of independent, identically distributed irregular terms such that $Eu_t = 0$, $Ex_tu_t = 0$, $Es_tu_t = 0$. It is customary to bound F between zero and unity; hence the model will change from $E(y_t|x_t) = x_t'\varphi$ to $E(y_t|x_t) = x_t'(\varphi + \theta)$ with the transition variable s_t. This feature makes it possible to capture different dynamics in different states of the economy.

The transition function of a kth order logistic smooth transition regression, LSTR(k), model is

$$F(s_t) = F(\gamma, c; s_t) = \left(1 + \exp\left\{-\gamma \prod_{i=1}^{k}(s_t - c_i)\right\}\right)^{-1}, \ \gamma > 0, c_1 \leq c_2 \leq ... \leq c_k \tag{4.2}$$

where the slope parameter (γ) determines how rapid the transition is and the vector of location parameters (c) decides where the transitions occur. The restrictions $\gamma > 0$ and $c_1 \leq c_2 \leq ... \leq c_k$ are identifying restrictions. If $\gamma = 0$ the nonlinear STR model (4.1) becomes linear and if $\gamma \to \infty$ the STR model (4.1) becomes a step function. For $k = 2$ and $c_1 = c_2$ the LSTR(2) model closely approximates the exponential STR (ESTR) model. The ESTR model is specified as (4.1) with

$$F(s_t) = F(\gamma, c; s_t) = 1 - \exp\left\{-\gamma(s_t - c)^2\right\}, \ \gamma > 0.$$

In what follows the ESTR will be regarded as a special case of LSTR(2). Moreover, note that when $s_t = t$ the STR model (4.1) may be interpreted as a linear model with

time varying parameters. This makes it possible to use the STR model to investigate the claims of unstable parameters in the DHSY model.

4.2. The modelling cycle for STR models

Following Teräsvirta (1998) the modelling cycle contains four stages.

1. Estimate a linear model with no error autocorrelation.
2. Perform the tests of linearity and parameter constancy in this model against an STR alternative. If either of the two hypotheses are rejected choose an STR model for further consideration.
3. Estimate the parameters of the STR model and reduce the size of the model if necessary.
4. Evaluate the final STR model using misspecification tests of no error autocorrelation, parameter constancy and no additive nonlinearity.

In the second step the null hypothesis of linearity is tested. The alternative is specified using several different variables as potential transition variables. If more than one of the null hypotheses are rejected, one starts by specifying an STR model using the transition variable which was used in the most forcefully rejected null hypothesis. The rationale behind this decision rule is discussed in Teräsvirta (1994). The initial choices of potential transition variables are provided by economic theory.

The third step consists of estimating the STR model. This will be done by nonlinear least squares (NLS) which is equivalent to conditional maximum likelihood estimation in the case of normal homoscedastic errors. The estimation is sensitive to the choice of starting values. The slope parameter γ is therefore standardized through division by the standard deviation of the transition variable for an LSTR(1) model and by the variance for an LSTR(2). It may be mentioned that when γ is large its estimate easily gets an inflated standard error estimate in small and even moderate samples. The reason for this is that an accurate estimation requires a sufficient number of observations of s_t in a small neighborhood of the location parameter, see e.g. Teräsvirta (1998). The parameters in the estimated full model will be reduced according to their explanatory power using the t-values as a guidance, except for $\hat{\gamma}$ and $\hat{c}$.

The last step of the modelling consists of model evaluation. The assumptions of no error autocorrelation and parameter constancy used in the parameter estimation are tested. Furthermore, it is interesting to know whether or not all nonlinearity of STR type is captured by the estimated model, so this should be checked as well. A test of no additive nonlinearity is available for this purpose. The misspecification tests are described in Section 4.4. If the model fails the test of no error autocorrelation, respecification of the model is the only solution. If the STR model fails the test of no additional nonlinearity or parameter constancy there are two different solutions. Respecification is again one of them; the other alternative is to introduce another nonlinear component of the type that the alternative hypothesis indicated. However, this can only be considered for large samples.

4.3. Testing linearity

If the true data generating process is nonlinear and a linear model is used for describing the nonlinear process, the estimated model will be underidentified and the dynamics will be misspecified. On the other hand, if the true data generating process is linear and a nonlinear parameterisation is applied then the nonlinear model will overfit the data in the sample. In the post-sample period the model might perform badly, even if it satisfies all the diagnostic tests and variance dominates the linear model, see Granger and Teräsvirta (1992). Testing linearity before fitting a model helps to avoid such misspecifications.

The linearity test presented below follows Teräsvirta (1994). The hypothesis of linearity is tested against an LSTR model of order k. In order to derive the test some additional assumptions are necessary. The stochastic variables $x_{1t}, .., x_{mt}$ are assumed to be stationary. The transition variable s_t is also assumed to be stationary or to be a time trend (t), and all cross-moments $Ex_{it}x_{jt}$, $Ex_{it}s_t^m$, $Ey_{t-i}s_t^m$ and $Ey_{t-i}x_{jt}$, $m \leq 3$ are assumed to exist. The linearity test is derived under the assumption of constant variance and is not robust against conditional heteroscedasticity.

The LSTR(k) model becomes linear if $\gamma = 0$ so that the transition function (F) equals $\frac{1}{2}$. For notational simplicity the transition function in equation (4.1) is replaced by $\tilde{F}(\gamma, c; s_t) = F(\gamma, c; s_t) - \frac{1}{2}$. This implies $\tilde{F}(0, c; s_t) = 0$, and $H_0 : \gamma = 0$ is a natural hypothesis of linearity. Assuming normality of u_t, $t = 1, .., T$, the conditional

log likelihood becomes

$$L = \sum_{t=1}^{T} \ell(\varphi, \theta, c; y_t | x_t, s_t) = -\frac{1}{2}\left(T \ln(2\pi) + T \ln \sigma^2 + \frac{1}{\sigma^2} \sum_{t=1}^{T} u_t^2\right). \tag{4.3}$$

There is a small caveat though: the STR model is not identified under the null hypothesis due to the nuisance parameters θ and c. That is, under the null θ and c can take on any values without affecting the value of the log-likelihood (4.3). In order to circumvent this difficulty the nonlinear component of (4.1) is Taylor-expanded around $\gamma = 0$. Using a kth order Taylor expansion leads, after rearranging terms, to the STR model

$$y_t = x_t' \beta_0 + \sum_{j=1}^{k} \left(\tilde{x}_t s_t^j\right)' \beta_j + v_t \tag{4.4}$$

where $\tilde{x}_t = (x_{1t}, ..., x_{mt})'$ and $v_t = u_t + R(\gamma, \theta, c; x_t)$. The parameter vector β_j is a function of γ such that $\beta_j = 0$, $j = 1, .., k$, when $\gamma = 0$. The new null hypothesis becomes $H_0' : \beta_j = 0, j = 1, .., k$. The remainder term $R(\gamma, \theta, c; x_t)$ equals zero under the null hypothesis and does not affect the asymptotic distribution theory when the test is based on the LM-principle. The LM-type test will have an asymptotic χ^2 distribution with $k * \dim(\tilde{x}_t)$ degrees of freedom under the null. In small samples an F-version of the test is preferable to the χ^2 variant since it has good size and power properties; see for example Granger and Teräsvirta (1993, Chapter 7). The test can be carried out in stages:

1. Regress y_t on x_t and compute the residual sum of squares $SSR_0 = \frac{1}{T} \sum_{t=1}^{T} \hat{u}_t^2$.
2. Regress $\hat{u}_t$ (or y_t) on x_t, $\tilde{x}_t s_t$, $\tilde{x}_t s_t^2$ and $\tilde{x}_t s_t^3$ and compute $SSR_3 = \frac{1}{T} \sum_{t=1}^{T} \hat{w}_t^2$.
3. Compute the F statistic $F = \frac{(SSR_0 - SSR_3)/3m}{SSR_3/(T-4m-1)}$. $\overset{approx}{\sim}$ $F(3m, (T - 4m - 1))$ under H_0.

If H_0' is rejected the conditional mean is assumed to be nonlinear and an STR model is specified and estimated. If the null hypothesis of linearity cannot be rejected the conditional mean is assumed to be linear and no further modelling will be done.

A specification procedure for STR models is outlined in Teräsvirta (1994). After rejecting the hypothesis of linearity (H_0') the order of the LSTR model is selected. In this paper $k \leq 2$, hence LSTR(1) and LSTR(2) are the two parameterisations that

will be used. The testing sequence is defined within equation (4.4) for $k = 3$:

$$\begin{aligned} H_{04} &: \beta_3 = 0, \\ H_{03} &: \beta_2 = 0 | \beta_3 = 0, \\ H_{02} &: \beta_1 = 0 | \beta_3 = \beta_2 = 0. \end{aligned}$$

Independently of whether a hypothesis is accepted or rejected it is still vital to carry out the whole sequence, since the choice between an LSTR(1) and an LSTR(2) specification which are the main alternatives is based on p-values, Teräsvirta (1994). If the rejection of H_{03} is strongest in terms of p-values an LSTR(2) model is selected to describe the nonlinearity. Consequently, if H_{04} or H_{02} has the lowest p-value an LSTR(1) model is the final choice. The reason for including $\beta_3 = 0$ in the testing procedure when the choice stands between an LSTR(1) and an LSTR(2) model is to manage the situation where the only nonlinear element is the intercept. Consider the case when s_t is an element of x_t and $\theta = (\theta_0, 0, ., 0)'$, $\theta_0 \neq 0$; then $\beta_1 = 0$ even under the alternative and the test has no power. To remedy this situation $\beta_3 = 0$ is included in the testing sequence.

A special case of nonlinearity is when the transition variable is a time trend (t). This can be viewed as a model with time-varying parameters. Because of changes in institutions it is reasonable to be prepared for the possibility that some parameters change over time when dealing with macroeconomic variables. A standard assumption in many parameter constancy tests is that if a parameter changes it will only change once, as is the case in the Chow breakpoint test. Lin and Teräsvirta (1994) applied the STR framework to obtain a more general test by allowing the alternative to parameter constancy to be a smooth deterministic change. The special case of a discrete structural break is nested within the procedure. The transition function equals (4.2) with t as transition variable. Following Lin and Teräsvirta (1994) $k = 3$ is chosen as the order of the LSTR(k) model. This yields a flexible non-monotonous transition function. As for the linearity test $\tilde{F}(t, \gamma) = F(t, \gamma) - \frac{1}{2}$ will be used to simplify the notation and $H_0 : \gamma = 0$ in equation (4.2) is the hypothesis for parameter constancy; the alternative is $H_1 : \gamma > 0$. Testing for parameter constancy is similar to testing for linearity, see Lin and Teräsvirta (1994). The Taylor approximation is used to circumvent the problem with unidentified parameters (γ, c) under the null

hypothesis. The Taylor expansion is applied to the STR model and after rearranging terms the auxiliary regression becomes

$$y_t = \delta_0^{'} x_t + \delta_1^{'} x_t t + \delta_2^{'} x_t t^2 + \delta_3^{'} x_t t^3 + v_t$$

The testing sequence is

$$\begin{aligned} LM3 &: \delta_1 = \delta_2 = \delta_3 = 0 \\ LM2 &: \delta_1 = \delta_2 = 0 \mid \delta_3 = 0 \\ LM1 &: \delta_1 = 0 \mid \delta_2 = \delta_3 = 0. \end{aligned}$$

As in the case of testing for linearity it is important to perform all tests before specifying a model. If the rejection of LM1 is strongest (measured by the p-value) an LSTR(1) model is selected, and if LM2 is the strongest rejected hypothesis an LSTR(2) model is the most appropriate. When dealing with time-varying parameters an even more flexible specification may be needed and thus if LM3 is the strongest rejected hypothesis an LSTR(3) model will be estimated.

4.4. Misspecification tests

The misspecification tests to be discussed here are those of (i) no error autocorrelation, (ii) constant parameters, and (iii) no remaining nonlinearity. These tests are presented in Eitrheim and Teräsvirta (1996) and Teräsvirta (1998). The model, under the alternative hypothesis, is specified as

$$y_t = G(\psi; w_t) + A(a; v_t) + u_t \qquad t = 1, .., T \tag{4.5}$$

where $G(\psi; w_t)$ is the estimated STR model and $\psi = (\varphi, \theta)'$ and $\{u_t\}$ is a sequence of independent standard normal variables. Different parameterisations of $A(a; v_t)$ yield the alternative hypotheses for the various types of misspecification tests that will be explored. The function $A(a; v_t)$ is assumed to be at least twice continuously differentiable for all a. The maximum likelihood estimates of ψ are assumed to be consistent and asymptotically normal under any null hypothesis to be considered. The

LM test can be applied for testing the null hypothesis of no misspecification, that is $H_0 : a = 0$, but the F-statistic is preferable because of its superior size properties.

4.4.1. Test of no error autocorrelation

To test the hypothesis of no serial dependence in the conditional mean, the alternative is stated as a remaining serial dependence of order q. This gives the extended model (4.5) with $A(a; v_t) = a'v_t$, where $a = (a_1, ..., a_q)'$ and $v_t = (u_{t-1}, ..., u_{t-q})'$. The null hypothesis of no error autocorrelation is equivalent to testing $H_0 : a = 0$. As mentioned in Section 4.4 the F-statistic is used. Under the null hypothesis the F-statistic is asymptotically distributed with q and $(T - \dim(\psi) - q)$ degrees of freedom.

4.4.2. Test of parameter constancy

The hypothesis of constant parameters is tested against the alternative of time-varying parameters of STR type. Under the alternative hypothesis the STR model becomes

$$y_t = \bar{x}_t'\tilde{\varphi}(t) + \tilde{x}_t'\tilde{\theta}(t)\tilde{F}(\gamma, c; s_t) + u_t \tag{4.6}$$

where $\tilde{\varphi}(t) = \tilde{\varphi} + \lambda_1\tilde{F}_1(\gamma_1, c_1; t)$ and $\tilde{\theta}(t) = \tilde{\theta} + \lambda_2\tilde{F}_1(\gamma_1, c_1; t)$. That is, all time varying variables are assumed to have the same structure. The transition function $\tilde{F}_1$ is defined as in (4.2) assuming $k = 3$ and $s_t = t$, and the null hypothesis of parameter constancy becomes $H_0 : \gamma_1 = 0$. The function $\tilde{F}_1(\gamma_1, c_1; t)$ is Taylor-expanded around $\gamma_1 = 0$ to circumvent the identification problem. Substituting it into equation (4.6) and rearranging terms yields

$$\begin{aligned} y_t &= \bar{x}_t'\beta_0 + (\bar{x}_t t)'\beta_1 + \left(\bar{x}_t t^2\right)'\beta_2 + \left(\bar{x}_t t^3\right)'\beta_3 + \\ &\quad \left[\tilde{x}_t'\beta_4 + (\tilde{x}_t t)'\beta_5 + \left(\tilde{x}_t t^2\right)'\beta_6 + \left(\tilde{x}_t t^3\right)'\beta_7\right]\tilde{F}(\gamma, c; s_t) + u_t^* \end{aligned} \tag{4.7}$$

where $u_t^* = u_t + (\bar{x}_t'\varphi + \tilde{x}_t'\theta)R(\gamma_1, c_1; t)$. The remainder term is equal to zero under the null and does not affect the asymptotic distribution theory. Model (4.7) can be regarded as a special case of model (4.5) with $A(a; v_t) = a'v_t$, where $a = (\beta_1, \beta_2, \beta_3, \beta_5, \beta_6, \beta_7)'$ and $\hat{v}_t = \left\{\bar{x}_t' t, \bar{x}_t' t^2, \bar{x}_t' t^3, \left(\tilde{x}_t' t, \tilde{x}_t' t^2, \tilde{x}_t' t^3\right)\tilde{F}(\gamma, c; s_t)\right\}'$. The null hypothesis can be rewritten as $H_0 : a = 0$ and the F-statistic is applied with $3\dim(a)$ and $(T - 4\dim(a) - 1)$ degrees of freedom.

4.4.3. Test of no remaining nonlinearity

The third misspecification test checks if all nonlinearity is captured against the alternative of additive nonlinearity of STR type. Under the alternative the model becomes

$$y_t = G(\psi; w_t) + \left(w_t^{'}\psi\right) F^*(r_t; \gamma^*, c^*) + u_t \tag{4.8}$$

This STR model has two additive nonlinear components: one included in the original LSTR model is denoted $G(\psi; w_t)$ and the other is $\left(w_t^{'}\psi\right) F^*(r_t; \gamma^*, c^*)$. The transition function F^* may be defined as equation (4.2) with $k = 1, 2$. For notational simplicity it is assumed that the transition variable r_t is an element of $\tilde{x}_t$. The function $F^*(r_t; \gamma^*, c^*)$ is Taylor-expanded around $\gamma^* = 0$ to circumvent the identification problem. Substituting the Taylor-expansion into equation (4.8) gives, after rearranging terms,

$$y_t = G(\psi; w_t) + (\tilde{x}_t r_t)' \beta_1 + \left(\tilde{x}_t r_t^2\right)' \beta_2 + \left(\tilde{x}_t r_t^3\right)' \beta_3 + u_t^* \tag{4.9}$$

where $u_t^* = u_t + R(\gamma_r, c_r; r_t)$ and $u_t^* = u_t$ under the null. Model (4.9) is a special case of model (4.5) with $A(a; v_t) = a'v_t$ where $a = (\beta_1', .., \beta_3')'$ and $v_t = \left(\tilde{x}_t' r_t, \tilde{x}_t' r_t^2, \tilde{x}_t' r_t^3\right)'$. The null hypothesis of parameter constancy becomes $H_0 : a = 0$. The F-statistic will have $3dim(a)$ and $(T - 4dim(a) - 1)$ degrees of freedom.

4.5. Encompassing

After specifying an STR model that passes all misspecification tests, it is necessary to find out if the model really is an improvement over the linear or previous nonlinear ones. The theory of encompassing seeks to resolve the problem of selecting among competing models that describe the same economic phenomenon. To find out if the STR model encompasses the linear model amounts to testing whether the former can explain the results of the latter. The reverse has to be checked too. If the models are not nested, a practical way of testing encompassing restrictions is to define a minimal nesting model (MNM; Mizon and Richard, 1986) that nests the competing models. If the STR model can explain the results of the MNM while the linear cannot, the former is said to encompass ($\mathcal{E}$) the latter. For a more comprehensive discussion of the theory of encompassing, see Hendry (1995, Chapter 14).

Here is an example of the encompassing test exploring whether an STR model encompasses a linear model and vice versa. The MNM is constructed by linearly completing the STR model with the variables that are included in the linear model (L) but do not enter the STR model. The encompassing test STR $\mathcal{E}$ L consists of testing the null hypothesis that these variables have zero coefficients in the MNM. A likelihood ratio test, see Teräsvirta (1998), or an F-test can be used to compare the rival models. The null hypothesis is "the variables in the MNM not included in the STR model have zero coefficients".

As discussed above, one may also check whether the linear model can explain the results of the STR. In this case the MNM is not identified under the null hypothesis: L $\mathcal{E}$ STR. This problem is solved as before using a Taylor series expansion to approximate the transition function and thus the MNM. The encompassing test is equivalent to a linearity test of the type discussed in Section 4.3.

5. UK consumption behaviour, 1958(2)-1970(4)

In this section the original DHSY model and a variant in first differences will be considered applying the STR framework. Seasonal dummies are used to capture the seasonality in the latter model. This transformation makes it possible to explicitly model time-varying seasonality as in Harvey and Scott (1994). The modelling procedure is as follows. One starts by specifying a linear model and performing the linearity and parameter constancy tests. If either of the hypotheses is rejected an STR model is estimated and evaluated.

5.1. Annual changes

When modelling annual changes, i.e. four-quarter differences of consumption, the starting-point is equation (45) in DHSY. It has the form

$$\begin{aligned}
\Delta_4 c_t &= \underset{(.0032)}{.007}\ \Delta_4 D_t^0 + \underset{(.038)}{.47}\ \Delta_4 y_t - \underset{(.048)}{.21}\ \Delta_1\Delta_4 y_t \\
&\quad - \underset{(.021)}{.097}\ (c-y)_{t-4} - \underset{(.070)}{.13}\ \Delta_4 p_t - \underset{(.13)}{.30}\ \Delta_1\Delta_4 p_t + \hat{u}_t \qquad (5.1)\\
T &= 51, R^2 = .77, \hat{\sigma}_l = .0059, LJB = .06\,(.97)\,, AIC = -10.16, \\
A(2) &= 1.25\,(.94)\,, A\,(4) = .91\,(.46)
\end{aligned}$$

where the standard deviations of the parameter estimates appear in parentheses, T is the number of observations, R^2 the coefficient of determination, $\hat{\sigma}_l$ the estimated residual standard deviation and AIC the Akaike Information Criterion. The errors of equation (5.1) seem to be normal (LJB is the Lomnicki-Jarque-Bera test statistic of normality; the p-value is in parenthesis), and the LM tests of no ARCH against ARCH of at most order q, denoted $A(q)$, do not indicate misspecification. There is a small discrepancy between the estimates in equation (5.1) and the original DHSY regarding the coefficient of the rate of change of inflation, which Harvey and Scott (1994) also obtained. Except for this the estimated values are the same as in DHSY.

In linearity tests against STR, all explanatory variables in equation (5.1) , except the dummy variable $\Delta_4 D_t^0$, appear as transition variables. The results can be found in Table 5.1 and $\Delta_4 p_t$ seems to be the preferred transition variable since the hypothesis of linearity is most heavily rejected using $\Delta_4 p_t$ as the transition variable, see Section 4.3 for details. As mentioned in Section 2, the results of the unit-root tests in Appendix A reveal that not all variables of equation (5.1) are stationary. A unit root cannot be rejected for $(c-y)_{t-4}$ during the observation period. Hence, the variables on the left- and right-hand sides of equation (5.1) are of different orders of integration and equation (5.1) is said to be unbalanced. Because of this, there is a small caveat concerning p-values since the tests require stationary variables under H_0. Nevertheless, the smallest p-value is so small that the apparent lack of balance hardly affects any conclusions. The results of the model specification test sequence point at the LSTR(1) model. Furthermore, it should be noted that the parameters of equation (5.1) seem to be constant. Estimation of an LSTR(1) model for $\Delta_4 c_t$ yields

$$
\begin{aligned}
\Delta_4 c_t &= \underset{(.0025)}{.009}\ \Delta_4 D_t^0 + \underset{(.038)}{.34}\ \Delta_4 y_t - \underset{(.034)}{.15}\ \Delta_1\Delta_4 y_t \\
&\quad - \underset{(.063)}{.18}\ (c-y)_{t-4} - \underset{(.076)}{.11}\ \Delta_4 p_t - \underset{(.20)}{.34}\ \Delta_1\Delta_4 p_t \\
&\quad + \left(\underset{(.0038)}{.016} + \underset{(.063)}{.18}\ (c-y)_{t-4} + \underset{(.20)}{.34}\ \Delta_1\Delta_4 p_t\right) \\
&\quad \left[1+\exp\left(\underset{(11.7)}{-12.4}\left(\Delta_4 p_t - \underset{(.0017)}{.022}\right)/\sigma_{(\Delta_4 p_t)}\right)\right]^{-1} + \hat{u}_t \qquad (5.2) \\
T &= 51, R^2 = .85, \hat{\sigma}_{nl} = .0049, \hat{\sigma}_{nl}/\hat{\sigma}_l = .84, LJB = .78\,(.68)\,, \\
AIC &= -10.47, A(2) = .97\,(.39)\,, A(4) = .57\,(.69)\,.
\end{aligned}
$$

where $\hat{\sigma}_{nl}$ is the estimated residual standard deviation of the model. The nonlinear model (5.2) variance dominates the linear model (5.1) ; its standard deviation is 84% of that of equation (5.1). In equation (5.2) , the linear and nonlinear coefficients of $(c-y)_{t-4}$ and $\Delta_1\Delta_4 p_t$ are restricted to having the same size but opposite signs. The data strongly support these restrictions. They make the impact of equilibrium correction and the change in inflation approach zero as the value of the transition function approaches unity. The linear intercept is restricted to zero as in the DHSY specification while the nonlinear intercept enters significantly. The model survives a battery of misspecification tests. The Lomnicki-Jarque-Bera test of normality of the errors does not reject normality. There is no evidence of autoregressive conditional heteroscedasticity. Furthermore, equation (5.2) satisfies the assumptions of no error autocorrelation, no remaining nonlinearity and constant parameters according to the results of the misspecification tests in Table 5.2.

The speed of transition from one extreme regime to the other is rather rapid ($\gamma = 12$). This is seen from Figure 5.1 where the values of the estimated transition function are plotted against its argument, $\Delta_4 p_t$. Each dot represents an observation. The transition function attains value unity when $\Delta_4 p_t$ is large ($\geqslant 3\%$) and zero when $\Delta_4 p_t$ is small ($\leqslant 1\%$). Figure 5.2 shows the time path of the transition function.

The next step is to find out whether the nonlinear model (5.2) can explain the empirical findings of its linear competitor (5.1), that is, find out whether (5.2) $\mathcal{E}$ (5.1). Since all variables of the linear model (5.1) enter linearly in the STR model (5.2), (5.2) $\mathcal{E}$ (5.1), see Section 4.5. The opposite has to be checked too, that is whether (5.2) $\mathcal{E}$ (5.1). Testing the null hypothesis of linearity of equation (5.1) against the alternative of an LSTR(1) model with a constant, $(c-y)_{t-4}$, $\Delta_1\Delta_4 p_t$ in the nonlinear component and $\Delta_4 p_t$ as transition variable is rejected (p-value = 0.0096). Hence, (5.2) $\mathcal{E}$ (5.1) while the reverse is not true.

The functioning of the model is best seen from Figure 5.3 which shows the contribution of the equilibrium correction to $\Delta_4 c_t$ over time (solid line). In model (5.2), the equilibrium correcting term equals

$$ec_t = -0.18(c-y)_{t-4} + \{0.016 + 0.18(c-y)_{t-4}\}\, F\left(\Delta_4 p_t; \hat{\gamma}, \hat{c}\right) \qquad (5.3)$$

where the combined intercept increases from zero to 0.016 when the value of the tran-

sition function changes from zero to unity. It thus compensates for the diminishing effect of the equilibrium correction as a function of $\Delta_4 p_t$. The figure shows how the mechanism equilibrium corrects when the inflation rate is low but does not (ec_t remains constant) when the inflation rate is sufficiently high. The dotted line depicts the equilibrium correcting component of the original DHSY specification (5.1) where $ec_t = -0.097(c-y)_{t-4}$. This component appears to be trending and thus nonstationary, as the augmented Dickey-Fuller unit root test indicates; see Appendix A. On the other hand, equation (5.3) does not have a trend so that, at least visually, the LSTR specification seems a balanced equation.

The equilibrium correction mechanism may be given the following interpretation. The assumption of $(c-y)_t$ being stationary within the observation period is not correct. If the analysis is extended to cover data until 1976(2) then it appears that $\Delta_4 p_t$ is nonstationary and cointegrated with $(c-y)_t$. For the shorter period until 1970(4) $\Delta_4 p_t$ is stationary. However, c_t and y_t can be thought of as being "locally cointegrated" with coefficients 1 and -1 when the rate of inflation is low. The relationship disintegrates when the inflation rate is high. This may be compared with the discussion in Stewart (1998). In that paper it is pointed out that adding an intercept into model (5.1) renders the equilibrium correction term insignificant. Adding an intercept may therefore be viewed as another way of achieving a balanced equation, but at the cost of "losing" the long run. It also turns out that testing linearity with a model containing both an intercept and the equilibrium correction variable weakens the evidence for nonlinearity in the data. The p-value corresponding to 0.0041 in Table 5.1 becomes 0.067. However, estimating the corresponding fully parameterized LSTR(1) model and reducing its size inevitably leads to equation (5.2). The locally equilibrium correcting LSTR(1) model (5.2) is thus an intermediate specification between the unbalanced DHSY with global equilibrium correction and a linear model containing an intercept but no equilibrium correction or long-run equilibrium. The LSTR(1) model may also be given an economic interpretation: it suggests that the long-run savings ratio should be a nonlinear function of the current inflation rate. Nevertheless, the above, mainly statistical, interpretation may be the most plausible one. The small number of observations in the estimation period does caution against strong economic interpretations of equation (5.2).

5.2. Quarterly changes and seasonal dummies

DHSY wrote that their choice of differencing was based on the fact that $\Delta_4 c_t$ "represents a sensible decision variable when different commodities are being purchased in different quarters of the year." Harvey and Scott (1994), who focused on modelling evolving seasonal patterns, used $\Delta_1 c_t$ as the regressand. In this section their example is followed. The idea is to find out how that change affects the description of the equilibrium correction mechanism. $\Delta_4 y_t$ is also replaced by $\Delta_1 y_t$ but the annual inflation rate $\Delta_4 p_t$ will be retained. This is partly because it seems to be the measure of inflation which most agents use in practice, but also because the annual inflation rate is a potential I(1) variable; see Appendix A. It turns out that a linear model for the period 1958(2)-1970(4) requires extra dynamics in the form of two lags of $\Delta_1 c_t$. Incorporating these into the equation and estimating the parameters yields

$$\begin{aligned}
\Delta_1 c_t &= \underset{(.011)}{.067} + \underset{(.0033)}{.007}\ \Delta_4 D_t^0 - \underset{(.0067)}{.14}\ S_1 - \underset{(.017)}{.035}\ S_2 - \underset{(.019)}{.073}\ S_3 + \underset{(.059)}{.20}\ \Delta_1 y_t \\
&\quad - \underset{(.073)}{.23}\ \Delta_4 p_t - \underset{(.054)}{.082}\ (c-y)_{t-1} - \underset{(.13)}{.36}\ \Delta_1 c_{t-1} - \underset{(.13)}{.41}\ \Delta_1 c_{t-2} + \hat{u}_t \\
T &= 50, R^2 = .99, \hat{\sigma}_l = .0067, LJB = .46\,(.80), AIC = -9.84, \qquad (5.4) \\
A(2) &= 0.51\,(.60), A(4) = .40\,(.81)
\end{aligned}$$

While R^2 is high (not comparable with that of equation (5.1)), the residual standard error is 10% higher than that of model (5.1). Equation (5.4) is unbalanced for the same reason as equation (5.1), that is $(c-y)_{t-1}$ consists of a unit root while the other variables are stationary. It should be noted that model (5.4) contains a strongly significant intercept whereas the coefficient estimate of the equilibrium correction term seems insignificant (with a caveat for the interpretation of the t-ratio). This accords with the consumption model in fourth differences in Stewart (1998), where the inclusion of a constant in the original DHSY specification makes the equilibrium correction term insignificant.

In testing linearity with model (5.4) the choice of potential transition variables is affected by the fact that the first differences are strongly seasonal. The aim is not to consider possible nonlinearity of the intra-year seasonal pattern. The main interest lies, as before, in nonseasonal nonlinearity. Thus, annual differences $\Delta_4 c_{t-1}$ and $\Delta_4 y_t$

are used as potential transition variables instead of $\Delta_1 c_{t-1}$ and $\Delta_1 y_t$.

The results of linearity and parameter constancy tests are found in Table 5.3. It is shown that linearity cannot be rejected for any potential transition variable whereas parameter constancy is strongly rejected. Further testing indicates that the rejection is due to nonconstancy of the coefficients of the stochastic explanatory variables. On the other hand, while the estimation results of Harvey and Scott (1994), see their Table 1, indicate stochastic variation in the seasonal pattern even in the original DHSY data, the parameter constancy tests here do not reject the null hypothesis of stable seasonal parameters. But then, their specification is not the same as the one used here.

The threesome of tests (LM1, LM2 and LM3) in Table 5.3 for testing the constancy of the coefficients of other than dummy variables point at an LSTR(2) model. Estimating this model yields an LSTR(2) model with $c_1 = c_2$. For ease of interpretation an ESTR model will be applied instead of the LSTR(2) (the transition function of an ESTR model is bounded between zero and one while in the LSTR(2) model it is bounded between d and unity; $0 \leq d \leq 1/2$). The estimated model has the form

$$
\begin{aligned}
\Delta_1 c_t &= \underset{(.0092)}{.065} + \underset{(.0027)}{.0066}\ \Delta_1 D_t^0 - \underset{(.0055)}{.14}\ S_1 - \underset{(.013)}{.044}\ S_2 - \underset{(.016)}{.093}\ S_3 \\
&\quad + \underset{(.047)}{.21}\ \Delta_1 y_t - \underset{(.046)}{.16}\ (c-y)_{t-1} - \underset{(.071)}{.37}\ \Delta_4 p_t - \underset{(.10)}{.39}\ \Delta_1 c_{t-1} \\
&\quad - \underset{(.11)}{.62}\ \Delta_1 c_{t-2} + \left\{ \underset{(.075)}{.16}\ \Delta_4 p_t + \underset{(.036)}{.15}\ \Delta_1 c_{t-2} \right\} \\
&\quad \left[1 - \exp\left\{ - \underset{(0.74)}{1.38} \left(t/T - \underset{(.022)}{.45} \right)^2 / \sigma^2_{t/T} \right\} \right] + \hat{u}_t. \qquad (5.5) \\
T &= 50, R^2 = .99, \hat{\sigma}_{nl} = .0055, \hat{\sigma}_{nl}/\hat{\sigma}_l = .82, LJB = 0.99\,(.61)\,, \\
AIC &= -10.19, A\,(2) = 0.07\,(.93)\,, A\,(4) = 0.21\,(.93)\,.
\end{aligned}
$$

where $\sigma^2_{t/T} = \left(t/T - \bar{t}/T\right)^2 / n - 1$ and $\bar{t} = \sum_{t=1}^{T} t/T$. The standard deviation of the ESTR model (5.5) is 82% of that of the linear model (5.4). The residuals seem to be normal and the hypothesis of no autoregressive conditional heteroscedasticity cannot be rejected. Equation (5.5) passes most misspecification tests; see Table 5.4. The test of parameter constancy for all parameters rejects the null hypothesis against another LSTR(2) component. However, the other tests do not give a strong indication about the source of the problem so that model (5.5) is tentatively accepted as an adequate

specification for $\Delta_1 c_t$.

The transition function of the ESTR model (5.5) plotted against time is shown in Figure 5.6. The residuals in Figure 5.7 show that the nonlinear model (5.5) outperforms the linear (5.4) during the period when the transition function is close to zero.

The next step is to check whether or not the nonlinear ESTR model (5.5) can explain the empirical findings of the linear equation (5.4) and vice versa. In the former case the MNM equals the ESTR model (5.5) since all variables of the linear equation (5.4) are included in the linear part of equation (5.5). Thus (5.5) $\mathcal{E}$ (5.4). To see whether the linear model (5.4) $\mathcal{E}$ the ESTR model (5.5), a parameter constancy test against the alternative of an ESTR is applied. The nonlinear part consists solely of $\Delta_4 p_t$ and $\Delta_1 c_{t-2}$ as in equation (5.5). This null hypothesis is rejected (p-value = 0.0080); hence the linear model (5.4) does not encompass the ESTR model (5.5).

Equation (5.5) contains several interesting features. First, both the intercept and the equilibrium correction term have significant coefficient estimates. Second, the equilibrium correction term enters the equation linearly, which was not the case in (5.2). Third, the negative effect of the inflation rate on the consumption seems to have changed over time, being strongest in the beginning and at the end of the period. To obtain an idea of the statistical implications of this result, the extended "nonlinear equilibrium correction" has to be considered

$$\begin{aligned} ec_t &= -0.16\,(c-y)_{t-1} - 0.37\Delta_4 p_t + \{0.16\Delta_4 p_t\} \\ &\quad \left[1 - \exp\left\{ -\underset{(0.74)}{1.38}\left(t/T - \underset{(.022)}{.45}\right)^2 / \sigma^2_{t/T} \right\}\right] \end{aligned} \tag{5.6}$$

The graph of variable (5.6) appears in Figure 5.8 together with that of $ec_t = -\underset{(.054)}{.082}\,(c-y)_{t-1} - \underset{(.073)}{.23}\,\Delta_4 p_t$. It seems that variable (5.6) is stationary, a conclusion supported by a unit root test of ec_t, see Table A.1. The nonlinear component thus neutralizes the nonstationarity of the equilibrium correction term $(c-y)_{t-1}$.

The common feature in equations (5.2) and (5.5) is that in each case, STR-type nonlinearity balances the equation. A plausible conclusion from both equations may be that the relationship between consumption, income and the annual rate of inflation in 1958-1970 appears nonlinear. One has to add, however, that the actual form of the relationship is still open to debate precisely because the two specifications are

so different. It seems that the number of observations in the estimation period is sufficiently small to leave room for different specifications.

6. UK consumption behaviour, 1958(2)-1992(2)

In this section the consumption models are estimated using the extended data set, 1958(2)-1992(2). Unfortunately the consumption models in this section are not directly comparable with the ones in the previous section since the data sets belong to different vintages. Some of the variables have been redefined and the variables of the two data sets are not even cointegrated, see Hendry (1994). Therefore, any comparisons between models estimated from the two samples have to be made with great caution.

As in Section 5 the consumption function will first be specified for annual changes, as in the original DHSY specification, and then for quarterly changes with dummy variables capturing the seasonality.

6.1. Annual changes

A linear model of nondurable consumption is estimated for the period 1958(2)-1992(2). The short-run dynamics are specified as annual changes (fourth differences) and the set-up is identical to the one in the original DHSY model. This specification results in a model with strong error autocorrelation and lagged values of the dependent variable are included, making the problem of autocorrelated errors vanish. The dummy D_t^0 captures the effects of changes in VAT in 1968, 1973 and 1979. The parsimonious linear model becomes

$$\begin{aligned}
\Delta_4 c_t &= \underset{(.0027)}{.014}\ \Delta_4 D_t^0 + \underset{(.035)}{.20}\ \Delta_4 y_t - \underset{(.012)}{.026}\ (c-y)_{t-4} - \underset{(.019)}{.041}\ \Delta_4 p_t \\
&\quad - \underset{(.068)}{.31}\ \Delta_1 \Delta_4 p_t + \underset{(.057)}{.70}\ \Delta_4 c_{t-1} + \hat{u}_t. \qquad (6.1)\\
T &= 137, R^2 = .85, \hat{\sigma}_l = .0085, LJB = 1.69\,(.43)\,, AIC = -9.50, \\
LMA\,(2) &= .87\,(.42)\,, LMA\,(4) = .81\,(.52)\,.
\end{aligned}$$

The residuals of equation (6.1) seem to be normal and there is no evidence of autoregressive heteroscedasticity in the error process. It can be noted that equation (6.1)

variance dominates the "structural model" presented in Harvey and Scott (1994) in the sense that its residual standard deviation is 96% of that of the latter model when the models are estimated from the same sample, that is 1958(2)-1992(2).

A comparison between the linear consumption functions (5.1) and (6.1) reveals no obvious similarities but several discrepancies. The inclusion of lagged consumption ($\Delta_4 c_{t-1}$) in equation (6.1) makes some of the other variables redundant, and the magnitudes of the estimated coefficients of variables included in both models are quite different. Furthermore, it is not only $(c-y)_{t-4}$ that is a unit root process in equation (6.1) but also $\Delta_4 p_t$; see Appendix A. The unit root test indicates nonstationarity for $\Delta_4 c_t$ too. A closer look shows that when excluding the last six observations in the sample, thus only including data until 1990(4), the hypothesis of a unit root in $\Delta_4 c_t$ is only rejected at a 10% significance level, and thus the series is treated as stationary. Because of the cointegrating relationship between $(c-y)_{t-4}$ and $\Delta_4 p_t$, see Table A.3, and the fact that $\Delta_4 c_t$ is stationary, equation (6.1) can be regarded as a balanced equation.

In the next step linearity and parameter constancy tests are performed. The results of the tests can be found in Table 6.1. It turns out that the linearity test cannot be forcefully rejected for any of the transition variables, while there is evidence of time-varying parameters. The estimated LSTR(1) model with t/T as transition variable has the form

$$
\begin{aligned}
\Delta_4 c_t &= \underset{(.0025)}{.012}\ \Delta_4 D_t^0 + \underset{(.033)}{.25}\ \Delta_4 y_t - \underset{(.012)}{.050}\ (c-y)_{t-4} - \underset{(.064)}{.26}\ \Delta_1 \Delta_4 p_t \\
&\quad + \underset{(.081)}{.45}\ \Delta_4 c_{t-1} + \left\{ - \underset{(.13)}{.23}\ \Delta_4 p_t + \underset{(.20)}{.37}\ \Delta_4 c_{t-1} \right\} \\
&\quad \left[1 + \exp\left\{ - \underset{(1.09)}{1.81} \left(t/T - \underset{(.18)}{.72} \right) / \sigma_{t/T} \right\} \right]^{-1} + \hat{u}_t. \qquad (6.2) \\
T &= 137, R^2 = .87, \hat{\sigma}_{nl} = .0081, \hat{\sigma}_{nl}/\hat{\sigma}_l = .95, LJB = .56\,(.76)\,, \\
AIC &= -9.58, LMA\,(2) = 2.14\,(.12)\,, LMA\,(4) = 1.35\,(.25)\,.
\end{aligned}
$$

The standard deviation of the LSTR(1) model (6.2) is 95% of that of the linear model (6.1) and 92% of that of the "structural model" in Harvey and Scott (1994). The residuals seem to be normal and the hypothesis of no autoregressive heteroscedasticity cannot be rejected. According to the results of the misspecification tests in

Table 6.2, equation (6.2) satisfies the assumptions of no additive nonlinearity and parameter constancy. However, the hypothesis of no error autocorrelation is rejected against fourth-order autocorrelation at the 5% significance level. The reason for the misspecification might be the transformation into fourth difference which implicitly assumes constant seasonals. Harvey and Scott (1994) strongly reject constant seasonality, which at least implicitly makes modelling four-quarter differences a plausible idea in the present context.

The transition function of model (6.2) plotted against its argument t/T is shown in Figure 6.1. The transition is very smooth and it does not attain unity during the sample period. The residuals of equations (6.1) and (6.2) are shown in Figure 6.2. The graph shows the improved fit of the LSTR(1) model (6.2) during the 1990s compared to the linear equation (6.1). Except for this, the differences between the models are negligible.

There are few similarities between the nonlinear consumption functions (5.2) and (6.2). First and foremost, equation (5.2) is a nonlinear model whereas model (6.2) has time-varying parameters. Furhtermore, the system is equilibrium correcting in all periods in model (6.2) while it is locally equilibrium correcting in model (5.2). However, it is impossible to say whether the difference between the models depends on a different dynamic structure or is a result of data revisions.

Also, one would like to find out whether or not the LSTR(1) model (6.2) can explain the outcomes of the linear model (6.1). The MNM is created by including $\Delta_4 p_t$ linearly in equation (6.2). Under the null hypothesis the estimated coefficient of $\Delta_4 p_t$ is equal to zero. The F statistic $= 0.44$ (p-value $= 0.51$) and the hypothesis that (6.2) $\mathcal{E}$ (6.1) cannot be rejected. In order to find out if (6.1) $\mathcal{E}$ (6.2) a parameter constancy test is performed. The variables included in the nonlinear part are $\Delta_4 p_t$ and $\Delta_4 c_{t-1}$; when estimating the LM1 tests it turns out that the hypothesis can be rejected (p-value $= 0.0014$). Hence, (6.2) $\mathcal{E}$ (6.1) while the reverse is not true.

The effect of the time-varying coefficients of $\Delta_4 p_t$ and $\Delta_4 c_{t-1}$ in the LSTR(1) model (6.2) needs further discussion. In the nonlinear specification (6.2) $\Delta_4 p_t$ and $\Delta_4 c_{t-1}$ enter nonlinearly and the coefficients are increasing over time (in absolute value). The time-varying coefficient of $\Delta_4 p_t$ has a mean value of -0.071 and it varies between -0.0028 and -0.20. Comparing it to the coefficient of the linear model (6.1) which equals -0.041, it turns out that the mean of the time-varying coefficient is

larger in absolute value. Hence, the negative impact on consumption due to $\Delta_4 p_t$ is stronger in the nonlinear model. Moreover, the positive coefficient of $\Delta_4 c_{t-1}$ is also increasing over time, reducing the negative impact of $\Delta_4 p_t$ on consumption. It has a mean of 0.56, and it varies between $(0.45; 0.76)$. The coefficient of $\Delta_4 c_{t-1}$ in the linear equation (6.1) equals 0.70, making the positive impact of $\Delta_4 c_{t-1}$ smaller in the nonlinear model (6.2). These two effects yield a larger overall negative effect on consumption in equation (6.2) which is compensated for by stronger equilibrium correction.

In the nonlinear specification (6.2) $\Delta_4 p_t$ only enters nonlinearly and the absolute value of the coefficient is increasing over time. However, neither the specification with a constant coefficient of $\Delta_4 p_t$, equation (6.1), nor the one with a time-varying coefficient, equation (6.2) yields a stationary cointegrating relationship for the observation period. Furthermore, the results of the Johansen cointegrating test, Table A.3, reveal that the linear cointegrating relationship suggested by Hendry *et al.* (1990) ceases to exist when the sample is extended beyond 1990(4).

Also, neither of the models (6.1) and (6.2) has an intercept. This corresponds to the specification of the original DHSY model but there are two major differences. By including an intercept in equations (6.1) or (6.2) the significance of the equilibrium correction term is not affected and the intercept does not enter significantly.

Finally, because of the different features of the consumption functions of the two samples it seems hazardous to draw any conclusions about consumption behaviour in the UK.

6.2. Quarterly changes and seasonal dummies

In this section the quarterly changes of consumption 1958(2)-1992(2) are modelled. As in Section 5.2 seasonal dummies are used to capture the seasonality, and only inflation enters the model as an annual change to complete the cointegrating relationship. As mentioned above, Harvey and Scott (1994) concluded that time-varying seasonality is the major reason for the failure of the DHSY model when using this sample. It is therefore interesting to see whether or not the STR approach leads to a similar conclusion. When estimating the DHSY model there is strong error autocorrelation and to remedy this extra dynamics, in the form of $\Delta_1 c_{t-4}$, have to be included in the specification. The inclusion of $\Delta_1 c_{t-4}$ renders some of the variables redundant and

the linear model becomes, after some reduction,

$$\begin{aligned}
\Delta_1 c_t &= \underset{(.0024)}{.0060}\,\Delta_1 D_t^0 - \underset{(.0071)}{.032}\,S_1 - \underset{(.0027)}{.0048}\,S_2 - \underset{(.0026)}{.0052}\,S_3 \\
&\quad + \underset{(.043)}{.16}\,\Delta_1 y_t - \underset{(.020)}{.082}\,(c-y)_{t-1} - \underset{(.020)}{.047}\,\Delta_4 p_t \\
&\quad - \underset{(.072)}{.40}\,\Delta_1\Delta_4 p_t + \underset{(.057)}{.71}\,\Delta_1 c_{t-4} + \hat{u}_t. \qquad (6.3) \\
T &= 137, R^2 = .98, \hat{\sigma}_l = .0095, LJB = 2.17\,(.34)\,, \\
AIC &= -9.24, LMA\,(2) = .17\,(.85)\,, LMA\,(4) = .26\,(.90)\,.
\end{aligned}$$

According to the diagnostic tests the residuals seem to be normal and there is no evidence of autoregressive conditional heteroscedasticity.

There are several differences between equations (5.4) and (6.3). Because its estimate was not significant the intercept is excluded from equation (6.3). Furthermore, the lag structure is different in the two equations. The differences in the specifications also result in discrepancies regarding the magnitude of the coefficients for the variables that are included in both models. As concluded in Section 6.1, it is not possible to determine whether these differences are due to altered dynamics in consumption or if they are a consequence of the different data vintages.

The next step is to test for linearity and parameter constancy of equation (6.3). The choice of transition variables is made exactly as in Section 5.2. The results appear in Table 6.3. Both linearity and parameter constancy can be rejected. Parameter constancy for the seasonals is the most strongly rejected hypothesis and the test sequence points at an LSTR(1) specification. The estimated model becomes

$$\begin{aligned}
\Delta_1 c_t &= \underset{(.0022)}{.0063}\,\Delta_1 D_t^0 - \underset{(.0095)}{.055}\,S_1 + \underset{(.0096)}{.0054}\,S_2 - \underset{(.0027)}{.012}\,S_3 + \\
&\quad \underset{(.037)}{.19}\,\Delta_1 y_t - \underset{(.022)}{.16}\,(c-y)_{t-1} - \underset{(.019)}{.056}\,\Delta_4 p_t - \underset{(.064)}{.31}\,\Delta_1\Delta_4 p_t \\
&\quad + \underset{(.064)}{.46}\,\Delta_1 c_{t-4} + \left\{ - \underset{(.0089)}{.017}\,S_1 - \underset{(.0089)}{.026}\,S_2 \right\} \\
&\quad \left[1 + \exp\left\{ - \underset{(1.82)}{2.38}\left(t/T - \underset{(.11)}{.44} \right)/\sigma_t \right\} \right]^{-1} + \hat{u}_t \qquad (6.4) \\
T &= 137, R^2 = .98, \hat{\sigma}_{nl} = .0087, \hat{\sigma}_{nl}/\hat{\sigma}_l = .91, LJB = 1.55\,(.46)\,, \\
AIC &= -9.39, LMA\,(2) = .70\,(.50)\,, LMA\,(4) = .58\,(.68)\,.
\end{aligned}$$

The residual standard deviation of the LSTR(1) model (6.4) is 91% of that of the linear model (6.3) and 99% of that of the "structural model" presented in Harvey and Scott (1994). The residuals seem to be well behaved in the sense that they pass the tests of normality and no autoregressive conditional heteroscedasticity. The results of the misspecification tests can be found in Table 6.4. Equation (6.4) satisfies the assumption of no error autocorrelation but the seasonals are still time-varying according to the parameter constancy tests. An LSTR(3) model would also be possible since the LM3 test in Table 6.2 is also heavily rejected. Estimating this yields a model where two of the location parameters get out of range, and excluding them yields the LSTR(1) model (6.4). Nevertheless, comparing the significance levels of the rejection of parameter constancy in Table 6.3 and Table 6.4, there is a distinct improvement. The hypothesis of no additional nonlinearity can be rejected at the 5% significance level when $\Delta_4 y_t$ is used as the transition variable. Except for this the nonlinearity seems to be captured by the nonlinear components of equation (6.4).

Figure 6.3 shows the transition function of model (6.4) plotted against time; the amplitude of the seasonal fluctuations is increasing over time as is shown in Figure 6.4. The residuals appear in Figure 6.5. It can be seen that the STR model (6.4) has a better fit than the linear model (6.3) overall and not just at some selected quarters.

In order to find out whether the LSTR(1) model (6.4) can explain the empirical results of the linear model (6.3) and vice versa, all variables of the linear equation (6.3) are also included linearly in equation (6.4) so that the specifications are nested, that is (6.4) $\mathcal{E}$ (6.3). The reverse has to be checked as well, that is to find out whether or not (6.3) $\mathcal{E}$ (6.4). This is done by testing if the parameters of the linear model (6.3) are constant against the alternative of time-variation of the LSTR(1) specification. The null hypothesis can be rejected and, hence, (6.4) $\mathcal{E}$ (6.3), while the reverse is not true.

Comparing the consumption functions specified for quarterly changes in equations (5.5) and (6.4), there are more differences than similarities. Equation (5.5) is characterised by time-varying parameters, while on the other hand equation (6.4) is characterised by time-varying seasonality. Except for this the lag structure is different, making a direct comparison between the parameters difficult.

The most interesting feature of the nonlinear equation (6.4) is that the impact of seasonality has been increasing over time. Consumption in the first and second

quarters has decreased compared to consumption in the third and fourth quarters. In the fourth quarter consumption is higher than in any other quarter. The results probably mirror the increase in spending around Christmas during the observed period. According to Figure 6.4 the time-variation of the coefficients wears off by the middle of the 1980s and seems to stabilise in the beginning of the 1990s. When allowing for time-varying seasonals, the negative impact of the first quarter is larger than it was in the linear model (6.3), which implies a systematic lower consumption if it was not compensated for by stronger equilibrium correction in all periods in equation (6.4).

Thus a general conclusion is that model (6.4) accords with the results of Harvey and Scott (1994), who also found evidence of time-varying seasonality when estimating the DHSY consumption function using the same sample.

7. Final remarks

In this paper four nonlinear specifications of the DHSY consumption function have been presented. When estimated for the original sample and four-quarter differences the DHSY model is nonlinear with a nonlinear equilibrium correction term. When data is specified in first differences the model is characterised by time-varying parameters and a constant equilibrium correction term. But, when specifying the DHSY consumption function for the extended sample, both the model in four-quarter and the one in first differences give similar results: the parameters are not constant. The model based on first differences accords with the findings of Harvey and Scott (1994), who claimed that the reason for the failure of the DHSY model is time-varying seasonality. The four-quarter difference model suggests a nonconstant equilibrium correction term when the equilibrium correction is assumed to include inflation which is changing over time. Putting these together, the findings of this paper support the idea that time-varying parameters are a possible explanation for the failure of the DHSY model.

The differences between the final model specifications are striking. One reason is probably due to the fact that the data used are from two different vintages which are not even cointegrated, see Hendry (1994). Another reason is that the choice of specifying the models in four-quarter or first differences does not seem to be innocuous. The features of the final models, when estimated for the same sample, are conspicuous.

The assumption of the equilibrium correction term being stationary is not satisfied within the original observation period, making the DHSY model unbalanced. However, specifying an STR model yields a balanced equation. This is also true for the ESTR model specified for first differences. The LSTR(1) model defined for four-quarter differences is locally equilibrium correcting, that is, when the inflation rate is low the mechanism equilibrium corrects but when the inflation rate is sufficiently high, making the equation stationary, it does not. When the linear DHSY model is specified for first differences the equilibrium correction term does not enter significantly but it is still kept in the model because of theoretic arguments. In the corresponding STR model, in which the time-varying coefficients are parameterised, the equilibrium correction term enters linearly and significantly. The inclusion of the equilibrium correction term could in principle make the model unbalanced but in the ESTR model the "nonlinear equilibrium correction" is stationary and, hence, the model is balanced. Again the STR specification transforms the unbalanced linear equation into a balanced one.

Finally, the choice of STR models is just one out of many nonlinear alternatives. This does not imply inferiority of other nonlinear specifications and there may be other nonlinear models that also shed light on consumption. However, in the STR framework one is able to investigate both nonlinearity and parameter constancy simultaneously.

Appendix A: Unit root tests

The results of the Augmented Dickey-Fuller (ADF) tests for a unit root are shown below. A constant and a deterministic trend are included for all variables. The main criteria for determining the number of lags included are the Ljung-Box Q-statistic and the Akaike Information Criterion. The results of the tests can be found in Table A.1 for the original data set and Table A.2 for the extended data set. According to Table A.1 it is only the equilibrium correction term $(c-y)_t$ that seems to be characterized by a unit root 1958(2)-1970(4). However, when the sample is extended to include data until 1976(2) the hypothesis of a unit root cannot be rejected for $\Delta_4 p_t$ either. This supports the results of Hendry, Muellbauer and Murphy (1990) where they find a cointegrating relationship between the equilibrium correction term and inflation, $(c-\alpha_1 y-\alpha_2 \Delta_4 p)_t$. In Table A.3 the results of the Johansen cointegration tests are shown for different samples.

According to Table A.2. the values of the ADF test statistics differ from those in Table A.1 for 1958(2)-1970(4), but the order of integration remains the same so the discrepancies in the test statistics do not change any conclusions. However, extending the sample up to 1992(2) a unit root cannot be rejected for $\Delta_4 c_t$ at the 10% significance level. The null hypothesis is rejected at a 10% significance level for $\Delta_4 y_t$ but not on 5%. Apart from this, $(c-y)_t$ and $\Delta_4 p_t$ are both nonstationary.

The results of the cointegration tests in Table A.3 show that there is a cointegrating relationship between $(c-y)_t$ for the whole sample and between $(c-y-\Delta_4 p)_t$ until 1990(4).

Exogeneity tests of the variables will not be performed. Davidson and Hendry (1981) examine the exogeneity of income and find that it cannot be rejected. Hendry, Muellbauer and Murphy (1990) test and reject weak exogeneity defined as in Engle, Hendry and Richard (1983).

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Figure 3.1: Logarithms of consumption and income 1958(2)-1976(2).

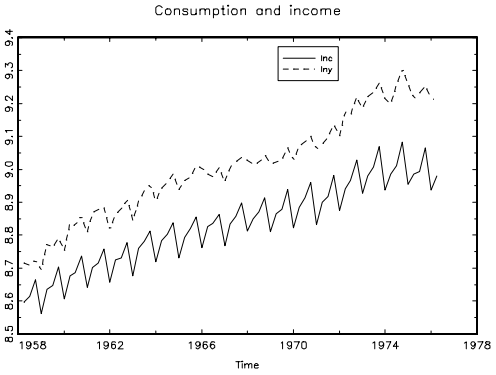

Figure 3.2: Logarithms of consumption and income 1958(2)-1992(2)

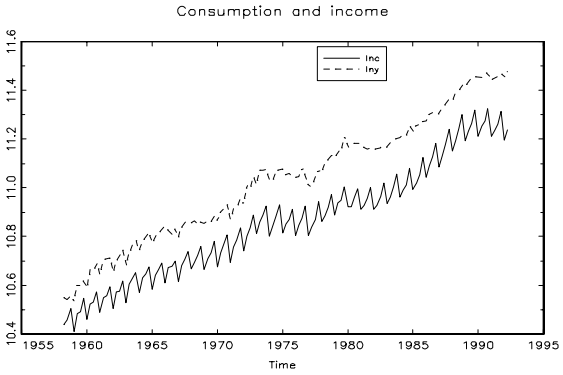

Figure 3.3: Annual changes of the logarithms of consumption and income 1958(2)-1976(2).

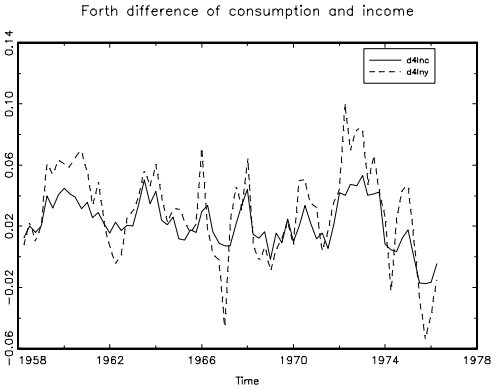

Figure 3.4: Annual changes of the logarithms of consumption and income 1958(2)-1992(2).

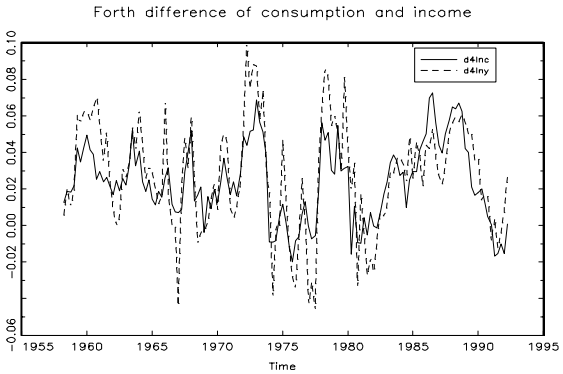

Figure 3.5: The equilibrium correction term for the two different datavintages,1958(2)-1976(2).

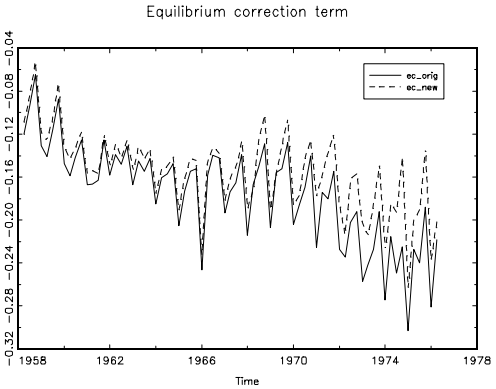

Figure 5.1: Values of the transition function of (5.2) plotted against the transition variable $\Delta_1 p_t$.

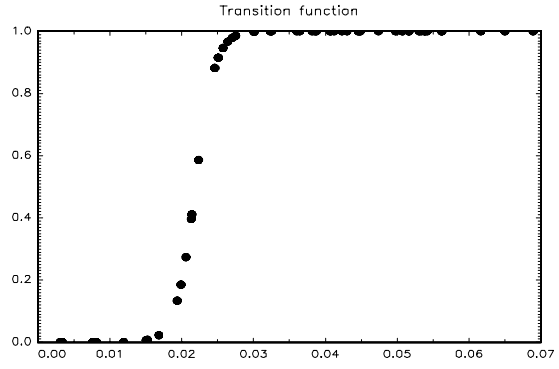

Figure 5.2: Transition function of (5.2) plotted against time, 1958(2)-1970(4).

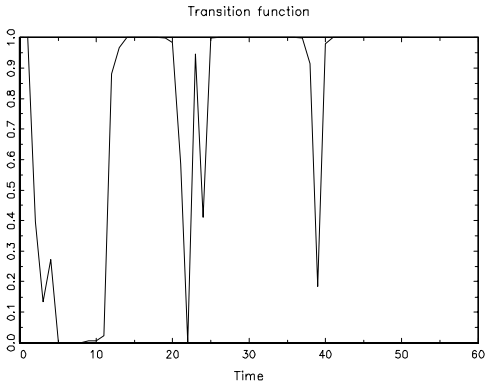

Figure 5.3: Linear and nonlinear equilibrium correction of (5.1) and (5.2) plotted against time, 1958(2)-1970(4).

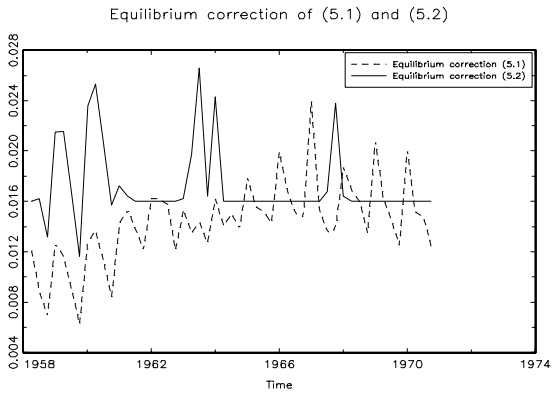

Figure 5.4: Residuals of (5.1) and (5.2), plotted against time, 1958(2)-1970(4).

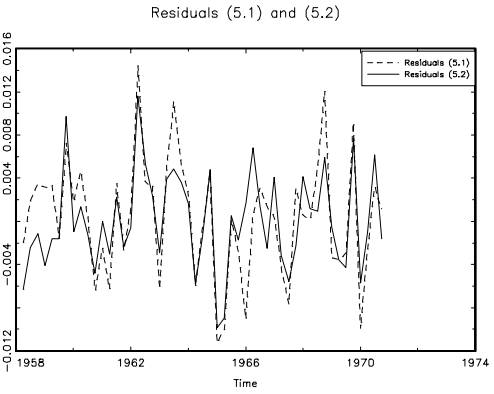

Figure 5.5: Differences in consumption between (5.1) and (5.2), 1958(2)-1970(4). The constant in the nonlinear part replace the joint effect of $(c-y)_{t-4}$ and $\Delta_1\Delta_4 p_t$.
Diff $= -.18 * (c-y)_{t-4} - .34 * \Delta_1\Delta_4 p_t - .016$

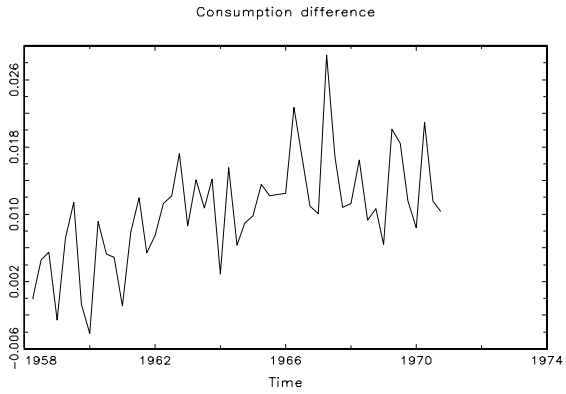

Figure 5.6: The transition function of the ESTR model (5.5), 1958(3)-1970(4).

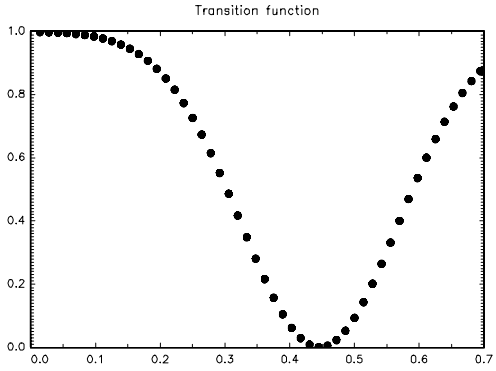

Figure 5.7: Residuals of (5.4) and (5.5), 1958(3)-1970(4).

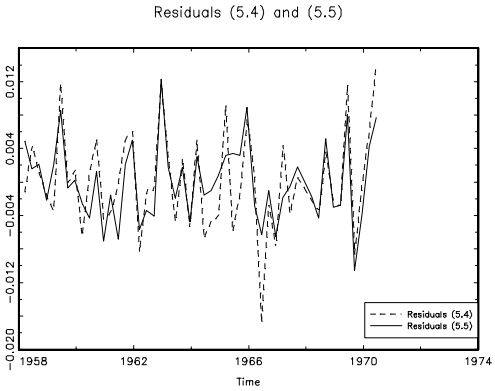

Figure 5.8: Linear and nonlinear equilibrium correction of (5.4) and (5.5) plotted against time, 1958(3)-1970(4).

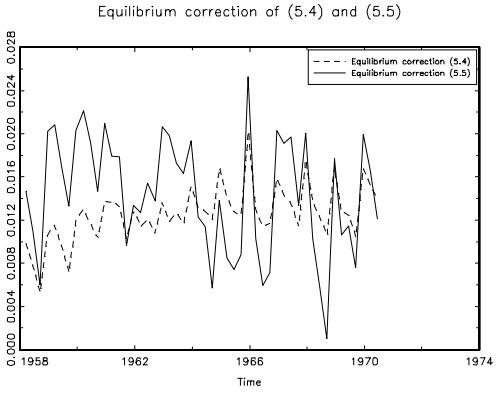

Figure 6.1: Transition function of (6.2) plotted against the transition variable $\Delta_4 y_t$, 1958(2)-1992(2).

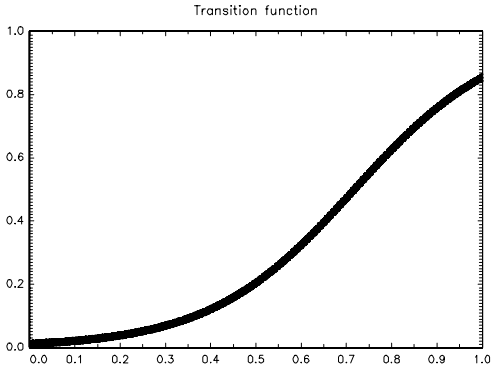

Figure 6.2: Residuals of consumption functions (6.1) and (6.2), 1958(2)-1992(4).

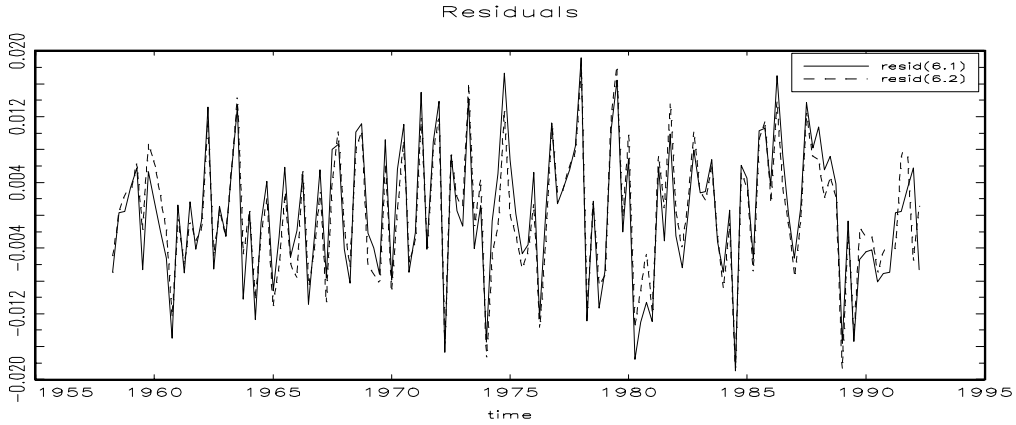

Figure 6.3: Transition function of the non-linear part of (6.4), where t/T is transition variable, 1958(2)-1992(2).

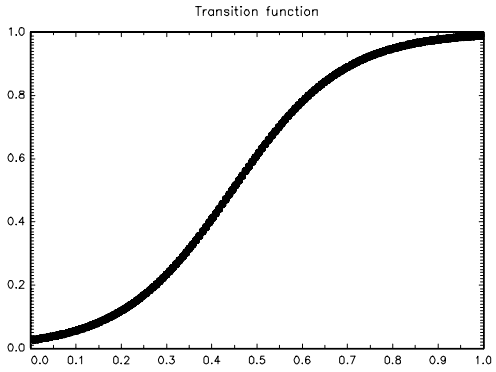

Figure 6.4: The time-varying seasonal pattern in the STR-model (6.4), 1958(2)-1992(2). (-.017s1-.026*s2)*tf.

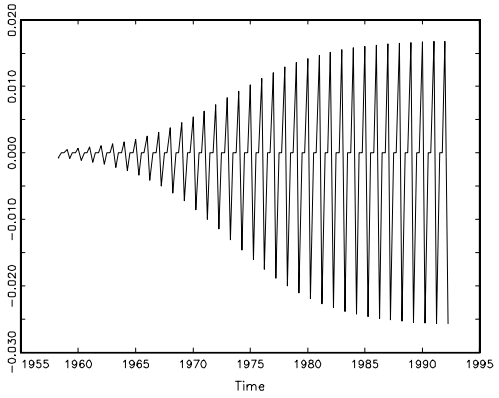
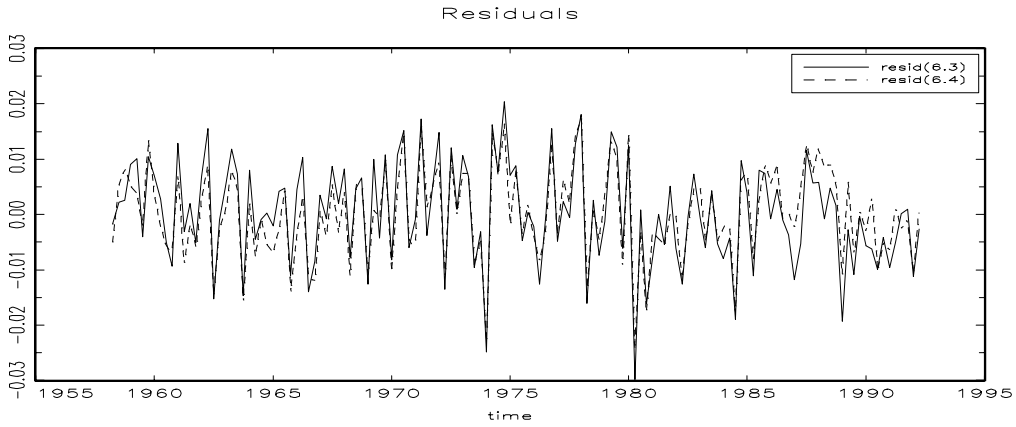
Figure 6.5: The residuals of (6.3) and (6.4), 1958(2)-1992(2).