Detecting equilibrium correction with smoothly time-varying strength

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Abstract

Simulations are used to check the probability of detecting a time-varying equilibrium correction by applying the existing tests of no cointegration and parameter constancy. Smooth transition regressions are chosen to describe the nonlinearity and the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test are applied. It turns out that both tests perform well separately but the joint power is quite low. The most notable result of this study is the high power when dealing with unrestricted cointegration, that is, when no cointegrating vector is estimated and the cointegrated varables freely enter the model in levels. The power of the parameter constancy test for the unrestricted cointegration is close to the power when the cointegrating vector is assumed to be known.

Keywords. Time-varying equilibrium correction, cointegration, parameter constancy, smooth transition regression.

JEL Classification Codes: C15, C50.

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1. Introduction

The concept of cointegration is based on the assumption that a pair of integrated economic variables are linked by a long-run stationary equilibrium relationship, see Engle and Granger (1987). It is implicitly assumed that the variables are cointegrated at all time periods and that the rate of adjustment towards the long-run equilibrium is constant over the sample. Recently, there have been several empirical studies where the equilibrium correction term enters the model nonlinearly, see e.g. Michael *et al.* (1996), Ericsson *et al.* (1998) and Teräsvirta and Eliasson (1998). This raises the question of the reliability of the existing tests for detecting cointegration and nonlinearities in a nonlinear equilibrium correction model. Two recent papers have studied similar questions. Balke and Fomby (1997) compare the performance of the Engle-Granger and the Johansen cointegration tests when the adjustment follows a threshold cointegration model, and van Dijk and Franses (1999) perform a study where the adjustment is of the smooth transition regression (STR) type. In the former study the cointegrating relationship is locally cointegrating, that is, the system may not be equilibrium correcting in all time periods. In the latter study the simulations are equilibrium correcting in all periods although the strength of the equilibrium correction varies. These studies conclude that both the Engle-Granger and the Johansen cointegration tests work well when the equilibrium correction is nonlinear.

This paper will consider the probability of detecting time-varying equilibrium correction by applying the existing tests of no cointegration and parameter constancy. The simulated series are locally cointegrated and the time-varying equilibrium correction is characterised by an STR model. There are no tests for simultaneously testing the joint hypotheses of no cointegration and constant parameters, and a two-step procedure will be applied. The investigator is assumed to follow a standard modelling procedure. First, the null of no cointegration is tested. The series are generated according to a two-dimensional equation system, and the Johansen cointegration test (Johansen, (1995)) is applied to test the hypothesis of no cointegration. If this is rejected, the cointegrating relationship is estimated. A lagged estimated relationship forms the equilibrium correction (EC) term, and from here on a single equation is used. The constancy of the coefficient of the EC term is tested. This is done as in Lin and Teräsvirta (1994), so that the alternative to parameter constancy is a smoothly changing parameter. The purpose of the paper is to find out how often the investigator reaches the correct conclusion that there exists a cointegrating relationship whose strength varies over time. Furthermore, the aim is to find out which factors affect the probability of arriving at this conclusion.

The results of the simulations show that the investigator rarely reaches the correct conclusion of a time-varying equilibrium correction. Considering the case where the sample size is large and the coefficient of the equilibrium correction term is high (both of which have positive effects on the power of both tests), the joint power of the two tests is still low and varies a lot, depending on the other parameter values. The power is at its lowest when the short-run dynamics are very strong since this makes it difficult to estimate the cointegrating relationship, and the estimated equilibrium correction term often seems redundant. The low significance level of the cointegrating term makes it difficult for the parameter constancy test to detect the time variation. However, by not estimating the cointegrating relationship and instead including the cointegrated variables in levels, the power of the equilibrium correction equation without a priori restrictions strongly increases and becomes almost as high as it is when the cointegrating relationship is assumed to be completely known a priori.

The outline of this paper is as follows. Section 2 reviews the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test. The simulation setup is also presented. In Section 3 the testing procedure and the results of the Monte Carlo simulations appear, and Section 4 concludes.

2. Methodology

2.1. Johansen cointegration test

A vector time series $\{y_t\}$ is said to be cointegrated of order b if each of the series taken individually is integrated of order d, $I(d)$, $d \geqslant 1$, whereas a linear combination of the series, $\beta' y$, is $I(d-b)$, $b > 0$, where β is the cointegrating vector. The vector is not unique since it can be multiplied by any non-zero scalar and still satisfy the cointegration condition.

There are several tests for no cointegration. The Johansen cointegration test, see Johansen (1995, Chapter 6) is one of the most popular tests in empirical studies and it is therefore chosen for this study. The testing procedure will be reviewed briefly.

Start by considering the structural VAR(p) model,

$$y_t = \sum_{i=1}^{p} \Pi_i y_{t-i} + \varepsilon_t, \quad t = 1, ..., T \tag{2.1}$$

where y_t is an $(n \times 1)$ vector, Π_i are $(n \times n)$ matrices and $\varepsilon_t \sim$nid$(0, \Omega)$. Each individual variable in y_t is assumed to be $I(1)$. The reduced form of the equilibrium correction model becomes

$$\Delta y_t = \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t, \quad t = 1, ..., T \tag{2.2}$$

where $\Gamma_1 = \Pi_1 - I_n$, $\Gamma_2 = \Pi_2 + \Gamma_1, ...,$ and $\Pi = I - \sum_{i=1}^{p-1} \Pi_i$. Under the null hypothesis $H(h)$ there are assumed to be exactly h linear combinations of y which are $I(0)$. Consequently, Π can be rewritten as $\alpha\beta'$ where α and β are $(n \times h)$. The alternative hypothesis is that there are n cointegrating relations, where n is the number of elements of y_t, which would imply that every linear combination of y_t is stationary and no restrictions would be imposed on Π. The likelihood ratio test of $H(h)$ against $H(n)$ is given by

$$\mathcal{L}_n - \mathcal{L}_h = -\frac{T}{2} \sum_{i=h+1}^{n} \ln\left(1 - \hat{\lambda}_i\right). \tag{2.3}$$

The eigenvalues $\hat{\lambda}_i$ of Π can be found by performing the first steps of Johansen's algorithm for the maximum likelihood estimator of β in $H(h) : \Pi = \alpha\beta'$. If equation (2.3) consisted only of stationary variables the test statistic would be distributed as a χ^2 asymptotically. However, under the null hypothesis the test statistic will depend on $(n - h)$ random walks, and the critical values are not the standard ones but can be found in Johansen (1995), Chapter 15.

2.2. Lin and Teräsvirta parameter constancy test

Consider the following single-equation STR model,

$$y_t = x_t'\varphi + x_t'\theta F(s_t) + u_t. \qquad t = 1, .., T. \tag{2.4}$$

where $\varphi = (\varphi_0, \varphi_1, .., \varphi_m)'$ is the parameter vector for the linear component and $x_t = (1, x_{1t}, .., x_{mt})' = (1, y_{t-1}, .., y_{t-p}, z_{1t}, .., z_{qt})'$ is the corresponding vector of stationary and ergodic explanatory variables. The nonlinear component is specified as a linear component multiplied by a nonlinear function, $x_t'\theta F(s_t)$ and $u_t \sim \text{nid}\big(0, \sigma^2\big)$ for simplicity. The nonlinear function $F(s_t)$ is the transition function which is continuous and bounded. It is customary to bound F between zero and unity. Hence, the model will change locally from $E(y_t|x_t) = x_t'\varphi$ for $F = 0$ to $E(y_t|x_t) = x_t'(\varphi + \theta)$ for $F = 1$ with the transition variable s_t. When $s_t = t$ the STR model can be interpreted as a linear model with time-varying parameters, which is the case that will be considered here. For elaborate discussions on STR models see Granger and Teräsvirta (1993) and Teräsvirta (1998).

The transition function of a kth order logistic smooth transition regression model, LSTR(k), has the form

$$F(t) = F(\gamma, c; t) = \left(1 + \exp\left\{-\gamma \prod_{i=1}^{k} (t - c_i)\right\}\right)^{-1}, \ \gamma > 0 \tag{2.5}$$

when a time trend (t) is used as the transition variable. The slope parameter (γ) determines how rapid the transition is and the vector of location parameters $c = (c_1, ..., c_k)'$ decides where the transitions occur. The STR model (2.4) becomes linear when $\gamma = 0$ so that the transition function $F(t) \equiv \frac{1}{2}$. For notational simplicity the transition function in (2.4) is replaced by $\tilde{F}(t, \gamma) = F(t, \gamma) - \frac{1}{2}$. This implies $\tilde{F}(t, 0) = 0$ and $H_0 : \gamma = 0$ becomes a natural hypothesis of parameter constancy, see Lin and Teräsvirta (1994). The alternative hypothesis is $H_1 : \gamma > 0$. There is one caveat though; the STR model is not identified under the null hypothesis because of the nuisance parameters θ and c. Thus the classical asymptotic distribution theory for testing the null hypothesis $\gamma = 0$ does not work. In order to circumvent this problem a Taylor series approximation to the transition function is used to obtain an appropriate test. Equation (2.6) shows the auxiliary regression for the tests when $k = 2$. In equations (2.7) and (2.8) the null hypotheses of the parameter constancy tests are presented.

$$y_t = \delta_0' x_t + \delta_1' x_t t + \delta_2' x_t t^2 + v_t \tag{2.6}$$

$$LM_2 : \delta_1 = \delta_2 = 0 \tag{2.7}$$

$$LM_1 : \delta_1 = 0 \mid \delta_2 = 0. \tag{2.8}$$

If $k = 1$, the test to be applied is LM_1; if $k = 2$ it is LM_2. Thus, size and power of the

LM_1 statistic are reported when an LSTR(1) model is used for simulating the series and LM_2 is used in simulations with the LSTR(2) model.

Testing the hypotheses of parameter constancy, a Lagrange multiplier (LM) type test statistic can be obtained by using the linear auxiliary regression (2.6). The test statistic will have an asymptotic χ^2 distribution under the null hypotheses. In practice, an F test is preferred to the χ^2-test since it has better small sample properties, see Lin and Teräsvirta (1994).

2.3. Simulation setup

The following two-dimensional system is used in the experiment:

$$\Delta y_t = \theta \Delta y_{t-1} + \delta \left(y_{t-1} - z_{t-1} \right) F\left(t\right) + u_t \tag{2.9}$$

$$\Delta z_t = \omega_t \tag{2.10}$$

where $F\left(t\right)$ is the transition function, $u_t \sim \text{nid}(0, 0.25)$, $\omega_t \sim \text{nid}(0, 1)$, u_t and ω_t are mutually independent. Equation (2.9) represents a time-varying equilibrium correction model and equation (2.10) generates a random walk. The magnitude of the short-run dynamics is allowed to vary, $\theta = \{0.2, 0.4, 0.9\}$ and so is the coefficient of the cointegration relationship, $\delta = \{-0.1, -0.4, -0.8\}$. In order to reduce the importance of the starting values, the first third of the observations are excluded from the reported results. That is, $T + \frac{1}{2}T$ observations are generated and the sample sizes used are $T = 100$, 200. Each test is performed $N = 10000$ times.

The simulated series are all generated by the time-varying equilibrium correction model (2.9) and (2.10). The transition function $F\left(t\right)$ is given by equation (2.5) with either $k = 1$ or $k = 2$. In the former case one has an LSTR(1) model and in the latter one has an LSTR(2) model.

The LSTR(1) model will be generated for different speeds of transition. When $\gamma = 1$ the transition is very smooth, when $\gamma = 10$ the transition function has the typical S-shape and for $\gamma = 100$ the transition is rather quick, see Figures 2.1-2.3. Setting $\gamma = 1$, the strength of attraction increases almost linearly over time and cointegration will be present in the whole observation period, see Figure 2.1. The location parameter c is also allowed to vary and the series are simulated for $c = \{0.25, 0.50, 0.75\}$. When

$\gamma = 10$ or $\gamma = 100$ the length of the observation period including cointegration will vary with c. For larger values of c the observation period including cointegration ($F > 0$) will be shorter, see Figures 2.2 and 2.3.

Generating an LSTR(2) model, the speed of transition is always high, $\gamma = 100$, see Figure 2.4. There are two location parameters in the LSTR(2) model, see equation (2.5) $k = 2$, and they are allowed to vary too. The series are generated with $c_1 = \{0.25, 1/3\}$ and $c_2 = 1 - c_1$. For the LSTR(2) model cointegration will be present in the beginning and the end of the sample. Hence, there will not be any cointegration ($F = 0$) in the middle of the sample, see Figure 2.4. The situation resembles that in threshold cointegration; the difference is that in this experiment time is the transition variable. One may order the observations in a threshold cointegration model according to the threshold variable and draw a graph of the strength of attraction. If this is done it turns out that the figure is analogous to that corresponding to an LSTR(2) model where $\gamma \to \infty$.

3. Monte Carlo simulations

In this section the testing procedure and the results of the simulations are presented. The purpose of the simulations is to find out how often a hypothetical researcher would arrive at the correct conclusion, that there exists a cointegrating relationship whose strength varies over time. As mentioned in the introduction, there are no tests for simultaneously testing the hypothesis of no cointegration and parameter constancy; hence two tests will be applied: the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test. The results of the simulations will be presented for the two tests separately, but also jointly since this is the major concern. For the sake of simplicity, the rejection frequencies of the tests will be referred to as "power" throughout the paper.

3.1. The testing procedure

The testing procedure starts by estimating the null hypothesis of no cointegration. If it is rejected, the procedure continues by exploring the constancy of the cointegrating parameter using the Lin and Teräsvirta parameter constancy test. It is also tested whether the cointegration term enters the linear equilibrium correction equation significantly. This will be referred to as the significance test.

Three different scenarios are considered when testing for parameter constancy: In Case 1 the cointegrating vector is assumed to be unknown, in Case 2 it is known and in Case 3 no cointegration relationship is defined. In addition, the size of the Johansen cointegration test and the parameter constancy test are explored.

A more detailed description of the testing procedure is as follows:

- **Size of Johansen cointegration test and the Lin and Teräsvirta parameter constancy test**. When the size of the first test is investigated the series are generated according to equations (2.9) and (2.10), keeping the coefficient of the cointegrating relationship δ equal to zero in (2.9). The results are reported as "J-size" in Table 3.1. This yields the empirical size of the Johansen test for the sample sizes $T = 100$ and $T = 200$. When the size of the parameter constancy test is examined the series are generated according to equations (2.9) and (2.10) assuming that the transition function $F(t)$ is equal to unity. Hence the model is linear. The hypotheses of linearity are tested and the size results for Case 1 are reported as "LT-size" in Tables 3.2-3.5. This yields the empirical size of the parameter constancy test for Case 1. The selected nominal size is 5% for both tests.

- **The Johansen cointegration test.** The numbers of rejections of the null hypothesis will be referred to as the power of the test and are reported as "Johansen test" in Tables 3.2-3.5. If the null hypothesis of no cointegration is rejected, and only then, the testing procedure continues with the parameter constancy test for three different scenarios named Case 1, 2 and 3 as follows.

- **Case 1: The cointegrating vector is unknown**. After testing and rejecting the hypothesis of no cointegration the cointegrating vector is estimated. The maximum likelihood estimator is given by the eigenvector corresponding to the h largest eigenvalues $\hat{\lambda}_1$, see Johansen (1995, Chapter 6), and is included in the linear model subject to testing which becomes

$$\Delta y_t = \theta \Delta y_{t-1} + \delta \left(y_{t-1} - \hat{\xi} z_{t-1} \right) + u_{1t}.$$

Existence and parameter constancy of the estimated cointegration coefficient $\hat{\delta}$

are tested; note that this is done only if the null of no cointegration is rejected. The number of times a false null hypothesis is rejected (power) is reported under "Case 1" in Tables 3.2-3.5. The only test results that will be reported for the parameter constancy tests are LM_1 for the LSTR(1) model and LM_2 for the LSTR(2) model.

- **Case 2: The cointegrating vector is known.** In this case the real cointegrating relationship is assumed to be known. This assumption is not realistic but the purpose is to create a case which can be used for comparisons with the more realistic scenarios. By doing this it is easier to detect where the major weaknesses in the testing procedure occur. The linear model subject to tests becomes

$$\Delta y_t = \theta \Delta y_{t-1} + \delta \left(y_{t-1} - z_{t-1} \right) + u_{2t}.$$

The power of the tests are reported under "Case 2" in Tables 3.2-3.5.

- **Case 3: Unrestricted cointegration**. In the third case the integrated variables are included without any restriction in the linear model. That is, the variables are still cointegrated but the cointegrating vector is not estimated. The estimated linear model becomes

$$\Delta y_t = \theta \Delta y_{t-1} + \delta_1 y_{t-1} + \delta_2 z_{t-1} + u_{3t}.$$

The former cointegrating coefficient will be divided into two separate estimates, one for each integrated variable. The estimated coefficients $\hat{\delta}_1$ and $\hat{\delta}_2$ ought to satisfy $\hat{\delta}_1 \approx -\hat{\delta}_2$ since data are generated according to equation (2.9). The hypothesis $\delta_1 = \delta_2 = 0$ and that of parameter constancy are tested. Note that the assumptions of the parameter constancy tests are violated, as the test requires stationarity of the regressors. But since the $I(1)$ variables are cointegrated, the combination of the two is still stationary. The power of the tests are reported under "Case 3" in Tables 3.2-3.5.

3.2. Simulation results

The simulation results of the LSTR(1) model can be found in Tables 3.2-3.4 and the results of the LSTR(2) model are shown in Table 3.5. Tables 3.2.A-C present the results when the coefficient of the lagged first difference Δy_{t-1} is small ($\theta = 0.2$), in Tables 3.3.A-C the coefficient is higher ($\theta = 0.4$), and in Tables 3.4.A-C the coefficient of Δy_{t-1} is $\theta = 0.9$. The capital letters in the tables' titles refer to the speed of transition. Thus, A denotes the smoothest transition $\gamma = 1$, B corresponds to $\gamma = 10$ and C is for a transition function close to a step function, $\gamma = 100$, see Figures 2.1-2.4.

The size and power results of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test are discussed below. The results of the latter test will be considered separately for each of the three different cases. The two tests are initially discussed separately for simplicity of exposure since some parameter changes affect the power of the two tests in opposite ways. The joint performance of the tests will be discussed in Section 3.2.5.

3.2.1. The size of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test

Before considering the power of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy tests (LM_1 and LM_2), the size of the tests are examined. It turns out that the size of the Johansen test varies between 3.3% and 5.7% and is usually slightly below the chosen significance level of 5%, see Table 3.1. The size of the Lin and Teräsvirta parameter constancy test, on the other hand, is in the close neighborhood of the nominal significance level of 5%. The smallest value equals 4.33% and the largest 5.40%, see Tables 3.2.A-C - 3.5. The size is also good when the basic assumption of stationarity is violated: Case 3, where two of the variables are characterised by a unit root. Obviously, this is because when the two I(1) variables are cointegrated a combination of the two variables is still stationary. This may affect the outcome even though the parameters of the cointegrating vector are not restricted. The basic models for the LM_1 and LM_2 tests are equations (2.4) and (2.5) with $k = 1, 2$.

3.2.2. The power of the Johansen cointegration test

The results of the simulations show that the power of the Johansen cointegration test is usually quite high when there is time-varying equilibrium correction. However, it is sensitive to some of the parameter changes and in this section the effect of parameter changes on the power of the Johansen cointegration test will be discussed in more detail.

When the magnitude of the cointegrating coefficient δ increases (in absolute value) the power increases for both the LSTR(1) and LSTR(2) models, see e.g. Tables 3.2.A and 3.5. An increase in δ makes the cointegrating relationship more distinct and therefore easier for the test to detect, and hence the power increases. For the LSTR(1) models the power of the Johansen test is also high when the speed of transition is low ($\gamma = 1$), which is not surprising considering that the test is developed for a linear model. Besides, the parameter change is almost a linear function, making the equilibrium correction present during the whole observation period. For a more rapid transition speed ($\gamma = 10, 100$) the cointegration only enters when $F > 0$ and the observation period during which the system equilibrium corrects is only a subperiod of the original one. Because of this, the power also depends on the location parameter c. A time series generated with a higher c will have lower power when $\gamma = 10, 100$ since there will be fewer observations available with information about the cointegration relationship, see e.g. Table 3.2.B. The same result also appears when the observations are simulated according to an LSTR(2) model, see Table 3.5. When the location parameter $c_1 = 1/3$ the period including equilibrium correction will be longer than when $c_1 = 0.25$, since it is assumed that $c_2 = 1 - c_1$, and the power of the test is higher. The power is also positively affected by the sample size for both the LSTR(1) and LSTR(2) models; hence the tests appear consistent.

More surprisingly, the power of the cointegration test increases when the strength of the short-run dynamics (θ) is high. This is especially noticeable when $\gamma = 100$, see Tables 3.2.C and 3.4.C. The power is at least three times as high in the latter table as in the former. A reason for this might be that the change in the strength of the equilibrium correction disturbs the test less when the relative weight of the cointegrating relationship is small. Besides, a previous experiment comparing the power of the Johansen and the Engel-Granger cointegration tests showed that the power of the Johansen test was higher when data was generated according to equations (2.4) and (2.5) instead of $\Delta y_t = \theta \Delta z_{t-1} + \delta \left(y_{t-1} - z_{t-1} \right) F(t) + u_t$ and (2.5). Hence, the Johansen test performs better when the short-run dynamics are given by lagged values of the dependent variable than when they are given by an exogenous variable. Keeping this in mind, the improved power of the cointegration test when $\theta = 0.9$ is less surprising.

3.2.3. The power of the Lin and Teräsvirta parameter constancy test for Case 1: The cointegrating vector is unknown

The performance of the Lin and Teräsvirta parameter constancy test is strongly dependent on the choice of parameter values. For most setups the power is fair but when the short-run dynamics are very strong ($\theta = 0.9$) it is close to zero. A more detailed discussion of how the parameter changes affect the power is given below.

When the model is generated by an LSTR(1) model an increase (in absolute value) in the coefficient of the cointegrating relationship (δ) improves the power, see e.g. Table 3.2.B. When the speed of transition increases ($\gamma > 1$) the nonlinearity becomes more distinct and the power of the test increases, see Table 3.2.A-C. However, there seems to be a peak when $\gamma = 10$ since the power is often slightly weaker when $\gamma = 100$. This is due to the fact that the test is conditional on the rejection of the hypothesis of no cointegration. Moreover, changing the location of the transition (c) does not generally seem to affect the power much. But in some cases the power is slightly smaller when $c = 0.75$ and γ is high, see Table 3.2.C. In this case, the period during which the equilibrium correction is operating is short. This might make it hard to estimate the parameters of the cointegrating vector and it becomes difficult for the test to detect the time-variation. An increase in the sample size improves the power, especially when δ is small, see e.g. 3.2.B.

Finally, increasing the coefficient of the short-run dynamics from $\theta = 0.2$ to $\theta = 0.9$ reduces the power sharply, see Table 3.3.A-C. The reason for the poor performance when $\theta = 0.9$ is that the dominating short-run dynamics make the cointegrating vector difficult to estimate. The power of the significance test is also very low for this scenario and if the cointegrating relationship does not enter significantly it is hardly surprising that the parameter constancy test has low power. When data is generated according to an LSTR(2) model the behaviour is similar. A stronger coefficient of the cointegrating relationship and a larger sample size improve the power of the parameter constancy test while it seems to be unaffected by the value of the location parameter, see Table 3.5.

3.2.4. The power of the Lin and Teräsvirta parameter constancy test for Case 2: The cointegrating vector is known

In this case the cointegrating vector is known and the overall performance of the Lin and Teräsvirta parameter constancy test is better than for Case 1. The power of the significance test also improves considerably. This demonstrates the importance of high quality estimates of the cointegrating vector for the second test.

Generating the series according to an LSTR(1) model, the power of the parameter constancy test increases for a higher coefficient of the cointegrating relationship, a larger sample size and when the speed of transition is high ($\gamma > 1$). The power is usually not affected by changes in the location parameter c. But, as in Case 1, the power is slightly smaller when $c = 0.75$ and γ is high, see e.g. Table 3.3.C. Moreover, an increase in the short-run dynamics from $\theta = 0.2$ to $\theta = 0.9$ really improves the power, which is quite puzzling. However, when the cointegrating relationship is known and the short-run dynamics are strong it might be easier for the test to separate the dynamics of the two components and the time variation is more easily found. This can be compared to the results of Case 1 where the power was close to zero for strong short-run dynamics, pointing at the importance of a correct estimate of the cointegrating vector when trying to detect the time-varying coefficient.

When data is simulated according to an LSTR(2) model the power properties are similar to those of the LSTR(1) model. A higher coefficient of the equilibrium correction term and a larger sample size has a positive effect on the power. The performance of the test when changing the location parameter (c_1) is puzzling. Because it is assumed that $c_2 = 1 - c_1$, increasing c_1 implies a longer operating period for the equilibrium correction. But, as is seen from Table 3.5, the power is not higher for higher values of c_1. This is due to the selection bias, since the test is only carried out if the null of no cointegration is rejected.

3.2.5. The power of the Lin and Teräsvirta parameter constancy test for Case 3: Unrestricted cointegration

In this scenario the cointegrating vector is not estimated and the long-run variables enter unrestricted into the linear model. Hence, since the cointegrating vector will not be estimated, there are two coefficients for the long-run variables. The power is measured as the number of times parameter constancy can be rejected for either one of the two parameters.

Considering the LSTR(1) model, the power properties for Case 3 are very similar to those in Case 2. Hence, the power increases when the cointegrating coefficient is stronger and the speed of transition increases. The power is quite sensitive to large values of the location parameter which, as discussed, makes the observation period with an equilibrium correction short. An increase in the sample size improves the power and very strong short-run dynamics have a positive effect on the power. The LSTR(2) model has very similar features to those of Case 2, also in the sense that an increase in the location parameter, which makes the period including the equilibrium correction longer, has a negative effect on the test.

3.2.6. How often will the conclusion be a time-varying equilibrium correction?

The overall power of the two tests, Johansen cointegration test and Lin and Teräsvirta parameter constancy test, is rather low. This might be surprising at first since both tests perform well separately. But since some parameter changes affect the power of the tests in opposite ways, the joint power will not be as high as for each test separately and it varies a lot depending on the parameter choices.

When the series are generated according to an LSTR(1) model, the highest joint power occurs when the transition speed is quite fast ($\gamma = 10$), the coefficient of $\Delta_1 y_{t-1}$ is moderate ($\theta = 0.2, 0.4$), the magnitude of the cointegrating coefficient is strong ($\delta < -0.1$), the location parameter is small ($c = 0.25$) and the sample size is large. However, the joint power deteriorates when any of the parameter values change. It is especially sensitive to changes in γ, c and θ. That is, when $\gamma = 1$ the power of the parameter constancy test is strongly negatively affected and when $\gamma = 100$ the performance of the Johansen cointegration test deteriorates and, hence, a change in either direction results in a decrease in the joint power. Moreover, a higher c implies a shorter period, including the cointegrating relationship which results in a sharp fall of the power of the Johansen test. Another variable that severely affects the joint power is a high θ, when considering Case 1. However, it turns out that by not estimating the cointegrating relationship, Case 3, and only including the cointegrated variables (in levels) in the equation, the joint power will almost be as high as for Case 2, where the cointegrating vector is assumed to be known.

When generating the series according to an LSTR(2) model the joint power is also generally low. In this case the power is most strongly affected by the value of the location parameter c. A high c implies a longer period, including the cointegrating relationship, and the Johansen cointegration test is especially sensitive to this, which makes the joint power smaller.

4. Conclusions

In this paper the probability of detecting a time-varying equilibrium correction by using the Johansen cointegration test and Lin and Teräsvirta parameter constancy test is explored. It turns out that the joint power of the two tests is quite low and the power varies a lot, depending on the choice of parameter values.

The most interesting result is perhaps the fact that the power of the parameter constancy tests strongly improves when the cointegrated variables enter without restrictions. This improves the power, which becomes almost as high as it is when the cointegrating vector is assumed to be known. This result may have important implications for empirical work.

Finally, based on the results above, it seems fair to conclude that the probability for a potential researcher to detect a time-varying equilibrium correction when using the general modelling strategy of testing for cointegration, estimating a cointegrating vector and detecting time variation of the equilibrium correction term, is low. This calls for a new test which simultaneously tests the joint hypothesis of no cointegration and parameter constancy, but that is beyond the scope of this paper.

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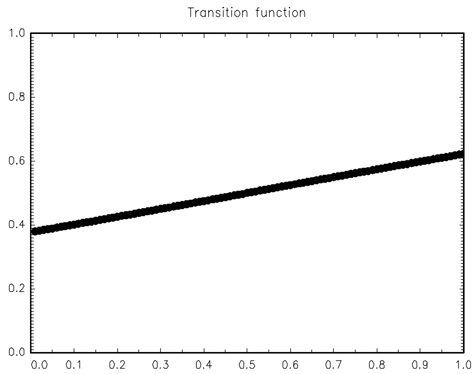

Figure 2.1: Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 1$.

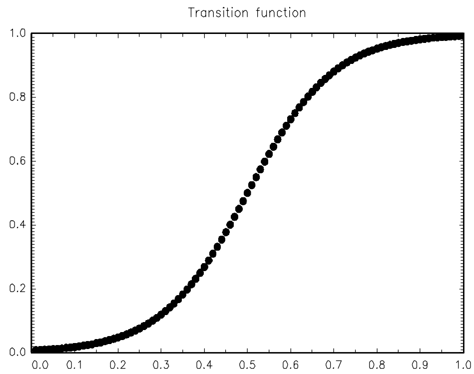

Figure 2.2: Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 10$.

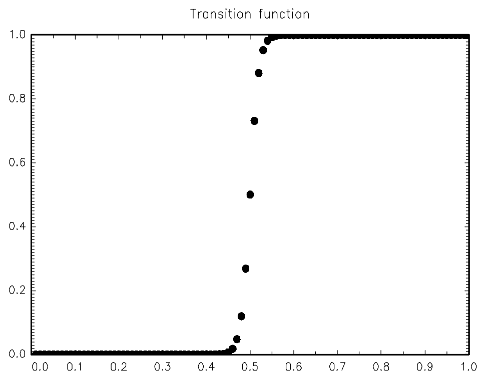

Figure 2.3: Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 100$.

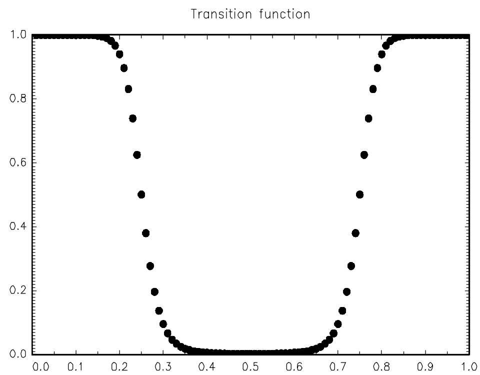

Figure 2.4: Transition function of an LSTR(2) model where $c_1 = 0.25$, $c_2 = 0.75$ and $\gamma = 100$.

Table 3.1 Size of the Johansen cointegration test. Rejection frequencies of the null hypothesis are presented in the table. N=10000.

T=100	**θ=0.2**	**θ=0.4**	**θ=0.9**
J-size	3.68	3.46	5.70
T=200	**θ=0.2**	**θ=0.4**	**θ=0.9**
J-size	3.54	3.33	4.67

Table 3.2.a: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.2, γ =1. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = 0.1**			**δ = 0.4**			**δ = 0. 8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	54.62	48.28	40.16	100.0	99.94	99.76	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	6.52	6.48	6.53	11.98	13.70	14.38	16.89	19.81	22.42
H_0: $\delta = 0$	88.50	89.23	89.74	74.22	75.77	78.08	51.76	57.39	62.36
Case 2:									
LM_1	6.74	7.33	6.80	14.50	16.11	17.00	30.34	32.12	33.66
H_0: $\delta = 0$	99.56	99.38	99.08	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	9.67	10.34	10.28	12.55	13.71	14.02	23.03	24.80	26.33
H_0: $\delta_1 = \delta_2 = 0$	71.84	67.77	64.42	99.98	99.95	99.79	100.0	100.0	100.0
LM-size	4.59	4.79	4.56	4.73	5.04	4.84	4.81	4.58	4.71
T = 200	**δ = 0.1**			**δ = 0.4**			**δ = 0..8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	96.96	93.41	88.30	100.0	100.0	100.0	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	8.19	8.71	9.17	21.24	22.94	24.49	28.98	34.68	38.33
H_0: $\delta = 0$	86.17	87.22	88.57	74.22	76.51	77.83	50.05	56.46	62.12
Case 2:									
LM_1	8.67	9.06	10.08	26.91	29.08	30.07	54.23	59.08	60.29
H_0: $\delta = 0$	99.99	99.98	99.90	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	9.86	10.28	10.55	21.66	22.40	23.64	44.61	48.12	50.08
H_0: $\delta_1 = \delta_2 = 0$	95.12	92.41	89.24	100.0	100.0	100.0	100.0	100.0	100.0
LM-size:	4.78	4.90	4.77	4.75	4.88	4.95	4.85	4.89	5.02

Table 3.2.b: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.2, γ=10. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test H_0: No cointegration	56.68	22.46	14.61	95.89	31.62	13.30	99.22	31.64	11.28
Parameter constancy test **Case 1**:									
LM_1	34.33	24.00	10.81	79.32	75.96	50.98	62.62	69.12	64.63
H_0: $\delta = 0$	91.95	89.76	88.36	81.08	81.63	85.41	61.37	69.43	77.30
Case 2:									
LM_1	48.71	61.98	70.77	96.73	99.34	93.53	99.90	100.0	99.82
H_0: $\delta = 0$	99.70	97.33	87.13	100.0	99.78	94.89	100.0	99.87	96.28
Case 3:									
LM_1	38.64	49.29	60.37	91.76	97.15	90.38	99.79	100.0	99.02
H_0: $\delta_1 = \delta_2 = 0$	75.72	52.05	49.08	99.76	87.98	59.62	100.0	92.95	77.04
LM-size:	4.87	4.48	5.08	5.04	4.77	5.23	4.60	4.87	5.19
T = 200	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test H_0: No cointegration	91.41	40.59	21.06	99.99	51.05	21.94	100.0	52.94	17.69
Parameter constancy test **Case 1**:									
LM_1	70.44	58.17	21.70	84.09	83.80	64.04	59.98	76.79	71.40
H_0: $\delta = 0$	95.26	93.27	90.50	83.31	86.33	87.33	58.65	76.20	83.15
Case 2:									
LM_1	80.10	89.53	79.77	99.89	100.0	99.32	100.0	100.0	100.0
H_0: $\delta = 0$	100.0	99.24	91.98	100.0	99.94	98.27	100.0	99.98	99.04
Case 3:									
LM_1	75.52	84.60	68.47	99.77	100.0	98.72	100.0	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	94.44	74.92	60.97	100.0	95.75	77.44	100.0	97.15	89.49
LM-size:	4.90	4.77	4.88	5.03	5.01	4.81	5.15	4 .57	4.72

Table 3.2.c: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ =0.2, γ =100. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	δ = **0.1**			δ = **0.4**			δ = **0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	32.44	11.55	6.33	29.91	8.82	3.26	23.41	6.40	3.06
Parameter constancy test									
Case 1:									
LM_1	28.61	26.75	18.00	63.66	64.06	38.04	39.85	46.09	42.16
H_0: $\delta = 0$	88.22	83.20	73.93	65.86	68.03	54.60	40.62	44.53	44.44
Case 2:									
LM_1	44.88	59.05	54.82	97.06	99.77	73.31	99.96	100.0	87.58
H_0: $\delta = 0$	99.32	92.29	71.41	99.67	95.80	70.55	99.32	95.16	67.65
Case 3:									
LM_1	41.25	47.79	33.18	96.05	99.09	61.04	99.87	99.84	79.08
H_0: $\delta_1 = \delta_2 = 0$	69.27	45.54	30.96	95.29	75.85	34.05	93.21	79.38	47.39
LM-size:	5.03	4.57	4.84	4.85	5.10	4.81	4.76	5.03	4.93
T = 200	δ = **0.1**			δ = **0.4**			δ = **0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	45.41	16.39	7.58	32.15	9.26	3.87	24.63	6.59	2.54
Parameter constancy test									
Case 1:									
LM_1	64.52	51.68	22.82	71.32	69.33	51.42	41.74	51.44	51.18
H_0: $\delta = 0$	91.96	87.49	81.00	71.51	74.08	63.82	41.58	50.22	53.15
Case 2:									
LM_1	83.97	88.90	65.17	100.0	100.0	89.95	100.0	100.0	92.52
H_0: $\delta = 0$	99.85	95.42	78.63	99.88	96.76	74.94	99.27	95.76	69.69
Case 3:									
LM_1	80.33	80.54	42.35	99.97	100.0	74.16	100.0	100.0	86.61
H_0: $\delta_1 = \delta_2 = 0$	85.40	63.09	40.63	96.98	84.88	46.77	94.48	82.25	46.46
LM-size:	4.85	4.67	4.68	4.78	4.66	5.16	4.71	5.27	5.21

Table 3.3.a: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.4, γ =1. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test H_0: No cointegration	52.43	43.60	36.04	100.0	99.99	99.88	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	6.29	6.08	6.60	7.88	9.56	9.92	6.82	8.84	10.31
H_0: $\delta = 0$	75.22	75.34	76.44	47.08	50.90	54.77	16.35	22.56	28.47
Case 2:									
LM_1	6.28	6.01	5.85	14.59	15.89	16.84	31.00	33.64	34.48
H_0: $\delta = 0$	99.31	99.54	98.97	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	8.98	8.62	9.46	12.52	13.74	14.20	24.13	26.61	26.56
H_0: $\delta_1 = \delta_2 = 0$	67.92	61.81	56.83	100.0	99.94	99.83	100.0	100.0	100.0
LM-size:	4.59	4.87	4.70	4.80	4.97	5.06	4.48	5.03	4.84
T = 200	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test H_0: No cointegration	97.55	93.74	87.84	100.0	100.0	100.0	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	7.36	8.05	8.06	12.62	15.69	17.38	8.51	13.69	17.35
H_0: $\delta = 0$	73.45	74.39	76.71	45.06	50.00	53.57	13.73	21.08	26.15
Case 2:									
LM_1	8.39	9.58	9.06	26.13	29.24	31.27	56.51	60.75	63.14
H_0: $\delta = 0$	100.0	99.99	99.94	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	9.48	10.36	9.56	20.73	22.62	24.34	46.17	51.21	52.52
H_0: $\delta_1 = \delta_2 = 0$	95.40	92.76	88.50	100.0	100.0	100.0	100.0	100.0	100.0
LM-size:	4.61	4.76	4.37	4.56	4.74	5.16	4.83	5.10	5.26

Table 3.3.b: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.4, γ =10. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test H_0: No cointegration	50.80	16.62	8.13	97.20	30.71	10.85	99.93	37.53	11.02
Parameter constancy test									
Case 1:									
LM_1	30.06	22.32	8.86	48.60	50.21	39.91	26.28	34.85	44.19
H_0: $\delta = 0$	75.93	76.47	73.92	48.57	52.33	60.18	26.13	34.51	48.00
Case 2:									
LM_1	45.33	53.67	49.08	96.69	99.67	90.14	99.92	100.0	99.46
H_0: $\delta = 0$	99.74	96.15	78.60	100.0	99.80	92.17	100.0	99.87	95.64
Case 3:									
LM_1	42.15	45.67	34.81	94.87	99.19	84.06	99.87	100.0	99.46
H_0: $\delta_1 = \delta_2 = 0$	70.63	43.32	28.91	99.80	88.70	57.97	100.0	92.97	81.03
LM-size:	4.60	5.10	4.51	4.74	4.65	4.81	4.93	5.17	5.16
T = 200	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test H_0: No cointegration	88.21	31.09	13.00	99.98	47.95	16.96	100.0	58.93	16.08
Parameter constancy test									
Case 1:									
LM_1	61.46	50.08	21.62	49.64	57.88	56.72	17.11	41.80	47.57
H_0: $\delta = 0$	79.75	78.39	81.77	48.74	56.87	67.39	16.77	41.08	49.56
Case 2:									
LM_1	80.18	85.46	70.62	99.90	100.0	99.46	100.0	100.0	100.0
H_0: $\delta = 0$	100.0	99.16	90.46	100.0	99.98	97.46	100.0	100.0	98.32
Case 3:									
LM_1	75.31	79.22	57.92	99.78	100.0	98.76	100.0	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	93.55	67.39	43.69	100.0	95.43	77.95	100.0	96.95	86.32
LM-size:	4.77	4.95	5.10	5.48	5.01	4.97	4.95	5.02	4.62

Table 3.3.c: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.4, γ =100. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test H_0: No cointegration	26.73	8.98	4.41	32.73	10.82	4.57	32.24	9.80	4.50
Parameter constancy test									
Case 1:									
LM_1	25.63	20.60	11.79	31.01	36.23	26.91	14.14	27.76	28.00
H_0: $\delta = 0$	70.37	67.15	56.92	31.38	35.03	35.23	15.79	21.33	24.22
Case 2:									
LM_1	42.76	52.56	31.52	97.68	99.72	76.59	100.0	100.0	90.89
H_0: $\delta = 0$	99.51	90.31	60.32	99.60	94.55	66.74	99.32	91.02	62.00
Case 3:									
LM_1	37.75	41.09	23.81	97.22	98.89	58.64	100.0	99.90	75.56
H_0: $\delta_1 = \delta_2 = 0$	66.55	39.64	24.04	94.29	76.34	29.76	92.56	76.12	42.22
LM-size:	4.87	4.96	4.55	4.89	4.71	4.81	4.64	5.13	4.79

T = 200	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test H_0: No cointegration	38.37	12.46	4.64	33.34	10.48	3.81	30.86	9.07	3.08
Parameter constancy test									
Case 1:									
LM_1	54.96	44.94	18.97	32.63	41.41	30.71	13.90	26.24	32.79
H_0: $\delta = 0$	72.58	70.55	62.07	31.97	41.22	41.21	16.14	23.59	30.52
Case 2:									
LM_1	83.89	87.40	48.06	100.0	100.0	77.43	100.0	100.0	87.34
H_0: $\delta = 0$	99.79	94.54	72.41	99.67	96.56	70.08	99.58	93.16	58.44
Case 3:									
LM_1	80.43	78.41	32.33	100.0	99.81	69.03	100.0	99.89	84.09
H_0: $\delta_1 = \delta_2 = 0$	84.15	59.95	26.50	95.71	83.78	39.37	94.49	80.50	42.53
LM-size:	4.90	4.37	4.77	4.72	4.58	5.28	4.94	4.63	5.23

Table 3.4.a: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.9, γ =1. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	98.58	96.93	94.25	100.0	100.0	100.0	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	3.04	3.19	2.98	1.43	1.51	1.89	1.02	0.97	0.94
H_0: $\delta = 0$	2.51	2.40	2.53	0.28	0.38	0.51	0.03	0.07	0.06
Case 2:									
LM_1	9.80	10.60	10.90	39.64	42.05	43.27	79.06	81.49	82.18
H_0: $\delta = 0$	99.98	99.98	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	9.65	10.74	11.05	31.49	33.69	34.58	71.42	74.15	74.35
H_0: $\delta_1 = \delta_2 = 0$	97.18	95.90	92.89	100.0	100.0	100.0	100.0	100.0	100.0
LM-size:	4.33	4.78	4.60	5.04	5.01	4.82	4.85	4.52	4.87
T = 200	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	100.0	100.0	99.97	100.0	100.0	100.0	100.0	100.0	100.0
Parameter constancy test									
Case 1:									
LM_1	1.08	1.00	1.10	0.39	0.50	0.48	0.35	0.32	0.36
H_0: $\delta = 0$	0.80	0.92	1.22	0.03	0.03	0.03	0.00	0.00	0.00
Case 2:									
LM_1	16.02	18.74	18.42	68.17	71.87	72.50	97.55	98.42	98.26
H_0: $\delta = 0$	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Case 3:									
LM_1	13.32	15.86	14.71	58.59	62.93	64.03	95.58	96.85	96.73
H_0: $\delta_1 = \delta_2 = 0$	100.0	100.0	99.97	100.0	100.0	100.0	100.0	100.0	100.0
LM-size:	4.88	4.51	4.68	4.95	5.09	5.23	5.00	5.40	5.09

Table 3.4.b: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.9, γ =10. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	97.79	45.96	20.26	100.0	74.29	28.26	100.0	87.21	25.75
Parameter constancy test									
Case 1:									
LM_1	3.82	3.76	5.82	1.07	2.53	4.21	0.58	2.00	3.81
H_0: $\delta = 0$	1.17	2.59	4.64	0.16	1.10	3.22	0.03	0.80	2.60
Case 2:									
LM_1	87.79	95.56	85.93	99.73	100.0	99.65	99.99	100.0	100.0
H_0: $\delta = 0$	100.0	99.56	92.20	100.0	99.97	96.89	100.0	100.0	97.71
Case 3:									
LM_1	85.09	92.93	80.45	99.57	100.0	99.50	99.99	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	95.46	65.67	47.14	100.0	77.90	62.81	100.0	84.53	65.86
LM-size:	5.11	4.61	4.70	5.07	4.72	4.61	4.88	4.93	4.76
T = 200	**δ = 0.1**			**δ = 0.4**			**δ = 0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	99.96	53.91	18.90	100.0	92.46	30.29	100.0	99.23	31.25
Parameter constancy test									
Case 1:									
LM_1	1.41	2.15	2.96	0.35	1.46	2.64	0.11	1.16	3.55
H_0: $\delta = 0$	0.49	1.67	3.49	0.02	0.61	1.85	0.00	0.35	2.24
Case 2:									
LM_1	97.91	99.91	94.23	100.0	100.0	100.0	100.0	100.0	100.0
H_0: $\delta = 0$	100.0	99.87	94.60	100.0	100.0	98.28	100.0	100.0	99.10
Case 3:									
LM_1	96.75	99.80	91.43	100.0	100.0	99.97	100.0	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	99.89	76.63	56.72	100.0	92.70	70.55	100.0	99.00	74.66
LM-size:	4.86	4.93	4.68	5.04	4.81	4.69	4.64	4.97	4.93

Table 3.4.c: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(1), θ=0.9, γ =100. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	δ = **0.1**			δ = **0.4**			δ = **0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	93.58	69.58	54.53	94.59	59.80	49.33	84.90	43.15	30.98
Parameter constancy test									
Case 1:									
LM_1	1.70	1.16	2.59	0.34	0.72	1.48	0.31	1.02	2.84
H_0: $\delta = 0$	0.56	0.82	1.82	0.25	0.48	0.75	0.26	0.76	1.71
Case 2:									
LM_1	95.91	99.17	97.21	99.95	100.0	99.74	100.0	100.0	99.71
H_0: $\delta = 0$	100.0	99.57	97.41	99.99	99.38	94.65	100.0	98.22	86.67
Case 3:									
LM_1	95.03	98.58	95.47	99.94	99.98	99.70	100.0	100.0	99.55
H_0: $\delta_1 = \delta_2 = 0$	91.12	79.72	72.82	88.56	73.51	70.18	77.13	65.17	54.84
LM-size:	4.50	4.58	4.84	5.21	4.99	4.72	5.31	5.04	5.08
T = 200	δ = **0.1**			δ = **0.4**			δ = **0.8**		
Test	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**	**c=0.25**	**c=0.50**	**c=0.75**
Johansen test									
H_0: No cointegration	89.49	59.39	48.20	78.43	40.01	27.79	57.49	25.31	12.90
Parameter constancy test									
Case 1:									
LM_1	0.77	0.96	1.45	0.32	1.32	2.37	0.37	2.92	4.96
H_0: $\delta = 0$	0.27	0.61	1.22	0.26	0.67	1.84	0.57	2.02	3.33
Case 2:									
LM_1	99.83	99.93	97.26	100.0	100.0	98.92	100.0	100.0	99.30
H_0: $\delta = 0$	99.99	99.29	95.35	100.0	97.88	87.51	99.98	94.19	75.47
Case 3:									
LM_1	99.68	99.76	95.75	100.0	99.93	98.38	100.0	99.92	97.21
H_0: $\delta_1 = \delta_2 = 0$	88.97	77.18	73.42	79.05	66.73	55.31	77.42	61.36	40.85
LM-size:	5.23	4.41	4.65	4.67	4.96	4.64	5.03	4.51	5.18

Table 3.5: Size and power of the Johansen cointegration test and the parameter constancy tests. Rejection frequencies of the null hypotheses are presented in the table. Model: LSTR(2), θ=0.2, γ=100, c_2 =1- c_1. Numbers of repetitions N = 10000. Note that δ_1 and δ_2 are the coefficients for y_{t-1} and z_{t-1} when no cointegrating relationship is estimated.

T = 100	**δ = 0.1**		**δ = 0.4**		**δ = 0.8**	
Test	**c_1=0.25**	**c_1=1/3**	**c_1=0.25**	**c_1=1/3**	**c_1=0.25**	**c_1=1/3**
Johansen test						
H_0: No cointegration	19.93	43.27	22.73	74.19	18.40	78.10
Parameter constancy test						
Case 1:						
LM_1	18.41	14.93	60.36	60.97	54.57	48.35
H_0: $\delta = 0$	80.63	82.20	66.74	67.52	54.57	47.63
Case 2:						
LM_1	48.86	28.15	99.16	94.33	100.0	99.92
H_0: $\delta = 0$	96.09	99.19	99.43	100.0	99.84	100.0
Case 3:						
LM_1	37.43	24.66	97.22	90.25	100.0	99.77
H_0: $\delta_1 = \delta_2 = 0$	51.73	74.79	89.75	99.77	97.34	100.0
LM-size:	4.37	4.86	4.37	4.88	5.03	4.77

T = 200	**δ = 0.1**		**δ = 0.4**		**δ = 0.8**	
Test	**c_1=0.25**	**c_1=1/3**	**c_1=0.25**	**c_1=1/3**	**c_1=0.25**	**c_1=1/3**
Johansen test						
H_0: No cointegration	34.91	76.79	31.66	96.11	24.79	98.41
Parameter constancy test						
Case 1:						
LM_1	45.92	40.19	72.33	71.11	59.06	50.56
H_0: $\delta = 0$	82.15	83.28	73.12	69.59	57.68	49.47
Case 2:						
LM_1	81.18	65.24	100.0	99.94	100.0	100.0
H_0: $\delta = 0$	98.85	99.95	99.84	100.0	100.0	100.0
Case 3:						
LM_1	71.50	56.75	100.0	99.77	100.0	100.0
H_0: $\delta_1 = \delta_2 = 0$	74.25	92.84	94.32	100.0	98.95	100.0
LM-size:	4.76	4.97	4.72	4.71	4.76	4.96