Is the short-run Phillips curve nonlinear? Empirical evidence for Australia, Sweden and the United States

Ann-Charlotte Eliasson

Stockholm School of Economics, Department of Economic Statistics

P.O. Box 6501, SE-113 83 Stockholm

E-mail: stace@hhs.se

Preliminary version: June 1999

Abstract

The Phillips curve has generally been estimated in a linear framework which implies a constant relationship between inflation and unemployment. Lately there have been several studies which claim that the slope of the Phillips curve is a function of macroeconomic conditions and that the relationship is asymmetric. If this is true the assumption of linearity is too restrictive. In this paper linear Phillips curves for Australia, Sweden and the United States is tested for linearity and parameter constancy. The nonlinear alternative is specified as a smooth transition regression model. It turns out that linearity is rejected for both Australia and Sweden while the Phillips curve for the United States is linear.

Keywords. Phillips curve, dynamic model, econometric model building, encompassing, parameter constancy, smooth transition regression.

JEL Classification Codes: C52, E31.

Acknowledgements. The author acknowledges support from Jan Wallander's and Tom Hedelius's Foundation for Social Research and from Riksbanken, the Central Bank of Sweden. This paper has benefitted from comments by Mikael Apel, Tor Jacobsson, Joakim Skalin, Timo Teräsvirta and Anders Vredin. The responsibility for any errors or shortcomings remains mine.

1. Introduction

The shape of the Phillips curve is central to the conduct of monetary policy. The Phillips curve has generally been estimated in a linear framework, see, e.g., Gordon (1970, 1975, 1977, 1983, 1997) but recent studies have allowed for a nonlinear relationship, see, e.g., Laxton *et al.* (1998), Callen and Laxton (1998), Debelle and Vickery (1997). These studies have in common that the nonlinearity is imposed on the Phillips curve without prior econometric testing. If the true relationship is nonlinear this should be reflected in the econometric specification since it has important implications for the effects of monetary policy, see, e.g., Isard *et al.* (1998) and Dupasquier and Rickets (1998a). On the other hand, if the true relationship is linear introducing nonlinearities yield an overparameterised model which will break down outside the sample period, see Granger and Teräsvirta (1992).

In this paper the expectations-augmented short-run Phillips curve will be reconsidered. The focus is on investigating whether the relationship between inflation and unemployment is nonlinear using the modelling strategy for smooth transition regression (STR) models outlined in Teräsvirta (1994, 1998). There are many theories suggesting a nonlinear relationship but the different theories yields different shapes and the STR methodology is making it possible to test for linearity and estimate a nonlinear model without making any a priori assumptions about the shape of the nonlinear relationship. The implications of the final model specification can be compared to the economic theories. It turns out that the Phillips curves for Australia and Sweden are nonlinear while the one for the United States appears to be linear.

The paper is organised as follows: After a historical and theoretical background in Section 2 the STR models are introduced in Section 3. Expectations-augmented short-run Phillips curves for Australia, Sweden and the United States are estimated and the results discussed in Sections 4, 5 and 6 respectively. Section 7 contains conclusions.

2. Background

In a seminal paper, Phillips (1958) found a negative relationship between unemployment and wage inflation in the United Kingdom, 1861-1957. Subsequent research found a similar relationship between unemployment and price inflation. This relationship has since been known as the Phillips curve and at the time there was strong empirical support for a stable inflation-unemployment trade-off. In the late 1960s this hypothesis was severely criticized from the theoretical side by Friedman (1968) and Phelps (1968) who argued that it was unreasonable to assume that nominal variables could affect real variables and that a shift by policy makers to expansionary policies would eventually change the way prices and wages are set. In the early 1970s the first empirical failures of the Phillips curve occurred when both inflation and unemployment increased simultaneously, primarily due to the oil-price shocks. The critique prompted the formulation of the expectations-augmented Phillips curve. According to this specification no policy can permanently lower unemployment below its natural rate unless expectations are highly irrational.

The Phillips curve is regaining interest after a period in neglect and there has been considerable theoretical work suggesting a nonlinear relationship between inflation and unemployment. The so called "new Phillips curves"; see Galí and Gertler (1998), are based on early studies of Taylor (1980) and Calvo (1983) using staggered nominal wage contracts and price setting by forward looking individuals and firms. There are other microeconomic theories that also give rise to an asymmetric relationship such as the theories dealing with, for example, capacity constraints, menu costs and nominal rigidities. However, the shape of the nonlinearity is ambiguous since the different theories yield different nonlinear relationships, see Section 2.2. The focus in empirical work has also shifted towards issues concerning the shape of the relationship, see, e.g. Yates (1998), Gruen *et al.* (1998) and Dupasquier and Rickets (1998b) for reviews on empirical studies of nonlinear Phillips curves.

2.1. The expectations-augmented short-run Phillips curve

The linear expectations-augmented short-run Phillips curve is typically assumed to be of the following form

$$\pi_t = \pi_t^e + \gamma \left(u^* - u_t\right) + \varepsilon_t \tag{2.1}$$

where π_t is inflation, π_t^e is inflation expectations, u^* is the non-accelerating inflation rate of unemployment, NAIRU, u_t is the unemployment rate and ε_t is the error term. There are two main difficulties when estimating this model empirically. They are due to measurement problems regarding inflation expectations and the NAIRU. Neither of these variables are directly observable and has to be approximated. In some studies inflation expectations are modelled as a simple weighted average of past inflation rates. But since expectation formations are sensitive to monetary policy inflation expectations based on past inflation rates might be inappropriate and survey measures of inflation expectations are getting more popular for capturing the forward-looking component. However, in empirical work inflation expectations are generally assumed to be a combination of backward- and forward-looking components. This can be viewed as a mixture between the traditional and the "new" Phillips curve according to Galí and Gertler (1998). Thus,

$$\pi_t^e = \lambda_1 \pi_t^f + \lambda_2 \pi_t^{imp} + (1 - \lambda_1 - \lambda_2) A(L) \pi_{t-i} \tag{2.2}$$

where π_t^f is the forward-looking component and $A(L)$ is a lag polynomial. The backward-looking component reflects the inertia in the inflation process and the forward-looking component mirrors public expectations. Equation (2.2) is extended to include inflation in imported goods, π_t^{imp}, which may be an influential component when modelling Phillips curves for small open economies. The other major difficulty estimating empirical Phillips curves is due to the NAIRU. By assuming that u^* is constant, model (2.1) can be rewritten as

$$\pi_t = \alpha + \pi_t^e + \gamma u_t + \varepsilon_t \tag{2.3}$$

where $\alpha = \gamma u^*$ in (2.3). This assumption is not innocuous. Several studies estimates the unobservable NAIRU by using the Kalman filter and they often conclude that the NAIRU is not stable over time, see, e.g., Apel and Jansson (1997), Gruen *et al.* (1998), Laxton *et al.* (1998) for studies on Sweden, Australia and the United States respectively. But the uncertainty concerning the estimates is considerable and the results are also highly dependent on the initial setup, see Staiger *et al.* (1996). Hence, the characteristics of the NAIRU will not be modelled in this paper and in the following the NAIRU is assumed to be constant.

2.2. Microfoundations for a nonlinear Phillips curve relationship

According to Section 2.1 the short-run trade-off between unemployment and inflation is assumed to be constant over time. However, many theoretical models of price-setting behaviour predict the slope of the Phillips curve to be a function of macro-economic conditions. There are several theories that may give rise to an asymmetric relationship but the precise form of the nonlinearity is ambiguous. A comprehensive review of the microfoundations can be found in Dupasquier and Rickets (1998a) and Yates (1998). Below is a brief review:

- The *capacity constraint model* is based on the assumption that increasing marginal costs and a fixed capacity in the short run is making it costly for the firms to increase output and employment in the short-run. Thus, inflation becomes more sensitive to output in times of excess demand and the short-run Phillips curve has a convex shape.

- The *signal extraction model*; Lucas (1972, 1973), suggests that the relationship between output and inflation arises because agents are unable to distinguish between aggregate and relative price shocks. The shocks are not directly observable and the relationship between output and inflation will depend on the variance of inflation. The more volatile the aggregate prices, the less a given price change will be attributed to a change in relative prices, and thus the smaller will be the output response. The short-run Phillips curve is linear but its slope will vary positively with the volatility of inflation.

- The *costly adjustment model*; see, e.g., Ball *et al.* (1988), implies that the relationship between output and inflation varies with the level of inflation. In the presence of menu costs only some firms will change their prices in response to a demand shock. But, the greater the number of firms that change their prices the more responsive will the aggregate price level be to demand shocks. The firms will increase the frequency and size of price adjustment as inflation rises so that aggregate demand shocks will have less effect on output and more effect on the price level. Another example of the costly adjustment model occurs when the wage contracts between firms and workers have long duration. The short-run Phillips curve is a convex function that becomes linear as inflation approaches zero.

- The *downward nominal wage rigidity model*; see, e.g. Stiglitz (1986), Fisher (1989), suggests that workers are more reluctant to accept a decrease in their nominal wages than a decrease in their real wages because of money illusion, institutional or behavioural factors. And, hence, a low inflation environment is more likely to create allocation inefficiencies. Provided that full adjustment to individual demand shocks eventually occurs this model has two implications for the shape of the short-run Phillips curve: The effects of nominal wage floors are more likely to be important at low rates of inflation. Second, if this is true excess supply will have less effect on inflation than excess demand resulting in asymmetries with respect to the output gap.

- The *monopolistically competitive model*; see, e.g. Stiglitz (1984), refers to the strategic pricing behaviour of firms in monopolistically competitive or oligopolistic markets. Producers might lower prices quickly in order to avoid being undercut by rivals. Furthermore, they might be reluctant to raise prices even in the face of generally rising prices, hoping to keep out potential new competitors. The short-run Phillips curve will in this case be concave.

2.3. Monetary policy and the Phillips curve

The main emphasis in monetary policy will differ depending on which of the above theories is the "right" one. The question of whether the Phillips curve is convex, concave or linear yields different effects of monetary policy. Yates (1998) and Dupasquier and Rickets (1998a) discuss the policy implications of the different theories thoroughly and the arguments will not be repeated here. The most important issue when dealing with asymmetric relationships is the importance of timing of the policy. If the central bank allows the economy to deviate from the target for some time a larger change in the monetary instrument may be necessary to achieve the desired effect. In a linear environment (setting credibility issues aside) it does not matter if there is one large or several small changes whereas in a nonlinear environment the effect of several small changes might be very different from that of a single large one.

3. Smooth transition regression model

Smooth transition regression (STR) models will be applied to describe the empirical expectations-augmented short-run Phillips curves of Australia, Sweden and the United States. This makes it possible to test for linarity and estimate a nonlinear model without making any a priori assumptions about the shape of the nonlinear relationship. In this section some basic features of the STR models are introduced and the modelling cycle will be reviewed together with the tests of linearity and parameter constancy.

Start by considering the following nonlinear model,

$$y_t = x_t'\varphi + x_t'\theta F(s_t) + e_t. \qquad t = 1, ..., T. \tag{3.1}$$

where $e_t \sim \text{nid}\left(0, \sigma^2\right), x_t = (1, y_{t-1}, ..., y_{t-k}; z_{1t}, ..z_{mt})' = (1, \tilde{x}_t')'$ with $p = k + m$ is the vector of stationary explanatory variables, some of which may be linear combinations of nonstationary variables. Furthermore, $\varphi = (\varphi_0, ..., \varphi_p)'$ and $\theta = (\theta_0, ..., \theta_p)'$ are parameter vectors. $F(s_t)$ is the transition function which is continuous in s_t and bounded between zero and unity. The transition variable s_t is either stationary or a time trend (t). The transition function of a kth order logistic smooth transition regression, LSTR(k) model is

$$F(s_t) = F(s_t; \gamma, c) = \left(1 + \exp\left\{-\gamma \prod_{i=1}^{k} (s_t - c_i)\right\}\right)^{-1}, \ \gamma > 0, \ c_1 \leq .... \leq c_k \tag{3.2}$$

where $k = 1$ yields the LSTR(1) model and $k = 2$ the LSTR(2) which are the two parameterisations of transition functions that will be considered in this paper. The slope parameter γ determines the speed of transition between the two extreme regimes and the vector of location parameters c determines the location of the transition. For derivations of the tests and an exhaustive description of STR models, see Granger and Teräsvirta (1993) and Teräsvirta (1994, 1998).

3.1. The modelling cycle for STR models

Following Teräsvirta (1994) the modelling cycle consists of four steps. Initially a linear model with no error autocorrelation is estimated. The tests of linearity and parameter constancy is performed in this model against the alternative of an STR parameterisation. If either of the hypotheses are rejected an STR model is estimated. The preferred STR specification is evaluated using misspecification tests of no error autocorrelation, parameter constancy and no additive nonlinearity, see Eitrheim and Teräsvirta (1996).

When testing for linearity the alternative is specified using several different variables as potential transition variable. If more then one of the null hypotheses are rejected the STR model is specified for the transition variable which was used in the most forcefully rejected hypothesis. The initial choice of possible transition variables is based on economic theory.

3.2. Testing linearity and parameter constancy

Testing linearity against the alternative of an LSTR(k) model amounts to testing if $\gamma = 0$ in equation (3.2). The model is not identified under the null hypothesis due to the nuisance parameters θ and c. A Taylor series approximation about $\gamma = 0$ is used as a substitute to circumvent this problem, and the tests are based on this transformed equation:

$$y_t = x_t'\beta_0 + \sum_{j=1}^{k} \left(\tilde{x}_t s_t^j\right)' \beta_j + e_t^* \tag{3.3}$$

where $e_t^* = e_t + \tilde{x}_t'\tilde{\theta} R(\gamma, c)$, but $e_t^* = e_t$ under H_0. The null hypothesis $H_0 : \gamma = 0$ in equation (3.2) where $k = 3$ implies

$$H_0' : \beta_1 = \beta_2 = \beta_3 = 0 \tag{3.4}$$

within (3.3) because $\beta_j = \gamma\tilde{\beta}_j$ where $\tilde{\beta}$ is a function of the parameters in the original STR specification. In order to decide between $k = 1$ and $k = 2$, one continues by carrying out the following test sequence within (3.3):

$$H_{04} \quad : \quad \beta_3 = 0, \tag{3.5}$$

$$H_{03} \quad : \quad \beta_2 = 0 | \beta_3 = 0, \tag{3.6}$$

$$H_{02} \quad : \quad \beta_1 = 0 | \beta_2 = \beta_3 = 0. \tag{3.7}$$

If the rejection of H_{03} is strongest (measured by the p-value of the test), the rule is to choose $k = 2$, otherwise one selects $k = 1$; for the reasoning behind this rule see

Teräsvirta (1994). Furthermore, it is shown that in small samples an F approximation to the LM-test statistic is preferable to the asymptotic χ^2-distribution because it has good size properties.

A special case of nonlinearity is when the transition variable is a time trend (t). The STR model can be viewed as a linear model with time-varying parameters, see Lin and Teräsvirta (1994). Following Lin and Teräsvirta (1994), the testing procedure becomes

$$\begin{aligned} LM_3 &: \beta_1 = \beta_2 = \beta_3 = 0, \\ LM_2 &: \beta_1 = \beta_2 = 0 | \beta_3 = 0, \\ LM_1 &: \beta_1 = 0 | \beta_2 = \beta_3 = 0. \end{aligned}$$

in (3.4). If the rejection of LM_1 is strongest (measured by the p-value) an LSTR(1) model is preferred and if LM_2 is the strongest rejected hypothesis an LSTR(2) model is the most appropriate. When dealing with time-varying parameters a more flexible specification may be needed and if LM_3 is the strongest rejected hypothesis the non-monotonous LSTR(3) model will be estimated.

In the following, the Phillips curve for each of Australia, Sweden and the United States is estimated and evaluated. A linear model is specified for each country and in a second step linearity is tested against a nonlinear alternative of smooth transition regression (STR) type. STR is not the only possible nonlinear alternative but an advantage of STR models is that there exists a well established specification and modelling cycle complete with linearity and misspecification tests, see Teräsvirta (1994). The tests have good small sample properties which makes them useful analysing macroeconomic relationships since the time-series often are short. The STR modelling procedure is making it possible to simultaneously test for linearity and parameter constancy. Time-varying parameters may be an issue especially since the NAIRU is assumed to be constant and due to large changes in monetary policies. If any of the linearity or parameter constancy hypotheses are rejected the corresponding STR model will be estimated and evaluated.

4. Australia

There have been numerous studies of the Australian Phillips curve starting with Phillips (1959). A review of old and recent studies of the Australian Phillips curve can be found in Gruen *et al.* (1998). The older studies usually estimate a linear relationship while the more recent ones often allow for either a time-varying NAIRU or a nonlinear specification. The common theme of the studies that allow for a nonlinear relationship is that nonlinearities are introduced without any initial tests of linearity. For example, Debelle and Vickery (1997) estimate and compare a linear and a nonlinear short-run Phillips curve with forward looking expectations using quarterly data from 1959(3) to 1997(1). They find that the nonlinear specification is more accurate than the linear since it has higher explanatory power. This does not necessarily imply that the true relationship is nonlinear although this possibility cannot be excluded. The reason is that the authors do not perform any initial test of linearity before estimating the nonlinear model. The true relationship may still be linear and the estimated nonlinear specification will in this case be over-parameterised.

In this section a linear Phillips curve will be estimated and the tests of linearity and parameter constancy will be performed against the alternative of an STR model. If any of the hypotheses are rejected a nonlinear model will be specified.

4.1. Data

Quarterly seasonally unadjusted series for 1977(1)-1997(4) are used for the estimation. The Phillips curve is defined for the annual inflation rate, that is, $\pi_t = \Delta_4 p_t$ where p_t is the consumer price index in logarithms, $\Delta_i = \left(1 - L^i\right)$ is the difference operator and L is the lag operator. There are several measures of inflation available. The underlying inflation series is used for the Australian study since this is the inflation measure currently targeted by the Reserve Bank of Australia. The underlying inflation rate is calculated as the total inflation net of fresh fruit and vegetables, mortgage interest and consumer credit charges, automotive fuel and health services. In the rest of Section 4, the term inflation always refers to underlying inflation unless otherwise is explicitly stated. The inflation expectation series used here is derived from bond yields by Debelle and Vickery (1997).

The CPI inflation and the underlying inflation is plotted against time in Figure

4.1. Inflation has been diminishing over the sample. It was very high in the 1980s but has decreased sharply in the beginning of the 1990s. One possible explanation is that the Reserve Bank of Australia announced an explicit inflation target of 2-3 % in December 1993 in targeting the underlying inflation rate. Inflation expectations are graphed together with the CPI inflation in Figure 4.2, and with underlying inflation in Figure 4.3. The most striking feature in both figures is that the inflation expectations series is systematically higher than both the actual and the underlying inflation rate. This can be interpreted as lack of credibility of the monetary policy in Australia. The systematic difference persists when the level of both inflation and inflation expectations decrease.

Since stationarity is a prerequisite for the modelling techniques used; see Section 3, it is necessary to determine the order of integration of the variables involved. The augmented Dickey-Fuller tests are applied to test the hypothesis of a unit root and the results can be found in Table 4.1. The hypothesis of a unit root is rejected for all series at the 1 % level of significance and, hence, the variables appear to be stationary.

4.2. Empirical results

The estimation procedure starts by specifying a linear expectations-augmented short-run Phillips curve for 1977(1)-1997(4). The initial setup is consistent with the theoretical arguments in Section 2. Economic theory does not give any guidance regarding the dynamics and all variables are initially included with several lags in order to achieve a model without error autocorrelation. Variables with poor explanatory power are excluded from the final specification using the t-values as a guidance. The parsimonious equation becomes

$$\begin{aligned}
\pi_t &= \underset{(0.0065)}{0.019} + \underset{(0.058)}{1.04}\ \pi_{t-1} - \underset{(0.099)}{0.49}\ \pi_{t-4} + \underset{(0.079)}{0.29}\ \pi_{t-5} + \underset{(0.025)}{0.091}\ \pi_t^f \\
&\quad + \underset{(0.0099)}{0.050}\ \pi_{t-1}^{imp} - \underset{(0.011)}{0.023}\ \pi_{t-2}^{imp} - \underset{(0.0084)}{0.030}\ u_{t-1} + \underset{(0.0088)}{0.021}\ u_{t-2} + \hat{e}_t \qquad (4.1) \\
T &= 84, R^2 = 0.99, \sigma_{lin} = 0.0035, AIC = -11.19, \\
LJB &= 0.22(0.90), A(1) = 0.18(0.67), A(4) = 0.88(0.48).
\end{aligned}$$

where π_t denotes the annual inflation rate, π_t^f is the expected annual inflation rate, π_t^{imp} is the annual inflation rate in import prices and u_t denotes the log of unemployment. The standard deviations of the coefficients appear in parentheses, T is the number of observations, R^2 is the coefficient of determination, σ_{lin} is the standard deviation of the linear model, and AIC is Akaike Information Criterion. The Lomnicki-Jarque-Bera (LJB) test does not reject normality of the error process; the corresponding p-value appear in parenthesis, and the LM test of no ARCH of order j, $A(j)$, does not indicate a misspecified model. The results of the test of no error autocorrelation can be found in Table 4.2. The hypothesis of no error autocorrelation against error autocorrelation of order one is rejected at the 5 % level significance (p-value = 4.7 %). However, since the hypothesis of no error autocorrelation against error autocorrelation of a higher order than one cannot be rejected and that the rejection level of no error autocorrelation of order one is close to the chosen significance level of 5 % the linear model (4.1) will be treated as if it fulfills the assumption of no error autocorrelation.

The linear specification (4.1) is estimated without any a priori restrictions of long-run neutrality of money. The coefficients of the inflation variables sums to 0.958 which is close to unity and imposing the restriction of long-run neutrality does not change any conclusions. The sum of the coefficients of the unemployment variables is negative indicating a negative relationship between inflation and unemployment. The intercept can be interpreted as a measure of the NAIRU and the linear equation (4.1) seems to be consistent with economic theory.

The tests of linearity and parameter constancy are performed and the results are shown in Table 4.3. Parameter constancy is the most strongly rejected hypothesis pointing at an LSTR(3) model. Estimating this yields a model with strong error autocorrelation and the null hypothesis of no additional nonlinearity is rejected for two of the transition variables. Moreover, the specification does not capture all time-variation in the parameters. Taking all these features together the model does not seem to perform well and will not be considered further. According to Table 4.3 linearity is also rejected. It is most strongly rejected when the change in contemporary unemployment $\Delta_1 u_t$ is used as transition variable and the testing procedure indicates an LSTR(2) model. Estimating this yields a model with non normal residuals and time-varying parameters. Moreover, the hypothesis of no additional nonlinearity is also rejected. But since there are very few observations that make the second transition necessary an LSTR(1) model is estimated instead. Excluding the variables with poor explanatory power the model becomes

$$
\begin{aligned}
\pi_t &= \underset{(0.0069)}{0.031} + \underset{(0.051)}{0.92}\ \pi_{t-1} - \underset{(0.092)}{0.35}\ \pi_{t-4} + \underset{(0.076)}{0.17}\ \pi_{t-5} + \underset{(0.022)}{0.12}\ \pi_t^f \\
&\quad + \underset{(0.0064)}{0.043}\ \pi_{t-1}^{imp} - \underset{(0.0090)}{0.039}\ u_{t-1} + \underset{(0.0094)}{0.026}\ u_{t-2} \\
&\quad + \left\{ -\underset{(0.042)}{0.13} - \underset{(0.33)}{1.08}\ \pi_{t-4} + \underset{(0.35)}{1.33}\ \pi_{t-5} + \underset{(0.057)}{0.11}\ \pi_t^f + \underset{(0.020)}{0.053}\ u_{t-2} \right\} \\
&\quad \times \left[1 + \exp\left\{ -\underset{(1.49)}{6.61} \left(\Delta_1 u_t - \underset{(0.0095)}{0.053} \right) / \hat{\sigma}_{\Delta_1 u_t} \right\} \right]^{-1} + \hat{e}_t \qquad (4.2) \\
T &= 84, R^2 = 0.99, \sigma_{nl} = 0.0032, \sigma_{nl}/\sigma_{lin} = 0.90, AIC = -11.35, \\
LJB &= 1.62(0.45), A(1) = 0.24(0.62), A(4) = 1.38(0.25).
\end{aligned}
$$

where σ_{nl} denotes the standard deviation of the nonlinear model (4.2). With a residual standard deviation that is 10 % less than that of (4.1), the LSTR(1) model (4.2) variance dominates the linear model (4.1). The assumption of normality and that of no ARCH is satisfied according to the diagnostic tests shown below the equation. There is no evidence of error autocorrelation which was close to being rejected in the linear specification (4.1), see Table 4.4. The hypothesis of no additional nonlinearity is rejected (p-value = 0.031) when using π_t^f as the transition variable, see Table 4.4. However, linearity cannot be rejected for any of the other transition variables which is an improvement compared to results of the initial linearity tests in Table 4.2. Parameter constancy of the whole set of parameters is still rejected (p-value = 0.0029) but equation (4.2) captures a major part of the time-variation compared to the parameter constancy tests presented in Table 4.3.

The transition function of equation (4.2) is plotted against its argument ($\Delta_1 u_t$) in Figure 4.4. When the transition function equals zero the linear component of equation (4.2) adequately describes the relationship and when the transition function equals one the complete model is necessary for capturing the features of the short-run Phillips curve. In Figure 4.5 the transition function is plotted against time. This graph shows when the complete model is working (the transition function equals one) and when only the linear part is necessary to describe the inflation in Australia. The residuals of equations (4.1) and (4.2) are plotted in Figure 4.6 and the nonlinear specification (4.2) outperforms the linear (4.1) for the observations where the complete nonlinear model (4.2) is working.

In order to find out whether the nonlinear model (4.2) explains the empirical results of the linear model (4.1) and the other way around, the encompassing tests are being computed. An introduction to encompassing tests can be found in Hendry (1995) and, for applications of encompassing tests in STR models see Teräsvirta (1998) and Teräsvirta and Eliasson (1998). A brief review can be found in Appendix A. Testing the hypothesis of whether the linear model (4.1) encompasses the nonlinear (4.2) amounts to testing the null hypothesis that the nonlinear part of (4.2) does not enter the linear equation (4.1). The hypothesis is strongly rejected ($F = 4.15$, p-value $= 0.000038$) which implies that the linear model (4.1) does not encompass the nonlinear (4.2). The next step is to find out if equation (4.2) encompasses (4.1), that is, whether the explanatory power of equation (4.2) is significantly improved by linearly including the explanatory variables of model (4.1) that does not enter equation (4.2) linearly. The null hypothesis is that the variables are superfluous and the hypothesis cannot be rejected ($F = 0.16$, p-value$= 0.67$). Hence, model (4.2) encompasses (4.1) while the reverse is not true.

The estimated nonlinear Phillips curve of Australia displays very interesting features. The unemployment variable plays a central role and it enters equation (4.2) both in levels and in first differences. Since $\Delta_1 u_t$ is used as the transition variable it determines which regime is most adequate in describing the Phillips curve for different observations. According to model (4.2) the relationship between inflation and unemployment is negative most of the time but for large increases in the unemployment rate the relationship turns positive. This suggests that the empirically observed failure of the Phillips curve might be a result of a nonlinear relationship and only mirrors a shift in the Phillips curve to a new level where the usual negative relationship again is valid. Moreover, equation (4.2) indicates that the NAIRU varies over time but the nonlinear regime does not prevail sufficiently long to permit any conclusions about the movements in the NAIRU.

The next issue is to find out whether equation (4.2) supports any of the theories of Section 2.2. The interpretation of the model is not straightforward since the nonlinearity of equation (4.2) is ruled by the rate of change of unemployment instead of the level of unemployment. The theories where the nonlinearity origins from demand factors are defined for excess demand or supply which can translated to unemployment or full employment but not as changes in unemployment. The capacity constraint model states that a low demand, or high unemployment, makes inflation less sensitive to unemployment. This is supported by the nonlinear equation (4.2) since the negative relationship between inflation and unemployment decreases (turns positive) in times of increasing of unemployment.

The downward nominal rigidity model suggests that excess supply will have less effect on inflation than excess demand. According to equation (4.2) the relationship appears to be the opposite. The coefficient of unemployment is large (and positive) in times of large increases in unemployment but the number of observation in the nonlinear regime are few and the size of the coefficient might change with a larger sample.

5. Sweden

In November 1992, Sveriges Riksbank, the Central Bank of Sweden, announced an inflation target regime of an annual inflation rate of 2 % with a tolerance band of 1 % from 1995 and onwards. Initially there was a severe credibility problem for the new target since inflation expectations exceeded the upper limit of 3 % and it seemed as if the target would be missed for 1995 and 1996; see Svensson (1995). However, the policy quickly gained credibility and was in the neighborhood of the target of 2 % by the end of the sample, see Figure 5.1.

There are few empirical studies of the Swedish Phillips curve allowing for a nonlinear trade-off. Yates (1998) investigates, using Swedish annual data for 1864-1938, whether or not the slope of the Phillips curve varies depending on if the prices are rising or falling. There is no significant evidence of any variation in the slope indicating asymmetries. Furthermore, he explores whether or not there is a kink in the Phillips curve which could indicate downward nominal rigidity. The conclusion is that there is no evidence of a kink.

In this section a Swedish linear Phillips curve will be estimated and evaluated with respect to linearity and parameter constancy where the alternative is specified as an STR model. If any of the hypotheses are rejected a nonlinear model will be specified.

5.1. Data

The time-series consists of quarterly seasonally unadjusted data for the period 1979(3)-1997(4). The inflation measure used is annual changes in total CPI, which is the inflation measure currently targeted by Sveriges Riksbank. The forward-looking component of the inflation expectations are approximated by a survey of the households' inflation expectations one year ahead gathered by the National Institute of Economic Research. The time-series of inflation and inflation expectations can be found in Figure 5.1. The series generally move closely together and there are no systematic differences.

Unit root tests are performed for all variables and the results are shown in Table 5.1. All variables seems to be integrated of order one and taking the first difference yields stationary variables[1]. Because of this, the Phillips curve will be estimated in differences instead of levels as in the original specification shown in Section 2. The rate of change of inflation is graphed against time in Figure 5.2. The salient features of the series are the oil-price shock in the late 1970s and the turbulence in the early 1990s.

5.2. Empirical analysis

Estimating a linear expectations-augmented short-run Phillips curve for Sweden 1979(3)-1997(4) an eight order lag structure is initially applied for all variables to capture the inertia in the inflation process. German inflation is also included as one of the explanatory variables in the original setup. This is done partly to capture inflation in imports, Germany being one of Sweden's major trading partners, and partly because of its leading role in the ERM. Besides that, Juselius (1992) shows that German inflation is influential describing the Danish inflation but it turns out that this is not the case for Sweden. The variables with poor explanatory power are excluded from the equation and the parsimonious linear model becomes

$$\Delta_1\pi_t = \underset{(0.088)}{0.31}\,\Delta_1\pi_{t-2} - \underset{(0.089)}{0.46}\,\Delta_1\pi_{t-4} + \underset{(0.15)}{0.49}\,\Delta_1\pi_t^f$$

[1] Skalin and Teräsvirta (1999) show that the unemployment series for Sweden is not a unit-root process. It is the asymmetries of the series that makes it appear like one. However, in this paper the series will be treated as an I(1).

$$
\begin{aligned}
& + \underset{(0.14)}{0.57}\ \Delta_1\pi^f_{t-1} - \underset{(0.0076)}{0.011}\ \Delta_1 u_{t-1} + \hat{e}_t \qquad (5.1)\\
T &= 74, R^2 = 0.47, \sigma_{lin} = 0.80, AIC = -0.38,\\
LJB &= 17.36(2\times 10^{-4}), A(1) = 0.23\,(0.63)\,, A(4) = 0.077\,(0.99)\,.
\end{aligned}
$$

The assumption of normality is rejected while the assumption of no ARCH is satisfied according to the diagnostic tests shown below equation (5.1). One way to deal with the lack of normality is to introduce dummies capturing the large changes in inflation due to the turbulence in early 1990s. But, by including dummy variables information about the dynamics of the process would be disregarded in these observations and, hence, dummy variables are not introduced at this stage. There is no evidence of any error autocorrelation according to the results in Table 5.2.

The most striking features of the linear model (5.1) is the strong dependence on inflation expectations. Note that the variables of inflation expectations are more influential than the variables of lagged inflation. The result stresses the important role of inflation expectations when the monetary policy is devoted to an inflation target. The relationship between inflation and unemployment is negative and there is no intercept in the model. The exclusion of the intercept is reasonable if the assumption of a constant NAIRU is true since equation (5.1) is specified for first differences.

The results of the linearity and parameter constancy tests can be found in Table 5.3. It is seen that linearity is most strongly rejected using either $\Delta_1\pi_{t-2}$ or $\Delta_1\pi^f_{t-1}$ as transition variable. Both models are estimated and there are many similarities between the two specifications. However, the LSTR(1) model where $\Delta_1\pi^f_{t-1}$ is used as the transition variable performs slightly better. It becomes

$$
\begin{aligned}
\Delta_1\pi_t &= \underset{(0.14)}{0.49}\ \Delta_1\pi_{t-2} - \underset{(0.080)}{0.46}\ \Delta_1\pi_{t-4} + \underset{(0.13)}{0.62}\ \Delta_1\pi^f_{t-1} - \underset{(0.018)}{0.039}\ \Delta_1 u_{t-1}\\
&\quad + \left\{ - \underset{(0.18)}{0.30}\ \Delta_1\pi_{t-2} + \underset{(0.17)}{0.67}\ \Delta_1\pi^f_t - \underset{(0.019)}{0.032}\ \Delta_1 u_{t-1} \right\}\\
&\quad \times \left[1 + \exp\left\{ - \underset{(28065)}{134.35} \left(\Delta_1\pi^f_{t-1} + \underset{(3.88)}{0.42} \right) / \hat{\sigma}_{\Delta_1\pi^f_t} \right\} \right]^{-1} + \hat{e}_t \qquad (5.2)\\
T &= 74, R^2 = 0.54, \sigma_{nl} = 0.77, \sigma_{nl}/\sigma_{lin} = 0.96, AIC = -0.42,\\
LJB &= 3.23(0.20), A(1) = 0.028\,(0.87)\,, A(4) = 0.60\,(0.66)\,.
\end{aligned}
$$

The nonlinear LSTR(1) model (5.2) variance dominates its linear equivalent (5.1) in the sense that its standard deviation is 96% of the latter one. The assumption of normality is satisfied in equation (5.2) which was a problem for model (5.1) and there is no indication of ARCH. The results of the misspecification tests can be found in Table 5.4. Equation (5.2) satisfies the assumptions of no error autocorrelation and parameter constancy. Linearity is still rejected using $\Delta_1 \pi_{t-2}$ as transition variable but the number of observations is not large enough to consider two nonlinear components and the remaining nonlinearity will not be considered further.

The transition function of equation (5.2) is plotted against its argument $\left(\Delta_1 \pi_t^f\right)$ in Figure 5.3. It is shown that the transition is very fast ($\gamma = 134$) and that the transition function is either equal to unity or to zero during this sample. In Figure 5.4 the transition function is graphed against time showing when the complete model is necessary to describe the Phillips curve relationship (the transition function equals unity). The full specification is necessary most of the time but for some observations the linear component is adequate for capturing the features of the rate of change of inflation. The residuals of equations (5.1) and (5.2) can be found in Figure 5.5. The residuals appear to be similar except in the early 1990s where the nonlinear model (5.2) performs better than the linear (5.1).

The next step is to find out whether the nonlinear model (5.2) encompasses the linear (5.1) and the other way around. Testing the hypothesis of whether equation (5.1) encompasses (5.2) the MNM; see Appendix A, is created by including the nonlinear part of equation (5.2) into model (5.1). The hypothesis is rejected ($F = 2.43$, p-value $= 0.019$) which implies that the explanatory power of model (5.1) would be improved by including the nonlinear component. The linear model (5.1) does not encompass the nonlinear (5.2). To test whether the nonlinear model (5.2) encompasses the linear (5.1), equation (5.2) is completed linearly with the variables included in (5.1) that does not enter the linear component of (5.2). The result of this test appear in Table 5.4 as "the test of no linear restrictions". The null hypothesis cannot be rejected and, hence, model (5.2) encompasses (5.1) while the reverse is not true.

In the nonlinear Phillips curve for Sweden (5.2) the central variable is $\Delta_1 \pi_t^f$ which not only enters the model with a high coefficient but also rules the transition between the extreme regimes. According to equation (5.2) the rate of change of inflation depends more strongly on inflation expectations than on lagged values of inflation.

The full model is usually needed for capturing the dynamic features of the rate of change of inflation. The coefficients of the inflation variables sums to 1.02 and the unemployment variables enters with a negative coefficient. However, for very large negative changes $\left(\Delta_1 \pi_t^f < -0.42\right)$ the linear component is sufficient for describing inflation and the influence of both the inflation variables and the unemployment variable is smaller. There is no intercept in equation (5.2) which may be interpreted as the assumption of a constant NAIRU seems plausible when dealing with Swedish data. Hence, by taking the first difference the intercept disappears and it does not enter the nonlinear component of equation (5.2) either.

None of the theories of Section 2.2 considers asymmetries due to inflation expectations. However, the costly adjustment model states that higher inflation makes it more costly for the firms to keep their prices fixed resulting in an overall higher price-level. By assuming that the firms are forward looking higher inflation expectations should result in a price increase.

6. United States

Many of the empirical models of the Phillips curve for the United States are heavily influenced by the work of Gordon, see, e.g., Gordon (1970, 1975, 1977, 1983, 1997). His preferred Phillips curve specification is linear with backward looking inflation expectations. It incorporates a long lag structure and there are several dummy variables included in the specification. In Gordon (1996) he allows for a kinked functional form and finds no significant evidence of nonlinearity and therefore concludes that the Phillips curve is resolutely linear. Despite that, there has been many nonlinear Phillips curves estimated for the United States. Clark *et al.* (1996) find significant nonlinearity when estimating the Phillips curve for 1964(1)-1990(4) allowing for a kinked functional form. Debelle and Laxton (1997) find that a nonlinear model fits data better when estimated for 1971(2)-1995(2) but they do not perform any test of linearity before estimating the models. There are many more studies for the United States and a review of empirical nonlinear Phillips curve studies can be found in Yates (1998) or Dupasquier and Rickets (1998b).

In this section an expectations-augmented short-run Phillips curve will be specified for the United States and the hypotheses of linearity and parameter constancy will be tested against a nonlinear alternative of STR type.

6.1. Data

The time-series are quarterly seasonally unadjusted quarterly data covering the period 1978(1)-1997(4). The inflation measure used is annual changes in total CPI and the inflation expectations are given by the Michigan survey measure of expected inflation over the next year held at period t. Monetary policy in the United States differs from the one in Australia and Sweden in the sense that the Federal Reserve has not chosen to announce an explicit inflation target or apply one. The Federal Reserve formally opened its disinflationary policy in October 1979 and inflation has been reversed and stabilized after the peak in 1981, see Figure 6.1. The time-series move closely together during the whole sample and there are no systematic discrepancies. A striking feature of the inflation series is the change in the level of inflation before and after 1982 when the disinflation policies became efficient and the inflation stabilized.

The results of the augmented Dickey-Fuller tests can be found in Table 6.1. The null hypothesis of a unit root is forcefully rejected for all the variables and the Phillips curve will be specified in levels.

6.2. Empirical analysis

Estimating a linear Phillips curve for the United States for the period 1978(1)-1997(4) a twelfth-order lag structure is initially applied. Excluding the variables with poor explanatory power using the t-values as guidance the linear Phillips curve becomes,

$$\begin{aligned} \pi_t &= \underset{(0.0064)}{0.0062} + \underset{(0.10)}{1.00}\,\pi_{t-1} - \underset{(0.15)}{0.39}\,\pi_{t-2} + \underset{(0.15)}{0.53}\,\pi_{t-3} - \underset{(0.16)}{1.11}\,\pi_{t-4} \\ &\quad + \underset{(0.17)}{0.87}\,\pi_{t-5} - \underset{(0.16)}{0.27}\,\pi_{t-6} + \underset{(0.15)}{0.35}\,\pi_{t-7} - \underset{(0.14)}{0.63}\,\pi_{t-8} \\ &\quad + \underset{(0.078)}{0.35}\,\pi_{t-9} + \underset{(0.056)}{0.44}\,\pi_t^f - \underset{(0.056)}{0.61}\,u_{t-1} + \hat{e}_t \qquad (6.1) \\ T &= 79, R^2 = 0.99, \sigma_{lin} = 0.0033, AIC = -11.27, \\ LJB &= 4.64(0.098), A(1) = 1.23\,(0.27)\,, A(4) = 0.61\,(0.66)\,. \end{aligned}$$

The errors appear normally distributed, and the tests of no ARCH do not indicate misspecification. The null hypothesis of no error autocorrelation cannot be rejected, see Table 6.2.

The linear Phillips curve of the United States (6.1) is characterized by strong inertia which supports earlier studies. The coefficient of inflation expectations is quite large and there is a negative relationship between inflation and unemployment. The intercept is not significant (p-value = 0.33) but it is still included for theoretic arguments since it is assumed to capture the NAIRU, see Section 2.1.

The results of the linearity and parameter constancy test can be found in Table 6.3. There is no sign of either nonlinearities or time-varying parameters and, hence, the United States Phillips curve appear to be linear when tested against an STR model.

7. Conclusions

The results of the paper support the assumption of a nonlinear Phillips curve in Australia and in Sweden while the Phillips curve in the United States appear to be linear. The empirical models agree with the original Phillips curve setup despite the differences in the final specifications. This is not so strange considering that economic theory is not explicit about the dynamics and the only thing that can be judged is the sign and to some extent the magnitude of the coefficients. The nonlinear models, when estimated, variance dominates and encompasses its linear equivalents.

The nonlinear Phillips curve of Australia has very interesting features. According to this specification the relationship between inflation and unemployment is negative most of the time but turns positive for large increases in the unemployment rate. This suggests that the empirically observed "failure" of the Phillips curve might be the result of a nonlinear relationship and only mirrors a shift in the Phillips curve to a new level where the usual negative relationship is valid. The model also indicates that the NAIRU varies over time.

The Swedish Phillips curve appear to be nonlinear too. The rate of change of inflation expectations seems to be the key variable in this model since it rules the nonlinearity and it is more influential in the estimated model than lagged values of the inflation variable itself. The specification emphasize the importance of inflation expectations which will have important implications for monetary policy.

The Phillips curve of the United States is characterized by strong inertia in the inflation process. The intercept is not significantly included in the final specification which is surprising since the intercept can be viewed as a measure of the NAIRU. There is no evidence of nonlinearity or parameter constancy in the final model specification and the Phillips curve appear to be linear in the United States when the alternative is defined as an STR model.

According to the results of this paper the Phillips curve appear to be nonlinear in both Australia and Sweden. It would be interesting to find out if similar conclusions could be reached by other nonlinear model specifications. Another topic for further research would be to define the Phillips curve with output gaps instead of unemployment variables. This is however beyond the scope of this paper and will be left for future research.

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Appendix

A. Encompassing tests using the Minimal Nesting Model (MNM)

A.1. Comparing the linear and nonlinear equations

Consider

$$y_t = \alpha_1' x_{1t} + u_{1t} \tag{A.1}$$

and

$$y_t = \alpha_2' x_{2t} + \alpha_3' x_{3t} G_1 \left(s_t; \gamma_1, c_1\right) + u_{2t} \tag{A.2}$$

where the x_{1t}, x_{2t}, x_{3t} do not contain common elements. The MNM becomes

$$y_t = \alpha_1' x_{1t} + \alpha_2' x_{2t} + \alpha_3' x_{3t} G_1 \left(s_t; \gamma_1, c_1\right) + u_{3t}. \tag{A.3}$$

The null hypotheses are:

$$\begin{aligned} H_{01}\left[(A.1) \text{ encompasses } (A.2)\right] &: \quad \alpha_2 = 0 \text{ and } \gamma_1 = 0 \text{ in } (A.3) \\ H_{02}\left[(A.2) \text{ encompasses } (A.1)\right] &: \quad \alpha_1 = 0 \text{ in } (A.3) \end{aligned}$$

The model is not identified under the null hypothesis due to the nuisance parameters α_2 and γ_1. As for the linerity tests in Section 3.2 this can be circumvented by using a Taylor series approximation about $\gamma_1 = 0$.

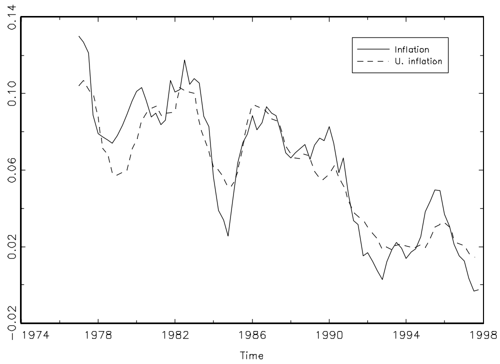

Figure 4.1: Inflation and underlying inflation for Australia, 1977(1)-1997(4).

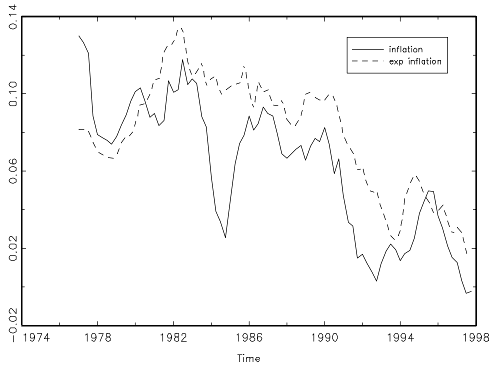

Figure 4.2: Inflation and inflation expectations for Australia, 1977(1)-1997(4).

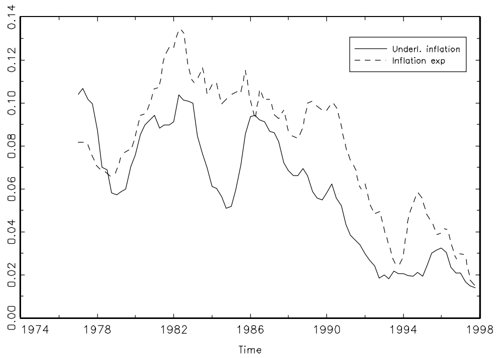

Figure 4.3: Underlying inflation and inflation expectationsfor Australia, 1977(1)-1997(4).

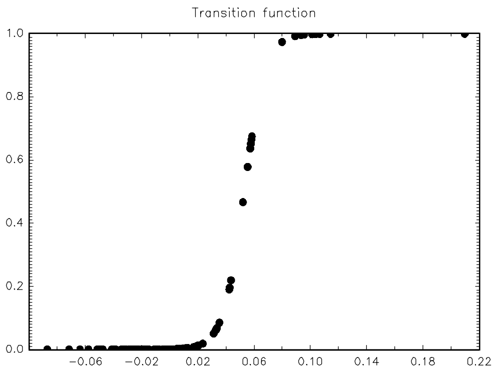

Figure 4.4: Transition function of (4.2) plotted against its argument, 1977(1)-1997(4).

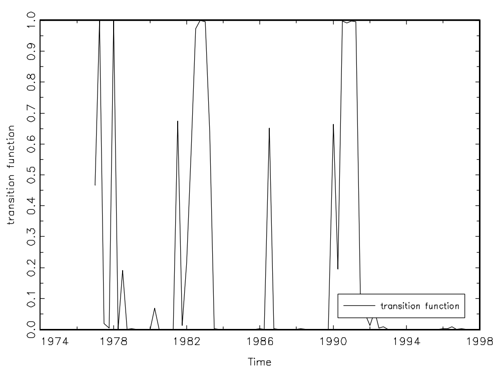

Figure 4.5: Transition function of (4.2) plotted against time, 1977(1)-1997(4).

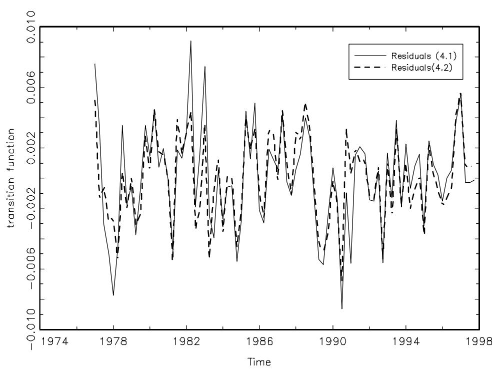

Figure 4.6: Residuals of (4.1) and (4.2), 1977(1)-1997(4).

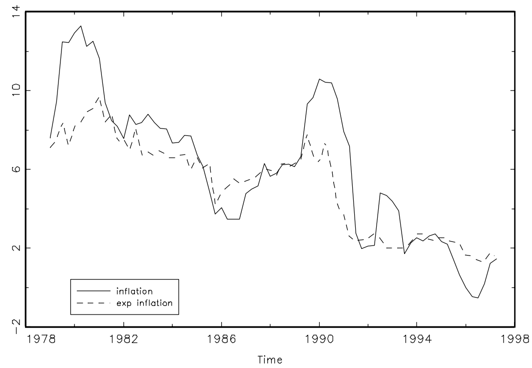

Figure 5.1: Inflation and inflation expectations for Sweden, 1979(2)-1997(4).

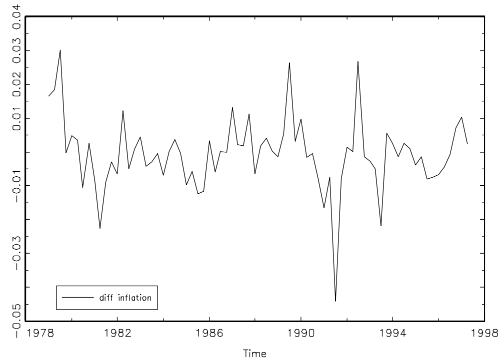

Figure 5.2: Rate of change of inflation for Sweden, 1979(2)-1997(4).

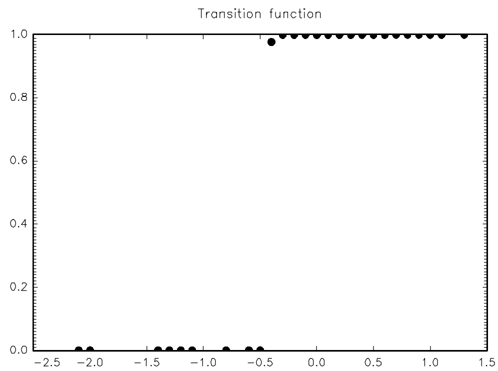

Figure 5.3: Transition function of (5.2) plotted against its argument, π_t^f, 1979(3)-1997(4).

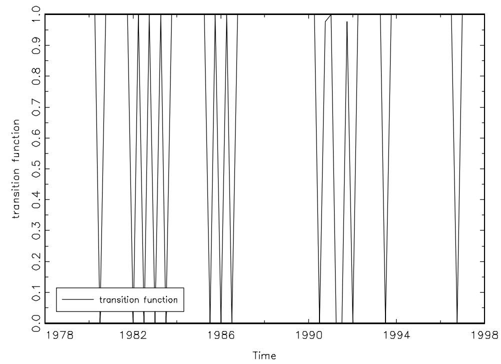

Figure 5.4: Transition function of (5.2) plotted against time, 1979(3)-1997(4).

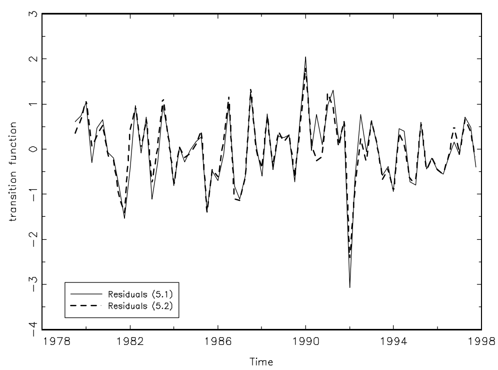

Figure 5.5: Residuals of (5.1) and (5.2), 1979(3)-1997(4).

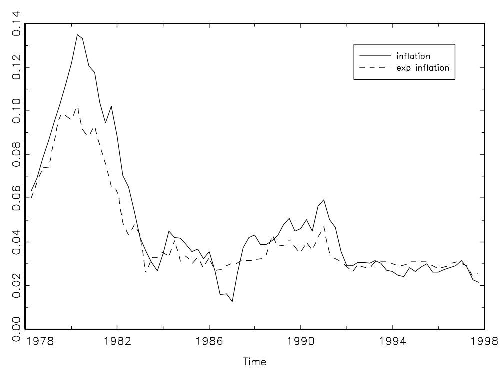

Figure 6.1: Inflation and inflation expectations for the U.S, 1978(1)-1997(4).

Table 4.1: Augmented Dickey-Fuller unit-root tests for Australian data 1977(1) - 1997(4)
H_0: The series has a unit root. Against the alternative of a stationary process.
*, **, *** indicates the 10%, 5%, 1% significance level of the rejection

	1977(1) - 1997(4)		
Variables	**Lag length**	**constant & trend**	**ADF**
Annual underlying inflation	10	c & t	-4.12^{***}
Expectation of annual inflation rate	0	---	-7.63^{***}
Annual inflation rate of import prices	11	c & t	-4.10^{***}
Log unemployment	8	---	-3.33^{***}

Table 4.2: *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the linear model (4.1) for the Australian inflation 1977(1)-1997(4).

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.047	0.13	0.24	0.062	0.063	0.080

Table 4.3. *p*-values of tests of linearity in the linear model (4.1) for a set of transition variables estimating the Australian inflation 1977(1)-1997(4).

Linearity test	Transition variables									
Null-hypothesis	π_{t-1}	π_{t-4}	π_{t-5}	$\Delta_1\pi_t^f$	$\Delta_1\pi_{t-1}^{imp}$	$\Delta_1\pi_{t-2}^{imp}$	u_{t-1}	u_{t-2}	$\Delta_1 u_t$	$\Delta_1\pi_{t-1}$
F	---	---	---	---	0.50	0.28	---	---	0.0014	0.77
F_{04}	---	---	---	---	0.68	0.59	---	---	0.74	0.59
F_{03}	---	---	---	---	0.89	0.71	---	---	0.000031	0.68
F_{02}	0.11	0.52	0.66	0.018	0.046	0.024	0.060	0.027	0.11	0.63

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

p-values of parameter constancy tests of the linear model (4.1) against STR type nonconstancy for the Australian inflation, 1977(1)-1997(4).

Tests of parameter constancy	Null hypothesis
Test	(1)
LM_3	0.000028
LM_2	0.0028
LM_1	0.030

Notes: The null hypotheses are:
(1): “All parameters are constant.”

The remaining parameters not under test are assumed constant in each case.

Table 4.4.Misspecification tests for the LSTR(1) model (4.2) for the Australian inflation 1977(1)-1997(4).

p-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.34	0.42	0.48	0.11	0.18	0.18

p-values of tests of no additive nonlinearity in (4.2) for a set of transition variables

Linearity test	Transition variable							
Null-hypothesis	π_{t-1}	π_{t-4}	π_{t-5}	$\Delta_1\pi_t^f$	$\Delta_1\pi_{t-1}^{imp}$	$\Delta_1\pi_{t-2}^{imp}$	u_{t-1}	u_{t-2}
F	---	---	---	---	0.41	0.38	---	---
F_{02}	0.30	0.61	0.66	0.031	0.16	0.057	0.080	0.18

Test of linear restriction
”Linear” coefficient of $\Delta_1\pi_{t-2}^{imp}$ = 0, *p*-value: 0.67

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

p-values of parameter constancy tests of the LSTR(1) model (4.2) against STR type nonconstancy.

Tests of parameter constancy		Null hypotheses	
Test	(1)	(2)	(3)
LM_3	0.0029	0.0090	0.84
LM_2	0.0029	0.0095	0.75
LM_1	0.030	0.16	0.76

Notes: The null hypotheses are:
(1): “All parameters of the linear part are constant.”
(2): “All parameters of the nonlinear part except the intercept are constant.”
(3): “The intercept in the nonlinear part is constant.”

The remaining parameters not under test are assumed constant in each case.

Table 5.1: Augmented Dickey-Fuller unit-root tests for Swedish data 1979(1) - 1997(4)
H_0: The series has a unit root. Against the alternative of a stationary process.
*, **, *** indicates the 10%, 5%, 1% significance level of the rejection

	1979(1) - 1997(4)		
Variables	**Lag length**	**constant & trend**	**ADF**
Annual inflation rate	4	c & t	–2.67
First difference	3	---	–6.43***
Expectation of annual inflation rate	4	c & t	–3.15
First difference	4	---	–3.60***
Annual inflation rate of German prices	7	c & t	–2.79
First difference	3	---	–4.66***
Log unemployment (u_t)	7	---	–0.22
First difference	6	---	–2.56**

Table 5.2. *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the linear model (5.1) for the Swedish inflation 1979(3)-1997(4).

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.42	0.18	0.34	0.33	0.21	0.20

Table 5.3. *p*-values of tests of no additive nonlinearity in the linear model (5.1) for a set of transition variables estimating the Swedish inflation 1979(3)-1997(4).

Linearity test	Transition variables				
Null-hypothesis	$\Delta_1\pi_{t-2}$	$\Delta_1\pi_{t-4}$	$\Delta_1\pi_t^f$	$\Delta_1\pi_{t-1}^f$	$\Delta_1 u_{t-3}$
F	0.0079	0.82	0.40	0.0062	0.24
F_{04}	0.029	0.71	0.55	0.13	0.15
F_{03}	0.011	0.49	0.33	0.015	0.11
F_{02}	0.66	0.77	0.29	0.090	0.99

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

p-values of parameter constancy tests of the linear model (5.1) against STR type nonconstancy for the Swedish inflation, 1979(3)-1997(4).

Tests of parameter constancy	Null hypothesis
Test	(1)
LM_3	0.71
LM_2	0.74
LM_1	0.97

Notes: The null hypotheses are:
(1): ”All parameters are constant.”

The remaining parameters not under test are assumed constant in each case

Table 5.4. Misspecification tests for the LSTR(2) model (5.2) for the Swedish inflation 1979(3)-1997(4).

p-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error.

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.31	0.082	0.17	0.17	0.11	0.12

p-values of tests of no additive nonlinearity in (5.2) for a set of transition variables

	Transition variables				
Null-hypothesis	$\Delta_1\pi_{t-2}$	$\Delta_1\pi_{t-4}$	$\Delta_1\pi_t^f$	$\Delta_1\pi_{t-1}^f$	$\Delta_1 u_{t-3}$
F	0.0053	0.37	0.65	0.057	0.13
F_{02}	0.037	0.32	0.16	0.34	0.55

Test of linear restriction
"Linear" coefficient of $\Delta_1\pi_t^f = 0$, *p*-value: 0.29

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

p-values of parameter constancy tests of (5.2) against STR type nonconstancy

Tests of parameter constancy		Null hypotheses	
Test	(1)	(2)	(3)
LM_3	0.35	0.39	0.94
LM_2	0.90	0.72	0.87
LM_1	0.92	0.87	0.72

Notes: The null hypotheses are:
(1): "All parameters are constant."
(2): "All parameters of the linear part are constant."
(3): "All parameters of the nonlinear part are constant."

The remaining parameters not under test are assumed constant in each case.

Table 6.1: Augmented Dickey-Fuller unit-root tests for US data 1978(1) - 1997(4)
H_0: The series has a unit root. Against the alternative of a stationary process.
*, **, *** indicates the 10%, 5%, 1% significance level of the rejection

	1978(1) - 1997(4)		
Variables	**Lag length**	**constant & trend**	**ADF**
Annual inflation rate	7	---	-3.78^{***}
Expectation of annual inflation rate	5	c	-3.24^{***}
Log unemployment	4	---	-3.84^{***}

Table 6.2. *p*-values of the LM test of no error autocorrelation against an AR(q) and MA(q) error process in the linear model (6.1) for the U.S. inflation 1978(1)-1997(3).

Test	Maximum lag q					
	1	2	3	4	5	6
No error autocorrelation	0.53	0.70	0.80	0.85	0.93	0.97

Table 6.3. *p*-values of tests of no additive nonlinearity in the linear model (6.1) for a set of transition variables estimating the U.S. inflation 1978(1)-1997(3).

Linearity test Null-hypothesis	Transition variables π_{t-1}	π_{t-2}	π_{t-3}	π_{t-4}	π_{t-5}	π_{t-6}	π_{t-7}	π_{t-8}	π_{t-9}	π_t^f	u_{t-1}
F	---	---	---	---	---	---	---	---	---	---	---
F_{04}	---	---	---	---	---	---	---	---	---	---	---
F_{03}	---	---	---	---	---	---	---	---	---	---	---
F_{02}	0.27	0.14	0.077	0.091	0.85	0.069	0.074	0.058	0.054	0.053	---

Notes: Linearity tests: F is the F-test based on a third-order Taylor expansion of the transition function. F_2 is based on the first-order Taylor expansion and is thus a test against LSTR(1).

p-values of parameter constancy tests of the linear model (6.1) against STR type nonconstancy for the U.S. inflation, 1978(1)-1997(3).

Tests of parameter constancy	Null hypothesis			
Test	(1)	(2)	(3)	(4)
LM_3	0.059	0.78	0.055	0.061
LM_2	0.034	0.48	0.027	0.028
LM_1	0.78	0.55	0.72	0.53

Notes: The null hypotheses are:
(1): ”The coefficient of inflation expectations is constant.”
(2): ”All parameters of inflation variables are constant.”
(3): ”The coefficient of the unemployment variable is constant.”
(4): “The intercept is constant.”

The remaining parameters not under test are assumed constant in each case