Efficient estimation of price adjustment coefficients

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Abstract

The price adjustment coefficient model of Amihud and Mendelson (1987) is shown to be suitable for estimation by the Kalman filter. A techique that, under some commonly used conditions, is asymptotically efficient. By Monte Carlo simulations it is shown that both bias and mean squared error are much smaler compared to the estimator proposed by Damodaran and Lim (1991) and Damodaran (1993). A test for the adeqacy of the model is also proposed. Using data from four minor, the nordic countries except Iceland, and one major, US, stock markets the results are that the markets under-react to new information, but for most of the nordic countries, the model is not adequate.
Keywords: Estimation; efficiency; price adjustment
JEL: C22; C52; G10

1 Introduction

The modeling of returns, of a variety of assets, is a growing area of economics. Most models rely on efficient market, i.e. a market that fully reflects all information, there is no transaction costs and so on. In the real world, markets are not fully efficient and the models are adjusted accordingly. In this paper we treat the price adjustment model of Amihud and Mendelson (1987), especially we are interested in estimation and testing the validity of the model. Estimation is proposed to be carried out by Kalman filter techniques which have advantages compared to previous estimators (proposed by Damodaran and Lim, 1991, and Damodaran, 1993), i.e. i) efficient estimation, ii) less assumptions, iii) valid and easy inference follows, iiii) an estimate of the underlying price is gained. The

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estimators is compared by a Monte Carlo simulation, showing that the Kalman filter is much more efficient. Further we develope a test-statistic to used to test if the price adjustment model is a valid approximation of the data generating process, i.e. the, unknown, process which generates the data and a model should, as far as possible, mimic.

The proposed estimator and test statistic are exemplified on four minor, the Nordic except Iceland, and one major, US, monthly stock indexes. The results are that the price adjustment coefficient are somewhat less than one for all countries except Denmark which is much lower, although it is only Norway and Sweden that are not significantly different from one. A more worrying result is that for Finland, Norway and Sweden the test result indicates that the price adjustment model is not a valid model.

In what follows, Section 2 discuss the price adjustment model and Section 3 estimation. Tests of the validity of the model is derived in Section 4 while the empirical example is in Section 5. A conclusion ends the paper

2 Price adjustment coefficients

Let the true price (value) and the observed price be denoted by V_t and P_t respectively. The observed returns is $R_t = P_t - P_{t-1}$. It is often assumed that the true price follows an random walk, $V_t = V_{t-1} + v_t$. Then the price adjustment model of Amihud and Mendelson (1987) is

$$\begin{aligned} R_t &= P_t - P_{t-1} = g_1 \left(V_t - P_{t-1} \right) + u_t && (1) \\ V_t &= V_{t-1} + v_t && (2) \end{aligned}$$

By the constraint $0 < g_1 < 2$, stationarity of R_t is imposed, see Amihud and Mendelson (1987) or Liu et al (1992). A full adjustment to news is represented by $g_1 = 1$, although the true price is observed with error. If also $\sigma_u^2 = 0$, the true and the observed price coincide, a model often used for testing the random walk hypothesis. The case $g_1 = 0$ gives no adjustment at all. The region $0 < g_1 < 1$ represents under-reaction and $1 < g_1 < 2$ over-reaction.

If n periods returns is considered, the model is

$$\begin{aligned} R_{t,n} &= P_t - P_{t-n} = g_n \left(V_t - P_{t-n} \right) + u_{t,n} && (3) \\ V_t &= V_{t-n} + v_{t,n} && (4) \end{aligned}$$

where the relation between g_1 and g_n is, see Theobald and Yallup (1998),

$$g_n = n g_1 \left[1 + (n-1) g_1 \right]^{-1} \qquad (5)$$

From this it is easily seen that $g_n \to 1$ when $n \to \infty$, i.e. eventually there is a full adjustment to the underlying price. The components of the variance of the returns, $V(R_t)$, may be given economic interpretations, see Amihud and Mendelson (1987), Damodaran and Lim (1991) and Damodaran (1993).

3 Estimation

A simple estimator of the price adjustment model of Amihud and Mendelson (1987) were presented by Damodaran and Lim (1991) and Damodaran (1993). Later Brisley and Theobald (1996) corrected a minor error. It was assumed that i) the true price follows an random walk, ii) the noise terms of the underlying price process and the observed price process is independent, iii) $g_n \to 1$ as $n \to \infty$. As noted above, the third one is not rely an assumption, but merely an consequence of the model. By taking a closer look at the variance of the n and the k $(> n)$ period returns and assuming that $g_k = 1$, g_n may be solved for,

$$g_n = \frac{2V(R_{t,n})/n + 2Cov(R_{t,k}, R_{t,k-1})/n}{V(R_{t,n})/n + V(R_{t,k})/k + 2Cov(R_{t,k}, R_{t,k-1})/k} \tag{6}$$

Common choices of k is 5, 10 and 20. Obviously, the need for a choice of k is a drawback of this estimator. The variance and the covariance are estimated by the non-overlapping estimators

$$V(R_{t,n}) = \frac{1}{(T/n-1)} \sum_{t=1}^{T/n} \left(R_{t,n} - \bar{R}_n\right)^2 \tag{7}$$

$$Cov(R_{t,k}, R_{t,k-1}) = \frac{1}{(t/k-2)} \sum_{t=2}^{T/k} \left(R_{t,k} - \bar{R}_k\right)\left(R_{t,k-1} - \bar{R}_k\right) \tag{8}$$

where $\bar{R}_n$ and $\bar{R}_k$ are the means of n and k periods returns respectively.

Recognizing that (2) and (4) is a state space form, estimation is easily done with Kalman filter, see Harvey (1989). One of the assumptions of estimation with Kalman filters is that the disturbances are multivariate normal distributed. If this is true then the Kalman filter is asymptotic efficient, it achieves the Cramér-Rao lower bound when the sample size grows to infinity, i.e. no other estimator could have lower variance, see e.g. Harvey (1989). Even when the normality conditions fails, some optimality properties still holds, see White (1982). Other advantages are that an estimate of V_t evolves and that u_t and v_t may be dependent.

Table 1 *in here*

In Table 1, a simulation study is displayed. Two sample sizes ($T = 500$, 1000) are used. When generating data, $T + 100$ observations are generated and the first 100 is discarded to gain stationarity of the data. To investigate if the performance of the estimators are sensitive to g_1, a grid of values is used, $g_1 = (0.50, 0.75, \ldots, 1.50)$. The number of replicates is 1000 and the evaluation criteria are bias and mean squared error (MSE). The program used is Gauss. The results are that, for the Kalman filter, the bias and MSE decreases significantly with g_1 and sample size. For all k considered are bias and MSE much larger and increase, in absolute value, with k. E.g. consider $g_1 = 1$ and

$T = 500$, the bias of the Kalman filter is 0.03 while for the other estimators it is 0.14, 1.14 and 12.5 for $k = 5, 10, 20$, indeed a large difference.

One reason for the high MSE of the Damodaran estimator is that it is a function of second order moments, hence the variance is a function of fourth order moments. This high variability also applies other estimators of this type, e.g. the variance ratio.

Table 2 in here

The size and power of the $t-$test of the hypothesis $g_1 = 1$ have also been investigated, see Table 2. The Monte Carlo setup is as above. The test is oversized but only marginally for the larger sample size. The power increase with sample size but only slightly and the power is much better for the higher values of g_1 than for the lower.

It should be noted that Liu et al (1992) use an ARMA(1,1) model to estimate g_1. Using an ARMA(1,1) would give results close to Kalman filter, depending on the choice of estimation method. The GMM estimator derived by Säfvenblad (1997) also suffers from a choice of a truncation parameter k. He reports results for the GMM estimator that lie between the Kalman filter and the by Damodaran and Lim (1991) and Damodaran (1993) proposed one.

4 Testing the adequacy of the model

If the Amihud and Mendelson's (1987) model is correct then the relation in equation (5) would hold. Testing this with the result that the null is rejected must be interpreted in the sense that the data generating process for the returns is not well approximated by this model. It is possible to derive a test using standard asymptotic theory, see e.g. Casella and Berger (1990) pp. 328-331. Let the function of interest be

$$f(\hat{g}_1, \hat{g}_n) = \hat{g}_n - n\hat{g}_1 \left[1 + (n-1)\hat{g}_1\right]^{-1} \tag{9}$$

A first order Taylor expansion yields

$$\begin{aligned} f(\hat{g}_1, \hat{g}_n) &= f(g_1, g_n) + f_1'(g_1, g_n)(\hat{g}_1 - g_1) \\ &\quad + f_n'(g_1, g_n)(\hat{g}_n - g_n) + \text{Remainder} \end{aligned} \tag{10}$$

where $f'(g_1, g_n)$ is the derative of $f(g_1, g_n)$. Taking expectations, while ignoring the remainder, gives

$$\begin{aligned} Ef(\hat{g}_1, \hat{g}_n) &\approx f(g_1, g_n) + Ef_1'(g_1, g_n)(\hat{g}_1 - g_1) \\ &\quad + Ef_n'(g_1, g_n)(\hat{g}_n - g_n) && (11) \\ &= f(g_1, g_n) && (12) \end{aligned}$$

where E denotes the expectations operator. The variance may be approximated by

$$\begin{aligned} V\left(f\left(\hat{g}_1, \hat{g}_n\right)\right) &\approx E\left(f\left(\hat{g}_1, \hat{g}_n\right) - f\left(g_1, g_n\right)\right)^2 \\ &\approx \left(f_1'\left(g_1, g_n\right)\left(\hat{g}_1 - g_1\right) + f_n'\left(g_1, g_n\right)\left(\hat{g}_n - g_n\right)\right)^2 \\ &= f_1'\left(g_1, g_n\right)^2 V\left(\hat{g}_1\right) + f_n'\left(g_1, g_n\right)^2 V\left(\hat{g}_n\right) \\ &\quad +2 f_1'\left(g_1, g_n\right) f_n'\left(g_1, g_n\right) Cov\left(\hat{g}_1, \hat{g}_n\right) \end{aligned} \tag{13}$$

Substituting in the deratives yields

$$\begin{aligned} V\left(f\left(\hat{g}_1, \hat{g}_n\right)\right) &= \left[\frac{n}{\left(1 + g_1 n - g_1\right)^2}\right]^2 V\left(\hat{g}_1\right) \\ &\quad +V\left(\hat{g}_n\right) - 2\frac{n}{\left(1 + g_1 n - g_1\right)^2} Cov\left(\hat{g}_1, \hat{g}_n\right) \end{aligned} \tag{14}$$

The test statistic is the usual

$$z_n = \frac{f\left(\hat{g}_1, \hat{g}_n\right)}{\sqrt{V\left(f\left(\hat{g}_1, \hat{g}_n\right)\right)}} \tag{15}$$

which is asymptotically standard Gaussian distributed under the null.

Asymptotic estimates of $V\left(\hat{g}_1\right)$ and $V\left(\hat{g}_n\right)$ may be gained by estimating equations (2) and (4). It remains to estimate $Cov\left(\hat{g}_1, \hat{g}_n\right)$. By using a bootstrap approach it is possible to estimate $V\left(\hat{g}_1\right), V\left(\hat{g}_n\right)$ and $Cov\left(\hat{g}_1, \hat{g}_n\right)$, see Efron and Tibshirani (1993) for a discussion of bootstrap in general. The bootstrapping scheme is

1. Estimate equation (2) on the data of length T.
2. With the parameter estimates from 1, generate B number of time series of length T.
3. For each of the B time series estimate both (2) and (4) and save $\hat{g}_1$ and $\hat{g}_n$.
4. Using the B $\hat{g}_1$ and $\hat{g}_n$, estimate $V\left(\hat{g}_1\right), V\left(\hat{g}_n\right)$ and $Cov\left(\hat{g}_1, \hat{g}_n\right)$.
5. Estimate the test statistic, $\hat{z}_n$, using (15).
6. If the absolute value of $\hat{z}_n$ is greater than 1.96 (for a test on the 5% level), reject the null.

A choice of n is needed. A to large number of n reduces the sample size to much with increased uncertainty of $\hat{g}_n$ as a consequence, so $n = 2$ is chosen.

5 Empirical example

Data are monthly stock index for Denmark (Cop. SE), Finland (HEX), Norway (Norway Share Prices/Total index), Sweden (AFGX) and US (S&P500) ranging from January 1957 to September 1996, i.e. there are 477 observations on each index. The data source is the database EcoWin. The data is transformed by first taking the logarithm and then the first difference. As the price adjustment model does not allow for drift, the returns are mean adjusted. A new price index series is derived by the cumulative sum of the mean adjusted returns.

Table 3 *in here*

In Table (3) the results of the empirical analysis are displayed. Bootstrapped values of $V(\hat{g}_1)$, $V(\hat{g}_2)$ and $Cov(\hat{g}_1, \hat{g}_2)$ are gained by the approach described above using 500 bootstrap replicates. The bootstrapped values may be seen as small sample estimates. The asymptotic standard errors do not, for this sample size and most countries, corresponds well to the small sample. This implies that for valid inference one should not use the asymptotic standard errors even for such large samples as in this case. Note that the covariances are relatively small. For every country, except Norway, the hypothesis $g_1 = 1$ is rejected. This indicate that the market do not immediately adjust to new information. A more striking feature is that although the model predicts that $g_2 > g_1$, this is only true for Denmark. With this result in mind it is not surprising that the z_2 test indicates that the price adjustment model is not valid for three out of the five series investigated, hence, other models should be search for.

6 Conclusion

The price adjustment model of Amihud and Mendelson (1987) has previously been estimated by inefficient methods. In this paper an efficient estimator is proposed by using the Kalman filter. A monte Carlo simulation indicates that the Kalman filter is much better, in terms of bias and MSE, than previous methods. Further, a test for the validity of the model is proposed and its asymptotic distribution is derived. Using data from four minor and one major stock market the results are that the market either is not fully efficient or that the price adjustment model is not a valid model to approximate the data generating process.

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Table 1: Bias and mean squared error of g_1

		g^{KF}		$g^D_{k=5}$		$g^D_{k=10}$		$g^D_{k=20}$	
T	g_1	$Bias$	MSE	$Bias$	MSE	$Bias$	MSE	$Bias$	MSE
500	0.50	-0.0319	0.125	0.420	0.379	0.131	2.18	-1.40	32.7
	0.75	-0.0491	0.0884	0.127	0.196	-0.0962	1.64	-1.24	21.0
	1.00	-0.0190	0.0288	-0.0536	0.138	-0.215	1.14	-1.03	12.5
	1.25	-0.00988	0.0101	-0.190	0.127	-0.300	0.757	-0.851	6.95
	1.50	-0.00709	0.00413	-0.329	0.166	-0.373	0.491	-0.711	3.44
1000	0.50	-0.0482	0.121	0.456	0.297	0.280	1.07	-0.379	10.1
	0.75	-0.0226	0.0556	0.158	0.105	0.0187	0.761	-0.484	7.17
	1.00	-0.00893	0.0150	-0.0285	0.0613	-0.131	0.543	-0.496	4.68
	1.25	-0.00475	0.00544	-0.172	0.0710	-0.240	0.383	-0.491	2.78
	1.50	-0.00285	0.00214	-0.318	0.128	-0.333	0.282	-0.490	1.45

Table 2: Size and power fo the test $g_1 = 1$

			g_1		
T	0.50	0.75	1.00	1.25	1.50
500	0.487	0.347	0.0925	0.646	0.991
1000	0.499	0.355	0.0725	0.901	0.998

Table 3: Empirical results using monthlystock price indexes for the period 5701-9609. index

	$\hat{g}_1$	$\hat{g}_2$	$Cov(\hat{g}_1, \hat{g}_2)$	z_2
Denmark	0.154*	0.324*	0.0146	0.277
	(0.0556)	(0.115)		
	[0.152]	[0.185]		
Finland	0.724*	0.207*	0.00170	−3.38*
	(0.0446)	(0.0988)		
	[0.0944]	[0.182]		
Norway	0.856	0.234*	0.00336	−2.22*
	(0.0460)	(0.252)		
	[0.193]	[0.295]		
Sweden	0.838	0.00969*	−0.000809	−3.23*
	(0.0455)	(0.0116)		
	[0.156]	[0.261]		
US	0.7215*	0.655*	0.003232	−1.03
	(0.0442)	(0.262)		
	[0.0990]	[0.178]		

Standard errors below, () denotes asymptotic ones and [] bootstrapped. A * denotes significant result, at the 5% level, when using bootstrapped values.