On Consumption Bunching under Campbell-Cochrane Habit Formation*

Lars Ljungqvist
Stockholm School of Economics and CEPR

and

Harald Uhlig
Tilburg University and CEPR

October 1999

*Ljungqvist: Stockholm School of Economics, Box 6501, SE-113 83 Stockholm, Sweden (e-mail lars.ljungqvist@hhs.se). Uhlig: CentER for Economic Research, Tilburg University, Postbus 90153, 5000 LE Tilburg, The Netherlands (e-mail uhlig@kub.nl). Our research on the policy implications of consumption externalities have benefited from comments by seminar participants at the Federal Reserve Banks of Chicago and Minneapolis, IIES at Stockholm University, Stanford University and London School of Economics. Ljungqvist's research was supported by a grant from the Bank of Sweden Tercentenary Foundation.

Abstract

Campbell and Cochrane (1999) propose a preference specification that can explain a wide variety of asset pricing puzzles such as the high equity premium. They augment the basic power utility function with a time-varying subsistence level, or "habit", which is in the spirit of "catching up with the Joneses" but with a novel nonlinear mapping of consumption into habit. This paper demonstrates a surprising implication of the Campbell-Cochrane preference specification: consumption bunching is desirable.

JEL: E21

Keywords: catching-up-with-the-Joneses preferences, habit, consumption bunching

1 Introduction

Campbell and Cochrane (1999) formulate a model that successfully explains a wide variety of asset pricing puzzles, including the high equity premium, the procyclical variation of stock prices, and the countercyclical variation of stock market volatility. These remarkable results are achieved by augmenting the standard power utility function with a time-varying subsistence level, or "habit", that adapts slowly and nonlinearly to the history of consumption. Given the achieved breakthrough in matching key asset pricing facts, it is all the more important to fully understand the implications of these choices. Here we will demonstrate another surprising implication of the Campbell-Cochrane preference specification: consumption bunching is desirable.

2 The Campbell-Cochrane Preferences

The utility function of the representative agent is

$$E \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \tag{1}$$

where δ is the subjective time discount factor and X_t is the level of habit. The habit is external to the individual because it is determined by the history of average consumption in the economy, but Campbell and Cochrane argue that most of their results are maintained under the alternative assumption of an internal habit that is determined by the history of individual consumption. Our findings on the desirability of consumption bunching apply to both specifications, so here we do not make any distinction between individual and per-capita variables.

A conventional specification for the evolution of X_t might be

$$X_t = \phi X_{t-1} + (1-\phi)(1-\bar{S})C_{t-1}, \tag{2}$$

for some constants $0 \leq \phi < 1$ and $0 < \bar{S} < 1$.

Campbell and Cochrane proceed differently. Instead of writing down a law of motion for the habit level, they postulate a process for the *surplus consumption ratio* $S_t \equiv (C_t - X_t)/C_t$. Using lowercase letters to indicate logs, they assume that s_t evolves as a heteroscedastic AR(1) process,[1]

$$s_t = (1-\phi)\bar{s} + \phi s_{t-1} + \lambda(s_{t-1})\left(c_t - c_{t-1}\right) \,, \tag{3}$$

where ϕ and $\bar{s}$ are parameters, and the function $\lambda(s)$ is given by

$$\lambda(s) = \begin{cases} \bar{S}^{-1}\sqrt{1-2(s-\bar{s})} - 1, & s \leq s_{max} \\ 0, & s \geq s_{max} \end{cases} \tag{4}$$

with $s_{max} = \bar{s} + \left(1 - \bar{S}^2\right)/2$. The parameter $\bar{s}$ is the logarithm of the steady-state surplus consumption ratio $\bar{S}$.

At any point in time, equation (3) can be used to back out the implied habit level X_t. To a first-order approximation around the steady state, the implied law for X_t can be shown to be given by (2). One may thus be led to believe that the differences to the linear formulation (2) may not matter much. We shall soon see that they do, however, and that the nonlinearities are crucial for whether or not consumption bunching is desirable.

3 Consumption bunching

To examine the effects of consumption bunching, we assume that the economy has access to the linear savings technology

$$K_{t+1} = R\left(K_t + Z - C_t\right), \tag{5}$$

where K_t and Z denote the capital stock and a constant exogenous endowment, and where we assume the return to equal the inverse of the discount

[1] Here we abstract from growth, i.e., we set Campbell and Cochrane's (1999) parameter g equal to zero.

factor, $R = \delta^{-1}$.[2]

Suppose that the economy is initially in a steady state where the representative agent has some capital stock $\bar{K}$. That is, let $\bar{C} = Z + (1 - R^{-1})\bar{K}$ and let the agent "start off" with $C_{-1} = \bar{C}$ and $S_{-1} = \bar{S}$. We now compare two feasible consumption paths. The first is the benchmark steady state, in which consumption remains unchanged at $\bar{C}$ forever. In the second, consumption is raised only in the initial period to $(1 + \Delta)\bar{C} \in (\bar{C}, Z + \bar{K}]$, and thereafter lowered permanently to the highest sustainable consumption level, which can be calculated to be $\hat{C} = (1 - (1 - R^{-1})R\Delta)\bar{C}$. Would agents be better off with this one-time deviation from a steady state path?

The answer to the question is trivially 'no' if the agents have standard concave preferences without habit formation. In this stationary environment with the return equal to the inverse of the subjective discount factor, agents prefer to remain on a smooth consumption path. The answer can also be shown to be 'no' under habit formation with the conventional formulation (2): in essence, the consumption increase would be valued at a diminishing marginal utility (given past habit) while the lower consumption thereafter would hurt extra much because of the higher habit arising from the one-period consumption hike.

In contrast, the answer to our question is not obvious for the Campbell-Cochrane preferences with a law of motion for the surplus consumption ratio, so we numerically calculate the welfare effects in Figure 1.[3] The x-axis shows the one-period percentage increase in consumption, $100 \cdot \Delta$, and the y-axis

[2] Campbell and Cochrane study an endowment economy with an exogenous stochastic process for consumption. Since their preferences imply a constant risk-free interest rate R, the authors suggest that the model can equally well be closed with the linear technology (5) and an exogenous stochastic process for Z. Our analysis demonstrates, that this invariance of the asset pricing implications to the introduction of a linear technology only applies to the assumption of an external habit. It does not extend to internal-habit formation, since agents would then choose to bunch consumption.

[3] All figures use Campbell and Cochrane's (1999, Table 1) parameter values; $\gamma = 2$, $\bar{S} = 0.057$, and the annualized values for $\delta = 0.89$ and $\phi = 0.87$.

depicts the welfare gain measured as a permanent percentage increase in the consumption stream $\bar{C}$ that would make individuals as well off as under the proposed temporary increase in consumption (with the subsequent permanent reduction in the future consumption level). It is clear from the figure that there are consumption perturbations that dominate the constant consumption path, and markedly so.

The welfare effects in Figure 1 are driven by the dynamics of the surplus consumption ratio in (3). Starting from a steady state with $s_{-1} = \bar{s}$, the consumption hike raises the logarithm of the surplus consumption ratio by the logdifference in consumption multiplied by the steady-state value of the λ-function. It is instructive to back out the implied effect on the habit level X_0 of such an increase in contemporaneous C_0. With a conventional habit formation as in equation (2), this cannot have any effect, since it is C_{-1} and not C_0 which is on the right hand side of that equation. As noted above, the same holds also to a first-order approximation for the Campbell-Cochrane specification. But the higher order terms in that approximation will turn out to be important, when we now calculate the exact response in X_0 to a contemporaneous deviation in consumption from $\bar{C}$. Figure 2 provides the result: it shows, perhaps surprisingly, that a consumption hike lowers the contemporaneous habit level. Indeed, for any positive (or negative) deviation in consumption away from the steady state, the contemporaneous habit level falls.[4]

In the period after the consumption hike, the logarithm of the surplus consumption ratio s_1 falls because it tends to revert to $\bar{s}$ and in response to the consumption decline. However, note that the negative effect of the con-

[4]To show this formally, note that equation (3) at the steady state, $s_0 = \bar{s} + \lambda(\bar{s})(c_0 - \bar{c})$, can be rewritten as

$$\log\left(\frac{C_0 - X_0}{\bar{C} - \bar{X}}\right) = \frac{\bar{C}}{\bar{C} - \bar{X}}(c_0 - \bar{c}).$$

Now consider a deviation in consumption away from the steady state, $C_0 = (1 + \Delta)\bar{C}$, and solve for the corresponding habit level $X_0 = X(\Delta)$ as implied by this expression. Then, compute the derivative of $X(\Delta)$, sumption decline is moderated by the fact that the λ-function is decreasing in the surplus consumption ratio of last period. In fact, the fall in consumption has no effect at all on s_1 if s_0 is greater than or equal to s_{max} in equation (4) when the λ-function becomes zero. But except for such high values of last period's surplus consumption ratio, we cannot say whether or not the surplus consumption ratio is above or below it steady-state value. Figure 2 backs out the implied habit level X_1. We see that the habit level in the period after the consumption hike is hump-shaped, increasing for small consumption deviations but decreasing for larger ones.

The described dynamics of the surplus consumption ratio and the implied habit levels explain why sufficiently large consumption deviations on the scale of Figure 1 are associated with welfare gains. Not only does the contemporaneous habit level X_0 fall in response to an increase in C_0, but next-period's surplus consumption ratio is also above its steady-state value. Welfare is then positively affected in all future periods by a slowly decaying surplus consumption ratio. In other words, the constant consumption level $\hat{C}$ yields higher utility than in the ultimate new steady state because the implied habit level is only gradually approaching its steady-state value from below. This outcome is illustrated in Figure 3 where we depict the evolution of the surplus consumption ratio and the habit level after a consumption hike of 14% in period 0 (with the subsequent lower sustainable level of consumption).

The welfare argument gets more complicated locally around a zero consumption perturbation where next-period habit level X_1 evolves as in equation (2): the initial consumption hike increases the habit level next period

---

$$X'(\Delta) = \bar{C}\left[1 - \frac{1}{1+\Delta}\exp\left(\frac{\bar{C}}{\bar{C}-\bar{X}}\log(1+\Delta)\right)\right] \begin{cases} <0, & \text{if } \Delta > 0; \\ =0, & \text{if } \Delta = 0; \\ >0, & \text{if } \Delta < 0; \end{cases}$$

where the inequalities exploit that $\bar{C}/(\bar{C}-\bar{X}) > 1$. Thus, given an initial steady state, we can conclude that $X_0 < \bar{X}$ for any $C_0 \neq \bar{C}$.

and decreases the surplus ratio at that date, as Figure 2 already shows for the habit level. Surprisingly, welfare increases in Figure 2 regardless of how small the initial consumption deviation is, and the argument must be different from the "large-deviation" argument given above. We provide a more detailed calculation in the appendix to analyze the "small-deviation" case as well. Crudely, the first-order conditions for utility maximization are satisfied at a consumption deviation of zero, so the consumption experiment will only have local second-order welfare effects. To calculate these second-order welfare effects, the first-order approximation of (2) is not sufficient: a second-order approximation is needed instead. When calculating the second-order approximation, it turns out that welfare is convex rather than concave along the dimension of the experiment performed here. The convexity is due to the strongly negative slope of the λ-function.

4 Conclusion

We have shown that consumption bunching is desirable under Campbell and Cochrane's (1999) habit formation. It is therefore clear that their suggestion that the endowment economy can alternatively be closed as a production economy with a linear technology cannot work for an internal-habit specification, because the agents would then seize on the opportunity to increase welfare by engaging in consumption bunching. In the case of an external habit, no individual agent would act in such a way but a benevolent government would like to intervene and induce consumption cycles in an otherwise stationary environment.

The drastically different implication that consumption bunching is desirable under Campbell and Cochrane's habit formation but not with a standard formulation of catching-up-with-the-Joneses preferences, is due to the postulated law of motion for the surplus consumption ratio which implies that habit can move negatively with consumption.[5] Thus, using the word 'habit' here might be a misnomer because, as pointed out by Campbell and Cochrane, "the notion of habit would be strained if we allowed habit to move in the opposite direction from consumption".

Appendix

Here we show that welfare rises even for very small consumption deviations $(1+\Delta)\bar{C}$ in our perturbation experiment. We restrict attention to $\gamma > 1$, and let $W(\omega)$ be the welfare resulting from the experiment where $\omega \equiv \log(1+\Delta)$. More precisely, let

$$\begin{aligned}
c_0(\omega) &= \bar{c} + \omega\,, \\
c_1(\omega) &= \bar{c} + \log\left(1 - \frac{1-\delta}{\delta}\left(e^{\omega} - 1\right)\right) = c_t(\omega)\,, \qquad \text{for all } t > 0\,; \\
s_0(\omega) &= \bar{s} + \bar{\lambda}\omega\,, \\
s_1(\omega) &= \bar{s} + \phi\bar{\lambda}\omega - \lambda(\bar{s} + \bar{\lambda}\omega)(\bar{c} - c_1 + \omega)\,, \\
s_t(\omega) &= (1-\phi)\bar{s} + \phi s_{t-1} = \bar{s} - \phi^{t-1}(\bar{s} - s_1)\,, \qquad \text{for all } t > 1\,; \\
W(\omega) &= \frac{\exp[(1-\gamma)(c_0 + s_0)] - 1}{1-\gamma} \\
&+ \sum_{t=1}^{\infty} \delta^t \frac{\exp[(1-\gamma)(c_1 + \bar{s} - \phi^{t-1}(\bar{s} - s_1))] - 1}{1-\gamma}\,,
\end{aligned}$$

where $\bar{\lambda} \equiv \lambda(\bar{s})$ and, to save notation, we usually suppress the argument ω.

[5] By contrast, Ljungqvist and Uhlig (1999) report on how welfare can be improved through consumption *stabilization* in a productivity-shock driven economy with standard catching-up-with-the-Joneses preferences. The consumption externality here calls for an optimal tax policy that affects the economy countercyclically via procyclical taxes, i.e., "cooling" down the economy with higher taxes in booms and lowering taxes in recessions to stimulate the economy.

The derivative of $W(\omega)$ is given by

$$\begin{aligned} W'(\omega) &= (1+\bar{\lambda})\exp[(1-\gamma)(\bar{c}+\bar{s}+(1+\bar{\lambda})\omega)] \\ &+ \sum_{t=1}^{\infty} \delta^t (c_1' + \phi^{t-1} s_1') \exp[(1-\gamma)(c_1 + \bar{s} - \phi^{t-1}(\bar{s} - s_1))] \,. \end{aligned}$$

Note that

$$\begin{aligned} c_1'(0) &= -\frac{1-\delta}{\delta} < 0\,, \\ s_1'(0) &= -\frac{1-\phi\delta}{\delta}\bar{\lambda} < 0\,. \end{aligned}$$

It follows that $W'(0) = 0$, which should be no surprise: at $\omega = 0$, all the marginal conditions for optimality are satisfied, so a first-order change in the consumption plan should have at most a second-order effect on welfare. However, we will now show that $\omega = 0$ constitutes a local minimum rather than a maximum. To do so, we need to show that

$$W'(\omega) \geq 0$$

locally in a neighborhood around 0 for $\omega \geq 0$ as well as $W'(\omega) \leq 0$ for $\omega \leq 0$. We will only show the first part of this claim since the proof for the second part is reasonably similar. Thus, assume that $\omega \geq 0$.

As a first step, use $\phi^{t-1}(\bar{s} - s_1) < \bar{s} - s_1$ to check that

$$W'(\omega) \geq g(\omega)\exp[(1-\gamma)(\bar{c}+\bar{s})],$$

where

$$g(\omega) = \frac{1}{\bar{S}}\exp\left(\frac{(1-\gamma)\omega}{\bar{S}}\right) + \left(\frac{\delta}{1-\delta}c_1' + \frac{\delta}{1-\delta\phi}s_1'\right)\exp[(1-\gamma)(c_1-\bar{c}+s_1-\bar{s})]\,.$$

To show that $W'(\omega) \geq 0$, it suffices to show that $g(\omega) \geq 0$ for $\omega \geq 0$ sufficiently small. And since $g(0) = 0$, it suffices to show that $g'(0) > 0$. Calculations yield

$$\begin{aligned} g'(0) &= \frac{1}{\bar{S}^2}\left(\frac{2}{1-\delta\phi} - (\gamma-1)\frac{1+\delta(1-\phi)}{\delta}\right) \\ &\quad -\frac{1}{\bar{S}}\left(\frac{1+\delta}{\delta(1-\delta\phi)} - (\gamma-1)(1-\phi)\right) - \frac{1-\phi}{1-\delta\phi}\,. \end{aligned}$$

For the numerical parameter values chosen by Campbell and Cochrane, one gets $g'(0) > 0$, i.e., a sufficient condition is satisfied for arbitrarily small consumption devations to increase welfare.

Obviously, the proof is "numerical": one can pick parameters such that $g'(0) < 0$. Closing down habit formation would be one rather drastic avenue, for example. One can now also see which features of the model are most "responsible" for the result that $g'(0) > 0$. Numerically, the far largest quantity in absolute terms is the positive term $\bar{S}^{-2} \cdot 2(1-\delta\phi)^{-1}$. Searching for its origin, one finds that it arises from the second derivative of s_1 with respect to ω, involving the first derivative of the λ-function. Put differently, the key reason for finding a welfare minimum at $\omega = 0$ rather than a maximum is that λ is extremely steep around the steady state, making s_1 extremely convex.

References

[1] Campbell, John Y. and John H. Cochrane (1999), "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy* 107 (2), 205-251.

[2] Ljungqvist, Lars and Harald Uhlig (1999), "Tax Policy and Aggregate Demand Management under Catching Up with the Joneses", *American Economic Review*, forthcoming.

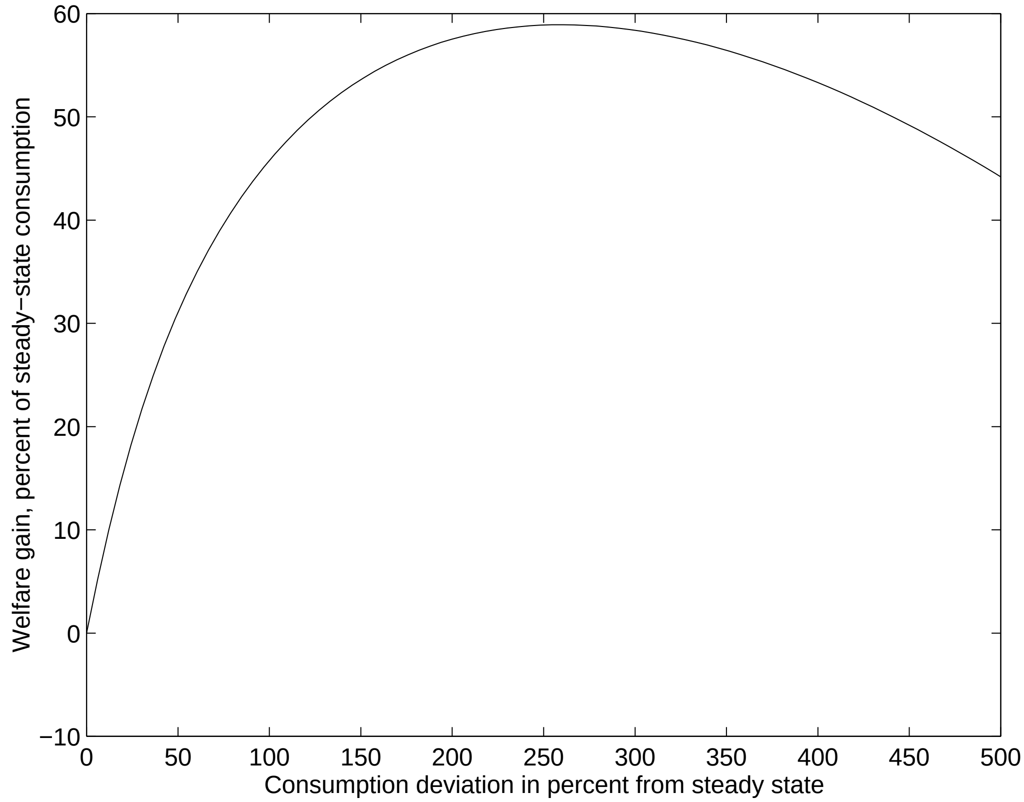

Figure 1: Welfare gain of a one-period consumption deviation from steady state, and thereafter a permanent reduction to the highest possible sustainable consumption level. The welfare gain is measured as a permanent percentage increase in the steady-state consumption stream that would make individuals as well off as under the proposed temporary consumption hike.

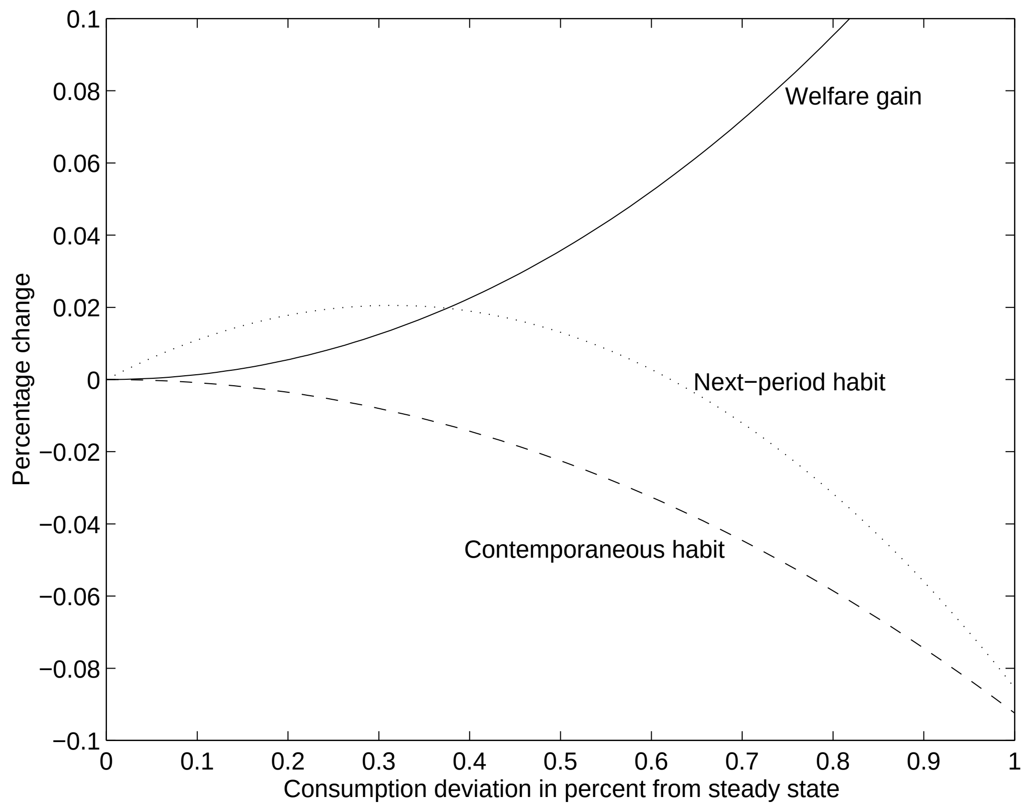

Figure 2: Change in contemporaneous as well as next-period habit level from a one-period consumption deviation from steady state (and thereafter a permanent reduction to the highest possible sustainable consumption level). Changes in habit are expressed as a percentage of the initial steady-state habit level. The depicted welfare gain is the same as in Figure 1.

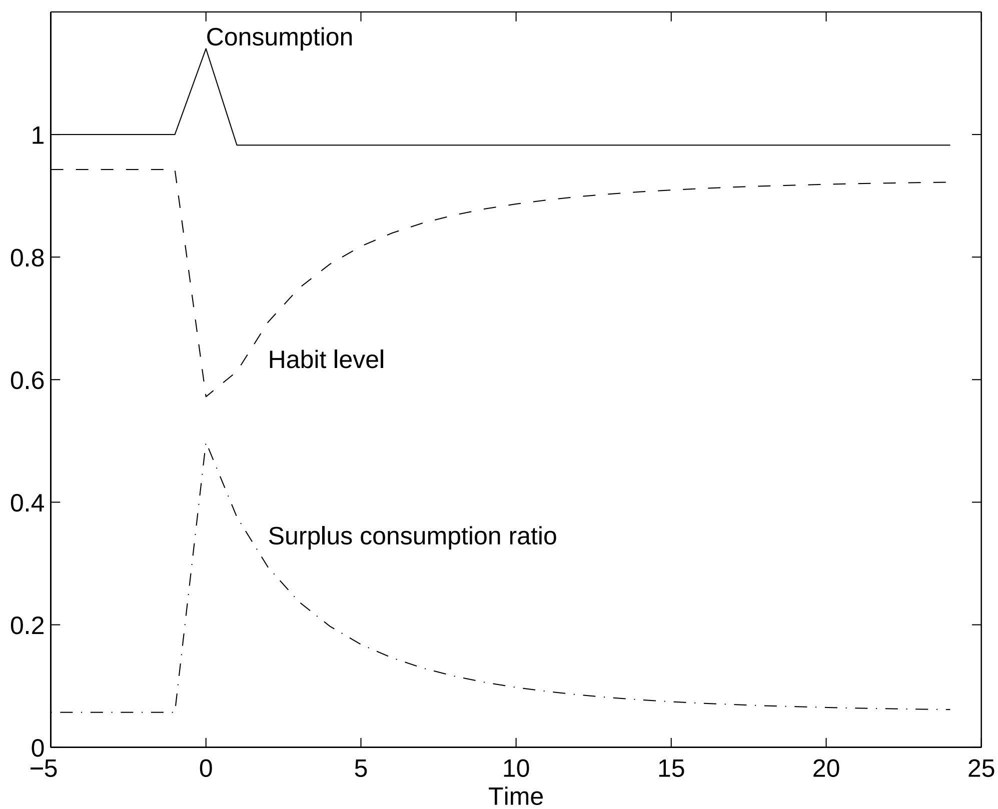

Figure 3: Evolution of the surplus consumption ratio and the habit level after a consumption hike of 14% from steady state in period 0, and thereafter a permanent reduction in consumption to the highest possible sustainable level.