FACTOR TAXATION WITH HETEROGENEOUS AGENTS*

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Abstract

We investigate the welfare implications of changing a proportional capital income tax for a model economy in which heterogeneous households face labor income risk and trade only one asset. Labor taxes are adjusted at the time of the reform to maintain long run budget balance. Our stochastic process for labor earnings is consistent with empirical estimates of earnings risk, and also implies a distribution of asset holdings across households closely resembling that in the United States.

We find that a vast majority of households prefers the *status quo* to eliminating capital taxes. This finding is interesting in light of the fact that this reform would be optimal if we abstracted from heterogeneity and assumed a representative agent. A second finding is that in the incomplete markets economy, a utilitarian government prefers the current calibrated U.S. capital income tax rate (39.7 percent) to any change in the capital tax rate. If markets were complete, on the other hand, average welfare would be maximized by reducing the capital tax rate to around 30 percent.

Keywords: Factor taxation; Redistribution; Heterogeneous agents
JEL classification: E6; H2; H3

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1. Introduction

This paper explores the relation between what is taxed and who is taxed. In the representative agent framework, a common finding is that the optimal tax program involves zero taxation of capital income in the long run (see Chamley 1986, Judd 1985, or the recent paper by Atkeson, Chari and Kehoe 1999). However, representative agent models abstract from the fact that in practice an increased reliance on labor taxation is likely to be regressive, since low income households receive a large fraction of their income from labor relative to the fraction they receive from asset income.[1] Thus reducing the tax rate on capital income will increase the poor's share of the tax burden initially, even though in the long run all households will benefit from the higher pre-tax income associated with an increase in the capital stock.

The goal of this paper is to quantitatively assess the distributional implications of tax reform within a calibrated model of the US economy. The model economy is populated by a large number of infinitely-lived households who face uninsurable idiosyncratic labor income risk and trade a single asset. Each household can achieve a path for consumption that is smoother than its path for labor income by adjusting its asset holdings in response to income shocks. Households have an incentive to accumulate a buffer stock of savings when their labor income is above average, since borrowing is ruled out by assumption. Because earnings shocks are uncorrelated across households, the distributions of income and wealth in the model are endogenous.

There is a government which finances constant expenditure by levying proportional taxes on labor and asset income, and by issuing debt. The tax reforms we consider are permanent unanticipated changes in the capital income tax rate. To ensure that these reforms are sustainable, the labor income tax rate is adjusted at the time of the reform so that the inter-temporal government budget constraint is satisfied.

The extent to which tax reforms redistribute the tax burden across households partly depends on the initial distribution of wealth, which in turn is a function of the process for earnings. Thus the parameterization of this process is critical for assessing the welfare implications of tax reform. We therefore calibrate the earnings process to satisfy the following two criteria: (i) the wealth distri-

[1] Diaz Gimenez, Quadrini and Rios-Rull 1997 give a breakdown of sources of income by income level for U.S. households in the 1992 Survey of Consumer Finances.

bution generated endogenously by the model closely resembles that observed in the United States, and (ii) the persistence and variance of earnings shocks are consistent with estimates from the Panel Study of Income Dynamics.

In order to understand the importance of our asset market structure for the effects of tax changes, we compare the predictions of our incomplete markets model economy to those of an economy in which markets are complete, but on which we impose the pre-reform wealth distribution from the incomplete markets economy. There are several reasons why the welfare effects of tax changes are sensitive to the assumed market structure.

First, the incomplete markets model generates mobility within the income and wealth distributions, since different households experience different paths for earnings.[2] If earnings shocks are non-permanent, then a household's expected productivity and wealth in the distant future converge to the economy-wide averages. Thus households which initially have little wealth expect to become richer in the long run, and may therefore favor reducing capital income taxation even if this means paying more labor income tax in the short run. By contrast, when markets are complete, the ranking of households by wealth is fixed through time.

Second, in the incomplete markets economy, households have a precautionary motive for saving. This means that the elasticity of savings with respect to the after tax interest rate is lower than in the complete markets economy. Consequently capital taxation is less distortionary in the incomplete markets economy, and the efficiency gains from reducing capital income taxes are smaller.

Related Literature

In a seminal paper, Judd (1985) studies tax reforms for an economy in which households differ in their initial capital holdings, under the assumption that asset markets are complete. He shows that agents with below average wealth will desire an immediate permanent capital income tax increase if the current tax rate is sufficiently low. Garcia-Mila, Marcet and Ventura (1995) consider capital tax reductions in a calibrated model with two types of household. They find that capital tax reductions typically leave the wealth-poor type worse off, since efficiency gains in term of increased production are too small to compensate these households for the higher labor taxes they must pay. However, Garcia-Mila et. al. also assume complete markets, and it is not clear whether their conclusions will still obtain in an economy which allows for mobility within the income and

[2] For data on wealth and earnings mobility in the U.S., see Dias-Gimenez et. al. 1997.

wealth distributions.

Aiyagari (1995) defines an optimal tax problem for an incomplete markets economy similar to ours in which he does not constrain tax rates to be time-invariant following the initial reform. He argues that if the optimal tax program converges, then the tax rate of capital income in that steady state is positive.[3] While the reforms we consider are unlikely to be solutions to the unconstrained optimal tax problem, our approach has the advantage that it enables us to explicitly characterize both transition following the reform and the steady state to which the economy eventually converges.

The redistributive effects of fiscal policy have been studied extensively within the overlapping generations (OLG) framework. For example, Auerbach and Kotlikoff (1987) investigate a range of tax reforms, including a switch from a general income tax to labor income tax. Conesa and Krueger (1999) consider a switch from a pay-as-you-go social security system to a fully funded one. In these models, as in ours, the distribution of wealth is endogenous. However, the redistribution resulting from shocks to policy is mostly inter-generational. We choose to abstract from inter-generational issues for two reasons: they are likely to be less important for the issue of factor taxation than for social security reform, and there is a large literature on optimal factor taxation in the infinite horizon setting that is a useful reference point for thinking about the welfare implications of tax reform.

Findings

The reform we primarily focus on involves moving from the current calibrated US capital income tax rate of 39.7 percent to a capital income tax rate of zero. Eliminating capital income taxation is a natural benchmark, since our assumption that labor is inelastically supplied means that this policy is in the class of optimal tax reforms for a representative agent economy. We compute the expected welfare gain for the representative agent in a complete markets economy, and find it to be equivalent to a permanent 1.07 percent increase in consumption. This is a large gain relative, for example, to Lucas' (1987) estimate of the likely benefits of eliminating business cycles.

When households differ, however, the welfare effects of the same policy change vary greatly depending on initial household wealth and productivity. None of

[3] Judd (1985) in an economy with heterogeneous agents but complete markets finds the optimal long run tax rate on capital income to be zero.

the tax changes we consider are Pareto improving.[4] Moreover the majority of households expect to lose from eliminating capital income taxation: 73 percent of households prefers the current tax system in the incomplete markets economy, while 72 percent do so in the complete markets economy. The average change in expected utility is equivalent to a permanent 0.95 percent fall in consumption in the incomplete markets economy. Households with higher initial wealth are more likely to be winners, and on average expect to gain more, in both economies.

In addition to eliminating capital income taxes, we also consider a range of possible new capital tax rates between 0 and 50 percent. The main finding here is that in the incomplete markets economy, a utilitarian government neither wants to reduce nor increase the capital tax rate. If markets are complete, on the other hand, average expected welfare is maximized by reducing the capital tax rate to around 30 percent. Comparing across markets structures, we find that capital tax reductions in the incomplete markets economy involve smaller efficiency gains since capital taxation is less distortive when there is precautionary saving. At the same time, capital tax reductions in the incomplete markets economy involve smaller redistributive losses since households are mobile within the income and wealth distributions.

The rest of the paper is organized as follows. Section 2 outlines the economic environment. Section 3 presents the results, and Section 4 concludes.

2. The Models

We consider two model economies: one in which households have access to a single savings instrument and face a no-borrowing constraint, and a second in which households can trade a complete set of state contingent claims.

Both economies are populated by a continuum of *ex ante* identical and infinitely lived households. Households supply labor inelastically and maximize the expected discounted utility from consumption. In aggregate, household savings decisions determine the evolution of the aggregate capital stock, which in turn

[4] This is not the case in Chamley (1998), who considers tax changes pre-announced far in advance, so that a household's expected position within the income / wealth distribution at the time of the tax change is independent of its current position. Thus Chamley is able to characterize tax reforms that leave all households better off. Note that households in our economy expect to be average roughly two hundred years into the future (see figure 5).

determines aggregate output and the return to saving.

There is a government which finances constant government spending by issuing one period debt and levying taxes. From the households' perspective, debt and capital are perfect substitutes, since the one period return to both is risk free, and there are no transaction costs. An equilibrium condition is that aggregate asset holdings at each date must equal the sum of the capital stock and the stock of outstanding government debt. To focus on the effects of tax changes, we abstract from aggregate productivity shocks or other sources of aggregate risk.

We assume that households face idiosyncratic labor productivity shocks. In the incomplete markets model economy, markets which in principle could allow complete insurance against this risk do not exist. Instead there is a single risk-free savings instrument which enables households to partially self-insure by accumulating precautionary asset holdings, as in Aiyagari (1994) and Aiyagari and McGrattan (1998). An important assumption is that no borrowing is permitted. This limits the ability of a low-wealth households to smooth consumption when faced with a fall in its disposable income.

In the complete markets economy, by contrast, households can perfectly insure against idiosyncratic productivity shocks, and choose complete insurance in equilibrium. Thus we can think of the complete markets economy as a world in which households make consumption and savings decisions as though household productivity were constant through time and identical across households. Since the momentary utility function is such that the Engel curve is linear in lifetime wealth, the evolution of aggregate variables in equilibrium does not depend on the distribution of wealth at the date of a tax change (see Chatterjee 1994). However, since we assume that households cannot insure against the (zero probability) event of a tax reform, the welfare implications of tax reform in our complete markets economy will be sensitive to the shape of the initial wealth distribution.

We now give a more formal description of the incomplete markets economy. The complete markets economy may be viewed as a special case of the incomplete markets economy in which all productivity levels are the same, and this economy-wide household productivity level is normalized to 1.

The environment

Each infinitely-lived household supplies $\overline{n}$ labor hours per period. A household's effective labor supply depends both on the hours it works and on its labor productivity, which is stochastic. At each date, household productivity takes one of $l < \infty$ values in the set E. Productivity evolves through time according to a first-order Markov chain with transition probabilities defined by the $l \times l$ matrix Π. The probability distribution at any date t over E is represented by a vector $p_t \in \mathrm{R}^l : p_t \geq 0$ and $\sum_{i=1}^{l} p_{it} = 1$. If the initial distribution is given by p_0 the distribution at date is given by $p_t = p_0 \Pi^t$. Given certain assumptions (which will be satisfied here) E has a unique ergodic set with no cyclically moving subsets and $\{p_t\}_{t=0}^{\infty}$ converges to a unique limit p^* for any p_0.

Let A be the set of possible values for household wealth (the endogenous individual state variable). We assume that a household's wealth at date zero, a_0, is non-negative and that households are unable to borrow. Thus $A = \mathrm{R}_+$. Let $(A, \mathcal{A})$ and $(E, \mathcal{E})$ be measurable spaces where $\mathcal{A}$ denotes the Borel sets that are subsets of A and $\mathcal{E}$ is the set of all subsets of E. Let $(X, \mathcal{X}) = (A \times E, \mathcal{A} \times \mathcal{E})$ be the product space. Thus X is the set of possible individual states.

Let $e^t = \{e_0, ..., e_t\}$ denote a partial sequence of productivity shocks from date 0 up to date t. Let $\left(E^t, \mathcal{E}^t\right)$, $t = 0, 1, ...$ denote product spaces, and define probability measures $\mu^t(x_0, \cdot) : \mathcal{E}^t \to [0, 1]$, $t = 0, 1, ...$ where, for example, $\mu^t(x_0, E^t)$ is the probability of history E^t given initial state $x_0 \in X$.

The household's problem

The timing convention is that e_t is observed before decisions are made in period t.[5] In period 0, given the initial state $x_0 = (a_0, e_0) \in X$, the household chooses savings for each possible sequence of individual productivity shocks. Let the sequence of measurable functions $s_t : E^t \to A$, $t = 0, 1, ...$ describe this plan, where $s_t(e^t; x_0)$ denotes the value for a_{t+1} that is chosen in period t if the history up to t is e^t, conditional on the individual state at date 0 being x_0. Let $c_t : E^t \to \mathrm{R}_+$ describe the associated plan for consumption.

Expected discounted lifetime utility is given by

$$\sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left(c_t\left(e^t; x_0\right)\right) \mu^t(x_0, e^t) \tag{2.1}$$

where β is the subjective discount factor and the momentary utility function is CRRA:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \qquad \gamma > 0. \tag{2.2}$$

[5] This means that for $Z \in \mathcal{E}^0$ $\mu^0(x_0, Z) = 1$ if $e_0 \in Z$ and 0 otherwise.

Let $t = 0$ denote the date of the tax change. At the start of period 0, a pair of new permanent proportional tax rates τ^k and τ^n are announced and implemented, where τ^k is the tax rate on asset income and τ^n the tax rate on labor income. The real return at t to one unit of the asset purchased at $t-1$ is r_t. The real return to supplying one unit of effective labor at date t is w_t.

The household budget constraints are therefore given by

$$\begin{aligned} c_t(e^t; x_0) + s_t(e^t; x_0) &= \left[1 + \left(1 - \tau^k\right) r_t\right] a_t + (1 - \tau^n) w_t e_t \overline{n} \\ \text{all } e^t &\in E^t,\ t = 0, 1, ... \end{aligned} \tag{2.3}$$

where $a_{t+1} = s_t(e^t; x_0)$.

Thus the solution to the household's problem is a set of choices $s_t(e^t; x_0)$ $\forall t$ and $\forall e^t \in E^t$ such that $s_t(e^t; x_0)$ maximizes 2.1 subject to 2.3 and $s_t(e^t; x_0) \in A$, taking as given sequences for prices $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$, tax rates τ^k and τ^n, and the initial household state $x_0 = (a_0, e_0)$.

Aggregate variables

From date 0 forward, each household's productivity evolves independently according to the Markov chain defined by Π. Thus we can interpret p_t as describing the mass of the population in each productivity state at date t, given a population of measure 1 and an initial distribution across types described by the measure p_0. Since the measure p converges to a unique limit, aggregate effective labor supply will therefore converge to a constant given by $\sum_{i=1}^{l} p_i^* e_i \overline{n}$. We assume that $p_0 = p^*$, and impose an appropriate normalization such that $\sum_{i=1}^{l} p_i^* e_i = 1$. Thus aggregate labor supply is equal to $\overline{n}$ for all t.

The distribution of these households across both individual wealth and individual productivity at time 0 is described by a measure $\lambda : \mathcal{X} \to [0,1]$. By integrating with respect to λ we can compute other aggregate variables. Let aggregate asset holdings at the start of period t be denoted A_t, where

$$A_0 = \int_X a_0 \lambda(dx_0). \tag{2.4}$$

$$A_t = \int_X \sum_{e^{t-1} \in E^{t-1}} s_{t-1}(e^{t-1}; x_0) \mu^{t-1}(x_0, e^{t-1}) \lambda(dx_0) \qquad t \geq 1. \tag{2.5}$$

Real per capita government spending is constant and equal to G. Government debt issued at date t is denoted B_{t+1} and is assumed to be risk-free; the government guarantees the one period real return between t and $t+1$ at the start of period t. Debt evolves according to

$$B_{t+1} + \tau^k r_t A_t + \tau^n w_t \overline{n} = \left[1 + \left(1 - \tau^k\right) r_t\right] B_t + G \qquad t \geq 0. \tag{2.6}$$

where B_0 is given.

Aggregate per capita output at t, Y_t, is produced according to a Cobb-Douglas technology from aggregate per capita capital at date t, K_t, and aggregate per capita labor supply:

$$Y_t = K_t^\alpha \overline{n}^{1-\alpha} \qquad t \geq 0 \tag{2.7}$$

where $\alpha \in [0,1]$.

Output can be transformed into future capital, consumption and government spending according to

$$C_t + G + K_{t+1} - (1-\delta)K_t = Y_t \tag{2.8}$$

where $\delta \in [0,1]$ is the rate of depreciation.

Product and factor markets are assumed to be competitive. This and the absence of aggregate productivity shocks implies a certain one period real return to saving in the form of capital.[6] Since the real one period return to debt is also known in advance (the government guarantees it), in equilibrium the two assets must pay the same real return. This is why it is not necessary to specify the division between capital and bonds in an individual's portfolio.

Equilibrium

We assume that conditions are satisfied which guarantee that a unique invariant measure λ^* on wealth and productivity exists for the initial constant tax rates and quantity of government debt, and that for any λ_0 the economy converges to λ^* (see Aiyagari 1994). Corresponding to λ^* and the constant fiscal policy are an initial steady state capital stock, value for government spending, and factor prices. We assume that at date 0, the economy is in the steady state associated with λ^*.

[6] Of course, prior to the tax reform households expectations over future after-tax interest rates are incorrect; we make the standard assumption in this type of exercise that the reform is assumed to be a zero probability event.

A post-reform equilibrium for this economy is a pair of constant tax rates τ^k and τ^n and sequences of pre-tax prices $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$ such that when all households take prices and taxes as given and solve their maximization problems, the markets for capital, labor and output clear, and government debt is stationary. A formal definition of equilibrium is given in appendix A.2.

2.1. Parameterization

The model period is one year. All parameter values used are reported in yearly terms in table 1. The parameters relating to aggregate production and preferences are set to standard values. Capital's share in the Cobb-Douglas production function is 0.36 and the depreciation rate is 0.1. The risk aversion parameter γ is set to 1, implying logarithmic utility, and the discount factor β is 0.96.

The household productivity process

The main question addressed in the paper is how the presence of heterogeneity changes the welfare implications of tax reform, and the approach taken is to generate heterogeneity endogenously as a consequence of households receiving uninsurable idiosyncratic productivity shocks. Thus the specification of the process for these shocks is critical, since the choices here will determine how different households are in equilibrium, and therefore how differently they experience changes in fiscal policy. Broadly speaking there are two desiderata for the income process. The first is that the labor income uncertainty households experience is consistent with empirical estimates from panel data, so that the model is able to deliver appropriate time series variability in household income and consumption, and plausible levels of aggregate precautionary saving. The second is that the model economy generates realistic heterogeneity in terms of the distributions of labor and capital income, so that the tax reform involves a realistic redistribution of the tax burden.

We assume that the set E has three elements, $E = \{e_l, e_m, e_h\}$, since we found this to be the smallest number of states required to match overall wealth concentration and at the same time reproduce the fact that in the data the wealth-poorest two quintiles hold a positive fraction of total wealth. To reduce the number of free parameters, we assume that households cannot move between the high and low productivity levels directly, that the fraction of high productivity households equals the fraction of low productivity households, and that the probabilities of moving from the medium productivity state into either of the others are the same. These assumptions constitute four restrictions on the transition probability matrix, π_e. Since each row must add up to 1, we are left with two independent transition probabilities, p and q, where $p = \pi_e(e_h, e_h)$ and $q = \pi_e(e_m, e_m)$, and where p and q jointly define π_e as follows.

$$\pi_e = \begin{bmatrix} p & 1-p & 0 \\ \frac{1-q}{2} & q & \frac{1-q}{2} \\ 0 & 1-p & p \end{bmatrix} \tag{2.9}$$

Assuming that average productivity equals 1, the total number of free parameters is four: transition probabilities p and q, and two of the three values for productivity.

Various authors have estimated stochastic AR(1) processes for logged labor productivity using data from the PSID. Such a process may be summarized by the serial correlation coefficient, ρ, and the standard deviation of the innovation term, σ. Allowing for the presence of measurement error and the effects of observable characteristics such as education and age, work by Card (1991), Flodén and Lindé (1999), Hubbard, Skinner and Zeldes (1995) and Storesletten, Telmer and Yaron (1999) indicates a ρ in the range 0.88 to 0.96, and a σ in the range 0.12 to 0.25.[7] We therefore impose two restrictions on our finite state Markov process for productivity: (i) that the first order autocorrelation coefficient equals 0.9, and (ii) that the variance for productivity is $0.05/(1-0.9^2)$, corresponding to a standard deviation for the innovation term in the continuous representation of 0.224.

To generate realistic heterogeneity, we require that the Markov process for productivity be such that when the model economy is simulated, on average it reproduces certain features of the wealth distribution recently observed in the United States.[8,9] Given the two restrictions above, the number of remaining free

[7] Heaton and Lucas (1996) allow for permanent but unobservable household-specific effects, and find a much lower ρ of 0.53, and a σ of 0.25.

[8] This approach was pioneered by Castaneda et. al. (1998).

[9] In an earlier version of the paper we experimented with including the Gini coefficient for earnings as one of our targets. We abandoned this approach for two reasons. First, while estimates of the wealth Gini are stable across different data sources, estimates of Gini coefficients for earnings and income differ substantially. For example, Quadrini (1999) reports a Gini coefficient for income of 0.45 using PSID data, compared to 0.57 using SCF data. Second, in the model we abstract from various types of observable heterogeneity, such as differences in education and age, that we believe are essential for explaining the observed distribution of earnings. This is why our model generates a Gini coefficient for earnings of only 0.21.

parameters is two, and we therefore seek to match two properties of the empirical asset holding distribution: (i) the Gini coefficient, and (ii) the fraction of aggregate wealth held by the two poorest quintiles of the population. The first criterion ensures a realistic overall wealth distribution. The second criterion is designed to capture the bottom tail of the wealth distribution, and we include it because we expect that the households most likely to lose from reducing capital taxation are those with below average wealth. Using data from the 1992 Survey of Consumer Finances, Diaz-Gimenez, Quadrini and Rios-Rull (1997) report a wealth Gini of 0.78, and find that the two poorest quintiles of the distribution combined hold 1.35 percent of total wealth.[10]

Then calibration procedure is described in more detail in appendix A.1. To our initial surprise, we were able to find parameter values that satisfy all four criteria. This finding is interesting in light of the debate as to whether uninsurable fluctuations in earnings can account for US households' wealth accumulation patterns (see Quadrini and Rios Rull 1997). Table 4 provides a detailed comparison between the asset holding distribution observed in the data, and the steady state pre-reform distribution implied by the calibrated incomplete markets model.

The values for productivity in the parameter set that matches our four targets are widely and asymmetrically spaced. The ratios between the productivity values in table 1 are

$$\frac{e_h}{e_m} = 5.09, \; \frac{e_m}{e_l} = 4.66. \tag{2.10}$$

The two transition probabilities are

$$p = 0.9, \; q = 0.988 \tag{2.11}$$

which imply that at any point in time 5.25 percent of households have the high productivity level and the same percentage have the low productivity level.

Fiscal policy parameters

All remaining parameters relate to fiscal policy. The initial tax rates are calibrated to match the actual tax rates in the US Since we are interested in the extent to which tax reform shifts the tax burden across households, we calibrate to average rather than marginal tax rates. Using the method outlined in Mendoza, Razin and Tesar (1994) we calculate average tax rates for the United States, the

[10] Kennickell and Woodburn (1999) report a wealth Gini of 0.788 for the 1995 SCF data.

United Kingdom, France and Germany using OECD data. These are presented in table 2. For the period 1990-96, the capital income tax rate in the US averaged 39.7 percent, while the labor income tax rate averaged 26.9 percent.

Constant government debt B in the pre-reform steady state is set to match the 67 percent debt / GDP ratio observed in post-war US data. Initial constant government spending G is set to ensure budget balance and is therefore not an independent parameter choice. However, the implied ratio of government spending to annual output is 0.20 (see table 3) which is close to the US average of 0.19 between 1990-96.

2.2. Solution method

While techniques for solving for steady states in models with incomplete markets and heterogenous agents are fairly well established, less work has been done on developing methods for solving for transition between steady states in economies with production and incomplete markets. Exceptions are Huggett (1997) and Conesa and Krueger (1999). We describe our approach in appendix A.3.

2.3. Welfare measures

Our measure of welfare gains and losses is standard, and we now describe it for the incomplete markets economy (the complete markets economy is treated analogously)[11]. Let $c_t^R(e^t; x_0)$ be equilibrium consumption after history e^t for a household with initial state $x_0 = (a_0, e_0)$ in the case in which there is a tax reform at date 0. Let $c_t^{NR}(e^t; x_0)$ be the same thing in the case in which there is no tax reform. The *welfare gain* for this household as a result of the reform is defined as the constant percentage increment in consumption in the no reform case that gives the household the same expected utility as when the reform is implemented. Thus the welfare gain is the Δ_{x_0} that solves the following equation:

$$\sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left(c_t^R\left(e^t; x_0\right)\right) \mu^t(x_0, e^t) = \sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left((1 + \Delta_{x_0}) c_t^{NR}\left(e^t; x_0\right)\right) \mu^t(x_0, e^t). \tag{2.12}$$

[11] In the complete markets economy, a household's welfare gain or loss is a known function of initial household wealth. In the incomplete markets case, due to idiosyncratic uncertainty we focus on expected welfare gains.

In table 5 we report the welfare gains from eliminating capital taxes for households with various initial combinations of wealth and productivity. These numbers are computed by first creating a large artificial population, each member of which starts out with the initial wealth and productivity level of interest. The economy is then simulated forward (using the appropriate equilibrium sequence for interest rates) under both scenarios for fiscal policy.

The *average welfare gain* for the whole economy as a result of the reform is defined as the constant percentage increase in consumption in the no reform case that gives a utilitarian planner the same utility as when the reform is implemented. Thus the average welfare gain is the Δ that solves the following equation:

$$\begin{aligned} &\int_X \sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left(c_t^R\left(e^t; x_0\right)\right) \mu^t(x_0, e^t) \lambda(dx_0) = \\ &\int_X \sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left((1+\Delta) c_t^{NR}\left(e^t; x_0\right)\right) \mu^t(x_0, e^t) \lambda(dx_0). \end{aligned} \tag{2.13}$$

We would like to be able to assess whether the changes in welfare that result from the tax reform occur because the reform affects the efficiency of production at the aggregate level, or because it involves a redistribution of existing resources.

To address this question, let $\widehat{c}_t(e^t; x_0)$ denote the hypothetical value for consumption in the case of reform if the household got to consume the same fraction of aggregate consumption as in the case of no reform. Thus

$$\widehat{c}_t(e^t; x_0) = \frac{c_t^{NR}(e^t; x_0)}{C_t^{NR}} C_t^R \tag{2.14}$$

where C_t^R $\left(C_t^{NR}\right)$ denotes aggregate consumption at date t in the case of reform (no reform).

The *efficiency gain* as a result of the reform for a household with initial state x_0 is defined as the $\Delta_{x_0}^e$ that satisfies

$$\sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left(\widehat{c}_t(e^t; x_0)\right) \mu^t(x_0, e^t) = \sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left((1+\Delta_{x_0}^e) c_t^{NR}\left(e^t; x_0\right)\right) \mu^t(x_0, e^t). \tag{2.15}$$

The *average efficiency gain*, Δ^e, is defined analogously to the average welfare gain:

$$\begin{aligned}&\int_X \sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left(\widehat{c}_t(e^t; x_0)\right) \mu^t(x_0, e^t)\lambda(dx_0) = \\ &\int_X \sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t u\left((1 + \Delta^e) c_t^{NR}\left(e^t; x_0\right)\right) \mu^t(x_0, e^t)\lambda(dx_0).\end{aligned} \tag{2.16}$$

The *distributional gain* for a household, $\Delta^d_{x_0}$, is defined as the difference between the welfare gain and the efficiency gain:

$$\Delta_{x_0} = \Delta^e_{x_0} + \Delta^d_{x_0}. \tag{2.17}$$

With logarithmic utility, the efficiency gains are the same for all households.

Proposition 2.1. *If $u(c) = \log(c)$, then for both market structures $\Delta^e_{x_0} = \Delta^e$ for all $x_0 \in X$.*

Proof. See appendix A.4 ■

A tax reform is efficient if the efficiency gain is positive, that is if $\Delta^e > 0$.[12]

3. Results

The tax reforms we consider involve moving from the current US capital income tax rate of 39.7 percent to a range of new capital tax rates between 0 and 50 percent. The welfare effects of tax reform are described in the tables and figures at the end of the paper. Our main findings are summarized in figure 6, which describes the efficiency, distributional, and overall welfare effects associated with the reforms.

The first thing to note is that the efficiency gains from reducing capital taxes are substantially smaller in the incomplete markets economy. In the complete markets economy, eliminating capital taxation maximizes efficiency, in which case the efficiency gain is equivalent to a 1.07 percent permanent increase in consumption. In the incomplete markets economy, the efficiency gain is maximized when the capital tax is reduced to around 20 percent, implying an efficiency gain equivalent to 0.23 percent of consumption.

[12] Of course, efficiency does not imply that the reform leaves everyone better off, since households typically do not consume the same fractions of aggregate consumption in the no refrom economy as in the reform economy: $\Delta^d_{x_0}$ is typically non-zero.

From the bottom left panel of figure 6 it is clear that reducing capital income taxes implies distributional losses. For all tax reforms we consider, the distributional effects of tax reform are smaller in the incomplete markets economy.

In terms of average welfare, the distributional losses associated with capital tax reductions dominate the efficiency gains. Thus the average welfare gain from eliminating capital taxes is negative: -0.44 percent of consumption in the complete markets economy versus -0.88 percent of consumption in the incomplete markets economy. An interesting finding is that in the incomplete markets economy, the average expected welfare gain is maximized when the capital tax is essentially unchanged. That is, a utilitarian planner prefers the current taxes to any permanent unanticipated change in the capital income tax rate. In the complete markets economy, on the other hand, a reduction to 29 percent maximizes the average expected gain.

Table 5 and figure 5 show that irrespective of market structure, the expected welfare gain for a particular household varies greatly depending on its initial wealth. The expected welfare gains of eliminating capital income taxes are strongly increasing in pre-reform wealth, while controlling for wealth, households with initially higher productivity are less affected one way or the other by the reform.

The last panel of figure 6 shows the fraction of households that *ex ante* prefer the various tax reforms to the *status quo*. When markets are complete, a household in the 72^{nd} percentile of the initial wealth distribution is indifferent between the current tax system and eliminating capital taxation. When markets are incomplete 73.2 percent of households face an expected loss. Thus in both market structures a substantial majority favors the current tax system over the elimination of capital income taxes.

A striking finding is that the number of households with a positive expected welfare gain is very similar for any reduction in capital taxes. Most households on the other hand expect to gain if the capital income tax is increased. The expected welfare gains are small, however, and the average expected welfare gain is negative.

3.1. Interpretation

To understand our results, we primarily focus on the case of eliminating capital taxes. This is a natural benchmark, since our assumption that labor is inelastically supplied means that this policy is in the class of optimal tax reforms for a representative agent economy.

Aggregate variables

The dynamics of aggregate variables are very similar across market structures.[13] Following the elimination of capital taxes, aggregate consumption falls and investment rises as households take advantage of the increase in the after-tax return to saving. In the long run, the capital stock, output, consumption and government debt all exceed the initial steady state values.

The fact that the capital stock is always larger when markets are incomplete reflects the fact that households accumulate precautionary savings when they are unable to purchase insurance. As the capital stock (and government debt) increases during transition, so do per capita asset holdings. Thus the typical household in the incomplete markets economy has more wealth to use to smooth consumption in response to income shocks, and the demand for precautionary savings falls. This is why the increases in the capital stock and in government debt are smaller in the incomplete markets economy.[14]

Efficiency

Given exogenous labor supply, eliminating capital income taxation maximizes efficiency when markets are complete (see the top right panel in figure 6). Under this policy, the capital stock converges to a level at which the household's intertemporal marginal rate of substitution between consumption at different dates is equated to the marginal rate of transformation in production between those dates. Because tax reforms have no distributional consequences for a household with mean wealth, the efficiency gain from any reform is equal to the welfare gain for this household (see tables 5 and 6).

[13] Ríos-Rull (1994) also finds that in calibrated model economies aggregate variables tend to behave in a similar manner under complete and incomplete market structures.

[14] Why does it take roughly 40 years for the capital stock to approach the new steady state level? One reason is that the total increase in the capital stock is large: 32 percent in the complete markets economy, for example. With an initial capital to output ratio of 2.13 this increase amounts to 68.2 percent of initial GDP, while initial aggregate consumption is only 58.4 percent of GDP (see table 3). With mild consumption smoothing (log-utility), the optimal plan for a household with average wealth is to gradually increase asset holdings such that only after about 10 years does consumption exceed the initial steady state level (see figure 1).

When markets are incomplete, the efficiency gains from reducing capital taxes remain positive, but are much smaller than in the complete markets case. There are several reasons for this. The precautionary savings motive means that the level of savings is less sensitive to the after tax interest rate than in the complete markets economy. Consequently, the increase in the capital stock is smaller in the incomplete markets economy. There is also an externality problem, in that households do not take into account the effect of their savings decisions on the equilibrium interest rate, and accumulate too much capital in aggregate from an efficiency point of view. This is the intuition behind Aiyagari's (1995) result that if the optimal tax program in this type of economy converges to a steady state, then the optimal tax rate on capital in that steady state is positive. Although our tax reform is not likely to be optimal, this finding helps explain why the efficiency-maximizing tax reform involves a positive tax rate on capital income.

Changing after tax factor prices

A second factor determining who gains and who loses is that emphasized by Garcia-Mila et. al. (1996). If households differ in the initial fractions of their income they receive from asset holdings versus labor supply, then reducing capital income taxation effectively shifts the burden of taxation away from households who receive a large fraction of their income from capital and towards those who receive a large fraction from labor.[15] This is clear from figure 2, which shows that immediately after the reform the after tax wage falls and the after tax return to capital rises. Subsequently, as the capital stock increases, wages rise and the return to capital falls, but even in the long run, after tax wages are below and the after tax return to capital above pre-reform levels.

Complete markets

In the complete markets economy this redistribution of the tax burden implies that the wealth-poorest households see the largest increase in their tax bills. In addition, there is a value for wealth such that all richer households benefit from the tax reform, while all poorer households lose (see figure 5). In particular, the

[15] An implicit assumption here is that markets for insurance against the redistributive effects of future tax changes do not exist; if they did all households would share equally in the efficiency gains associated with the reform. One might therefore argue that it is misleading to label the economy in which idiosyncratic labor income risk can be perfectly insured the "complete markets" economy.

amount of wealth a household requires to be indifferent amounts to 70.4 percent of mean per capita wealth.

We can now account for the finding that most households lose from eliminating capital taxation. Because the initial wealth distribution is so skewed, only 28 percent of households have more than 70.4 percent of mean wealth. This is why 72 percent of households would vote against the tax reform. The average welfare gain is negative because the reform hurts wealth-poor households with a high marginal utility of consumption, and benefits wealthy households whose marginal utility of consumption is relatively small.

Luck and mobility

In the incomplete markets economy, luck is one of several factors not present in the complete markets analysis that come into play when considering who gains and who loses from tax reform. As a consequence of uncertain idiosyncratic productivity, households expect to move around in the income and wealth distributions.[16] In the long run, however, all households are the same. Convergence of expected productivity is illustrated in the first panel of figure 3, and accounts for the observed convergence in expected consumption and wealth. This partly explains our finding that the distributional effects of tax reform are smaller in the incomplete markets economy than in the complete markets one (see the bottom left panel of figure 6).

Eliminating capital taxes: a good idea?

Although mobility reduces the distributional effects of eliminating capital taxes, distributional effects still swamp efficiency gains in the overall welfare calculus. Thus, the majority of households expect to be worse off following the elimination of capital taxes, and the average welfare gain is negative. There are three reasons for this.

First, the efficiency gains are smaller when markets are incomplete, as discussed above. Second, the idiosyncratic productivity shocks are very persistent relative to the households' rate of time preference. Third, the initial distribution of wealth is so skewed that the tax reform involves substantial redistribution even

[16] From figure 5 it is clear that there is considerable variation in the experienced welfare gain of households with identical initial wealth. For example, in our sample population with $9,600$ households, the poorest household to gain *ex post* had 6.6 percent of mean initial wealth, while the richest household to lose started with 200.5 percent of mean initial wealth.

in the short run. These last two points explain why a household's initial position in the income and wealth distributions is so important in determining its expected welfare gain from a tax reform (see figure 5).

Wealth versus productivity

Households with high initial labor productivity receive a larger fraction of their income from labor than equally wealthy households with lower productivity. This means that high productivity households face the largest initial tax increases following the elimination of capital income taxation (see the first panel in figure 4).

However, high productivity households want to increase their asset holdings, while low productivity households are typically dis-saving. This means that high productivity households are well placed to take advantage of the temporary increase in the after-tax return to saving.[17]

The two effects described above largely offset each other, so that the value for initial wealth such that a household is indifferent between eliminating capital income taxation and maintaining the initial tax system is similar for high and low productivity households (see the right panel of figure 5). For low values of wealth, however, the effect of the increase in the return to saving is particularly important, since high productivity households with low wealth have the highest marginal propensity to save, and accumulate wealth fastest. This partly explains our finding that low wealth households with high initial productivity expect smaller welfare losses than less productive households.

Other effects of market incompleteness

There are two additional factors that comes into play when markets are incomplete. First, in addition to shifting the tax burden, the increase in capital stock increases the share of capital income of total income. The post-tax asset to labor income ratio in the initial pre-reform steady state is 0.21, while in the post-reform steady state it is 0.28. Since asset income is riskless by assumption, the uncertainty households face about future income is reduced.

Second, the after-tax interest rate is higher in the post-reform steady state: 3.42 percent versus 3.23 percent in the initial steady state.[18] Thus the opportunity

[17] This increase is mostly temporary, because in the long run, the after tax return on capital falls towards its pre-reform level.

[18] Recall that in the incomplete markets economy, eliminating capital taxes increases

cost of accumulating a buffer stock of savings is reduced, and the no-borrowing constraint binds less frequently. In the pre-reform steady state 2.5 percent of households are borrowing constrained. During transition, the percentage of constrained households falls to 2.0 percent.

Which reform is best?

Figure 6 describes the welfare implications of a range of tax reforms in which the new tax rate of capital income is set to values between zero and 50 percent. The shapes of the figures for efficiency and distributional gains have been discussed above. The figure for average welfare gains is simply the sum of these two graphs. Note that distributional gains are approximately linear in the size of the tax change, while doubling the size of a tax cut does not double the efficiency gain. This explains why the graph for average welfare gains has an 'inverted u' shape.

The bottom right panel of figure 6 shows that approximately 30 percent of households expect to gain from reducing capital income taxes, irrespective of the size of the reduction. Consider, for example, a capital tax decrease to 25.6 percent, which is the average of our estimates of the French and German average tax rates on capital income in the 1990's (see table 2). Under this reform, households with less than median wealth expect large welfare losses (see table 5). The reason is that these households pay higher taxes following the reform, since they are heavily dependent on labor income. For example, when markets are complete, the median household by wealth has only 4.6 percent of mean wealth, whereas a household needs at least 47.1 percent of mean per capita wealth to be better off as a result of the capital tax reduction.

In contrast to capital tax reductions, most households are in favor of increasing the capital income tax rate. However, average welfare gains are negative in both economies. The intuition is that the efficiency costs of capital tax increases are very large. Consider, for example, an increase in the capital income tax rate to the UK level of 47.7 percent (see tables 5 and 6). This reform is associated with an efficiency loss under incomplete markets equivalent to a permanent 0.73 percent fall in consumption, and an average welfare loss of 0.42 percent of consumption. In the complete markets economy, all households with more than 12.3 percent of mean wealth are worse off as a result of this reform.

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the total stock of assets in the economy. To induce households to absorb this increase in the stock of assets, the after-tax interest rate must rise.

Why does the utilitarian planner want to reduce taxes in the complete markets economy but leave them more or less unchanged when markets are incomplete? This is because capital taxation is much more distortive when markets are complete, while current US tax rates happen to be such that in the incomplete markets economy the efficiency gains from capital tax reductions are exactly offset by the distributional losses.

3.2. Alternative parameterizations

In the benchmark parameterization, productivity shocks are very persistent relative to households' rate of time preference. We therefore recompute the effects of eliminating capital income taxes using a less persistent productivity process. We adopt the estimates of Heaton and Lucas (1996) which suggest an autocorrelation coefficient of 0.53 and a variance for productivity of $0.251^2/(1-0.53^2)$.[19]

With this process for productivity, we are unable to reproduce the degree of wealth concentration observed in the US. We therefore space the values for the productivity shocks evenly, and assume that the fractions of households in each state are as in the benchmark parameterization. The wealth Gini in the pre-reform steady state is then 0.45, and the poorest 40 percent of households hold 11.5 percent of total wealth. Thus the model now generates much less wealth concentration than under the benchmark process.

In the incomplete markets economy, the efficiency gains from eliminating capital taxation are now almost as large as when markets are complete (see table 7). This reflects lower precautionary saving when idiosyncratic shocks are less persistent, which implies smaller aggregate differences across market structures. Lower initial wealth inequality combined with greater earnings mobility imply that the distributional losses associated with eliminating capital taxes are smaller than under the benchmark parameterization.[20]

Overall, we find that in both economies approximately 50 percent of house-

[19] In contrast to the studies cited in section 2.1, Heaton and Lucas (1996) allow for permanent but unobservable household-specific effects when estimating the process for logged labor productivity.

[20] In the complete markets economy, the evolution of aggregate variables is independent of the distribution of wealth. Thus differences in the welfare effects of tax reform across the two parameterizations are entirely due to the fact that initial wealth is more evenly distributed under the Heaton and Lucas parameterization. This effect is important (see table 7).

holds expect to gain from eliminating capital income taxes under this parameterization. The average change in welfare is now positive.

4. Conclusions

The main conclusion we take from this paper is that changing the balance between capital and labor income taxation is likely to have very large distributional implications. Reducing taxes on capital income in our model *does* stimulate investment, raising output and consumption for all households in the long run. However, the short run cost in the form of higher labor taxes is too heavy a price to pay for all except the wealth-richest households. This finding survives even if markets are complete and idiosyncratic earnings risk is fully insurable.

In a representative agent economy eliminating capital income taxation is optimal. In a parameterization which endogenously reproduces the highly concentrated distribution of wealth observed in the US, over 70 percent of households expect to lose from this reform. Thus our quantitative modelling exercise suggests that heterogeneity is important for understanding the welfare implications of tax reform.

One interesting finding is that in the incomplete markets economy, a utilitarian government neither wants to reduce nor increase the capital tax rate. Reducing capital taxation is welfare-reducing since it effectively redistributes towards a few wealthy households whose expected marginal utility from consumption is typically low. Increasing capital taxation also reduces average welfare since capital taxation becomes increasingly distortionary. In contrast to this result, when markets are complete, average expected welfare is maximized by reducing the capital tax rate to around 30 percent.

In future work we plan to consider a switch from the current tax system to one based on consumption taxation. Our expectation is that this alternative reform will imply a more even distribution of the large efficiency gains from reducing tax distortions.

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A. Appendix

A.1. Calibrating the productivity process

Consider the following AR(1) process for labor productivity

$$\ln e' = \rho \ln e + \varepsilon' \qquad \varepsilon \sim N(0, \sigma^2). \tag{A.1}$$

and note that

$$\rho = \frac{cov(\ln e', \ln e)}{var(\ln e)} \tag{A.2}$$

and

$$var(\ln e) = \frac{\sigma^2}{1-\rho^2} \tag{A.3}$$

Equations resembling (A.1) have been estimated on panel data. Our goal to approximate equation (A.1) by a 3-state Markov chain, preserving the estimated autocorrelation and variance of log productivity. Let e_i, $i = 1, 2, 3$ denote the three productivity levels in our Markov chain, and let π_i denote the constant proportion of households with each productivity level in the ergodic distribution associated with the transition probability matrix, π_e. Thus $\sum_i \pi_i = 1$. The matrix itself, reproduced here, defines the probabilities of moving between productivity levels as functions of two parameters, p and q.

$$\pi_e = \begin{bmatrix} p & 1-p & 0 \\ \frac{1-q}{2} & q & \frac{1-q}{2} \\ 0 & 1-p & p \end{bmatrix} \tag{A.4}$$

Given the symmetry of π_e, $\pi_1 = \pi_3$, and π_1 is related to p and q as follows.

$$\begin{aligned} \pi_1(1-p) &= \pi_2 \frac{1-q}{2} \\ &= (1-2\pi_1)\frac{1-q}{2} \end{aligned} \tag{A.5}$$

To enable comparison with the estimated process for log productivity, assume that mean (natural) log productivity equals 1.

$$\overline{\ln e} = \sum_i \pi_i \ln e_i = 0 \tag{A.6}$$

The variance and covariance of log productivity are given by

$$var(\ln e) = \sum_i \left(\ln e_i - \overline{\ln e}\right)^2 \tag{A.7}$$

and

$$cov(\ln e', \ln e) = \sum_i \left(\ln e_i' - \overline{\ln e}\right)\left(\ln e_i - \overline{\ln e}\right) \tag{A.8}$$

Let π_1 and e_2 be such that when the model economy is simulated, on average it reproduces the two chosen moments characterizing the wealth distribution as discussed in section 2.1. Once values for these parameters have been chosen, the goal is to adjust the remaining free parameters so that the process for log productivity inherits the properties estimated in the data. During this second stage, π_1 and e_2 are treated as exogenously fixed.

Since $\pi_3 = \pi_1$, and $\sum_i \pi_i = 1$, (A.5) can be rearranged to express q as a known function of p.

$$q = \frac{\pi_2 - 2\pi_1(1-p)}{\pi_2} \tag{A.9}$$

Equation (A.6) can be rearranged to give an expression for $\ln e_3$

$$\ln e_3 = -\frac{\pi_1 \ln e_1 + \pi_2 \ln e_2}{\pi_1} \tag{A.10}$$

Given π_1 and e_2, and expressions (A.9) and (A.10), the only remaining free parameters are p and e_1.

From (A.3) and (A.7), equating the variances of the discrete and continuous processes for log productivity implies that.

$$\sigma_e^2 = \left(1-\rho^2\right)\left(\pi_1 (\ln e_1)^2 + \pi_2 (\ln e_2)^2 + \pi_1 (\ln e_3)^2\right). \tag{A.11}$$

Substituting (A.10) into (A.11) then implies

$$2 (\ln e_1)^2 + 2k \ln e_1 \ln e_2 + k (1+k) (\ln e_2)^2 - \frac{\sigma^2}{(1-\rho^2)\,\pi_1} = 0 \qquad \text{where } k = \frac{\pi_2}{\pi_1} \tag{A.12}$$

This is a quadratic equation that can be solved for $\ln e_1$. The relevant root is

$$\ln e_1 = \frac{-2k \ln e_2 - \sqrt{(2k \ln e_2)^2 - 4 \times 2 \times \left(k\,(1+k)\,(\ln e_2)^2 - \frac{\sigma^2}{(1-\rho^2)\pi_1}\right)}}{2 \times 2} \tag{A.13}$$

From (A.2), (A.7) and (A.8), equating the autocorrelation of the discrete and continuous processes for log productivity implies that

$$\rho = p + \frac{(-1+p)\,(\ln e_2)^2}{\pi_1\,(\ln e_1)^2 + \pi_2\,(\ln e_2)^2 + \pi_1\,(\ln e_3)^2}. \tag{A.14}$$

Substituting in equation (A.11) this simplifies to

$$\rho = p + \frac{(-1+p)\,(1-\rho^2)\,(\ln e_2)^2}{\sigma^2} \tag{A.15}$$

Equation (A.15) can then be used to solve for p

$$p = \frac{\rho + \frac{(1-\rho^2)(\ln e_2)^2}{\sigma^2}}{1 + \frac{(1-\rho^2)(\ln e_2)^2}{\sigma^2}}. \tag{A.16}$$

A.2. Definition of equilibrium

We now describe the conditions that jointly characterize the equilibrium path of the incomplete markets economy following a tax reform at date $t = 0$.

An equilibrium is a pair of constant tax rates τ^k and τ^n and sequences of decision rules $\{s_t(e^t; x_0)\}_{t=0}^{\infty}$ and $\{c_t(e^t; x_0)\}_{t=0}^{\infty}$ $\forall x_0 \in X$ and $\forall e^t \in E^t$, probability measures $\{\mu^t(x_0, Z)\}_{t=0}^{\infty}$ $\forall x_0 \in X$ and $\forall Z \in \mathcal{E}^t$, prices $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$, values for aggregate capital, debt and asset holdings $\{K_t\}_{t=0}^{\infty}$, $\{B_t\}_{t=0}^{\infty}$ and $\{A_t\}_{t=0}^{\infty}$, and a measure $\lambda(D)$ $\forall D \in \mathcal{X}$ describing the initial distribution across individual states such that $\forall e^t \in E^t$:

1. $\forall x_0 \in X$, $s_t(e^t; x_0)$ solves the household maximization problem (described in the text) given $\{r_t\}_{t=0}^{\infty}$, $\{w_t\}_{t=0}^{\infty}$, the sequence of measures $\{\mu^t(x_0, \cdot)\}_{t=0}^{\infty}$, and the pair of constant tax rates $\left\{\tau^k, \tau^n\right\}$.

2. $\forall x_0 \in X$, the sequence of measures $\{\mu^t(x_0, \cdot)\}_{t=0}^{\infty}$ is consistent with the with transition probability matrix Π in that for any $Z = Z_0 \times ... \times Z_t \in \mathcal{E}^t$

$$\mu^t(x_0, Z_0 \times ... \times Z_{t-1} \times Z_t) = \sum_{i: e_i \in Z_{t-1}} \mu^{t-1}(x_0, Z_0 \times ... \times e_i) \sum_{j: e_j \in Z_t} \Pi_{ij} \tag{A.17}$$

3. The market for savings clears.

$$K_0 + B_0 = \int_X a_0 \lambda(dx_0) = A_0. \tag{A.18}$$

$$K_t + B_t = \int_X \sum_{e^{t-1} \in E^{t-1}} s_{t-1}(e^{t-1}; x_0)\mu^{t-1}(x_0, e^{t-1})\lambda(dx_0) = A_t \qquad t = 1, 2, ... \tag{A.19}$$

4. Factor markets clear.

$$r_t = \alpha K_t^{\alpha-1} \overline{n}^{1-\alpha} - \delta \qquad t = 0, 1, ... \tag{A.20}$$

$$w_t = (1-\alpha) K_t^{\alpha} \overline{n}^{-\alpha} \qquad t = 0, 1, ... \tag{A.21}$$

5. The government budget constraint is satisfied and debt remains bounded.

$$B_{t+1} + \tau^k r_t A_t + \tau^n w_t \overline{n} = \left[1 + \left(1 - \tau^k\right) r_t\right] B_t + G \qquad t = 0, 1, ... \tag{A.22}$$

$$B_t \in [0, \infty) \qquad t = 0, 1, ... \tag{A.23}$$

where B_0 is given.

6. The goods market clears.

$$C_t + G + K_{t+1} - (1-\delta)K_t = Y_t \qquad t = 0, 1, ... \tag{A.24}$$

where

$$C_t = \int_X \sum_{e^t \in E^t} c_t(e^t; x_0)\mu^t(x_0, e^t)\lambda(dx_0). \tag{A.25}$$

A.3. Solution algorithm

1. Solve for the initial steady state given the initial capital tax rate as follows.
    1. Guess a value for the capital stock (and thus implicitly for output).
    2. Compute the government spending G, such that given the labor tax τ^n, government debt B remains constant at the target ratio for debt to GDP.
    3. Simulate the economy to compute a stationary asset holding distribution.
    4. Check that aggregate household savings decisions equal aggregate capital plus aggregate debt.

5. Adjust the guess for the capital stock and iterate until the market for savings clears.

2. Choose a new value for the capital tax τ^k. Assume this is announced before households make decisions in period 1.

3. Assume that the economy converges to a new steady state and that it is in this steady state in period T.

4. Guess a sequence $K_2 ... K_{T-1}$ for capital during transition.

5. Solve for the new proportional tax on labor τ^n such that given $K_2 ... K_{T-1}$ and τ^k, government debt is unchanged between $T-1$ and T. Compute the associated path for government debt, $B_2 ... B_T$.

6. Solve for the final steady state using the same procedure outlined in step one, taking as given tax rates τ^k and τ^n and G and B_T. Compute the capital stock in the new steady state, K_T.

7. Solve for household savings decisions in transition as follows.

   1. Start in period $T-1$.
   2. Assume that:
      1. capital today is K_{T-1} and capital tomorrow is K_T.
      2. consumption tomorrow (in period T) is given by the consumption function in the new steady state, $c_T(\cdot)$.
   3. Solve for the consumption decision rule at $T-1$ across the grid on individual wealth and productivity, $c_{T-1}(a, e : K_{T-1}, K_T, c_T(\cdot))$.
   4. Move back one period to $T-2$, and solve for $c_{T-2}(a, e : K_{T-2}, K_{T-1}, c_{T-1}(\cdot))$.
   5. Continue moving back until we have decision rule functions $c_i(a, e : K_i, K_{i+1}, c_{i+1}(\cdot))$, $i = 1 ... T-1$.

8. Now start updating the path of capital. The procedure below is a Gauss Seidel algorithm. The basic problem we have is one of finding a sequence of capital stocks such that when households optimize markets clear at every date and government debt eventually stabilizes at a finite level. A Newton Raphson approach would start by computing excess demand at every date before updating any values for capital in the sequence. The advantage of the Gauss Seidel method is that we update continuously.

1. Take the initial steady state distribution over wealth and productivity and use $c_1(a, e : K_1, K_2, c_2(\cdot))$ to compute the implied joint distribution in period 2.
2. Compute the value for aggregate capital in the second period of transition, $\widehat{K_2}$ that is implied by $c_1(a, e : K_1, K_2, c_2(\cdot))$. This is given by aggregate savings minus B_2.
3. Compare K_2 (the value for capital in period 2 that was used to compute household savings decisions) and compare it to $\widehat{K_2}$. Set $K_2 = K_2 + \phi\left(\widehat{K_2} - K_2\right)$ where $0 < \phi < 1$.
4. Recompute τ^n and the sequence for government debt.
5. Recompute $c_2(a, e : K_2, K_3, c_3(\cdot))$ and $c_1(a, e : K_1, K_2, c_2(\cdot))$.
6. Using the initial steady state distribution over wealth and productivity, simulate the economy forward two periods with savings rules given by $c_1(a, e : K_1, K_2, c_2(\cdot))$ and $c_2(a, e : K_2, K_3, c_3(\cdot))$.to compute the implied value for $\widehat{K_3}$.
7. Given $\widehat{K_3}$, adjust K_3, and recompute τ^n, the sequence for government debt, and $c_3(\cdot)$, $c_2(\cdot)$ and $c_1(\cdot)$.
8. Iterate forward until we have updated $K_2 ... K_{T-1}$,

9. If the new sequence for capital is the same as the old, we have found the equilibrium path. Otherwise go back to step 5, resolve for the new labor tax given the updated capital sequence, and proceed.

10. Once the sequence for capital has converged, check whether T is sufficient by increasing T and checking whether the equilibrium path is affected. In all experiments T has been set to 80, implying that the aggregate capital stock converges to its new steady state level with 80 years.

A.4. Efficiency

In this appendix we prove proposition 2.1. Beginning with the case of an individual household, let $\Delta^e_{x_0}$ satisfy equation (2.15) given $\{\widehat{c}_t(e^t; x_0)\}_{t=0}^{\infty}$ and $\left\{c_t^{NR}(e^t; x_0)\right\}_{t=0}^{\infty}$. Substituting equation (2.14) into (2.15) gives

$$\begin{aligned} &\textstyle\sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t \log\left(\frac{c_t^{NR}(e^t; x_0)}{C_t^{NR}} C_t^R\right) \mu^t(x_0, e^t) = \\ &\textstyle\sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t \log\left((1 + \Delta^e_{x_0}) c_t^{NR}(e^t; x_0)\right) \mu^t(x_0, e^t). \end{aligned} \tag{A.26}$$

which may be rewritten as

$$\begin{array}{l} \sum\limits_{t=0}^{\infty} \sum\limits_{e^t \in E^t} \beta^t \log(c_t^{NR}(e^t;x_0)) + \sum\limits_{t=0}^{\infty} \beta^t \log\left(\frac{C_t^R}{C_t^{NR}}\right) = \\ \sum\limits_{t=0}^{\infty} \sum\limits_{e^t \in E^t} \beta^t \log(c_t^{NR}(e^t;x_0)) + \sum\limits_{t=0}^{\infty} \beta^t \log(1+\Delta_{x_0}^e) \end{array} . \tag{A.27}$$

Now, consider the aggregate efficicency gain. Let Δ^e be such that equation (2.16) is satisfied given $\{\widehat{c}_t(e^t;x_0)\}_{t=0}^{\infty}$ and $\left\{c_t^{NR}(e^t;x_0)\right\}_{t=0}^{\infty}$ and aggregate consumption streams $\left\{C_t^R\right\}_{t=0}^{\infty}$ and $\left\{C_t^{NR}\right\}_{t=0}^{\infty}$. Then for all x_0, substituting equation (2.14) into (2.16) gives

$$\begin{array}{l} \int_X \sum\limits_{t=0}^{\infty} \sum\limits_{e^t \in E^t} \beta^t \log\left(\frac{c_t^{NR}(e^t;x_0)}{C_t^{NR}} C_t^R\right) \mu^t(x_0,e^t)\lambda(dx_0) = \\ \int_X \sum\limits_{t=0}^{\infty} \sum\limits_{e^t \in E^t} \beta^t \log\Big((1+\Delta^e) c_t^{NR}(e^t;x_0)\Big) \mu^t(x_0,e^t)\lambda(dx_0). \end{array} \tag{A.28}$$

which can be rewritten as

$$\begin{array}{l} \int_X \sum\limits_{t=0}^{\infty} \sum\limits_{e^t \in E^t} \beta^t \log(c_t^{NR}(e^t;x_0))\mu^t(x_0,e^t)\lambda(dx_0) + \sum\limits_{t=0}^{\infty} \beta^t \log\left(\frac{C_t^R}{C_t^{NR}}\right) = \\ \int_X \sum\limits_{t=0}^{\infty} \sum\limits_{e^t \in E^t} \beta^t \log(c_t^{NR}(e^t;x_0))\mu^t(x_0,e^t)\lambda(dx_0) + \sum\limits_{t=0}^{\infty} \beta^t \log(1+\Delta^e) \end{array} . \tag{A.29}$$

Comparing equations (A.27) and (A.29) we see that

$$\sum_{t=0}^{\infty} \beta^t \log(1+\Delta_{x_0}^e) = \sum_{t=0}^{\infty} \beta^t \log(1+\Delta^e). \tag{A.30}$$

Thus for all x_0, $\Delta_{x_0}^e = \Delta^e$.

Table 1: Parameter values (yearly basis)

		Market structure: Incomplete	Market structure: Complete
Aggregate production	α	0.36	
	δ	0.1	
Individual productivity	e_h	4.334	1.0
	e_m	0.852	1.0
	e_l	0.183	1.0
	$\pi(e_h\|e_h)$	0.900	
	$\pi(e_m\|e_m)$	0.988	
	$\pi(e_l\|e_l)$	0.900	
Preferences	γ	1.0	
	β	0.96	
Government	B/Y	0.67	
	τ^n	0.269	
	τ^k	0.397	

Table 2: Average tax rates (percent)*

	United States	United Kingdom	France	Germany
1965-1996				
Consumption tax	5.7	15.0	21.3	15.7
Labor income tax	23.6	26.4	42.7	37.8
Capital income tax	40.1	54.1	24.1	26.6
1990-1996				
Consumption tax	5.4	16.8	19.4	16.5
Labor income tax	26.9	24.3	48.8	42.1
Capital income tax	39.7	47.7	25.0	26.2

* These figures are computed using the method described by Mendoza, Razin and Tesar (1994) and OECD (1999) data.

Table 3: Aggregate properties of initial and final steady states
New $\tau^k = 0$

		Market structure	
		Incomplete	Complete
τ^k	initial	0.397*	0.397*
	final	0.000*	0.000*
τ^n	initial	0.269*	0.269*
	final	0.334	0.343
G/Y	initial	0.200	0.203
	final	0.186	0.183
B/Y	initial	0.670*	0.670*
	final	0.823	0.858
K/Y	initial	2.34	2.13
	final	2.68	2.54
C/Y	initial	0.565	0.584
	final	0.546	0.562
Y	initial	0.528	0.500
	final	0.570	0.553
r (% post-tax)	initial	3.23	4.17
	final	3.42	4.17
post-tax asset to labor income ratio	initial	0.21	0.25
	final	0.28	0.34

* Starred values indicate exogenous parameters.

Table 4: Distributional properties of initial and final steady states
New $\tau^k = 0$

	Data[*] U.S. 1992	Market structure Incomplete	Complete
Asset holding distribution in initial steady state			
Gini	0.78	0.78	0.78
99-100%	29.6	11.6	11.6
90-100%	66.1	60.2	60.2
80-100%	79.5	83.9	83.9
0-40%	1.35	1.35	1.35
Earnings Gini	0.63	0.21	0.00
Wealth – earnings correlation	0.23	0.34	0.00
Asset holding distribution in final steady state			
Gini	0.78	0.74	0.72
99-100%	29.6	10.1	10.8
90-100%	66.1	55.4	56.5
80-100%	79.5	79.0	79.2
0-40%	1.35	1.81	4.21
Earnings Gini	0.63	0.21	0.00
Wealth – earnings correlation	0.23	0.31	0.00

[*] The data column is taken from Diaz-Gimenez et. al. (1997) whose data source is the 1992 Survey of Consumer Finances.

Table 5: Expected gain from tax reforms – particular households

			Wealth		
		Productivity	Zero	Median	Mean
New τ^k = 0	Incomplete Markets	Low	-3.49	-2.96	1.50
		Medium	-3.39	-3.16	0.51
		High	-1.58	-1.46	0.72
	Complete Markets		-3.18	-2.95	1.07
New τ^k = 25.6	Incomplete Markets	Low	-1.08	-0.88	0.73
		Medium	-1.05	-0.97	0.36
		High	-0.35	-0.31	0.48
	Complete Markets		-0.80	-0.72	0.72
New τ^k = 47.7	Incomplete Markets	Low	0.45	0.33	-0.61
		Medium	0.43	0.38	-0.39
		High	0.00	-0.03	-0.48
	Complete Markets		0.13	0.08	-0.73

Table 6: Aggregate welfare effects of tax reforms

Ave. gain with incomplete markets (% of period consumption)	*New τ^k = 0*	*New τ^k = 25.6*	*New τ^k = 47.7*
Welfare gain	-0.95	-0.15	-0.10
Efficiency gain	0.12	0.22	-0.30
Distributional gain	-1.07	-0.37	0.40
Fractions in favor of reform:			
Low productivity	20.4	19.6	77.0
Medium productivity	23.6	25.7	70.5
High productivity	86.9	93.4	0.0
Entire population	26.8	28.9	67.1
Ave. gain with complete markets			
Welfare gain	-0.52	0.17	-0.42
Efficiency gain	1.07	0.72	-0.73
Distributional gain	-1.59	-0.55	0.31
Fraction in favor of reform	28.3	31.8	58.8

Table 7: Alternative parameterizations

New $\tau^k = 0$

Ave. gain with incomplete markets (% of period consumption)	Benchmark	Heaton & Lucas
Welfare gain	-0.95	0.53
Efficiency gain	0.12	0.93
Distributional gain	-1.07	-0.40
Fraction in favor of reform	26.8	50.5
Ave. gain with complete markets		
Welfare gain	-0.52	0.60
Efficiency gain	1.07	1.07
Distributional gain	-1.59	-0.47
Fraction in favor of reform	28.3	52.0

Figure 1: Paths for agg. consumption, investment, and the capital stock. New τ^k=0

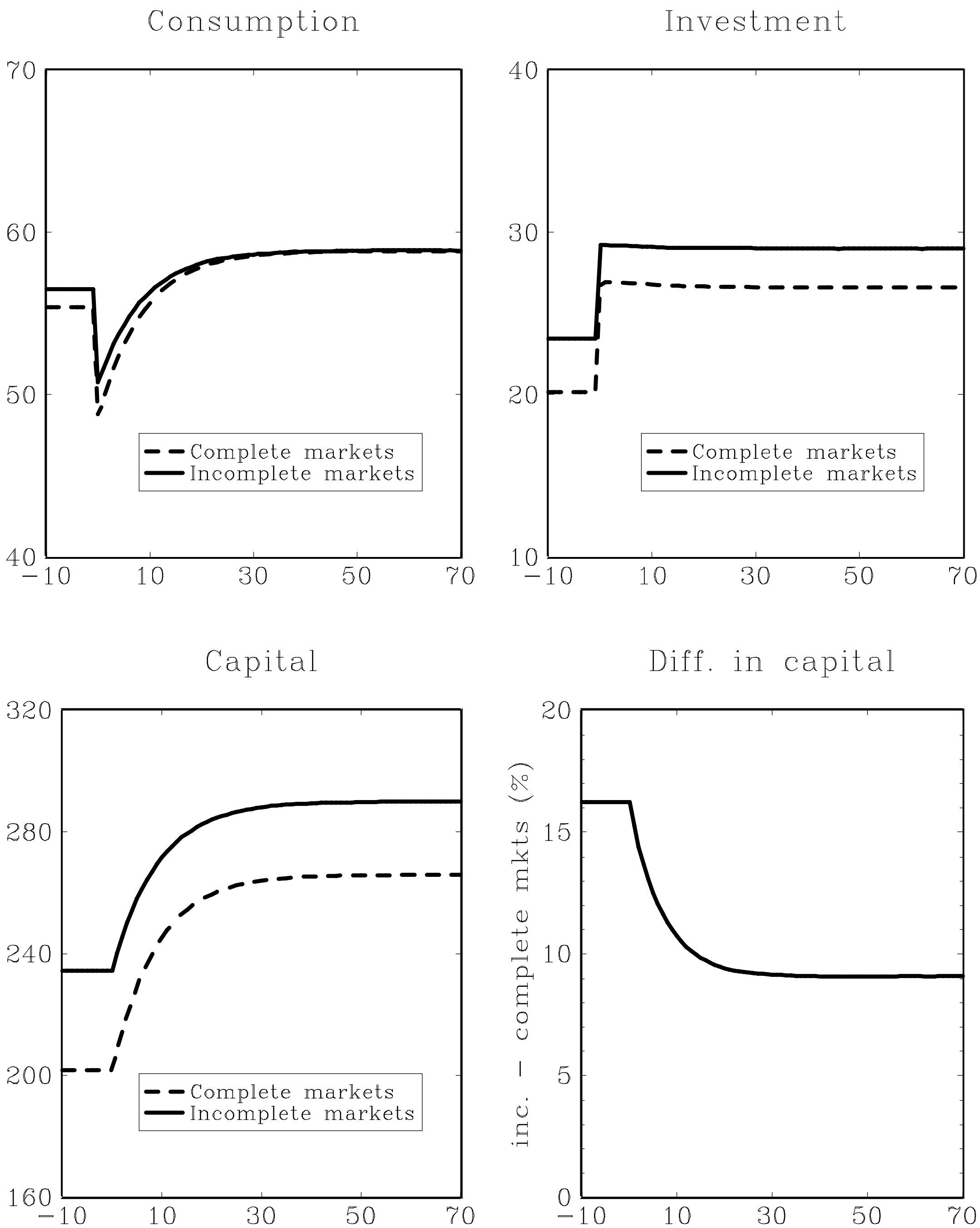

Figure 2: Paths for agg. debt, tax revenue and factor prices. New τ^k=0

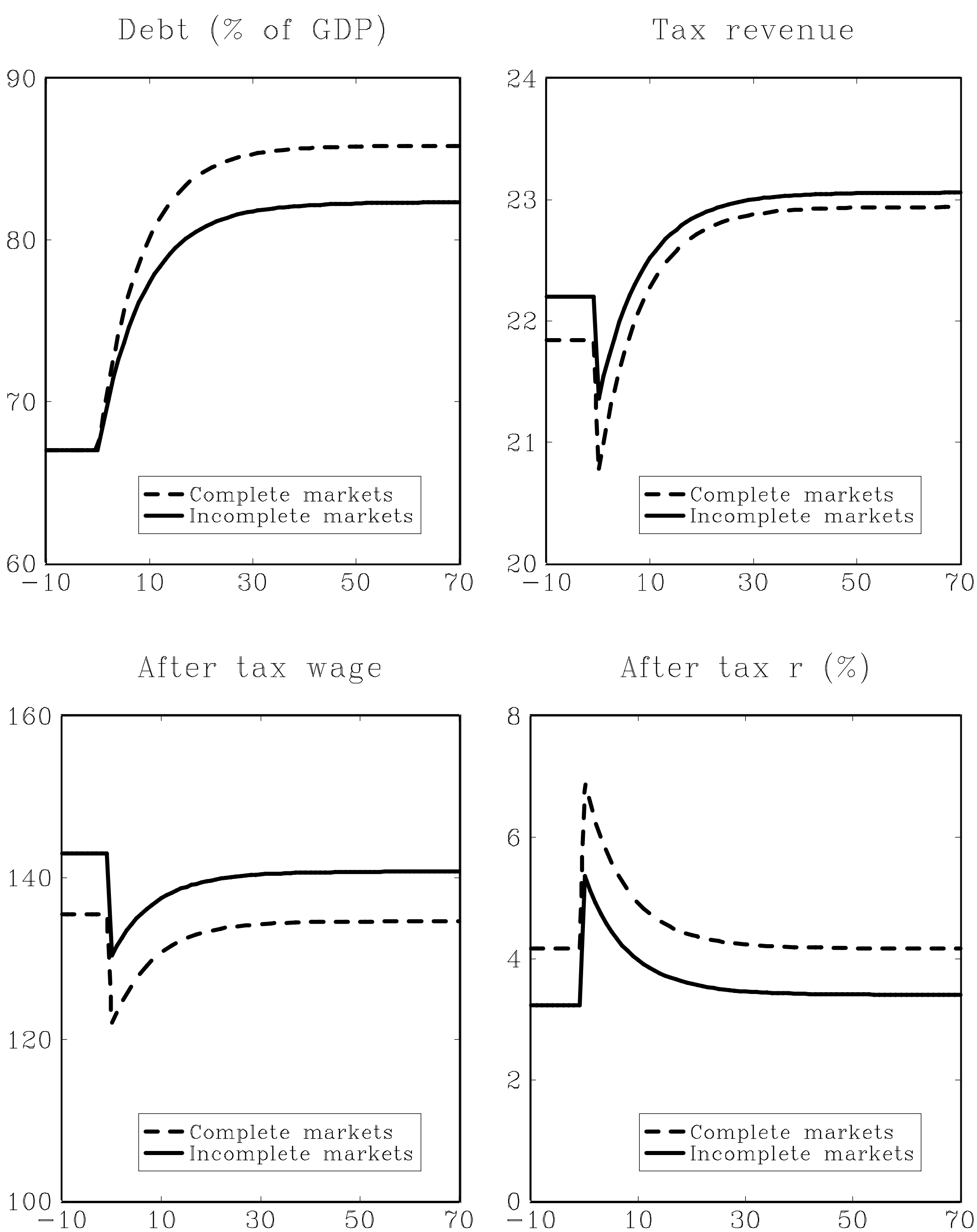

Figure 3: Household behavior with no tax reform (incomplete markets)

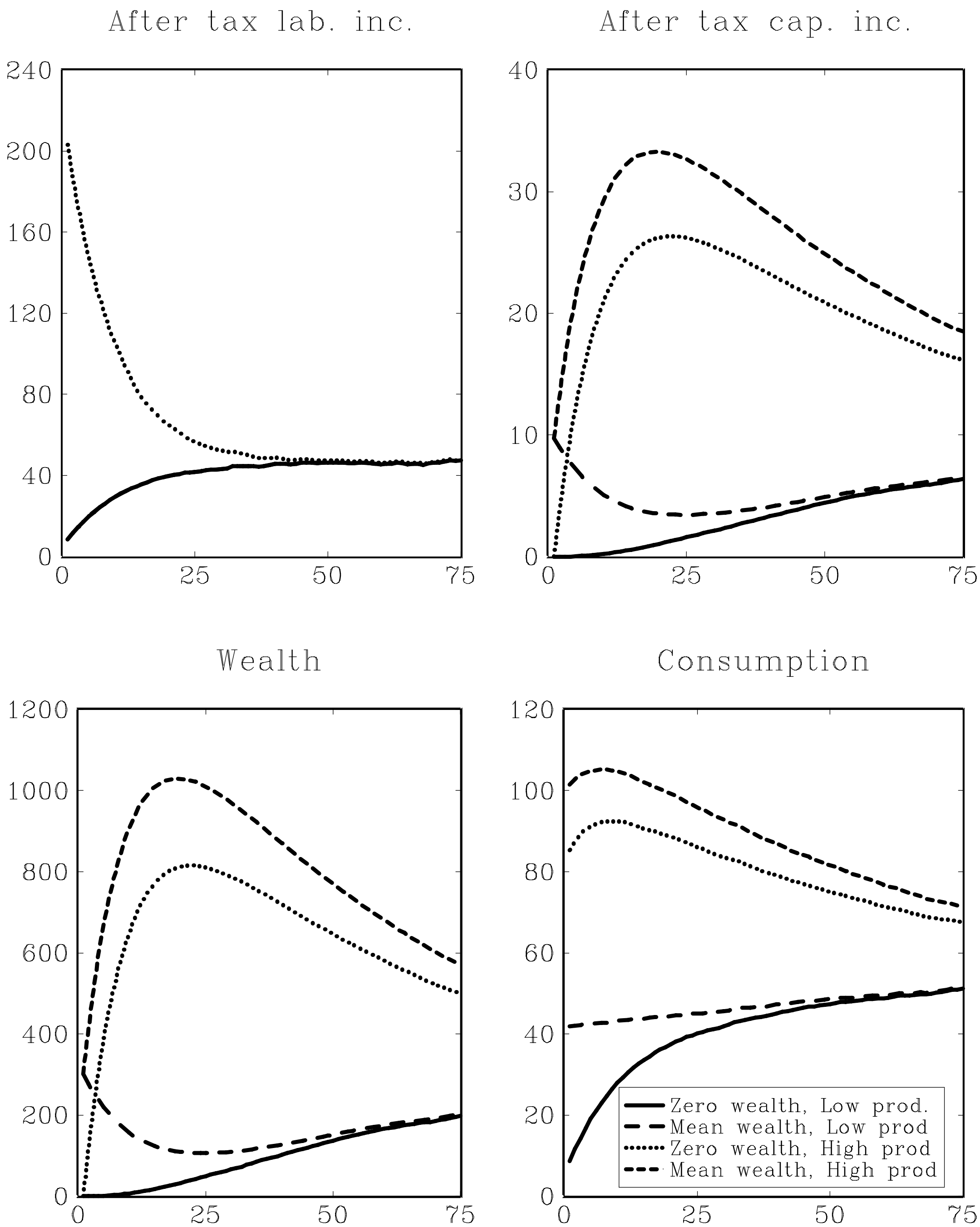

Figure 4: Household behavior: value w/ new τ^k=0 minus value w/o reform (incomplete markets)

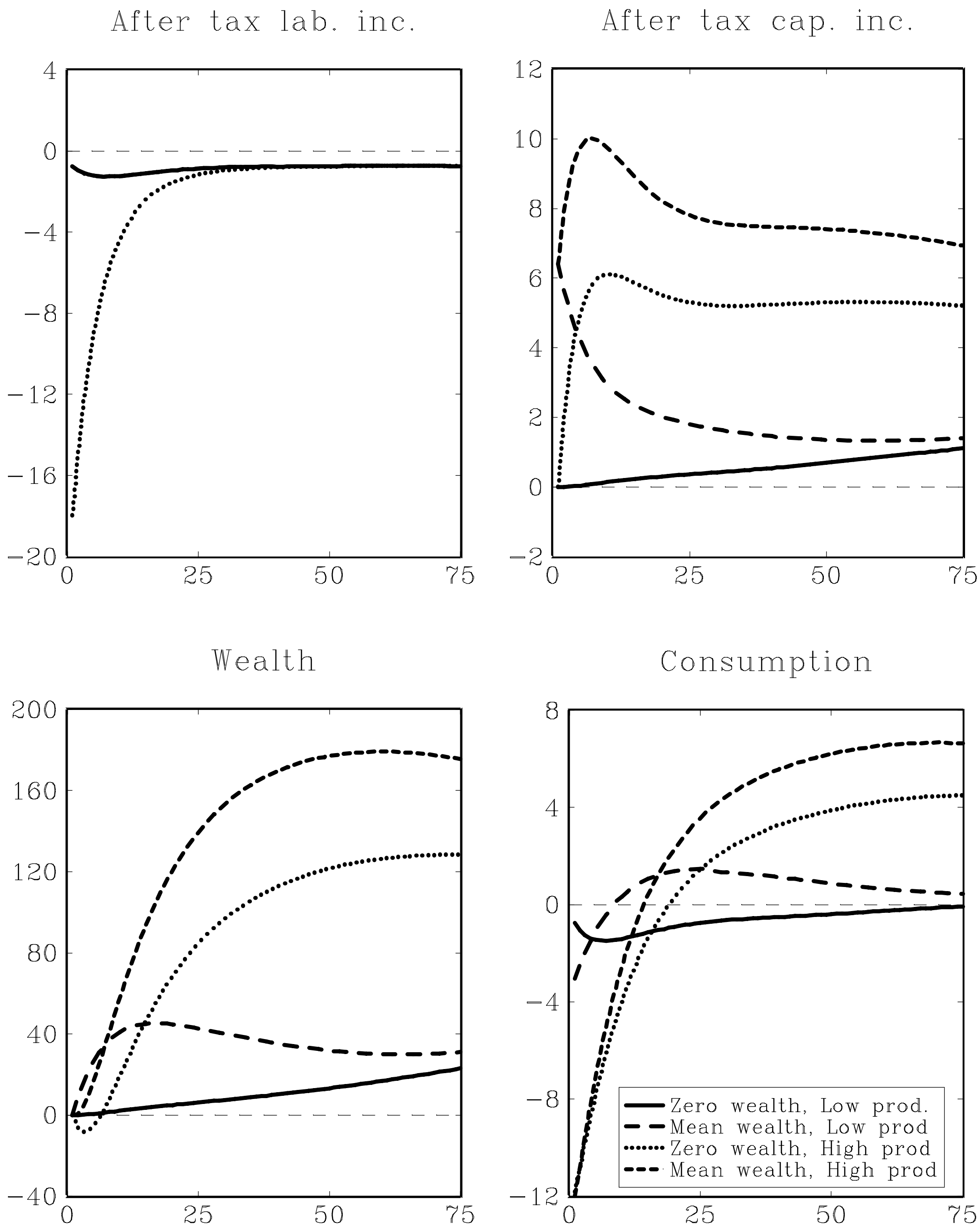

Figure 5: Dist. of gains and losses as equivalent % change in period consumption. New τ^k=0

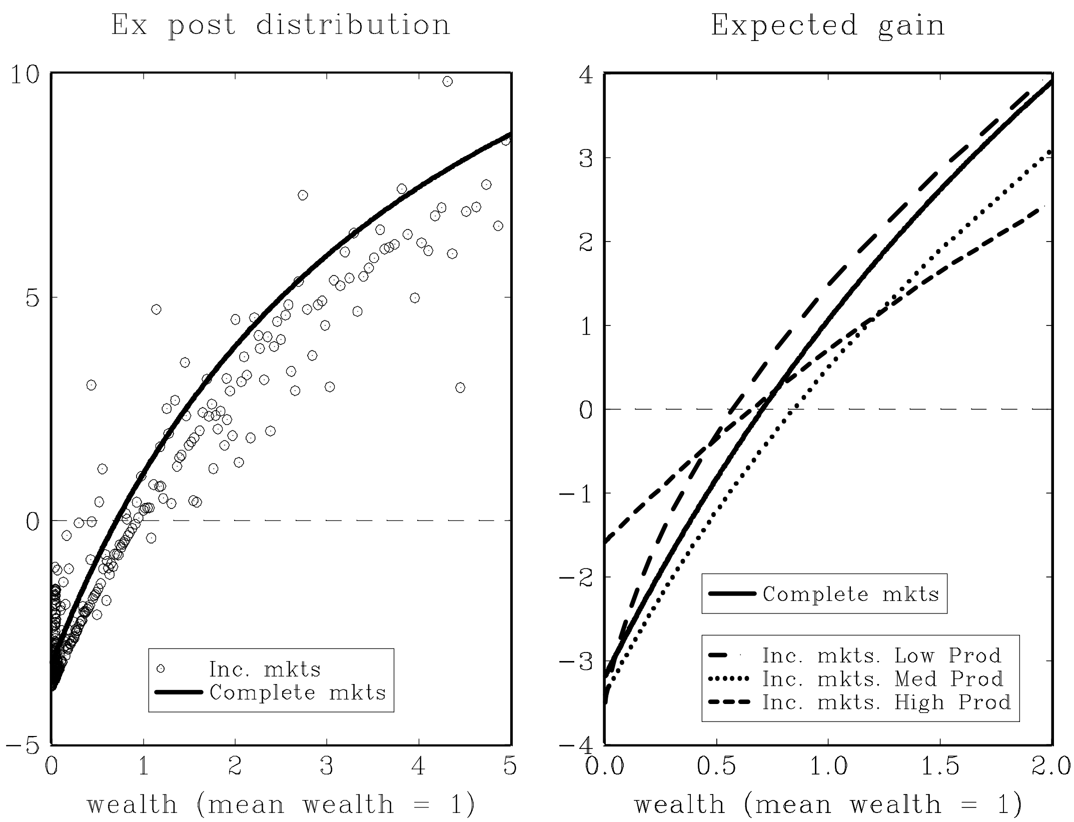

Figure 6: Welfare and efficiency gains
(% change in consumption)