The Distributional Component of the Price of the Tax Avoidance Service.

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Abstract

The traditional avoidance literature undeservedly neglects tax base distribution as a factor affecting the avoidance price, and generally assumed to be equal to the avoidance cost. In reality, avoidance providers are usually either high-skilled specialists or insiders. The strong collusion thus, naturally seems to be an assumption of the behavior of avoidance providers. Within such a framework, income distribution, which forms an avoidance demand together with tax codes, plays a very essential roll for the outcome of both avoidance price and quantity. My article models an economy with a monopolistic avoidance provider and imperfect information, and illustrates possible consequences of tax base changes. The paper examines the relationship between inequality and a government's ability to collect tax revenue, and also considers the possible outcome of a tax base broadening. Furthermore, it provides an additional explanation for the secession decision.

Journal of Economic Literature Classification: C72; D31; D42; D43; D69; D82; E61; E65; F15; G28; G29; H21; H24; H25; H31; H32; J61; K34; L12; O17

Key words: Tax avoidance, optimal taxation, income distribution, endogenous prices, inequality, tax base broadening, secession.

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1 Introduction

Tax avoidance is a problem great importance for many countries. Taxes do not collect themselves and taxpayers may try to avoid their legal liabilities[1]. Thus, according to Slemrod and Yitzhaki (2000), the U.S. government spends about 10% of the total tax revenue on tax enforcement. Furthermore, the Internal Revenue Service estimates that about 17% of income tax are not paid. The figure for most other countries is probably even higher. The government's inability to collect a sufficient tax revenue was the main reason for the Russian financial crisis in 1998. The tax avoidance practice forces the government to increase the tax burden on non-avoiding tax payers, which also leads to an increase in deadweight loss. For example, Feldstein (1995) estimates that due to tax avoidance, the deadweight loss from income tax in the U.S. is more than ten times larger than would otherwise be the case. Since rich individuals usually use avoidance practice, the government levies heavier taxes on the poor who have larger marginal utility. This also slows down the development of small business and economic growth. Moreover, avoidance behavior may cause shifts in the real economy, thereby affecting occupational choice, human capital investment and labor supply[2].

My particular research has been motivated by observing the Russian government's attempts to improve tax collection in 1996-1997. The government broadened the tax bases, introduced new sources as subject to tax, and finally, organized a new emergency committee to enforce tax collection. The efforts gave no results, however, which motivated me to design a model where the tax avoidance supply is adjusted to changes in tax avoidance demand. In particular, I investigate an economy where the avoidance provider endogenously defines the price for the tax avoidance service. An avoidance provider's ability to react not only to a new tax code, but also to the changes in the tax base distribution, creates an additional problem for the government. My paper introduces the distribution component of price for the tax avoidance services, for the first time.

Studying the existing avoidance literature, I find that it neglects the tax base distribution as a factor affecting the avoidance price. The authors assume either perfect competition between the provider's of the tax avoidance service or the ability to reveal the clients' income by an avoidance provider. In the first case, the price a householder pays for the avoidance service equals the cost, while in the second case, the avoidance provider receives an ability for the perfect price discrimination. In both cases, aggregate demand for the tax avoidance service has no impact on the avoidance price formation. Cross and Shaw (1992), Atkinson and Stiglitz (1980) and Slemrod (1998) are representative examples

[1] In this paper, I use the term "avoidance" to denote all transactions motivated by the desire not to pay tax. They might be illegal, but non-revealable or non-punishable due to imperfections in the legal system.

[2] For a detailed overview of the main problems related to tax compliance see Andreoni, Erard and Feinstein (1998). For examples of popular ways of carrying out tax avoidance in developed countries, see Slemrod (1995).

of the first class of the literature. In Cross and Shaw (1992), the tax avoider pays a fixed exogenous price. In Atkinson and Stiglitz (1980), the avoidance price depends on the avoidance amount and the government's effort to enforce tax collection. Slemrod (1998) proposes that initially richer individuals pay a lower price for the same amount of income saved from taxation. Hindriks, Keen and Muthoo's (1999) paper belongs to the second class of the literature. This paper provides the model where a tax inspector audits the true individual's income, but can be bribed by the householder to misreport the audit result to the government. In this model, the tax inspector is considered to be a monopolistic avoidance provider with perfect monitoring ability. Although, the price he sets for providing the tax evasion service is defined endogenously, it does not depend on the aggregate demand for tax avoidance. Demand becomes important for price setting after the introduction of an imperfect information structure, which reduces the possibility to monitor and price discriminate.

Some degree of market power seems to be a reasonable assumption for avoidance provision. In reality, the tax avoidance service is provided by high-skilled intermediaries, such as accountants or legal consultants, who could also be insiders with access to specific information. In developing countries, tax avoidance services are often provided by the firms enjoying especially favorable legal treatment, or by corrupted officials. Therefore, avoidance provision should not be considered as a business easily entered into, and a high degree of monopolistic power is rather a rational assumption. Together with the assumption of imperfect information the latter gives avoidance demand a more important role in price setting. In turn, the demand for tax avoidance depends on tax base distribution to at least the same extent as on the tax code set by the government. Hence, changes in the tax base distribution should not be neglected in a tax avoiding economy, when the government decides to broaden a tax base or implement a new immigration policy, or considers regional issues.

To show certain possible consequences of various policies related to the changes in the tax base distribution, I make my model as illustrative as possible. The model assumes monopolistic power of the agency, or a strong collusion among the institutions providing tax avoidance services. Another assumption is imperfect information: both the government and the agency know the distribution of a tax base but not the amount owned by any particular individual. The avoidance service is indivisible, the agency either helps hide the entire income, or none at all. Moreover, the householder's income is not revealed to the agency. Therefore, the only option for the agency is to set a fixed price for its service. I make such strong assumptions to simplify the model, the main purpose of which is to illustrate the possible consequences of avoidance price reactions to changes in demand. All conclusion can hold for an economy characterized by a lower degree of market power and some ability to price discriminate.

Due to the prevalence of tax avoidance, my model has a very wide range of applications, and it considers several of those which I find to be among the most interesting. In particular, I analyze the relation between inequality and a government's ability to collect tax revenue. It might be that maximum revenue grows with inequality, while inefficiency due to avoidance purchase falls. Hence, this paper contributes to the dilemma of a trade-off between the advantages of equity and the disadvantages related to the unpleasantness that will occur due to a government's disability to collect taxes when inequality is small. Even when the government can collect a target revenue, a higher degree of equality will increase inefficiency due to higher avoidance payments. The government can only operate effectively in an economy with a small inequality if an agency's cost, or the cost of the access to the avoidance service (the avoidance cost), is high enough. Moreover, the cost of additional tax revenue collection in terms of the funds flowed to the avoidance provider decreases with inequality.

Another important application is the analysis of a broadening of the tax base. On the one hand, it might be an efficient measure of the increase in the tax avoidance cost when the avoidance technology reclassifies a particular business to a class subject to tax exemption. In this case, the elimination of tax exemption would force the agency to spend additional time and money on developing and realizing new tax avoidance schemes: an effect which has most likely been observed after the implementation of the Tax Reform Act (TRA 86). On the other hand, a tax base broadening might also lead to a reduction in revenue collection, due to the change in the tax base distribution and the ensuing adjustment of the avoidance price.

I also investigate the issues of unification and secession, since those acts lead to crucial changes in the shape of income distributions. Secession is favorable for the tax avoidance providers because it gives them an additional possibility for price discrimination. Maximizing the profit tax, avoidance providers may set the lower tax avoidance cost for the poorer country, which creates the possibility of tax avoidance for the regional elite and provides an important reason for secession of the poorer region. On the other hand, the elite in the wealthier region could expect less progressiveness and therefore smaller avoidance expenditures after the secession. Despite the desistance of secession incentives, the government operates more efficiently in a unified economy: collecting the same tax revenue it can set a tax code allowing a larger average after tax/avoidance income.

The paper is organized as follows. Section 2 presents a general model and provides technical solutions for a specific case of a risk neutral householder's utility function. Section 3 investigates the relationship between inequality and tax collection. Section 4 extends the set-up of the model by introducing an agency cost or the cost of access to the avoidance service. Section 5 considers a wide range of applications including tax base broadening, immigration and unification issues. Section 6 concludes.

2 General Model

2.1 The Players

There are three players in the modeled economy: householders, the government and the agency providing tax avoidance services. There is a continuum of householders, normalized to 1. Each householder exogenously obtains some economic item x refereed to as "income", which the government taxes in order to collect its revenue. $F(x)-$is a cumulative distribution of x, known to everybody, while the particular x belonging to each householder is private information. The government sets up a tax code on the level $\tau(x)$ in order to finance its target expenditure, $\overline{g}$. If the government is not able to collect $\overline{g}$, it simply maximizes a revenue[3]. Otherwise, it collects revenue $\overline{g}$ by setting a tax code maximizing the social welfare function W equal to householders' average utility. The agency helps a householder avoid a tax by hiding his personal income x, and it has a monopoly on tax avoidance services. Since the agency is unable to monitor individual income, before or after providing assistance, it sets a fixed price T for its service. The shape of income distribution $F(x)$ is known to the agency. Let me consider the case where the agency has a zero cost per client and access to its service is free.

The assumption about not revealing one's income to the agency is probably too strong, but it is supported by evidence. For instance, some regions (Panama, the Bahamas, British Virgins Islands, etc.) declared as off-shore zones set up special rules for non-resident international business companies. According to these regulations, a firm registered in these regions pays no regional taxes and is not required to present an annual report, it only pays registration and fixed annual fees. Why does the agency provide such services without demanding a statement of income? The main reason might be confidentiality: by providing financial privacy, the agency can attract a larger number of clients. Moreover, once the agency can reveal a household's income, it has the option of asking the householder for much higher payment by threatening to report both his income and his avoidance practice to the government. The reasons for providing avoidance services at a fixed price are worth considering in a separate paper, but the existence of such practice supports the above assumption.

Moreover, if my assumptions about either the monopolistic power or the disability of the agency to discriminate clients were weaker, a tax base distribution would remain important for avoidance price formation.

2.2 Formalization

After the government has set up a tax rule $\tau(x)$, the agency chooses a price T for its service. Then, each individual decides either to declare his income x and pay $\tau(x)$ to the government or use an avoidance service, paying T to the agency and nothing to the government. Since an individual maximizes his after tax/avoidance earnings, he prefers to pay tax if $\tau(x) < T$.

If $F(x)$ is a continuous function, strictly-monotonic on the set $\{x,\ 0 < F(x) < 1\}$,

[3]It is a question of what actions a Government should take if it is not able to collect the target revenue. Further, I will show that maximizing revenue collection might be very inefficient, due to the high level of the marginal rate of avoidance expenditure. This assumption is motivated by Russian events. The Russian parliament has set up inexecutable revenue targets during 1996-1999 and then, the executive authorities have tried to do their best in collecting taxes. The attempt to implement expenditure sequestering in 1997 was unsuccessful.

and householders are either risk-averse or risk-neutral, then $\tau(x)$ can be selected from non-decreasing functions without loss of generality. This is formulated in more detail in the next proposition:

Proposition 1 *If $F(x)$ is a continuous function, strictly monotonic on the set $\{x,\ 0 < F(x) < 1\}$ and householders are risk-averse or risk-neutral,*

then $\forall\ \tau(.)$, the government can choose another tax code $\tau_1(.)$ such that:

1) $\tau_1(x)$ is non-decreasing;

2) the government will collect the same amount of revenue;

3) the welfare function, which is equal to the householder's average utility function, is not smaller than under the initial tax code $W(\tau(.)) \le W(\tau_1(.))$;

The formal proof is given in Appendix 7. The intuition is very simple: if a richer individual pays less tax than a particular householder, the government can simply exchange their tax duties, which would lead to the same revenue collection and the same tax avoidance, but might improve the social objective function.

Based on proposition 1, I will further consider the only non-decreasing tax codes for continuous monotonic distributions. When the government and the agency have made their decision, all householders are separated into three groups: those preferring to be taxed, those who are indifferent and tax avoiders. For simplicity, I assume that indifferent individuals would avoid taxation (since the agency has the option of decreasing its price by an infinitely small amount, thereby attracting all indifferent individuals).

Let $\widehat{x}$ be a solution of equation (1)

$$\widehat{x} = \inf_x \{\tau(x) = T\}. \tag{1}$$

Then, taxpayers whose income is larger than or equal to $\widehat{x}$ will use the agency's service, while the others will prefer to declare their income and pay tax $\tau(x)$ to the government. Further, I will call $\widehat{x}$ an income "breakdown". Thus, an agency's revenue is calculated by formula (2)

$$T\,(1 - F(\widehat{x})) = \tau(\widehat{x})(1 - F(\widehat{x})). \tag{2}$$

The agency maximizes its profit m by solving problem (3):

$$m \stackrel{d}{=} \max_x \{\tau(x)(1 - F(x))\}; \tag{3}$$

where $\tau(x)$ is a non-decreasing function, defining a tax level. There should exist some $\widehat{x}$ so that $\lim\limits_{x\to\widehat{x}} \tau(x)(1-F(x)) = m$. Solving this problem, the agency would set price $T = \tau(\widehat{x}) = \frac{m}{(1-F(\widehat{x}))}$.

The solution of the agency's problem is illustrated in figure 1, where a thin line represents an agency's isoprofit curve $\frac{m}{(1-F(\widehat{x}))}$, expressing the largest possible profit m under the given tax code $\tau(x)$, drawn as a solid line. Income

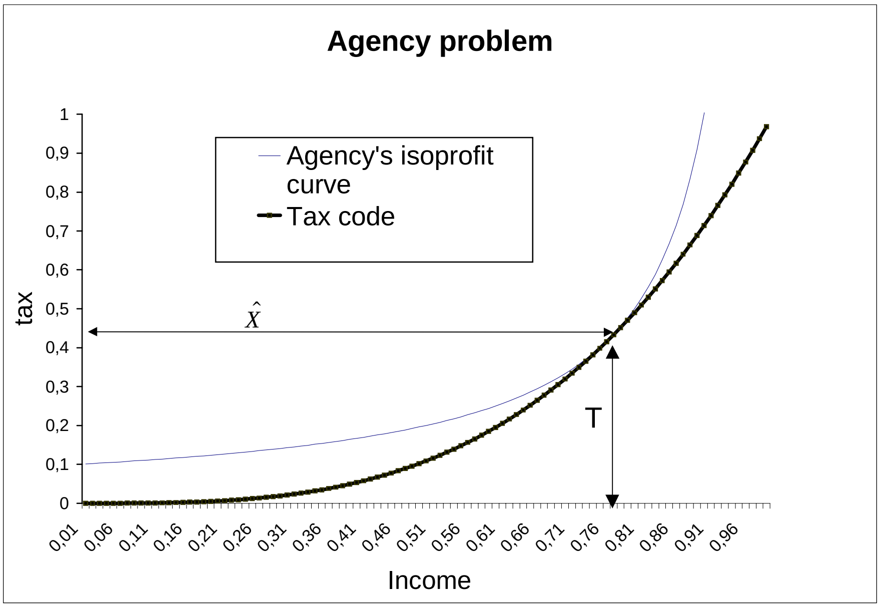

Figure 1:

breakdown, $\widehat{x}$, is defined as the maximum of a set of incomes, where the best available agency's isoprofit curve equals the tax code [4].

Once $\widehat{x}$ has been chosen, government revenue is calculated by formula (4):

$$\int_0^{\widehat{x}} \tau(x)dF(x) = g. \tag{4}$$

The government needs to collect a target revenue $\overline{g}$ setting $\tau(x)$, which maximizes householders' average utility, depending on the after-tax/avoidance income.

Generally, the problem can be reformulated in the following way:

[4] To simplify the presentation, I assume the agency to be government friendly and in case of indifference, the agency chooses the income "breakdown" that is the best for the government. Otherwise, the government can enforce this by slightly reducing a tax code.

$$\begin{aligned}
\max_{\tau(.)} W(\tau(.)) &= \int_0^{\widehat{x}} u(x-\tau(x))dF(x) + \int_{\widehat{x}}^{1} u(x-T)dF(x); &(5)\\
\widehat{x} &= \arg\max \tau(\widehat{x})(1-F(\widehat{x})); &(6)\\
\tau(\widehat{x}) &= T; &(7)\\
\int_0^{\widehat{x}} \tau(x)dF(x) &= \overline{g}; &(8)\\
\tau(x) &\leq x \text{ for any } x \in [0,1], &(9)
\end{aligned}$$

where formula (5) represents the government objective function. Householders whose income x is less than $\widehat{x}$ pay tax $\tau(x)$ consuming $x-\tau(x)$, while richer tax payers purchase a tax avoidance service paying price T and consuming $x-T$ good. Expressions (6-9) represent the budget constraints. Formulas (6) and (7) describe the mechanism through which the agency sets its price, while equation (8) expresses the necessity to collect a certain amount of government revenue, and condition (9) shows that the tax level cannot be higher than income.

I am particularly interested in finding an upper limit for the government's ability to collect revenue. The maximum tax revenue G can be found in equations $(10-12)$ and does not depend on householders' preferences.

$$\begin{aligned}
G &= \max_{\tau(.)} g(\tau(.)) = \int_0^{\widehat{x}} \tau(x)dF(x); &(10)\\
st. \qquad \widehat{x} &= \arg\max_{x} \tau(x)(1-F(x)); &(11)\\
\tau(x) &\leq x, \; for \; any \;\; x \in [0,1] &(12)
\end{aligned}$$

2.3 Risk Neutrality

In this subsection, I will consider a risk neutral utility function, $u(x) = x$. Since this assumption does not only simplify the model but also allows me to catch the effects non-related to risk aversion, it is worth considering as an extreme and interesting case.

If the government sets $\tau(x)$ to maximize the average utility function, which is proportional to after-tax income, formula (5) can be simplified as (13)

$$\begin{aligned}
\max_{\tau(.)} \;\; W(x) &= \int_0^{\widehat{x}} (x-\tau(x))dF(x) + \int_{\widehat{x}}^{1} (x-T)dF(x) = &(13)\\
&= \int_0^{1} xdF(x) - \overline{g} - T(1-F(\widehat{x})).
\end{aligned}$$

Expression (13) shows that maximizing the social welfare function is equivalent to minimizing agency revenue. Therefore, problem (5) can be rewritten as problem (14)

$$\underset{\tau(.)}{min} T(1 - F(\widehat{x})) \tag{14}$$

while constraints (6-9) remain unchanged.

2.4 Solution Under Different Restrictions

This section provides the solution to problems $(14, 6 - 9)$.

2.4.1 No Restrictions

It might be easier to consider a dual problem, where the government first decides how large a profit, m, it wants to give to the agency, and then maximizes its revenue choosing the optimal tax code $\tau(x, m)$ from the set of tax codes, thereby providing the agency with revenue m

$$\tau(., m) = \underset{\tau(.),\ m(\tau)=m}{\arg\max}\ g(\tau). \tag{15}$$

Solving equation (15), I construct $g(m) = g(\tau(., m))$. For any level of $\overline{g}$, I find the minimum value of m, such that $g(m) = \overline{g}$. Those m, $\tau(., m)$, $\widehat{x}$ would represent the solution to an original problem $(14, 6 - 9)$.

Now, I will describe the solution to a dual problem in more detail. According to conditions (6) and (7), an agency's isoprofit curve is represented by the expression $\frac{m}{1-F(x)}$, where m is an agency's profit. Accounting for restriction (9), the government should thus set up $\tau(x) = \min(x, \frac{m}{1-F(x)})$ (see Figure 1). The agency is indifferent to which point $\widehat{x}$ to choose on its isoprofit curve, while the government prefers the largest $\widehat{x}$.

As a widely applied example, let me consider a case when the isoprofit curve does not cross a 45 degree line more than twice [5]. Let (x_1, x_2) be all solutions of equation $x = \frac{m}{1-F(x)}$ and $x_1 < x_2$. Then, the highest value of the government revenue equals $g(m) = \int\limits_0^{x_1} x dF(x) + \int\limits_{x_1}^{x_2} \frac{m}{1-F(x)} dF(x)$; and it might be simplified as formula (17).

[5] This is true for a wide class of distribution functions. For example; if

$$F'' + 2F'^2 - F''F \geq 0, \tag{16}$$

the agency's isoprofit curve is convex and it either does not cross, tangents or crosses a 45 degree line at two points. Although an isoprofit curve is not convex for the lognormal distribution, it nevertheless crosses a 45 degree line at two points.

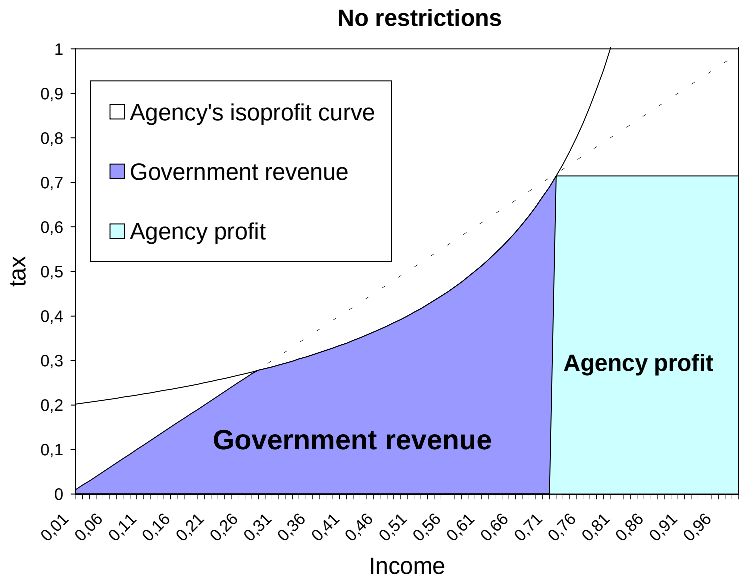

Figure 2:

$$g(m) = \int_0^{x_1} x dF(x) + m \ln\left(\frac{1 - F(x_1)}{1 - F(x_2)}\right). \tag{17}$$

Figure 2 represents solution (17) graphically.

Therefore, the optimal tax code in the assumption of risk neutrality is $\tau(x) = \min(x, \frac{m(\overline{g})}{1-F(x)})$; where $m(\overline{g}) = \min_{g(m)=\overline{g}} m$.

Additional restrictions do not principally change the algorithm, they only modify function $g(m)$ by adding the corresponding restriction to formula (15).

Figure 2 shows that a non-restricted government would tax the poorest individuals very heavily. A tax code shaped like that in Figure 2 could be observed in Russia, where target revenue was too high in 1995 - 1999, when the Russian income tax code had five different brackets. Although formally progressive, the frequent usage of wage and pension arrears could be interpreted as a heavier tax for the poor.

2.4.2 Progressiveness and Marginal Tax Restrictions

All the logics in this subsection is applicable under the assumption of convexity of agency's isoprofit curve. I will now consider the case when the government is forced to set a progressive tax for some political reason. Under this restriction, the government should use a proportional tax for householders whose income is less than x_t, where x_t[6] is the income level characterized by equivalence between marginal and average tax rates. For those people whose income is larger than x_t,[7] the tax level should be the same as in the previous case: $\tau(x) = \min(x, \frac{m}{1-F(x)})$.

The value of x_t is shown in the next calculation. Let α be the average tax rate for people whose income is less than x_t

$$\alpha = \frac{m}{1 - F(x_t)} / x_t. \tag{18}$$

At the same time, α is equal to a marginal tax rate and presents a slope of the agency's isoprofit curve at point x_t

$$\alpha = \frac{m}{(1 - F(x_t))^2} * F'(x_t). \tag{19}$$

Equations (18) and (19) define x_t

$$x_t F'(x_t) = 1 - F(x_t). \tag{20}$$

If a householder has the option to throw away his income, then the government has an additional restriction: the marginal tax rate should not exceed 1. Further, I will consider this restriction as given. Now, the breakdown point of income $\widehat{x}$ is equal to x_{mr}, found by the condition that marginal tax is equal to one, in other words, x_{mr} is a point where the agency's isoprofit curve has a 45 degree slope

$$\frac{d}{dx}\left(\frac{m}{1 - F(x_{mr})}\right) = \frac{mF'(x_{mr})}{(1 - F(x_{mr}))^2} = 1. \tag{21}$$

Given both restrictions, a government revenue is calculated as $g(m) = \alpha \int_0^{x_t} x dF(x) + \int_{x_t}^{x_{mr}} \frac{m}{1-F(x)} dF(x)$; or more simply:

$$g(m) = \frac{m}{x_t (1 - F(x_t))} \int_0^{x_t} x dF(x) + m \ln\left(\frac{1 - F(x_t)}{1 - F(x_{mr})}\right). \tag{22}$$

Figure 3 gives a graphical illustration.

[6] Here, I should mention an interesting property of x_t, which would be a "breakdown" point for any proportional tax code independent in the tax rate. It means that if avoidance is costless, a change of the rate in the proportional tax code would not affect avoidance behavior, but the avoidance price, and the loss for the economy would be proportional to the tax rate and tax revenue.

[7] Although, such x_t may neither exist nor be unique in general, but all the further reasoning are true for the case of logarithmic distribution or when the agency's isoprofit curve is convex on the interval $[x_1; x_2]$. The convexity of a cumulative distribution function on this interval is sufficient condition for the convexity of an isoprofit curve. This follows from condition (16).

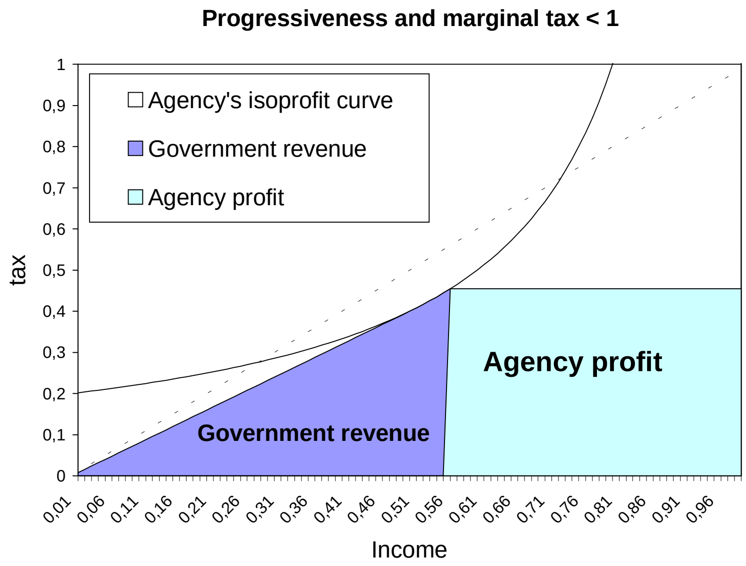

Figure 3:

Appendix 8 provides a detailed solution for a uniform distribution under different restrictions.

2.5 A Discrete Case

In reality, the information about income distribution has never been collected with perfect precision. Income statistics separate society into about 20 groups, distinguished by their respective income levels. Such approximation gives a reason for considering a discrete income economy.

2.5.1 Formalization

There are N income groups in an economy, where x_i, τ_i, n_i represent the income, the tax level and the size of group i, respectively.

Agency problem. Let the groups be sorted by a tax level. The optimal policy for the agency is to choose a price $T = \tau_{i^*}$

$$i^* = \max \left\{ i : argmax \left(\tau_i * \sum_{j \geq i} n_j \right) \right\}. \tag{23}$$

Government revenue. Formula (23) defines a government revenue calculated by expression (24)

$$g = \sum_{j<i^*} n_j \tau_j. \tag{24}$$

Distribution $F(x)$ is no longer continuous - and the assumptions of Proposition 1 are violated. It might be the case that the optimal tax code is not non-decreasing.

Example 2 ***"When a non-monotonic tax code must be implemented to increase a revenue"'***

An economy consists of 3 *groups with sizes* 4, 2 *and* 1 *and income levels* 0.3, 0.4 *and* 1. *The maximal government revenue can be collected with tax levels* 1/5, 1/7, *and* 1.

3 Inequality and Tax Revenue

In this section I investigate the relationship between inequality and tax revenue collection. First, I show that tax revenue as a percentage of total income grows with inequality in assumption of lognormal distribution if the government sets probational tax code. Then I consider the economy where government is unable to collect target revenue and tries to collect as much as possible. Along the way, I show the inefficiency of such a policy: small decrease in target revenue compare to the maximum available allows the householders to save large income part from being paid for the tax avoidance services. Finally, I demonstrate that higher inequality lower "cost of funds". In other words, if inequality is bigger, increasing in target revenue leads to the lower increasing in householders' loses due to paying for tax avoidance services.

3.1 The Possibility of Positive Correlation Between Tax Revenue and Inequality Under Stable Tax Code.

First, let me provide stylized facts observed in the US economy.

Figure 4 shows a positive correlation between the Gini coefficient and tax revenue[8] before and after the Economic Recovery Tax Act imposed in 1982. Such a relation might be explained by the willingness of the government to redistribute more when inequality grows. This explanation requires a positive correlation between inequality and non-interest government expenditure, which

[8] The data for the Gini coefficient are taken from the United Nation University date base. http://www.wider.unu.edu/wiid/wiid.htm. I choose Family as the Reference Unit, and Brandolini 1998 as the source. The data represent the inequality of monetary income.

IMF: Government Financial Statistics is a source for budget indicators.

Stylized facts: US

before and after the Economic Recovery Tax Act imposed in 1982

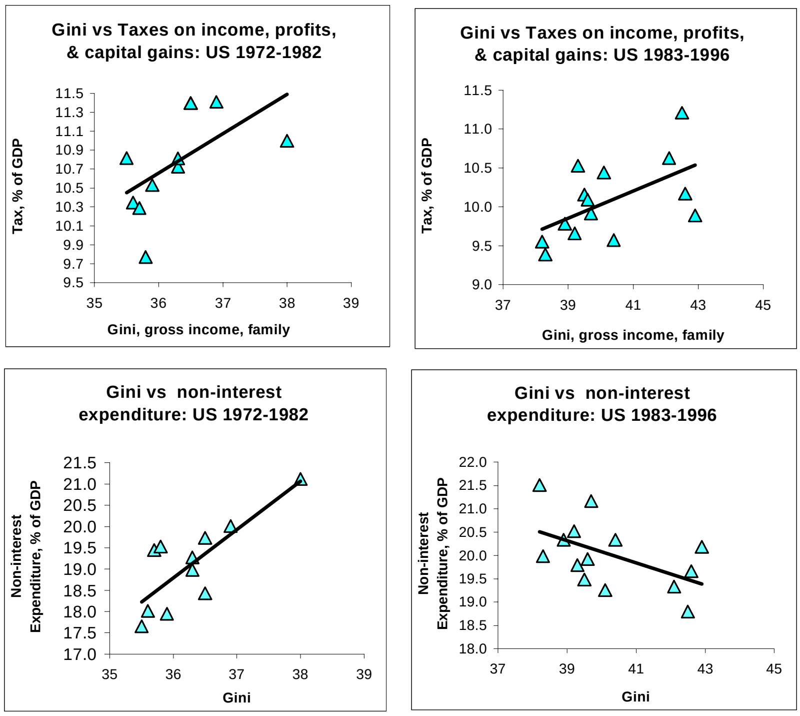

Figure 4:

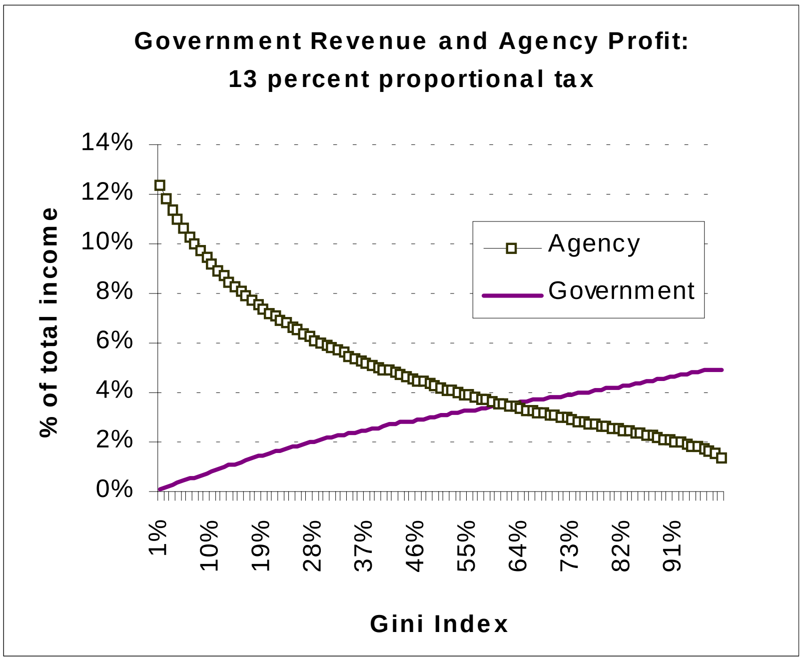

Figure 5:

has been observed during 1972-1982. The relation has then been negative since the government has used revenue to reduce outstanding debt. My paper provides an additional explanation, showing that tax revenue as a percentage of GDP might grow with inequality when the tax code is not significantly changed. To show this, let me consider a flat tax rate in an economy with lognormal income distribution.

Figure 5 illustrates the growing government revenue, and the decreasing agency profit with inequality under the stable flat tax rate.

3.2 When The Government Maximizes Its Revenue

Now, I will show that the government could not survive in an economy with small inequality if tax avoidance is costless. This is intuitively understandable; whatever tax the government sets, the agency would ask for a slightly lower price T for its service, and all householders would avoid paying the tax. In an egalitarian world with costless avoidance, the government should charge no tax at all, since householders would suffer from a loss of income equal to the tax level, but they would not benefit from the consumption of any public goods provided by the government, the revenue of which would be zero. If the economy is egalitarian and the agency has zero costs, all attempts by the government to

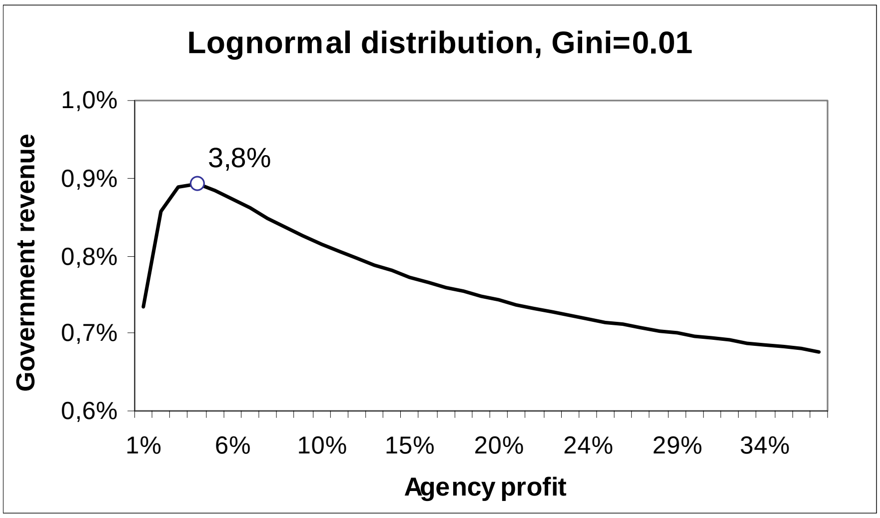

Figure 6:

increase the tax revenue would only lead to larger agency profits. Although the arguments I use for an egalitarian case can also be applied to an economy with perfect concentration, there are two particular distributions for which I find that the government is able to collect large revenues when the Gini index is huge, while tax collection remains negligible if inequality is small.

When an economy has a small inequality, the government can collect a small tax revenue, giving some profit to the agency. This profit depends on the shape of the distribution function $F(x)$ and the government target revenue, $\overline{g}$. Figure 6 represents the "Agency profit/government revenue" graph for lognormal and $F(x) = x^{\gamma}$ distributions with Gini=0.01.

If the government tries to collect as much revenue as possible, facing income distribution $F(x) = x^{\gamma}$, the agency profit would be nearly 87% of total income. In the case of lognormal distribution, it would be quite low (3.8%). The maximum government revenue is small in both cases: 0.9% for power distribution and 1.8% for lognormal distribution (see figures 6 and 7). Figure 7 supports the idea that maximizing the collected revenue might be a very irrational policy, since the government could save half the national income from going to the agency by reducing the target tax revenue by less than 0.05% .

3.2.1 The Distribution $F(x) = x^{\gamma}$ ("power" distribution)

This subsection will consider an economy with a cumulative distribution $F(x) = x^{\gamma}$, normalized in such a way that the income of the richest person equals 1. Further, I will call this a "power" distribution. Parameter γ is the measure of inequality for "power" distribution. The lower is γ, the larger is income

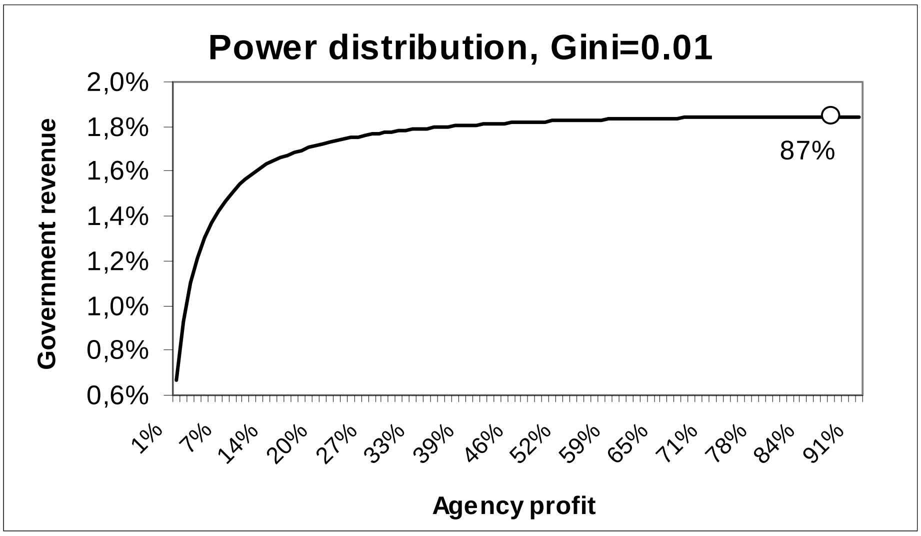

Figure 7:

concentration in the economy. If $\gamma = 0$, almost all individuals have zero income but the richest one has an income equal to 1, which means perfect concentration. If $\gamma = \infty$, almost all individuals have an income equal to 1, which means equality. The correspondent Gini coefficient is represented by formula (25)

$$Gini = 1 - 2\int_0^1 L(n)dn = 1 - \frac{2}{2 + \frac{1}{\gamma}} = \frac{1}{1 + 2\gamma}. \tag{25}$$

The government's ability to collect a tax grows with the Gini index. Solving the problem numerically, I found that the maximum amount the government can collect grows with inequality; see figure 8.

Inefficiency related to avoidance decreases with inequality. I have found that agency profit decreases with inequality, when the government maximizes revenue, as shown by figure 9. A progressiveness restriction increases agency profit if the target government revenue remains unchanged, and it also reduces the maximum amount the government is able to collect.

Householders' after tax/avoidance income is larger when the government is not restricted by progressiveness. The total amount that householders should pay both to the government and the agency is lower when the government is not restricted by progressiveness, as shown in figure 10.

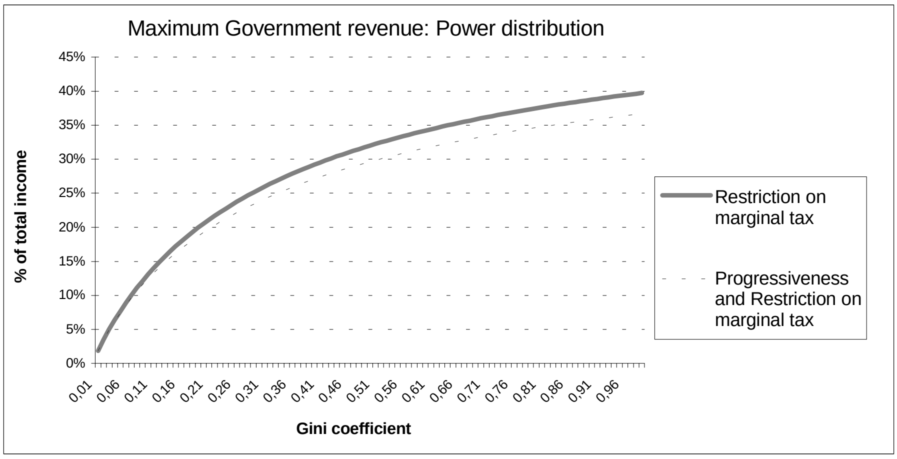

Figure 8:

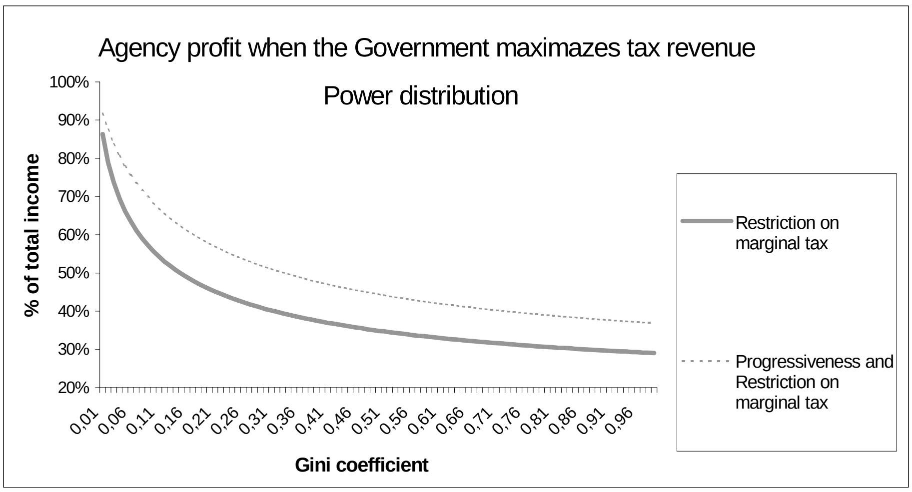

Figure 9:

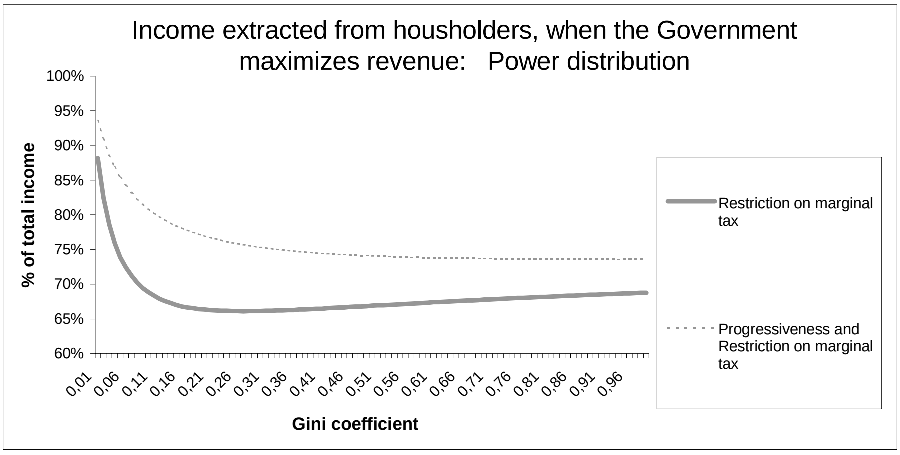

Figure 10:

3.2.2 Lognormal Distribution

Lognormality is the most common assumption for income distribution and is therefore of interest. If x has a lognormal distribution, $x = \exp(z)$, where z is normally distributed: $z \div N(a, \sigma)$; then the Gini coefficient could be calculated in formula (26)

$$Gini = 1 - 2 \int_{-\infty}^{-\frac{\sigma}{\sqrt{2}}} N(0,1) = \int_{-\frac{\sigma}{\sqrt{2}}}^{\frac{\sigma}{\sqrt{2}}} N(0,1). \tag{26}$$

Figure 11 demonstrates the relationship between government revenue and agency profit for a lognormal distribution with $Gini = 0.33$.

When the government maximizes a revenue, its collection increases with inequality as in the case of "power" distribution, while agency profit has a principally different shape: it is not strictly decreasing, after achieving a maximum level when the Gini index is equal to 24%, it falls.

The total amounts collected from householders by both the agency and the government grow with inequality, but they are lower when the government has a progressiveness restriction. This is due to the fact that such a restriction raises agency profit by relatively small amounts, compared to the reduction in government revenue collection. Under lognormality, the agency receives less profit if the Gini index is smaller than under "power" distribution.

Figure 12 presents the shapes of government revenue, agency profit, and the sum of both, depending on the Gini index, in the case of lognormal distribution when the government collects the maximum revenue.

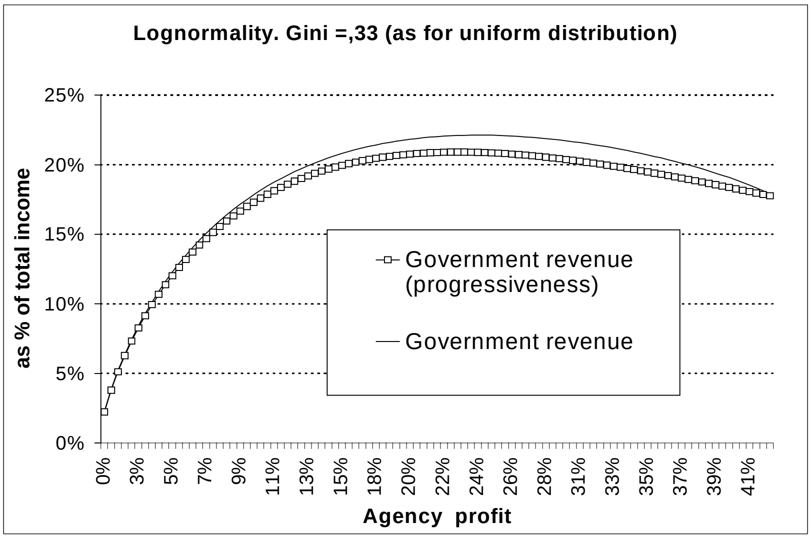

Figure 11:

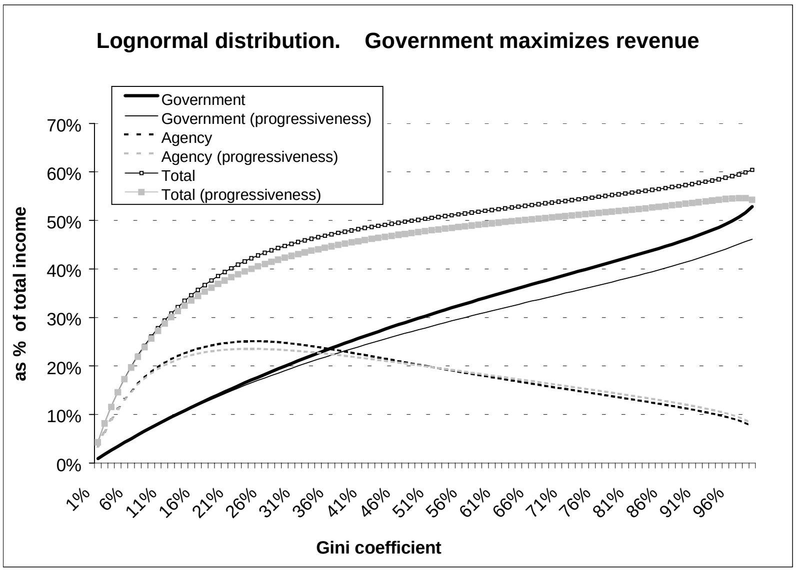

Figure 12:

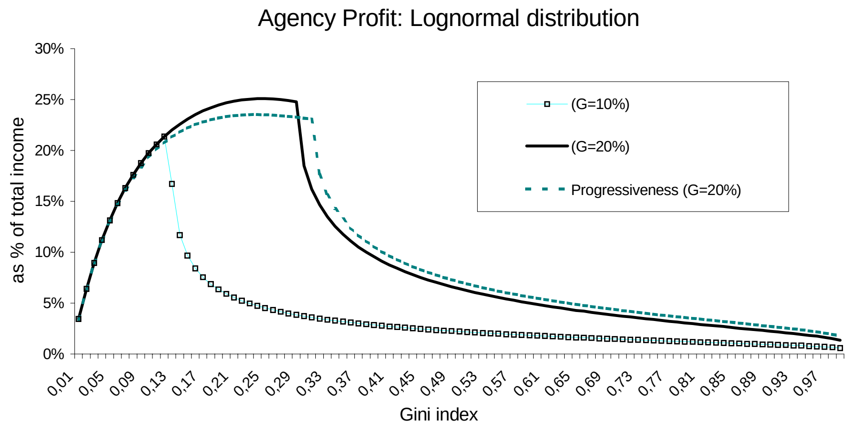

Figure 13:

3.3 When the Government Can Collect More than Target.

When the government can collect more than its target, inefficiency related to avoidance falls drastically. Figures 13 and 14 show what happens when the government becomes able to collect a target revenue under some rate of inequality. The transition from the state when the government collects as much as possible to the situation when a target revenue could be achieved, dramatically decreases the profit of an agency, which is considered as a loss for the economy. The crucial turns to decreasing trends correspond to that transition point.

Figures 13 and 14 also show that it is not only important to know the Gini index, but also the form of distribution, in order to estimate the total loss for an economy when the government is claimed to set a progressive tax. In the case of "power" distribution, the loss is larger than in lognormal one. Moreover, higher inequality implies lower marginal cost of additional revenue. In other words, government's decision to increase revenue is less profitable for the agency when inequality is high. Figure 15 demonstrates this.

To conclude, I want to repeat that a high level of inequality is profitable for the government and unprofitable for the agency. In a risk-neutral world, inequality thus makes society better off, because disparity allows the government to collect more revenue or give the agency less profit under the same revenue level.

4 When an Avoidance Supply is Costly

I will now extend my analysis by introducing a positive cost c per individual to the agency's technology, which may also describe the costly access to an avoidance service. Following the logic of Section 2.4, it can be found that an

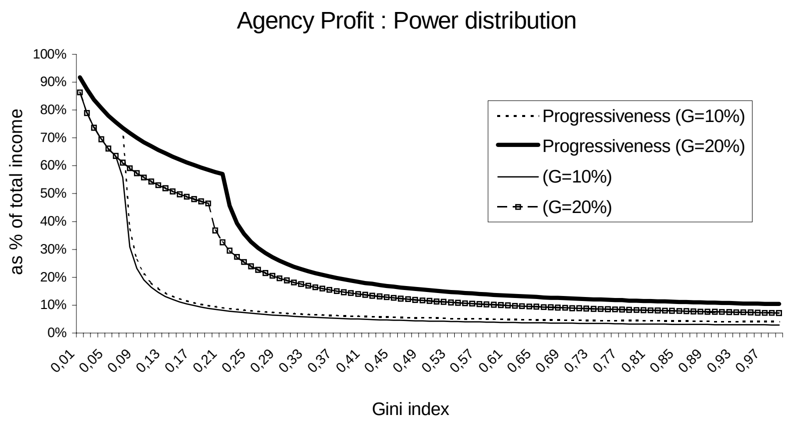

Figure 14:

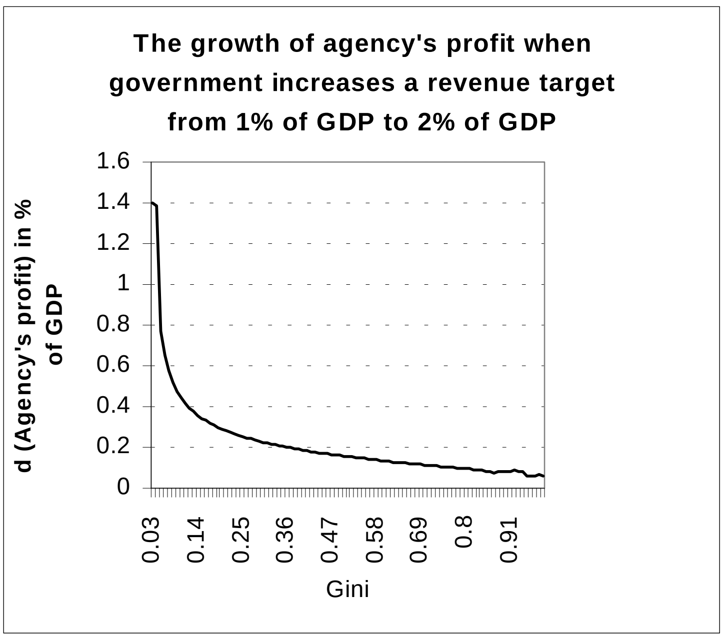

Figure 15:

agency's isoprofit curve is illustrated by formula (27):

$$\frac{m}{1-F(x)}+c. \tag{27}$$

Another option: no inefficiency due to avoidance. If the avoidance cost, c, is relatively large compared to the target revenue $\overline{g}$, the government could set a tax level lower than the agency cost for all householders. In that case, an economy would operate without loss, since the agency could not make any profit. The maximum amount the government could collect, when giving nothing to the agency, corresponds to the tax code in formula (28):

$$\tau(x)= \begin{matrix} x \text{ if } x<c \\ c \text{ if } x\geq c \end{matrix}. \tag{28}$$

In this case, government revenue is calculated by formula (29):

$$g_c=\int_0^c xdF(x)+(1-F(c))c. \tag{29}$$

If $g_c>\overline{g}$, the government does not need to give anything to the agency, while in case of a small cost, the government might prefer to collect more revenue by taxing somebody by more than the avoidance cost. In the latter case, the optimal tax code is presented in formula (30).

$$\tau(x)=\min(x,\frac{m}{1-F(x)}+c) \tag{30}$$

Two alternative tax codes, given in formula (30) and formula (28), are illustrated in figure 16 .

The government might prefer the first choice only if area A, measured by dF, is larger than area B.

In a low-cost case, B is smaller than A and the shape of the tax code (30) looks very similar to a zero-cost case, but the government is able to collect more tax, giving the agency the same revenue, since the agency's isoprofit curve is more favorable for the government.

When an Avoidance Cost depends on Government enforcement. Following Atkinson and Stiglitz [3] and Slemrod [22], I will introduce a positive relation between an agency cost $c=c(e)$ and government expenditures on enforcement, e. Extending the government problem $(5-9)$, the following formulas $(31-33)$ are obtained:

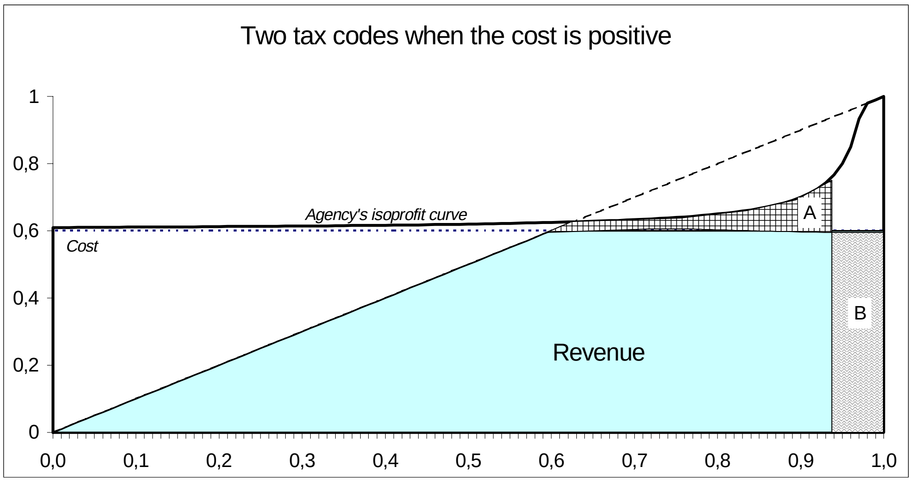

Figure 16:

$$\max_{\tau(.),e} W(\tau(.)) = \int_0^{\widehat{x}} u(x-\tau(x))dF(x) + \int_{\widehat{x}}^1 u(x-T)dF(x); \qquad st: \tag{31}$$

$$\int_0^{\widehat{x}} \tau(x)dF(x) = \overline{g} + e; \tag{32}$$

$$\widehat{x} = \left\{ \begin{array}{c} \arg\max_x [\tau(x) - c(e)] * (1 - F(x));\ if\ \exists x, \tau(x) > c(e) \\ 1, \text{otherwise} \end{array} \right\} \tag{33}$$

If the government is not efficient ($c(e)$ does not grow quickly enough), the optimal policy might be not to struggle against the agency at all ($e = 0$).

5 Applications

In this section, I will demonstrate that despite its simplicity, the model could be applied to a wide range of economic issues, such as tax reform and immigrational and regional policies.

5.1 Tax Base Broadening as a Measure of Increasing Revenue

This subsection examines if, and when, tax base broadening leads to improved tax revenue collection. There is a large amount of literature discussing tax base broadening versus tax rate increases. In particular, Baer and Silvani [5] argue that the simplification of a tax system, including a base broadening and the reduction of exemptions, makes administration much easier and increases the level of compliance. Moreover, according to Pillarisetti [19], the complexity of tax codes and high tax rates could also result in corruption.

Some other studies argue against tax base broadening. For example, Piggott and Whalley [20] show that the Canadian VAT base broadening on commodities and services, which are difficult to tax when provided by small scale suppliers, induced substitution into relatively inefficient household production, and stimulated underground activities.

My model considers the effects of tax base broadening related to the change in base distribution and the adjustment of the avoidance price. I will illustrate that tax base broadening has at least two indirect effects. The first, related to the change in the tax base distribution function, could cause both growth and reduction in collecting revenue. The second relates to an avoidance cost which might increase after the tax base broadening.

5.1.1 When an Avoidance Cost does not Depend on Tax Exemptions

Tax base broadening might lead to a change in the tax base distribution function, causing a reduction in the government's ability to collect a tax revenue.

A new income source as a tax subject. The first type of tax base broadening is carried out by including a new income source in the tax base. Example 3 presents a case where such a policy leads to decreasing revenue.

Example 3 ***"When the tax of a new income source leads to decreasing tax revenue"***

Let me consider an economy with two sources of income: farming x and fishing y, which might be interdependent; the extreme case is when $y_i = 1 - x_i$ for any householder i. Initially, the government only taxed farming income x_i, thereby granting fishing income y_i a tax exemption. Tax base broadening means that the government decides to tax total income $z_i = x_i + y_i = 1$. In this particular case, tax base broadening leads to complete tax avoidance and zero tax revenue.

A new tax payer. The second type of tax base broadening appears when the government adds another, new householder, as a tax subject. This involves no risk as long as the government can distinguished him by income level. Otherwise, a new tax subject might cause a decrease in tax revenue.

Example 4 ***"When a new tax payer causes a reduction in tax revenue"***

Let an initial economy consist of eight householders, with incomes 1, 1/4, 1/4, 1/4, 1/8, 1/8, 1/8, *and* 1/8, *respectively The government can set a tax code exactly equal to the income. In this case, only the richest householder would avoid tax, if spending his total income in payment for avoidance service. Total government revenue would be* $\frac{3}{4}+\frac{4}{8}=\frac{10}{8}=\frac{450}{45*8}$.

Let a new individual with income 1/4 *be included as a tax subject. The government could not tax the poorest at more that* 1/9, *otherwise* 1/9 *would be an avoidance price and all householders would avoid paying tax. For the same reason, the government could not tax medium-income householders at more than* 1/5, *which means that total government revenue could not exceed* $\frac{4}{5}+\frac{4}{9}=\frac{56}{45}=\frac{448}{45*8}$, *which is lower than previously.*

Tax rate reduction as a necessary complement to tax base broadening. Example 4 also demonstrates the necessity to decrease the tax levels together with tax base broadening. Although this example seems very complicated, there are plenty of simple examples showing that tax base broadening must be accompanied by a reduction in tax rates, to sustain the same revenue level as before tax base broadening took place.

Example 5 ***"When tax base broadening nay require a tax rate reduction"***

An initial economy consists of two householders, with incomes 1 *and* 0.8 *respectively. The government sets tax codes* 1 *and* 0.5, *collecting* 0.5 *as a revenue.*

Let a new householder earning 0.8 *be included to the tax base. If the tax remains unchanged, the agency will set a price of* 0.5 *and everybody will avoid paying tax. For a positive revenue collection, a new tax level for the poorer individuals should not exceed* 1/3.

5.1.2 If Avoidance Technology Uses Tax Exemptions

In many cases, a tax avoidance technology includes a reclassification of income. For example, the Russian government decision to cancel VAT for goods imported from Belarus in late 1996 led to a tremendous growth in imports from Belarus, while Ukrainian imports declined by approximately the same amount[9]. Since the changes in export activities did not lead to considerable variations in real economies, they can be explained by a tax avoidance practice through reexports of Ukrainian goods as described in the IMF publication [16].

Another example of tax avoidance by using a tax grant was observed in the early 1990's when the Russian government granted tax exemptions instead of financing the activity of some non-profit organization. As a result, these

[9] According to the IMF Direction of Trade Statistics, Russian imports from Belarus and Ukraine amounted to USD 2795 mn. and USD 6256 mn. in 1996. In 1997, exports from Ukraine amounted to USD 3981 mn. only, while Belarus exports rose tremendously and reached USD 4627 mn.

organizations started to share tax grants with other firms, thereby reducing the tax base.

In both cases, tax avoidance occurs due to the existence of tax exemptions in the Russian tax code. Eliminating these would thus induce an increasing avoidance cost, due to the necessity of an avoidance technology adjustment. If the cost is positively correlated with tax exemptions, a tax base broadening entails an additional component, causing a tax revenue increase. Accounting for the above, my model confirms that it might be better to abolish tax exemptions and give financial support to non-profitable organizations, rather than providing them with tax grants. This point was clarified in detail by IMF(1995) Policy Analysis and Assessments paper [15], which refers to the experience of Albania, Poland, Romania, Russia, Kazakhstan, and Uzbekistan.

More Evidence from Russia. Tax collection continues to be a question of survival for the Russian government. All economic reforms can be stopped because of the shortage of government revenues. The main reason for unsuccessful tax collection is high tax rates and consequently, a high disposition for tax avoidance. Pavel Kuznetsov, Gregory Gorobetz and Alexander Fominykh [17] provide some concrete examples of the popular tax evasion schemes implemented by financial industrial groups in Russia. Main financial transactions between the groups are conducted through off-shore accounts which are hidden, or not taxable, while the accounts of the industrial plants are always empty and thus, not taxable. There were no instruments for punishing such financial industrial groups since the links are not officially registered: the accounts of an offshore company are not public, neither can they be monitored or punished, while the implementation of bankruptcy legislation on industrial plants with empty account has a very high political cost. According to Ustinov [27], the Russian government was thus not able to collect even half the prescribed revenue expressed as a percentage of GDP. High tax rates and avoidance practices create further distortions, such as inter-enterprise and tax arrears. Because an economy provides an avoidance service for the rich, the government levies heavier taxes on the poor in order to collect the necessary tax revenue, which, in turn, slows down small business development and economic growth. Wages, pensions and other widespread budget arrears in Russia are an informal way of taxing the poor heavily as predicted by the model. In addition, the model shows the inefficiency of the excessive revenue target set by the Russian budget in the last 5 years and, as a consequences, the government tried to collect as much as possible but failed. Social welfare would improve a great deal if the government set the target revenue somewhat lower than the maximum possible target, because of the significant reduction in resources wasted on payment for avoidance services. This is illustrated by figures 7, 6, 11, 13 and 14.

The consequences of the introduction of a new 13% income tax might be ambiguous. Figure 17 demonstrates two possible cases. D is a breakdown point for a flat tax code or x_t defined in formula (20), and X is a

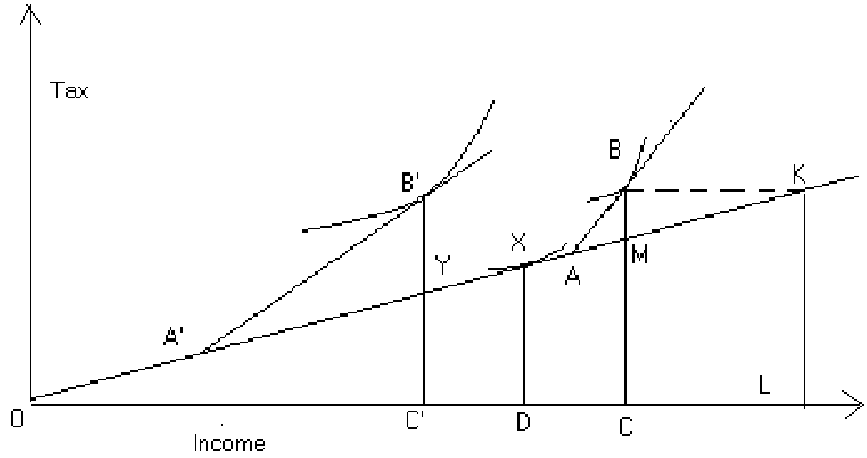

Figure 17:

corresponding tax level. A and $A\prime$ represent the tax payment by the richest householder in minimal tax brackets, defined by an initial tax code. C and $C\prime$ correspond to the incomes of the richest non-avoiding householders, while B and $B\prime$ illustrate their tax duties. Lines OAC and $OA\prime C\prime$ correspond to two possible initial tax codes. If the Russian tax code was initially similar to line OAC, more precise, A was allocated to the right side of X, the introduction of a flat tax would lead to a loss in tax revenue equal to $DXABC$. However, if the householder with income D initially paid more than 13 percent, the government would lose $A\prime B\prime Y$, but obtain $YXDC\prime$. The avoidance price would fall in case OAC and change ambiguously in case $OA\prime C\prime$, depending on the relation between $|C\prime B\prime|$ and $|DX|$. In both cases, the agency profit would be lower. I should say that in the real world, the agency might need some time to learn and react. In this case, the Government would lose ABM and receive $MKLC$ immediately after the introduction of a new code, but then breakdown income L will drift to D, and the maximum level of paying tax K will drift toward X.

The Tax Reform Act of 1986 and the Model The model will now attempt to describe the consequences of the Tax Reform Act of 1986 (TRA86) in the US. I do not wish to question other explanations, however, I only want to provide an additional one.

TRA86[10] reduced the number of marginal tax rate brackets and compressed the marginal rate structure so that the sharpest decline in marginal tax rates was experienced by high-income individuals, whose rate fell from 50% to 28%.

[10] A very detail overview of the related literature is provided by Auerbach and Slemrod in [4].

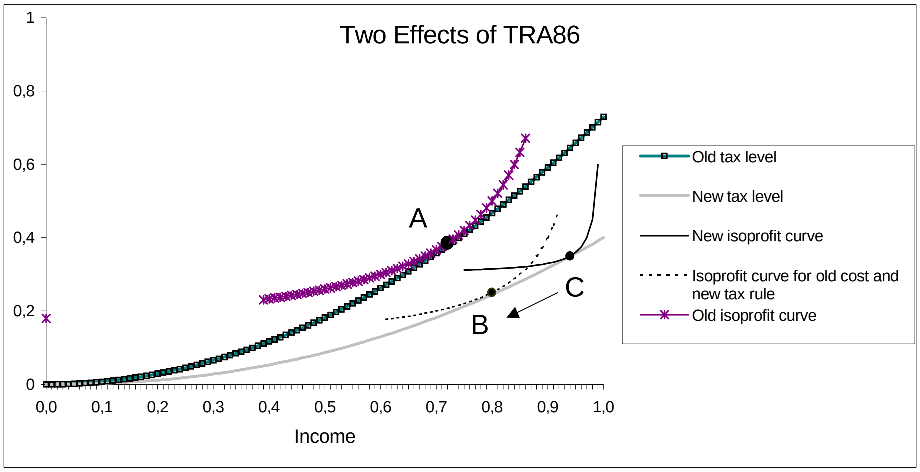

Figure 18:

TRA86 dramatically reduced the top individual and corporate tax rates and broadened the tax base at both the individual and corporate levels. Reported incomes of high-income taxpayers began to increase rapidly at the time of TRA86. The phenomenon was summarized by Feenberg and Poterba [9], who calculated the share of adjusted gross income reported by the top 0.5 percent of the tax payers, ordered by adjusted gross income. After remaining almost flat at about 6% from 1970 to 1981, it began to increase continuously in 1982 and reached 7.7% percent in 1985, and then jumped sharply to 9.2% in 1986. There was a slight increase to 9.5% in 1987, then another sharp rise to 12.1% in 1988. After 1988, there was a decline to 11.2 % in 1989 and 10.9% in 1990. Such changes might be explained by the impact of a tax code modification on an avoidance and evasion behavior, rather than by the movement of a real economy causing the change in inequality, since the considered time period is very short.

The possible explanation of the above dynamics of reported income after the implementation of TRA86 is illustrated by figure 18.

Once TRA86 reduced the income margin tax rate for everyone, it might have led to the shift from a tax rule represented by line A to a tax rule drawn as line BC. If this were the only change, the avoidance breakdown point A would be shifted to point B. Even this change could lead to an increase in the maximum reported income and augment the number of individuals who prefer paying tax, rather than using an avoidance technology, which could lead to an increase in the reported income concentration. (This could happen but might not necessary be the case. If a new tax code were much more regressive compared to the old one, this would lead to tax avoidance by some people paying taxes under the old tax rule. But agency revenue would be lower in any case; see Figure 19). The second impact of TRA86 on an economy is a broadening of the tax base,

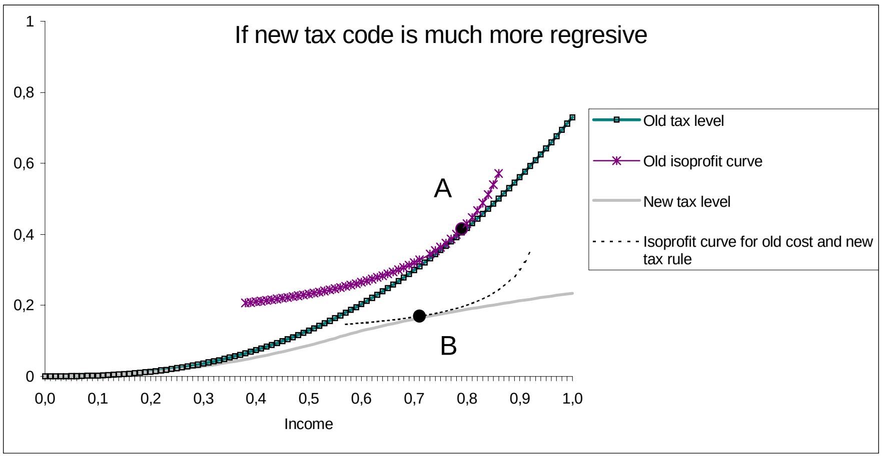

Figure 19:

which can be interpreted as the increase in an agency's cost, as described in subsection 5.1.2. After the implementation of TRA86, the agency must spend more time and money on uncovering and effecting the legal reduction in taxable income as well as the outright evasion of tax liability, and the expenses for camouflaging this. The effect of such a change in taxes is shown by the shift of the "breakdown" point from B to C in figure 18. In the following years, tax avoidance technology has been developed and the tax avoidance cost has decreased. This has led to the drift of a "breakdown" point from C toward B along the tax rule line, but B has not been achieved.

5.2 Immigration Policy

Another important issue causing the change in income distribution is the immigration policy. Example 4 can also be applied to immigration. Although that case illustrates the loss of revenue due to the entrance of a new immigrant, there are plenty of examples illustrating growth of revenue when the government uses an immigration policy together with tax rate reduction; see example 6.

Example 6 ***"Immigration and increasing in tax progressivity"*** *Let an economy initially consist of two householders, with incomes* 1 *and* 2/5*. The government can set a tax code* 4/5 *and* 2/5 *respectively and collect the maximum possible revenue* 2/5 *from the poorest householder. In this case only the richest householder would avoid tax, spending* 4/5 *on payment for the avoidance service. Now, a new immigrant with income* 2/5 *arrives. The government could set tax levels equal to* 1, 1/3 *and* 1/3 *and collect* 2/3 *from the two poorer householders while the richer householder would avoid paying his entire income.*

Example 6 provides the case when a tax base broadening increases both the ability to collect revenue and social welfare, but this policy redistributes after tax/avoidance incomes in such a way that the rich suffer. Therefore, an immigration could be blocked if the rich have larger bargaining power. The example ones more illustrates that tax base broadening must sometimes be implemented together with decreasing tax levels for the poor and perhaps increasing levels for the rich. If the government does not change the tax code 4/5, 2/5, 2/5, the agency would lower its price to 2/5, thereby causing complete avoidance.

5.3 Regional Policy.

The appearance of new independent states and the European unification attracts an increasing research interest in the secession/unification problem. The distribution of income inside the united regions, as well as a tax policy, plays an essential roll for break-up or integration decision. For example, Bolton and Roland (1997) by investigating a democracy where the median voter takes a decision about a flat tax rate, show that "a region with very low income inequality may want to break away from a nation with high income inequality and high tax rates in order to impose lower tax rates, and vice versa a region with high income inequality may want to separate in order to impose more redistribution than in the unified country". In this section, I will emphasis some regional issues related to tax avoidance.

5.3.1 Unification Could both Reduce and Increase the Government's Ability to Collect a Tax Revenue.

The effect of a change in the tax base distribution would appear after the unification of two states. Generally, unification improves the government ability to collect taxes, since it revokes the agency's option in price discrimination. Unification might also increase the efficiency of anti-avoidance measures, since the new government will use the anti-avoidance skills of both regions. But the change in income distribution could contribute negatively, as shown in example 4, which could be applied to the unification issue, if we consider a new tax payer as a new region collecting no tax before the unification.

5.3.2 The Elite of the Poorer Region Can Gain From a Secession.

Although the average tax burden might be lower after unification, rich householders in the poorer region might have considerable incentives to stay separate. Before unification, or after secession, they could enjoy an avoidance ability, or a lower tax level than in the united economy. Incentives are larger if the unified government has a higher target revenue.

5.3.3 The Rich in the Richer Country Might Have Incentives for Separation.

On the other hand, the rich in the richer region might also be interested in secession, especially if a unified government sets a high target revenue. In a union, a smaller proportion of rich might cause a more progressive tax. Example 7 shows that in an economy with two types of agents, the rich gain if their proportion is extended. Collecting tax from the poor, the government sets a minimum tax for avoiding rich just to insure necessary agency revenue to draw the agency away from the poor. That means that the government is only concerned with the sum of the tax duties of the rich, and that the individual tax is inversely related to the number of rich. Thus, the rich in the region with a higher proportion of rich would be interested in secession.

Example 7 ***"Two householders, economy"*** *It is interesting to consider an economy with two types of individuals as the simplest of heterogeneous agents. Let us assume that there are n_1 poor and n_2 rich householders earning an income of x and 1, respectively. If the government sets a tax level $\tau_1 < \tau_2$, then the agency will set*

$$T = \begin{cases} \tau_2; & \text{if } \tau_1(n_1+n_2) \leq \tau_2 n_2 \\ \tau_1; & \text{otherwise} \end{cases} \tag{34}$$

and the optimal tax code will be

$$\tau_1 = \frac{\tau_2 n_2}{n_1+n_2}. \tag{35}$$

The government will collect $\tau_1 n_1 = \overline{g}$ if $\overline{g} \leq min\left(n_1 x; \frac{n_1 n_2}{n_1+n_2}\right)$; otherwise the government maximizing its revenue will set $\tau_2 = 1$. Householders' consumption will be $x - \tau_1$ and $1 - \tau_2$ for the poor and the rich respectively. Social welfare is represented in formula (36)

$$W = n_1(x-\tau_1) + n_2(1-\tau_2) = I - \overline{g}\left(2 + \frac{n_2}{n_1}\right). \tag{36}$$

Using the same logic for a decreasing tax code, it might be concluded that if $n_1 > n_2$, the government will choose an increasing tax code, otherwise τ_2 could be less than τ_1.if $\overline{g} \leq \frac{n_1 n_2}{n_1+n_2}x$. Table 1 provides a summary solution for all possible cases of the parameters

	T	τ_1	τ_2	U_1	U_2
case 1	$\frac{\overline{g}(n_1+n_2)}{n_1 n_2}$	$\frac{\overline{g}(n_1+n_2)}{n_1 n_2}$	$\frac{\overline{g}}{n_2}$	$x - \frac{\overline{g}(n_1+n_2)}{n_1 n_2}$	$1 - \frac{\overline{g}}{n_2}$
case 2	$\frac{\overline{g}(n_1+n_2)}{n_1 n_2}$	$\frac{\overline{g}}{n_1}$	$\frac{\overline{g}(n_1+n_2)}{n_1 n_2}$	$x - \frac{\overline{g}}{n_1}$	$1 - \frac{\overline{g}(n_1+n_2)}{n_1 n_2}$
case 3	$\frac{x(n_1+n_2)}{n_2}$	x	$\frac{x(n_1+n_2)}{n_2}$	0	$1 - \frac{x(n_1+n_2)}{n_2}$
case 4	1	$\frac{n_2}{n_1+n_2}$	1	$x - \frac{n_2}{n_1+n_2}$	0

where case 1 represents the only situation when the government decides to set *a decreasing tax rate, that is when* $n_1 < n_2$ *and* $\overline{g} \leq \frac{n_1 n_2}{n_1+n_2}x$. *Case 2 corresponds to a situation when the government is able to collect a target tax* $\overline{g} \leq \min(\frac{n_1 n_2}{n_1+n_2}; n_1 x)$ *but when the conditions of case 1 are not satisfied. Cases 3 and 4 agree with the events when the government is not able to collect a target revenue, due to the lack of income of the poor (case 3,* $x \leq \frac{n_2}{n_1+n_2}$*) or the rich (case 4).*

5.4 Should Growth in Inequality Reduce Tax Progressivity?

Slemrod and Bakija [25] analyze the links between increasing inequality in pre-tax incomes and decreasing tax progressivity. The authors provide an overview of the optimal tax literature and consider a possibility of different causalities, including the likelihood that both trends are caused by the same third factor. My model provides an additional factor contributing to the considered phenomena. By changing parameters (x, n_1) in Example 7, both a positive and a negative correlation between inequality and progressivity can be reached.

n_1	50	150	150
n_2	50	50	50
x	0.3	0.15	0.6
inequality[11]	*bench mark*	*larger*	*smaller*
progressivity $= \frac{\tau_2}{\tau_1}$	2	$\frac{4}{3}$ *smaller*	$\frac{4}{3}$ *smaller*

6 Conclusion

The most essential property of the model is its ability to endogenously solve an avoidance price, depending on a tax code and a tax base distribution. Although the model neglects a large number of issues, rather considering the specific case of exogenous income, it is still capable of capturing many real world properties. It illustrates the necessity to account for changes in a tax base distribution and the possibility of decreasing the avoidance price when the government sets a tax code, makes decisions about tax base broadening, implements an immigration policy, or solves regional problems.

7 Appendix

In this section, I will prove Proposition 1 on page 7.

Proof. I will prove Proposition 1 by providing the algorithm of construction $\tau_1(.)$, giving $\tau(.)$. Simple intuition is shown in figure 20. A new tax code is simply constructed by shifting a high tax burden from poor to rich tax payers, so that for any tax level T, there is equality $\int\limits_{A(T)} dF = \int\limits_{A1(T)} dF$; where $A(T) =$

[11] I choose the parameters so that the Lorenz curves do not intersect. According to Atkinson [2] such a construction guarantees that the order of inequality does not depend on the measure used.

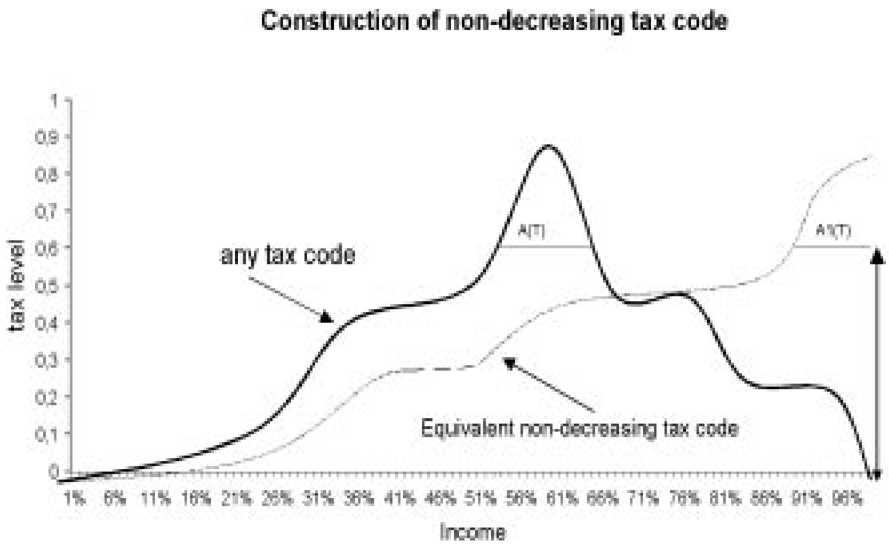

Figure 20:

$\{x, x \in X, \tau(x) \geq T\}$; $A1(T) = \{x, x \in X, \tau_1(x) \geq T\}$; $\tau(x)-$original tax code, $\tau_1(x)-$new tax code. Moreover, $A1(T)$ is interval and $F(\sup A1(T)) = 1$; I will now give the formal construction and proof. Let X be the domain of $\tau()$ or the support of income. For any $y \in \tau(X)$, $\exists\ x$ such that $1 - F(x) = \int\limits_{A(y)} dF$ because $F(x)$ is a continuous function. Such x is unique, because $F(x)$ is strictly monotone. If $y_1 < y_2$, then $A(y_1) \supseteq A(y_2)$ and $\int\limits_{A(y_1)} dF \geq \int\limits_{A(y_2)} dF$; so that I can define function $x = h(y) = F^{-1}\left(1 - \int\limits_{A(y)} dF\right)$ for any $y \in \tau(X)$; where $h(y)$ is non-decreasing. It means that $\forall x \in X$, $\exists y \in \tau(X)$, such that $\forall \varepsilon > 0, h(y+\varepsilon) \geq x$; $h(y-\varepsilon) \leq x$. Such y is the new tax level for individuals with income x.1) Now I will prove that $\tau_1(x)$ is non-decreasing; Let $x_1, x_2 \in X$; $x_1 \leq x_2$; Let $y_1 = \tau_1(x_1)$; $y_2 = \tau_1(x_2)$; then $\forall \varepsilon > 0$, $h(y_2 + \varepsilon) \geq x_2 \geq x_1 \geq h(y_1 - \varepsilon)$; Because $h(y)$ is non-decreasing, $y_2 + \varepsilon \geq y_1 - \varepsilon$, or $y_1 - y_2 \leq 2\varepsilon$, which means that $y_1 \leq y_2$;and $\tau_1(x)$ is non-decreasing;2) the government will collect the same amount of revenue;2a) the agency will choose the same price for its service $T(\tau) = T(\tau_1)$; the agency's profit is $T * F(A(\tau, T))$,where $A(\tau, T) = \{x, x \in X, \tau(x) > T\}$ .By the construction $F(A(\tau, T)) = F(A(\tau_1, T))$ for any T. So, $\arg\max T * F(A(\tau, T)) = \arg\max T * F(A(\tau_1, T))$. Government revenue equals $g(\tau) = \int\limits_{B(\tau,T)} \tau(x)dF$; where T is the avoidance price and $B(\tau, T) =$

$\{x, x \in X, \tau(x) \leq y\}$; $g(\tau) = \int\limits_{B(\tau,T)} \tau(x)dF(x) = \lim\limits_{n\to\infty} \frac{T}{n} \sum\limits_{m=0}^{n} F(A(\tau, \frac{mT}{n}))$. By the construction $F(A(\tau,T)) = F(A(\tau_1,T))$ for any T. So, $g(\tau) = g(\tau_1)$. 3) The value of a welfare function, which equals average utility, will not be lower than previous $W(\tau(.) \leq W(\tau_1(.))$; $W(\tau(.)) = \int\limits_{B(\tau,T)} u(x-\tau(x))dF(x) + \int\limits_{A(\tau,T)} u(x-T)dF(x)$; This expression can be simplified by introducing a new function $\widehat{\tau}(x) = \max(T, \tau(x))$. Then, $W(\tau(.)) = \int\limits_X u(x-\widehat{\tau}(x))dF(x)$; From above we know that the government would collect the same amount of a revenue $\int\limits_X \widehat{\tau}(x)dF(x) = \int\limits_X \widehat{\tau}_1(x)dF(x)$; and a new tax code $\widehat{\tau}_1(x)$ is a non-decreasing function. Now, I will prove the lemma I need for continuing:

Lemma 8 *Let:*

1. *$u(x)$ is a concave increasing function*
2. *$(x_1 < x_2 < ... < x_n)$;*
3. *$\widehat{\tau}_1(x_i)$ be a non-decreasing function, constrained from $\widehat{\tau}(x_i)$ such that $\forall i, i \in I_n, \exists$ unique $k_i \in I_n$, such that $\widehat{\tau}(x_i) = \widehat{\tau}_1(x_{k_i})$; where $I_n = (1,2,3,...,N)$;*

then $\sum\limits_{i=1}^{N} u(x_i - \widehat{\tau}(x_i)) \leq \sum\limits_{i=1}^{N} u(x_i - \widehat{\tau}_1(x_i))$;

Proof. *prove the Lemma by induction on n. If $N=1$ then $\widehat{\tau}_1(x_1) = \widehat{\tau}(x_1)$; and the lemma is obviously true. Let the lemma be true for $N-1$; And let $x_k = \arg\max\limits_{i\leq N} \widehat{\tau}(x_i)$; which means that $\widehat{\tau}_1(x_n) = \widehat{\tau}(x_k)$ by definition of $\widehat{\tau}_1(x_n)$, then $\sum\limits_{i\neq k}^{N} u(x_i - \widehat{\tau}(x_i)) + u(x_k - \widehat{\tau}(x_k))$; By the induction assumption $\sum\limits_{i\neq k}^{N} u(x_i - \widehat{\tau}(x_i)) \leq \sum\limits_{i=1}^{N-1} u(x_i - \widehat{\tau}_1(x_i))$; because $u(x)$ is concave $u(x_k - \widehat{\tau}(x_k)) \leq u(x_n - \widehat{\tau}(x_k)) + u(x_k - \widehat{\tau}(x_n)) - u(x_n - \widehat{\tau}(x_n))$ and because $u(x)$ is an increasing function $u(x_k - \widehat{\tau}(x_n)) - u(x_n - \widehat{\tau}(x_n)) \leq 0$ The lemma is thus proved.* ■

Now, I will continue to prove the proposition. By definition $\int\limits_X u(x-\widehat{\tau}(x))dF(x) = \lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{i=1}^{N} u(x_i - \widehat{\tau}(x_i))$, where $(x_1 \leq x_2 ... \leq x_{n+1})$ is devising of X, such that $F(x_k, x_{k+1}) = \frac{1}{N}$; $x_1 = \inf X$. Let $\widehat{\tau}^N(x)$ be the function constrained from $\widehat{\tau}(x)$ by the next way: $\widehat{\tau}^N(x) = \left\{ \begin{array}{c} \widehat{\tau}(x_{k_N}) \text{ if } x_n < x \leq x_{n+1}, \text{ where } x_{k_N} = \arg\max\limits_{i\leq N} \widehat{\tau}(x_i) \\ \widehat{\tau}(x_{k_{N-1}}) \text{ if } x_{n-1} < x \leq x_n, \text{ where } x_{k_{N-1}} = \arg\max\limits_{i\neq k_N} \widehat{\tau}(x_i) \\ ... \\ \widehat{\tau}(x_{k_1}) \text{ if } x_1 < x \leq x_2, \text{ where } x_{k_1} = \arg\min\limits_{i\leq N} \widehat{\tau}(x_i) \end{array} \right\}$

By the lemma, proved above, $\frac{1}{N}\sum_{i=1}^{N}u(x_i-\widehat{\tau}(x_i)) \leq \frac{1}{N}\sum_{i=1}^{N}u(x_i-\widehat{\tau}^N(x_i))$; By the construction $\lim_{N\to\infty}\widehat{\tau}^N(x) \to \widehat{\tau}_1(x)$, because $\widehat{\tau}^N(x)$ is non-decreasing and $\lim_{N\to\infty}F(A(\widehat{\tau}^N,T)) = F(A(\widehat{\tau},T))$ for any T, but $\widehat{\tau}_1(x)$ is the only function, with such properties. $\int_X u(x-\widehat{\tau}(x))dF(x) = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}u(x_i-\widehat{\tau}(x_i)) \leq \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}u(x_i-\widehat{\tau}^N(x_i)) = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}u(x_i-\widehat{\tau}_1(x_i)) = \int_X u(x-\widehat{\tau}_1(x))dF(x)$; End of proof ■

8 Appendix. Calculation for Uniform Distribution

Example 9 *Uniform distribution.* $F(x)=x$; *no restrictions. The agency's isoprofit curve is* $\frac{m}{1-x}$; *the solution of equation* $x=\frac{m}{1-x}$ *or* $x^2-x+m=0$ *is* $\left\{x_1=\frac{1}{2}-\frac{1}{2}\sqrt{(1-4m)}\right\}, \left\{x_2=\frac{1}{2}+\frac{1}{2}\sqrt{(1-4m)}\right\}$; *using the general formula (17), I obtain the next expression:* $g(m)=\frac{1}{4}-\frac{1}{4}\sqrt{(1-4m)}-\frac{m}{2}+m*\ln\left(\frac{1+\sqrt{(1-4m)}}{1-\sqrt{(1-4m)}}\right)$. *In the case of uniform distribution,* $g(m)$ *is concave and has the following quantitative characteristics* $\max g(m)=0.25$; $\arg\max g(m)=0.16$.

Example 10 *Uniform distribution.* $F(x)=x$; *progressive tax. Following the algorithm described above, I find that* $x_t=\frac{1}{2}$; $\alpha=4m$; $x_2=\frac{1}{2}+\frac{1}{2}\sqrt{(1-4m)}$. *Applying formula (22), I get:* $g(m)=4m\int_0^{1/2}xdx+m\left(\ln\frac{1}{2}-\ln(\frac{1}{2}-\frac{1}{2}\sqrt{(1-4m)}\right)$; $g(m)=\frac{m}{2}-m\ln(1-\sqrt{1-4m})$.

Example 11 *Uniform distribution.* $F(x)=x$; *restriction on marginal tax.* x_{mr} *should be found from equation* $\frac{mF'(x_{mr})}{(1-F(x_{mr}))^2}=\frac{m}{(1-x_{mr})^2}=1$; $x_{mr}=1-\sqrt{m}$; $x_1=\frac{1}{2}-\frac{1}{2}\sqrt{(1-4m)}$. *From formula 22:* $g(m)=\frac{1}{4}\left(1-\sqrt{(1-4m)}\right)-\frac{m}{2}+m*\ln\left(\frac{1+\sqrt{(1-4m)}}{2\sqrt{m}}\right)$.

Example 12 *Uniform distribution* $F(x)=x$; *progressiveness and restriction on marginal tax.* $x_{mr}=1-\sqrt{m}$; $x_t=\frac{1}{2}$. *From formula 22:* $g(m)=m\left(\frac{1}{2}+\ln\left(\frac{1}{2\sqrt{m}}\right)\right)$.

Figure 20 summarizes all four cases of restrictions.

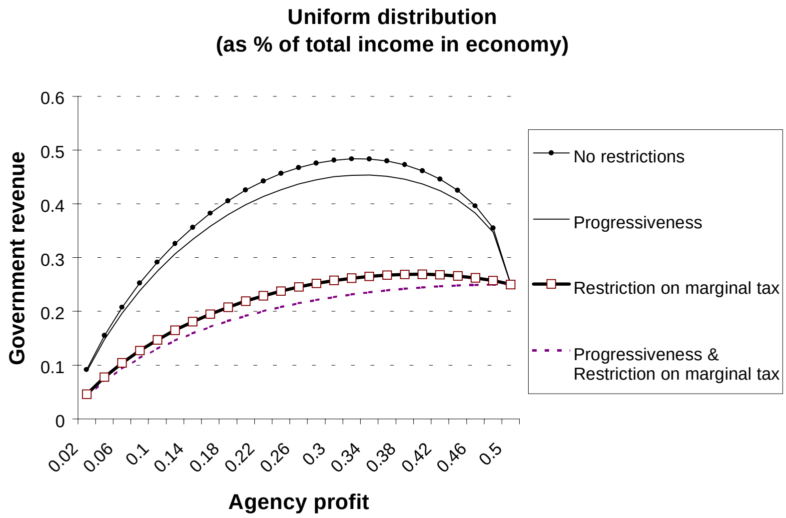

Figure 20:

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