Appendix to "Endogenous market segmentation and the law of one price". Should the Home market be closed?

Richard Friberg and Kaj Martensen
Stockholm School of Economics

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As noted in Appendix B, we need to check whether firms will want to sell on both markets. We model this as a two stage game, where the second stage is the segmentation-quantity game treated in the paper. In the first stage, a firm can choose whether to close the Home market. For simplicity, we assume that a closed market is also segmented (at zero cost). In this extended (a first and a second period) game, we call a strategy profile an equilibrium if it is subgame perfect. Hence, we use backwards induction to find the equilibria of the extended game. Given the profits in the second stage, we analyze a stage one game of the following form:

		Foreign	
		Sell on both markets	Close Home market
Home	Sell on both markets	1	2
	Close Home market	3	4

Each numbered square represents the equilibrium profits in the stage two game given the strategies in the first stage. These various profits are calculated in the document "profits". In this document we only compare the differences. The outcome of the game in square 1 is crucial for the comparisons and we therefore reproduce the outcome of that game here. The figure below gives the strategy profiles, given that both firms are forced to keep both markets open.

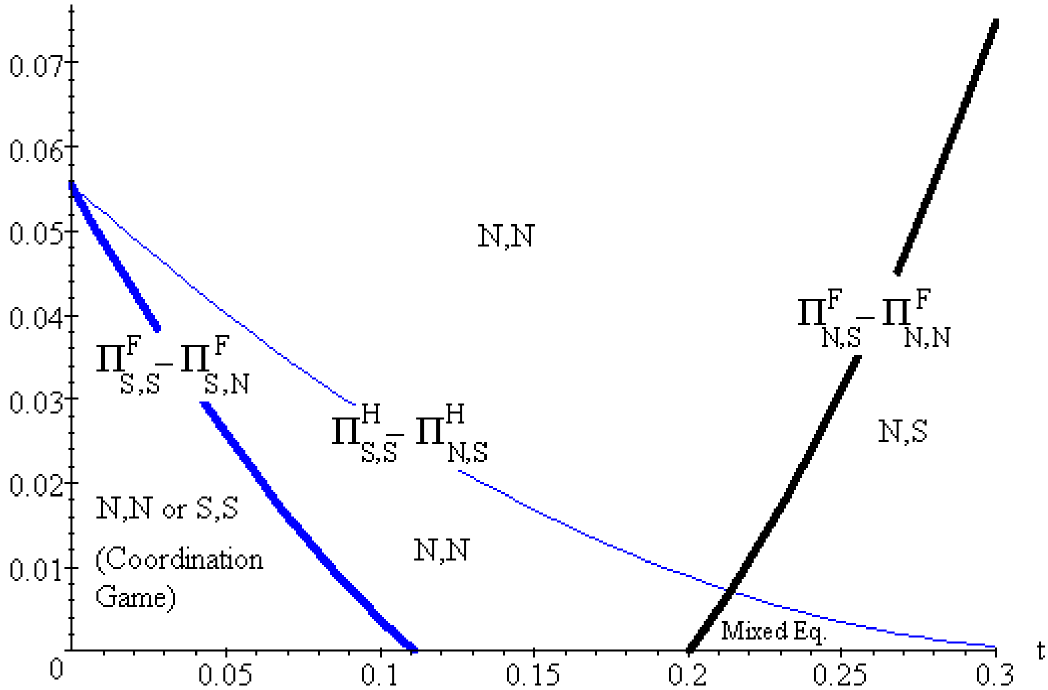


Figure 1. Equilibria for $\gamma \to 1$.

For $t > 0.2$ there will be no pure strategy equilibria for low K and for higher K there will be an (N, N) or (N, S) equilibrium.

We start with the comparisons for the Home firm, where the relevant comparisons are between squares 1 and 3 and 2 to 4. The exercise is done for $A = 2$ and $\gamma = 0.99$ (we have done the same exercise for $A = 2$ and $\gamma = 0.5$ as well, as expected it was not optimal for any of the firms to close the home market). We will refer to the second period game as the segmentation game.

1 The Home firm's problem

1.1 Home: comparison between squares 1 and 3

Given that the Foreign firm sells on both markets, what is the optimal strategy for the Home firm? Since the Home firm's first choice is whether to sell on both markets, we need to compare the outcome of the segmentation game (where (N, N), (S, S) or (N, S), respectively, can be equilibria), with the Home firm's profits of shutting down the Home market. The payoffs in square 3 will be dependent on whether Foreign integrates or segments and we will compare with both those cases. We will show

$$\Pi^H_{\bullet,\bullet} - \Pi^H_{C,\bullet} \geq 0,$$

which means that the Home firm prefers any outcome over shutting down its own market. For example, if

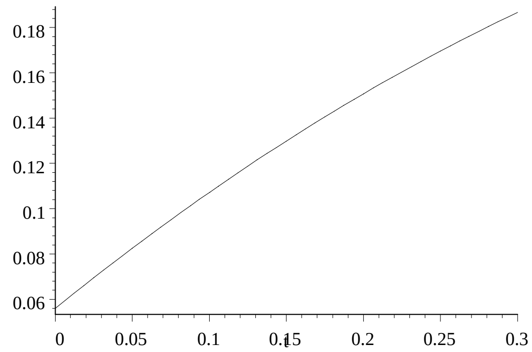
$$\Pi^H_{N,N} - \Pi^H_{C,S} > 0,$$
$$\Pi^H_{N,N} - \Pi^H_{C,N} > 0,$$

then the Home firm prefers the (N,N) equilibrium over shutting down its own market.

We start by comparing this (N,N) equilibrium to the outcome of the game in square 3. The difference in profits

$$\Pi^H_{N,N} - \Pi^H_{C,S} = \frac{1}{2}\frac{(A-t+1)^2}{(\gamma+2)^2} - \frac{(-2A+\gamma A+2t)^2}{(-4+\gamma^2)^2},$$

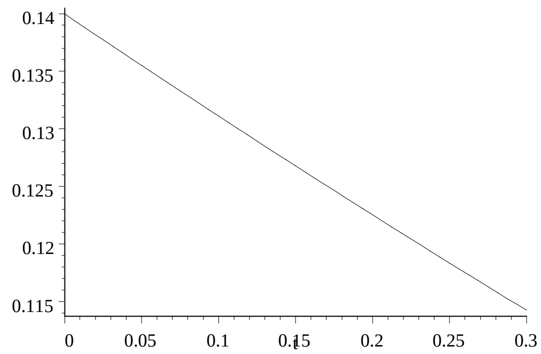
gives the figure below when plotted for $t \in [0, 0.3]$



So $\Pi^H_{N,N} > \Pi^H_{C,S}$ for the entire range. Further,

$$\Pi^H_{N,N} - \Pi^H_{C,N} = \frac{1}{2}\frac{(A-t+1)^2}{(\gamma+2)^2} - \left(\frac{(-4A+3\gamma A-3\gamma t-\gamma+4t)^2}{(-8+3\gamma^2)^2}\right),$$

gives the following figure.

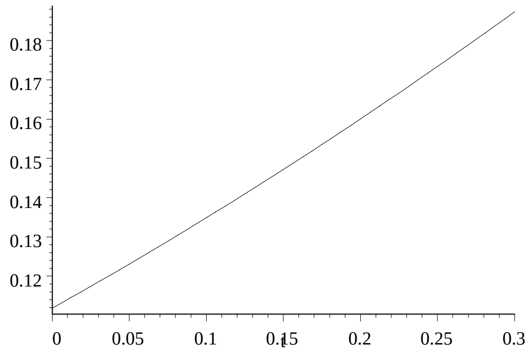


So, here as well, $\Pi^H_{N,N} > \Pi^H_{C,N}$. In these cases, segmentation costs do not enter the problem, thus we know that unambiguously the Home firm prefers the (N,N) outcome of the game in 1 over playing the game in 3. That is, it will prefer not to close its domestic market.

For low t we have a coordination game so that $\Pi^H_{S,S}$ is also relevant for comparison, typically we will have $\Pi^H_{S,S} > \Pi^H_{N,N}$, but let us check. The difference

$$\Pi^H_{S,S} - \Pi^H_{C,S} = \frac{1}{2}t\frac{-2(A-1)-\gamma t-2t}{(\gamma+2)(2-\gamma)} + \left(\frac{1}{4}\frac{(2A-t)^2}{(\gamma+2)^2} + \frac{1}{2}\frac{t^2}{(\gamma-2)^2} + \frac{1}{4}\frac{(t-2)^2}{(\gamma+2)^2} + \frac{1}{2}\frac{t^2}{2-\gamma} - K\right) + \\ +K - \left(\frac{(-2A+\gamma A+2t)^2}{(-4+\gamma^2)^2}\right),$$

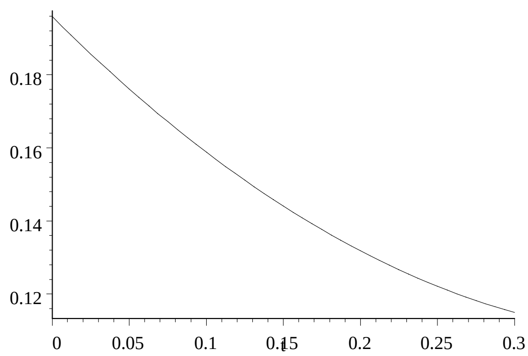
gives the figure below.



Thus $\Pi^H_{S,S} > \Pi^H_{C,S}$. This has to weighted against the cost of segmentation K, however. In the area below the line the Home firm will segment, since this profit difference is greater than the profit differential under which we can get the (S,S) outcome in the game in 1, as seen in Figure 1 above. We also compare

$$\Pi^H_{S,S} - \Pi^H_{C,N} = \frac{1}{2}t\frac{-2(A-1)-\gamma t-2t}{(\gamma+2)(2-\gamma)} + \left(\frac{1}{4}\frac{(2A-t)^2}{(\gamma+2)^2} + \frac{1}{2}\frac{t^2}{(\gamma-2)^2} + \frac{1}{4}\frac{(t-2)^2}{(\gamma+2)^2} + \frac{1}{2}\frac{t^2}{2-\gamma} - K\right) + \\ +K - \left(\frac{(-4A+3\gamma A-3\gamma t-\gamma+4t)^2}{(-8+3\gamma^2)^2}\right),$$

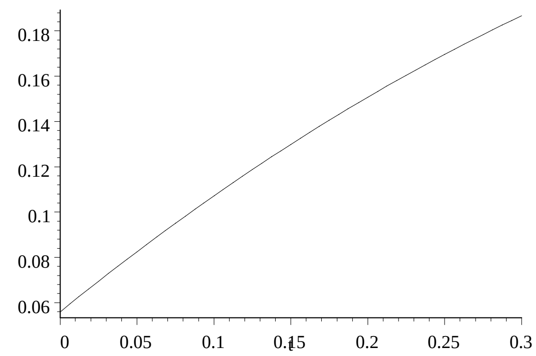
which gives the following figure.



Where by analogous reasoning, the firm will prefer the outcome of square 1. Finally, we need to compare with the outcome of (N, S).

$$\Pi^H_{N,S} - \Pi^H_{C,S} = \frac{1}{2}\frac{(t - A - 1)^2}{(\gamma + 2)^2} - \left(\frac{(-2A + \gamma A + 2t)^2}{(-4 + \gamma^2)^2}\right),$$

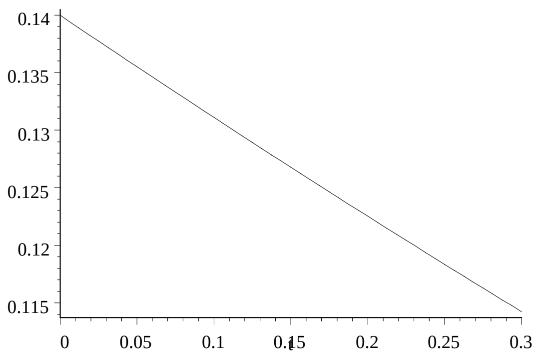
which gives the following figure.



So, $\Pi^H_{N,S} > \Pi^H_{C,S}$, the profits from keeping both markets open are higher than what is achieved if the Home market is closed. Finally,

$$\Pi^H_{N,S} - \Pi^H_{C,N} = \frac{1}{2}\frac{(t - A - 1)^2}{(\gamma + 2)^2} - \left(\frac{(-4A + 3\gamma A - 3\gamma t - \gamma + 4t)^2}{(-8 + 3\gamma^2)^2}\right),$$

is plotted in the following figure.



So, in all the cases square 1 yields higher profit than square 3 for the Home firm.

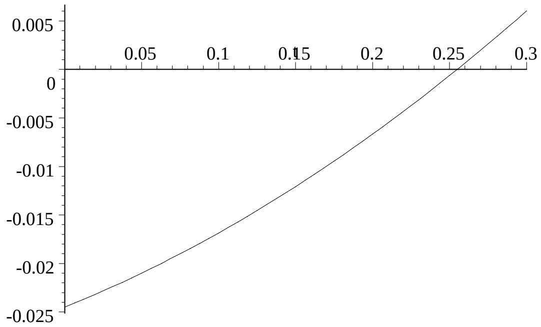
Since we rule out mixed strategies, we do not compare with the region where no pure strategy equilibria exist. Given that the expected value of a game where firms randomize over strategies gives a weighted average of the different pure strategy equilibria, and all of these dominate the square 3 outcomes, this is not worrisome.

1.2 Home: comparison between squares 2 and 4

Given that Foreign only sells on the Foreign market, what is the optimal strategy for Home? Now we compare

$$\begin{aligned}\Pi^H_{S,C} - \Pi^H_{N,C} &= \frac{1}{4} + \frac{2A-\gamma A-2t}{4-\gamma^2}\left(A - \frac{2A-\gamma A-2t}{4-\gamma^2} - \gamma\frac{2A-\gamma A+\gamma t}{4-\gamma^2}\right) - \\ &\quad -t\frac{2A-\gamma A-2t}{4-\gamma^2} - K - 2\frac{\left(-2-2A+\gamma A+\gamma^2+2t\right)^2}{\left(-8+3\gamma^2\right)^2} + K,\end{aligned}$$

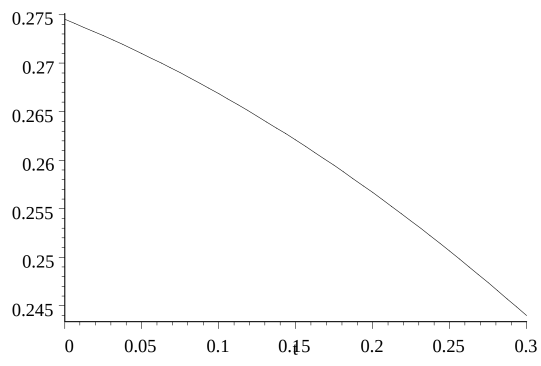
which gives the following figure.



For values of K and t such that we are below the line, the Home firm will segment markets. So, for most of the relevant range Home will choose not to segment. Only where K is extremely low and $t > 0.25$ will Home segment. So when comparing squares 2 and 4 the main comparison is

$$\Pi^H_{N,C} - \Pi^H_{C,C} = 2\frac{\left(-2-2A+\gamma A+\gamma^2+2t\right)^2}{\left(-8+3\gamma^2\right)^2} - \left(\frac{\left(-2A+\gamma A+2t\right)^2}{\left(-4+\gamma^2\right)^2}\right),$$

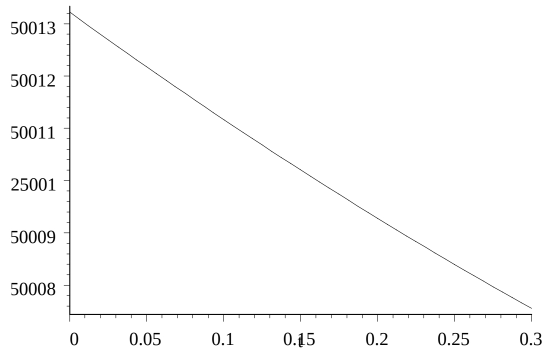
which gives the following figure.



Hence, $\Pi^H_{N,C} > \Pi^H_{C,C}$ over the relevant range. The Home firm will prefer to keep both markets open. We also need to check the case where Home segments (even though its a small area). The difference

$$\Pi^H_{S,C} - \Pi^H_{C,C} = \frac{1}{4} + \frac{2A - \gamma A - 2t}{4-\gamma^2}\left(A - \frac{2A-\gamma A - 2t}{4-\gamma^2} - \gamma\frac{2A-\gamma A+\gamma t}{4-\gamma^2}\right) - -t\frac{2A-\gamma A - 2t}{4-\gamma^2} - K + K - \left(\frac{(-2A+\gamma A+2t)^2}{(-4+\gamma^2)^2}\right),$$

gives the following figure.



For values of K (very high) and t such that we are below the line will the Home firm segment and profits in consequence be higher when both markets are open.

Also when it is optimal for the Home firm to segment are its profits higher when operating on both markets. So, given that Foreign does not sell on the Home market, the Home firm will prefer to sell on both markets. So Home prefers 2 over 4.

1.3 Conclusions Home

From the above we conclude that it is a dominant strategy for Home to sell on both markets for the parameter values given. Given this we can limit ourselves to comparing square 1 with square 2 for Foreign.

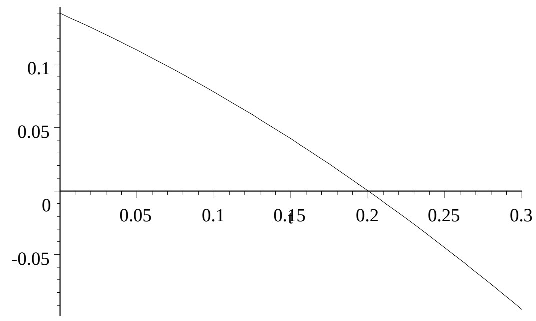
2 The Foreign firm's problem

2.1 Foreign: comparison between squares 1 and 2

This is the restriction that is most likely to be binding, not only is the Home market the weak market, it is also of less importance for the Foreign firm due to the transport cost. For much of the area in figure 1, (N,N) is the unique Nash equilibrium when both markets are open and Home will choose not to segment if Foreign closes the Home market (see previous comparison). Hence, the main comparison is

$$\Pi^F_{N,N} - \Pi^F_{N,C} = \left(\frac{1}{2}\frac{(A-t+1)^2}{(\gamma+2)^2} + t\frac{A-1-t}{\gamma+1}\right) - \frac{(-4A+3\gamma A-3\gamma t-\gamma)^2}{(-8+3\gamma^2)^2},$$

which gives the following figure.

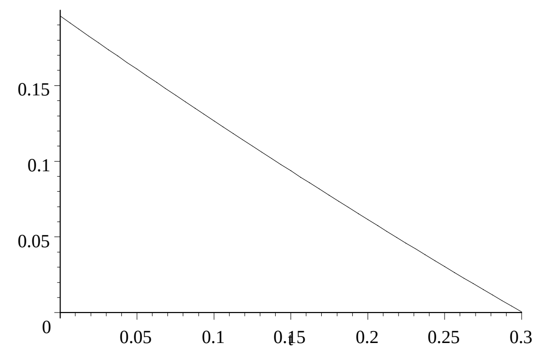


Since neither of these cases involve a comparison with K, all we need to examine is if the difference is positive. We see that for $t < 0.2$, profits are greater for the Foreign firm if it operates on both markets (the exact solution is $t = 0.20021$).

We should also compare with S,S.

$$\Pi^F_{S,S} - \Pi^F_{N,C} = \frac{1}{2}t\frac{2(A-1)-\gamma t-2t}{(\gamma+2)(2-\gamma)} + \left(\frac{1}{4}\frac{(2A-t)^2}{(\gamma+2)^2} + \frac{1}{2}\frac{t^2}{(\gamma-2)^2} + \frac{1}{4}\frac{(t-2)^2}{(\gamma+2)^2} + \frac{1}{2}\frac{t^2}{2-\gamma} - K\right) + \\ +K - \frac{(-4A+3\gamma A-3\gamma t-\gamma)^2}{(-8+3\gamma^2)^2},$$

giving the following figure.



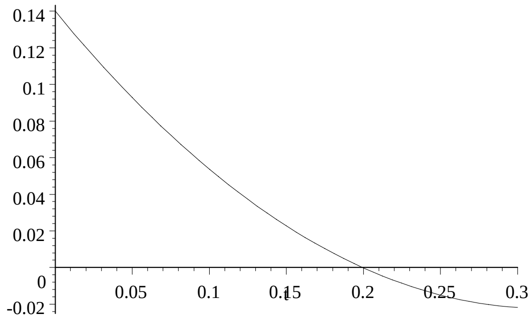
In the region below the line, K and t are such that $\Pi^F_{S,S} - \Pi^F_{N,C} > K$ and the firm will prefer the outcome of square 1 (since the region in which $\Pi^F_{S,S} - \Pi^F_{N,C} > K$ is greater than the region for which an (S,S) equilibrium can arise in Figure 1. Thus, for $t < 0.2$ Foreign will sell on both markets.

As seen above in the comparison $\Pi^F_{N,N} < \Pi^F_{N,C}$ when $t > 0.2$ so that the Foreign firm would rather close the Home market than keep it open and let pricing on the Foreign market be constrained.

What about the comparison with $\Pi^F_{N,S}$? The difference

$$\Pi^F_{N,S} - \Pi^F_{N,C} = \frac{1}{2}\frac{(t-A-1)^2}{(\gamma+2)^2} + \frac{1}{2}\frac{((A-1)(1-\gamma)+\gamma t+t)^2}{(2-\gamma^2)^2} - \frac{(-4A+3\gamma A-3\gamma t-\gamma)^2}{(-8+3\gamma^2)^2},$$

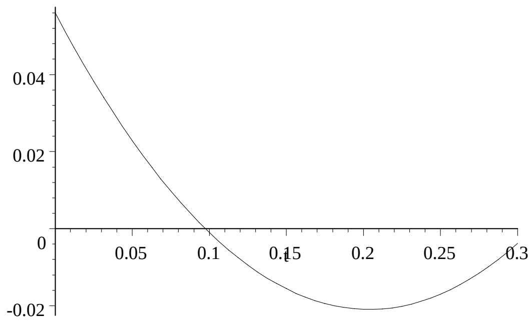
gives the figure below.



Hence, here too, it is better to close the Foreign market when $t < 0.2$. Again, K does not enter this comparison. Exact solution is $t = 0.19947$. Last case to check is the case where Home segments (which it will do for very low K and $t > 0.255$). The difference

$$\begin{aligned}\Pi^F_{N,S} - \Pi^F_{S,C} &= \frac{1}{2}\frac{(t-A-1)^2}{(\gamma+2)^2} + \frac{1}{2}\frac{((A-1)(1-\gamma)+\gamma t + t)^2}{(2-\gamma^2)^2} - \\ &\quad - \frac{2A-\gamma A+\gamma t}{4-\gamma^2}\left(A - \frac{2A-\gamma A+\gamma t}{4-\gamma^2} - \gamma\frac{2A-\gamma A-2t}{4-\gamma^2}\right),\end{aligned}$$

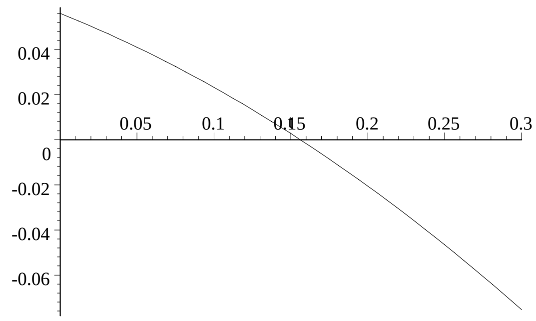
gives the following figure.



Thus, the difference this is negative when $t > 0.255$, and the Foreign firm would prefer to close the Home market rather than get the (N, S) outcome of square 1. We should also compare with the (N,N) case. The difference

$$\Pi^F_{N,N} - \Pi^F_{S,C} = \left(\frac{1}{2}\frac{(A-t+1)^2}{(\gamma+2)^2} + t\frac{A-1-t}{\gamma+1}\right) - \frac{2A-\gamma A+\gamma t}{4-\gamma^2}\left(A - \frac{2A-\gamma A+\gamma t}{4-\gamma^2} - \gamma\frac{2A-\gamma A-2t}{4-\gamma^2}\right),$$

gives the following figure.



Hence, also here the difference this is negative when $t > 0.255$. So, the results are clear. The case that we haven't explicitly checked is the small area in Figure 1 where no pure strategy equilibria exist.

3 Conclusions

The exercise is done for $A = 2$ and $\gamma = 0.99$. Home firm will always choose to operate on both markets. Foreign will operate on both if $t < 0.2$, when $t > 0.2$ it will quit selling to the Home market.