Taxation, Inequality and the Allocation of Talent

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Abstract

This paper examines the implications of income redistribution on human capital accumulation and income inequality, presenting a model where human capital investment is indivisible and agents differ in economic opportunity as well as intellectual ability. It is shown that the impact of redistribution is ambiguous on the income distribution as well as on human capital accumulation. In particular, while redistributive policy is likely to be successful both in terms of efficiency and equity in low-tax societies, it may be highly detrimental in both respects if the rate of redistribution is already moderate or high.

1. Introduction

The desirable degree of equality in the distribution of income and the efficiency and equitableness of redistributive tax policy are indeed among the most delicate issues on the economic-political agenda of the modern welfare state. The traditional view that redistributive taxation entails a conflict between efficiency and equity typically stems from the neo-classical analysis with complete markets and representative agents. Nevertheless, this result has to some extent been invalidated in the modern analytical framework, where it is allowed for market imperfections as well as individual differences. Particularly, in recent work on accumulation and distribution in the absence of perfect credit markets (e.g., Saint-Paul and Verdier [1993], Glomm and Ravikumar [1992], Durlauf [1996], Bénabou [1996], Perotti [1993], Loury [1981], Galor and Zeira [1993]), it is stressed that if individuals are credit constrained, redistribution is conducive to growth to the extent that it allocates resources more efficiently, but detrimental in the sense that it distorts incentives for investment.

In this paper, it is argued that while redistributive income taxation need not be damaging to aggregate economic activity, neither does it necessarily give rise to a more even distribution of income. Hence, redistributive taxation may not only be twice beneficial, but also twice detrimental in terms of efficiency and equity.

The paper introduces a simple model where individuals are differentiated by their intellectual ability, or talent, as well as their economic opportunity. Human capital investment is indivisible, thus requiring a minimum initial wealth, while the returns to schooling depend on individual talent, hence making investment more attractive to talented than to untalented agents. Further, income is taxed proportionally and redistributed lump-sum. However, in addition to equalizing after-tax income, redistributive taxation affects individuals' opportunities as well as their incentives to invest in education, and thus alters both the pre-tax and the after-tax distribution of income. Consequently, the implications of an increase in the rate of income redistribution are uncertain on the level of per capita income as well as the degree of income inequality.

The analysis indicates, firstly, that redistributive taxation need not induce a trade-off between efficiency and equity, and secondly, that the effects of increasing the rate of redistribution are highly dependent on the initial tax burden of the economy. In particular, while increases in the rate of redistributive taxation are quite likely to be twice beneficial, that is to enhance efficiency as well as equity, in low-tax societies, increases in the tax rate may very well be twice harmful in high-tax societies. One implication of these results is that class societies may arise in high-tax as well as in low-tax economies, while another is that in some cases, gains in terms of equity as well as efficiency might be achieved by lowering, rather than increasing the rate of income taxation.

The paper is structured as follows. The next two sections outline the theoretical model and its equilibrium properties. Section 4 analyzes the effects of increasing the rate of income redistribution on human capital investment and income inequality. Finally, Section 5 concludes.

2. The Model

2.1. Population, Preferences and Technology

Consider a one-sector economy with missing asset markets. The economy is populated by a constant-sized continuum of infinitely-lived dynasties. Individuals live for two periods in overlapping generations, and are differentiated within as well as across generations by their initial wealth, ω, and their talent, ξ. An agent's initial wealth is bequeathed upon her by her parent, and can take on two levels, $\underline{\omega}$ and $\overline{\omega}$, where $\overline{\omega} > \underline{\omega}$. Further, an individual's talent reflects her innate learning ability (cf Fershtman et al. [1996]), and can take on two levels, $\underline{\xi}$ and $\overline{\xi}$, where $\underline{\xi} < \overline{\xi}$. Talent is heritable to the extent that

$$E(\xi) = \begin{cases} \rho\overline{\xi} + (1-\rho)\underline{\xi} & \text{if } \xi_{-1} = \overline{\xi} \\ \rho\underline{\xi} + (1-\rho)\overline{\xi} & \text{if } \xi_{-1} = \underline{\xi} \end{cases}$$

where $E(\xi)$ and ξ_{-1} are the agent's expected talent and her parent's talent, respectively, and ρ is the degree of intergenerational persistence in talent. Further, ρ must belong to the interval $\left[\frac{1}{2}, 1\right)$, where the endpoints refer to the cases where talent is completely random and completely persistent, respectively.[1] It follows that there are four types of individuals, which are characterized by their talent on the one hand, and their initial wealth on the other. Thus, type α is described by $\left\{\overline{\xi}, \overline{\omega}\right\}$, type β by $\left\{\overline{\xi}, \underline{\omega}\right\}$, type φ by $\left\{\underline{\xi}, \overline{\omega}\right\}$ and type γ by $\left\{\underline{\xi}, \underline{\omega}\right\}$, where $\alpha + \beta + \varphi + \gamma = 1$.

In the first period of their lives, agents either acquire human capital through education or remain idle, while in the second period, they supply their labor inelastically and give birth, allocating their income between consumption and bequests to their offspring. All agents have identical preferences, which are defined over consumption, c, and bequests to their descendants, b, and are formally represented as

$$u = c_1 + \lambda c_2^{\theta} b^{1-\theta}$$

[1] The concept of individual talent is typically related to intelligence, or IQ, the heritability of which has been frequently debated in theoretical as well as empirical work (REFERENCES, JEcPersp). Moreover, in empirical studies, IQ has been estimated as being dependent on genetic factors only (as in xx [yy]), as well as entirely on social factors (as in yy [xx]). See Jencks [1972] for an overview and discussion.

where the discount factor λ falls below 1.[2]

Labor income, w, is a function of human capital, h. Moreover, human capital can take on two levels, $\hbar$ and 0, thus there are two levels of labor income, y and x, where $y \equiv f(\hbar)$, $x \equiv f(0)$ and $y > x$. At the beginning of life, after the realization of their talent, individuals decide whether to accumulate human capital by acquiring education or to remain idle.[3] There are two levels of education, $e = 0$ and $e = 1$, the latter requiring an investment of q.[4] In turn, human capital is determined by individual talent as well as educational achievement, thus talented agents acquire $\hbar$ with certainty if they invest in education, and with probability π otherwise, while non-talented agents acquire $\hbar$ with probability μ if they invest in schooling, and not at all otherwise.[5] In other words, the probability of accumulating non-zero human capital, $p[\hbar(\xi, e)]$, is given by

$$p[\hbar(\xi,e)] = \begin{cases} 1 & \text{if } \{\xi, e\} = \{\overline{\xi}, 1\} \\ \pi & \text{if } \{\xi, e\} = \{\overline{\xi}, 0\} \\ \mu & \text{if } \{\xi, e\} = \{\underline{\xi}, 1\} \\ 0 & \text{if } \{\xi, e\} = \{\underline{\xi}, 0\} \end{cases}$$

where $\pi \in (0,1)$, $\mu \in (0,1)$, $\pi + \mu < 1$ and $\mu > \pi$ The condition $\pi + \mu < 1$ implies that the expected return on formal education is higher to a talented than to

[2]In this context, the "warm-glow" bequest motive typically yields similar implications as the intertemporal case (cf Loury [1981]), where individuals care about future generations' well-being, rather than the size of bequests (cf Glomm and Ravikumar [1992] and Galor and Zeira [1993]). Particularly, as the absence of credit markets prevents poor families from saving for their descendants' education, the basic results of the analysis (see Section 2 and 3) should be practically the same in in the infinite-horizon case as in the present framework.

[3]It can be shown that all results remain intact in the case where agents decide on their education level knowing only their expected, but not their actual talent.

[4]In the case of human capital investment, the assumptions of credit constraints and non-convexity are fairly reasonable. The reason is, firstly, that human capital is typically considered as poor collateral for borrowing, and secondly, that most educational programs require a minimum investment or effort in terms of for example years of study or number of credits.

[5]For empirical evidence in favour of a positive relationship between earnings and intellectual ability, see for example Behrman et al [1981]. Further, evidence in support of a positive effect of schooling on earnings is provided by Card [1998].

a non-talented agent.[6] Moreover, the condition $\mu > \pi$ implies that there is a positive signalling effect of education, so that the probability of getting a high-income job is higher for educated than for uneducated agents, regardless of individual talent.

Finally, there is a government, that taxes income proportionally at the rate τ and redistributes the proceeds lump-sum. In what follows, z and τz represent the tax base and the transfer per head, respectively.

2.2. The Individual's Problem

In the first period of life, each individual chooses whether to invest in education or not, while in the second period she decides how much of her income to consume and bequeath, respectively. Thus, her problem is represented as

$$\max_{e,b} \; c_1 + \lambda c_2^{\theta} b^{1-\theta}$$

subject to the conditions

$$\begin{aligned} e &\in \{0,1\} \\ c_1 &= \omega - eq \\ c_2 &= (1-\tau)\, w + \tau z - b \\ E(w) &= \begin{cases} y & \text{if } \{\xi, e\} = \{\overline{\xi}, 1\} \\ \pi y + (1-\pi)\, x & \text{if } \{\xi, e\} = \{\overline{\xi}, 0\} \\ \mu y + (1-\mu)\, x & \text{if } \{\xi, e\} = \{\underline{\xi}, 1\} \\ x & \text{if } \{\xi, e\} = \{\underline{\xi}, 0\} \end{cases} \end{aligned}$$

In order to retain notational simplicity, it is convenient to define the following constants.

Definition 1. *Define the constants* $\Theta \equiv \theta^{\theta}(1-\theta)^{1-\theta}$ *and* $\Lambda \equiv \lambda\left(\frac{\theta}{1-\theta}\right)^{\theta}$. ■

Solving the maximization problem yields three decision rules. The first is that agents bequeath a fixed share of their after-tax income, thus $b = (1-\theta)\left[(1-\tau)\, w + \tau z\right]$. Parental bequests are equivalent to children's initial wealth endowments, thus in the

[6] This interpretation follows directly from the incentive compatibility constraint of talented and non-talented agents, which is derived in the next sub-section.

cases where parental labor income equals y and x, b is equivalent to $\overline{\omega}$ and $\underline{\omega}$, respectively. The second rule is that agents find it worthwhile to acquire education only if the expected utility of doing so exceeds the expected utility of remaining idle, that is if

$$\begin{array}{ll} (1-\tau)\,\lambda\Theta\,(1-\pi)\,(y-x)\geq q & \text{if } \xi=\overline{\xi} \\ (1-\tau)\,\lambda\Theta\mu\,(y-x)\geq q & \text{if } \xi=\underline{\xi} \end{array} \tag{2.1}$$

or in other words, if the rate of income taxation, τ, falls below $\frac{\lambda\Theta(1-\pi)(y-x)-q}{\lambda\Theta(1-\pi)(y-x)}$ and $\frac{\lambda\Theta\mu(y-x)-q}{\lambda\Theta\mu(y-x)}$. In what follows, (2.1) is referred to as the incentive compatibility constraint for talented and non-talented individuals, respectively. The incentive compatibility constraint is slack in the case of zero taxation. The third rule is that agents can afford investment in education only if their initial wealth net of taxes is at least as high as the education fee, that is if

$$\begin{array}{ll} (1-\theta)\,((1-\tau)\,y+\tau z)\geq q & \text{if } \omega=\overline{\omega} \\ (1-\theta)\,((1-\tau)\,x+\tau z)\geq q & \text{if } \omega=\underline{\omega} \end{array} \tag{2.2}$$

or in other words, if $\tau<\frac{(1-\theta)y-q}{(1-\theta)(y-z)}$ and $\tau>\frac{q-(1-\theta)x}{(1-\theta)(z-x)}$. Henceforth, (2.2) is referred to as the participation constraint for rich and poor individuals, respectively. The participation constraint exists for two reasons, the first being the absence of credit markets and the second being the indivisibility of investment in schooling. Hence, unless the rate of income redistribution is sufficiently high, that is unless $\tau>\frac{q-(1-\theta)x}{(1-\theta)(z-x)}$, the participation constraint implies that education is not affordable to poor individuals. However, the participation constraint also indicates that in the case of excessive income equalization, that is any case where $\tau>\frac{(1-\theta)y-q}{(1-\theta)(y-z)}$, neither rich nor poor agents are wealthy enough to invest in education.

The first of the assumptions below implies that in the case of zero taxation, education is affordable to rich individuals only. Moreover, the first part of the second assumption implies that the tax rate at which education becomes unaffordable to the rich exceeds the tax rate at which education becomes affordable to the poor, and falls short of the tax rate at which education becomes unaffordable to the poor. The second part of the second assumption, in turn, implies that there exists an equilibrium in which all individual types acquire education. Finally, the third assumption implies that the cost of investment is in excess of the expected utility of getting a low income with certainty after having acquired education.

Assumption 1. Assume that $(1-\theta)\,y > q$ and that $q > (1-\theta)\,x$. ■

Assumption 2. Assume that $\frac{q-(1-\theta)x}{(1-\theta)(z-x)} < \frac{(1-\theta)y-q}{(1-\theta)(y-z)}$ in any equilibrium where at least some agents choose $e=1$, and that $\frac{q-(1-\theta)x}{(1-\theta)(z-x)} > \frac{(1-\theta)y-q}{(1-\theta)(y-z)}$ otherwise. Furthermore, assume that $\frac{q-(1-\theta)x}{(1-\theta)(z-x)} < \frac{\lambda\Theta\mu(y-x)-q}{\lambda\Theta\mu(y-x)}$ in any equlibrium where only rich agents can afford education. ■

Assumption 3. Assume that $q > \lambda\Theta x$. ■

3. Equilibrium

The model generates four stationary equilibria, the existence of which is proved in Appendix B.[7] Henceforth, z refers to the tax base in the first of the equilibria described below.

3.1. Equilibrium 1

In the first equilibrium, the rate of income redistribution is sufficiently low not to destroy individuals' incentives to acquire education, but not high enough to make education affordable to poor agents. Hence, in this equilibrium, all individuals prefer schooling to remaining idle, but only those who are born rich can afford its acquisition.[8] The sufficient and necessary condition for this equilibrium is $\tau < \frac{q-(1-\theta)x}{(1-\theta)(z-x)}$, while the expected

[7] For individuals' transition matrices and definitions and calculations of per capita income and income inequality in each of the equilibria, see Appendix A.

[8] This equilibrium shares the properties of the case in Galor and Zeira [1993] to a large extent. In particular, credit market imperfections and indivisibility of investment imply that poor agents are excluded from the opportunity to invest in education, which in turn suggests that there may be an efficiency case for income redistribution. However, while the steady state distribution of income is determined by its initial shape in Galor and Zeira, it is time-independent in this context. The reason is that in the present framework, human capital may be acquired not only through education, but also by market luck, that is with probability π and 0 for talented and untalented agents, respectively.

per capita income and degree of income inequality are given by

$$E(Y_1) = \frac{\pi(1-2\rho\mu+\mu)}{2(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)}y + \frac{(1-\mu)(2\rho\pi-2\rho+2-\pi)}{2(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)}x$$

and

$$E(\sigma_1) = (1-\tau)\frac{\sqrt{\pi(1-\mu)(1-2\rho\mu+\mu)(2\rho\pi-2\rho+2-\pi)}}{2(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)}(y-x)$$

3.2. Equilibrium 2

In the second equilibrium, the rate of income redistribution is sufficiently high to make education affordable to anyone, but not to distort any agent's incentives to invest in education. Thus, in this equilibrium all agents acquire schooling, regardless of their talent and initial wealth. The sufficient and necessary conditions for this equilibrium are $\frac{q-(1-\theta)x}{(1-\theta)(z-x)} \leq \tau \leq \frac{\lambda\Theta\mu(y-x)-q}{\lambda\Theta\mu(y-x)}$, while the expected per capita income and degree of income inequality are given by

$$E(Y_2) = \frac{1+\mu}{2}y + \frac{1-\mu}{2}x$$

and

$$E(\sigma_2) = (1-\tau)\frac{\sqrt{(1-\mu)(1+\mu)}}{2}(y-x)$$

3.3. Equilibrium 3

In the third equilibrium, the rate of redistributive taxation is sufficiently high to make education affordable to anyone, but also to destroy non-talented individuals' incentives to acquire schooling. Hence, in this equilibrium, only talented agents find it worthwhile to invest in education. The sufficient and necessary conditions for this equilibrium are $\frac{\lambda\Theta\mu(y-x)-q}{\lambda\Theta\mu(y-x)} \leq \tau \leq \frac{\lambda\Theta(1-\pi)(y-x)-q}{\lambda\Theta(1-\pi)(y-x)}$, while the expected per capita income and degree of income inequality are given by

$$E(Y_3) = \frac{1}{2}y + \frac{1}{2}x$$

and

$$E(\sigma_3) = \frac{1}{2}(1-\tau)(y-x)$$

3.4. Equilibrium 4

Finally, in the fourth equilibrium, the tax rate is high enough to discourage not only non-talented, but also talented agents from acquisition of education. Consequently, in this equilibrium all individuals choose to remain idle. The sufficient and necessary condition for this equilibrium is $\tau \geq \frac{\lambda\Theta(1-\pi)(y-x)-q}{\lambda\Theta(1-\pi)(y-x)}$, while the expected per capita income and degree of income inequality are given by

$$E(Y_4) = \frac{\pi}{2}y + \frac{2-\pi}{2}x$$

and

$$E(\sigma_4) = (1-\tau)\frac{\sqrt{\pi(2-\pi)}}{2}(y-x)$$

4. The Implications of Redistributive Income Taxation

In this section, we consider the effects of increasing the rate of income redistribution. Henceforth, τ_0 and τ' refer to initial and current tax rates, respectively, while $\sigma[\tau]$ refers to the degree of income inequality at the tax rate τ. According to the first of the definitions below, tax increases that induce agents to alter their educational choices and thus cause the economy to jump from one equilibrium to another, are referred to as *non-marginal*, while tax increases that leave individual allocations unaffected are referred to as *marginal*. Moreover, according to the second definition, non-marginal tax increases are classified as *small*, *medium* or *large* depending on the extent to which it affects individuals' allocations.

Definition 2. *Define the following tax rates;* $\tau^* \equiv \frac{q-(1-\theta)x}{(1-\theta)(z-x)}$, $\tau^{**} \equiv \frac{\lambda\Theta\mu(y-x)-q}{\lambda\Theta\mu(y-x)}$ *and* $\tau^{***} \equiv \frac{\lambda\Theta(1-\pi)(y-x)-q}{\lambda\Theta(1-\pi)(y-x)}$. *If* $\tau_0 < \widetilde{\tau}$ *and* $\widetilde{\tau} < \tau'$, *where* $\widetilde{\tau} \in \{\tau^*, \tau^{**}, \tau^{***}\}$, *then* $\tau' - \tau_0$ *is a non-marginal tax increase, while if* τ_0 *and* τ' *both belong to one of the intervals* $[0, \tau^*]$, $[\tau^*, \tau^{**}]$ *or* $[\tau^{**}, \tau^{***}]$, *then* $\tau' - \tau_0$ *is a marginal tax increase.* ■

Definition 3. *A non-marginal tax increase is small if it causes the economy to jump from equilibrium* j *to equilibrium* $j+1$, *medium if it causes the economy to jump from* *equilibrium j to equilibrium $j+2$, and large if it causes the economy to jump from equilibrium j to equilibrium $j+3$.* ■

Note that any marginal increase in redistributive taxation causes inequality to decrease, but leaves per capita income unaffected. In what follows, the implications of non-marginal tax increases are analyzed. The proofs of all propositions in this section are gathered in Appendix C.

The proposition below establishes that if the economy is initially in an equilibrium in which education is affordable to anyone, an increase in the rate of redistribution causes per capita income to fall, while if the economy is initially in an equilibrium in which only rich agents can invest in education, the effect of further redistribution on per capita income is ambiguous and depends on the magnitude of the tax increase. Particularly, if the increase in the tax rate makes education affordable to poor agents without discouraging talented agents from undertaking investment, then redistribution generates a rise in per capita income. However, if the tax rate increases to the extent that all agents refrain from acquiring education, redistribution produces a decline in per capita income.

Proposition 1. *A non-marginal increase in the rate of redistributive taxation generates a rise in per capita income if $\tau_0 < \tau^*$ and $\tau' < \tau^{***}$, and a decline otherwise.*

An important implication of Proposition 1 is that income redistribution need not be harmful to human capital accumulation, the reason being that redistributive taxation generates two counter-acting effects on agents' investment decisions. The first is the *facilitation effect*, according to which poor individuals are given the opportunity to invest in education. This effect stems from the assumptions of credit constraints and non-convexities in human capital investment. The second is the *distortion effect*, which refers to the adverse implications of redistribution on individuals' incentives to invest in education. While the facilitation effect is beneficial to human capital accumulation and per capita income, the distortion effect is clearly unfavorable.

Clearly, Proposition 1 indicates that the facilitation effect is stronger than the distortion effect only in the presence of wealth constraints and for sufficiently small increases in taxation, that is if only rich individuals can afford schooling and if redistribution facilitates investment for the poor without destroying incentives for acquiring education. However, if wealth constraints are absent, the distortion effect is the strongest, thus further redistribution merely weakens incentives to invest in schooling, which in turn slows down aggregate economic activity. In other words, the facilitation effect dominates the distortion effect in the case where a non-marginal tax increase causes the economy to jump from equilibrium 1 to equilibrium 2 or 3, but not in any other case (see Figure 1 for an illustration). These implications are similar to those of Bénabou [1996] and Perotti [1993].[9]

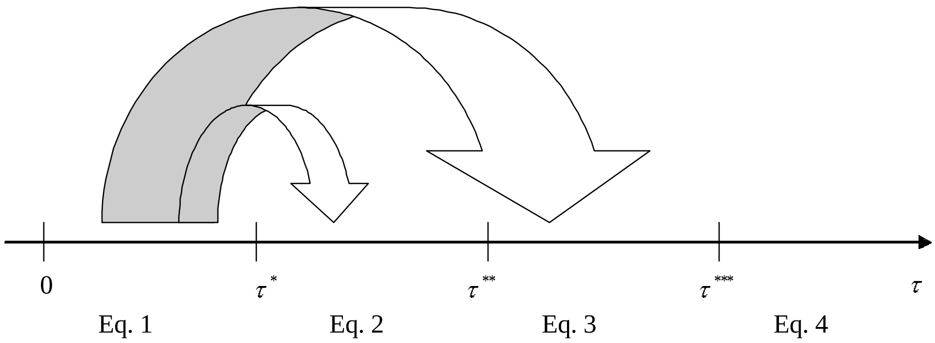

Figure 1. The cases where a tax increase implies a rise in aggregate income.

Another implication of Proposition 1 is that the allocation of talent matters to aggregate economic activity (cf Murphy et al. [1991], Fershtman et al. [1996] and Hassler and Rodríguez-Mora [2000]). In particular, per capita income seems to be higher in equilibria where only talented individuals acquire education, than in equilibria where only rich agents choose to do so, although the rate of income taxation is higher in the former case. In other words, talented agents are more productive than rich agents.[10]

[9] See also Saint-Paul and Verdier [1993] and Fernández and Rogerson [1996], who show theoretically and empirically, respectively, that redistribution through public provision of education is beneficial to growth if individuals are differentiated by economic opportunity.

[10] In an alternative setting, an equilibrium in which all agents invest in schooling need not be the most efficient outcome. For instance, in Fershtman et al [1996], the demand for social status may induce untalented and rich (type φ) agents to acquire education, thus increasing the supply of high-skill labour and "crowding out" talented and poor (type β) agents by weakening their incentives to invest in schooling. In turn, as talented and poor agents are typically more productive than untalented

The next proposition indicates that the effect of a non-marginal tax increase on after-tax income inequality depends on the degree of pre-tax income dispersion as well as the degree of intergenerational persistence in talent. In particular, the proposition establishes that if the pre-tax dispersion of income is narrow enough and if the degree of heritability of talent is not too high, then redistribution of income generates a decline in inequality if the increase in the tax rate is sufficiently large to remove untalented, but not talented individuals' incentives for investment in education, and a rise otherwise.

Proposition 2. *Assume that* $y < \frac{7}{3}x$ *and* $\rho < \frac{2}{3}$. *Then a non-marginal increase in the rate of redistributive taxation,* τ*, generates a rise in inequality if* $\tau^{**} < \tau' < \tau^{***}$*, and an ambiguous effect on inequality otherwise.*

An interesting implication of Proposition 2 is that redistribution clearly need not produce a more equal income distribution. This is because redistribution gives rise to two separate, and sometimes counter-acting, effects on the distribution of income. The first effect is the *equalization effect*, which refers to the reduction of after-tax inequality that is generated by redistributive taxation. The second is the *allocation effect*, which stems from the assumption of heterogeneity in individual talent and refers to the change in the pre-tax income distribution that arises to the extent that redistribution induces agents to alter their investment decisions. If the allocation effect is positive and in excess of the equalization effect, inequality rises, rather than declines, in response to income redistribution. Note that the wider is the dispersion of pre-tax income, the more likely is the equalization effect to offset the allocation effect.

Indeed, Proposition 2 implies that if incomes are not too widely dispersed and if the degree of heritability is sufficiently low, then the allocation effect is larger than the equalization effect if the tax rate is raised to the extent that only talented agents benefit from investment in education. In other words, the allocation effect dominates the equalization effect only in the case where a non-marginal tax increase causes the

and rich individuals, economic growth is discouraged.

economy to jump from equilibrium 1 to equilibrium 2 (see Figure 2 for an illustration).

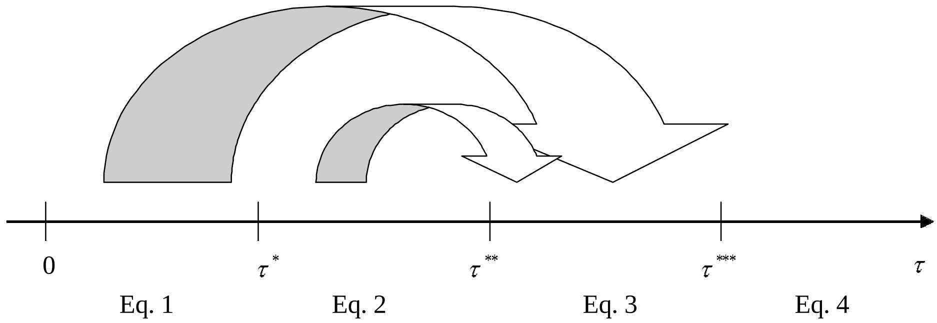

Figure 2. The cases where a tax increase implies a rise in inequality.

Suppose that redistribution is considered efficient to the extent that it increases per capita income and equitable to the extent that it reduces income inequality.[11] Then Proposition 1 and 2 imply that while redistributive income taxation need not be inefficient, neither is it necessarily equitable. In other words, there is no unambiguous relationship, and particularly not a negative one, between the effects of redistributive policy on efficiency and equity. Further, Proposition 1 and 2 imply that the implications of an increase in the rate of income redistribution depend on the initial rate of taxation in the economy as well as the magnitude of the tax increase. The reason is, for one thing, that the implications of redistribution are determined by the relationships between the facilitation and distortion effects on the one hand, and the allocation and equalization effects on the other, and for another, that the relative strength of these effects seems to be highly variable with respect to the rate of taxation. Consider the case where $y < \frac{7}{3}x$ and $\rho < \frac{2}{3}$, that is the case analyzed in Proposition 2. Table 1 shows how inequality and per capita income respond to non-marginal increases in the rate of redistribution, depending on which of these effects are in dominance. Furthermore, Figure 3 depicts the consequences of non-marginal increases in the rate of redistributive taxation in terms of efficiency and equity, depending on the initial tax rate and the size of the tax increase.

	Facilitation effect	Distortion effect
Allocation effect	$\sigma \uparrow, Y \uparrow$	$\sigma \uparrow, Y \downarrow$
Equalization effect	$\sigma \downarrow, Y \uparrow$	$\sigma \downarrow, Y \downarrow$

[11] It is easy to show that the results remain intact in the case where efficiency is measured as aggregate income less the aggregate cost of education.

Table 1.

Clearly, Figure 1 indicates that the effects of a non-marginal tax increase are highly dependent of the initial rate of taxation as well as the magnitude of the tax increase. Consider first the case where education is affordable only to rich agents, that is the case where the economy is in equilibrium 1. In this case, a small (non-marginal) increase in the tax rate, that is an increase that causes the economy to jump to equilibrium 2, is conducive to equity as well as efficiency (cf the lower right corner of Figure 3). Furthermore, a medium tax increase, that is an increase that causes the economy to jump to equilibrium 3, seems to enhance efficiency at the expense of a decline in equity (cf the upper right corner of Figure 3). Finally, a large tax increase, that is an increase that causes the economy to jump to equilibrium 4, gives rise to declining efficiency and rising equity (cf the lower left corner of Figure 3). Consider now the case where education is affordable to everyone, that is the case where the economy is in equilibrium 2, 3 and 4. If the economy is initially in equilibrium 2, a small increase in the tax rate, that is an increase that causes the economy to jump to equilibrium 3, turns out to be inefficient as well as unequitable (cf the upper left corner of Figure 1). In any other case, non-marginal tax increases give rise to declining efficiency and rising equity (cf the lower left corner of Figure 3).

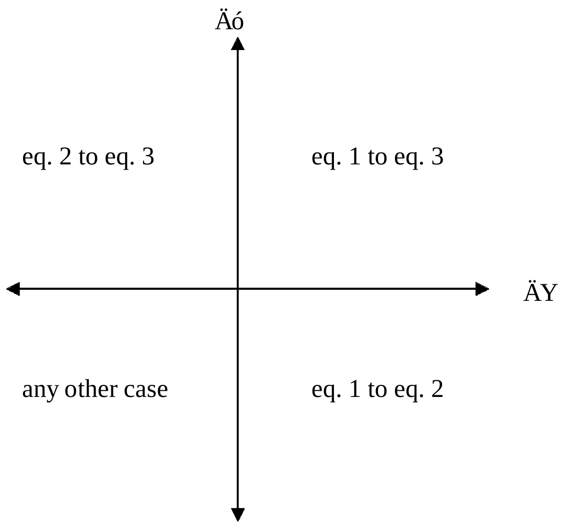

Figure 3.

It follows that the effects of taxation are highly dependent of the initial rate of taxation as well as the magnitude of the tax increase. In particular, a small non-marginal tax increase is twice beneficial in terms of efficiency and equity if the economy is initially in equilibrium 1, but twice harmful if the economy is initially in equilibrium 2. Nevertheless, any other tax increase induces a trade-off between equity and efficiency.

In turn, this result yields two additional implications. The first is that the optimal design of economic policy depends to a large extent on the initial state of the economy. In particular, the appropriate response of policy to a highly unequal income distribution should not necessarily be to increase the rate of redistributive taxation. In the case where high income inequality is a consequence of lacking economic opportunity to invest in education, as in equilibrium 1, a moderate increase in taxation clearly enhances equity as well as efficiency. However, in the case where income inequality is due to lacking intellectual ability to undertake investment, as in equilibrium 3, the optimal policy response is apparently to cut, rather than increase the rate of taxation.

The second implication is that class societies may arise at high as well as low rates of redistribution. We define a class society as an equilibrium where individuals are differentiated by their level of education. Hence, class societies arise in equilibrium 1 and 3, that is when $\tau < \tau^*$ and $\tau^{**} < \tau < \tau^{***}$, respectively, but not in equilibrium 2 or 4, that is when $\tau^* < \tau < \tau^{**}$ or $\tau > \tau^{***}$. Thus, while a moderate tax increase in equilibrium 1 eliminates the class society by facilitating investment for poor individuals, larger increases in redistributive taxation might create even larger class differences by destroying incentives to invest for the less talented.

It should be noted that the channel of redistribution is not particularly critical to the results. Suppose that the proceeds from taxation is used to subsidize the education fee, rather than to finance a lump-sum transfer. Under this policy, the equalization effect of redistribution turns out to be reversed in equilibria where only a fraction of the population undertake investment in education. This effect arises as all agents' incomes are subject to taxation, while only those who acquire schooling are entitled to the subsidy. Thus, in equilibria in which the investment subsidy is either too small to make education affordable to poor agents, or too large to make education attractive to non-talented agents, that is equilibria in which either only rich individuals or only talented individuals acquire schooling, redistributive taxation implies a transfer of resources from the poor to the rich.[12] Hence, as opposed to the case of lump-sum transfers, in this case marginal tax increases in equilibrium 1 and 3 gives rise to increasing, rather than decreasing income inequality. Recall that Proposition 2 establishes that income inequality is higher in equilibrium 1 and 3 than in equilibrium 2 and 4 in the case of proportional taxation with lump-sum transfers. Clearly, this result remains intact in the present case, since the de-equalizing effects of marginal tax increases in equilibrium 1 and 3 imply that inequality in these equilibria is even higher in the case of investment subsidies than in the case of lump-sum transfers. It follows that replacing general transfers by investment subsidies does not significantly alter the effects of redistributive taxation on either per capita income or the distribution of income.[13]

5. Concluding Remarks

In this paper, I have shown, firstly, that redistributive taxation need not induce a trade-off between efficiency and equity, and secondly, that the effects of increasing the rate of income redistribution are highly dependent on the initial tax burden of the economy. In particular, while redistributive policy is likely to be beneficial to society both in terms of efficiency and equity in low-tax economies, it may be highly damaging in both respects if the rate of redistribution is already moderate or high. Evidently, a variety of topics remain to be analyzed within the present framework. On the empirical side, an obvious first step would be to confront the model with an appropriate data set. Particularly, it would be interesting to estimate the real world cut-off tax rates, at which investment in education is made affordable to the poor, and at which individuals' incentives to invest

[12] Note that in this case, transfering from the poor to the rich is equivalent to transfering from the untalented to the talented.

[13] In contrast, Bénabou [2001] argues that redistribution through investment subsidies reduces inequality more efficiently, that is with less distortions, than does redistribution through progressive income taxation.

are destroyed.

Furthermore, on the theoretical side, a promising extension would be to reconsider the modelling of individual talent as well as the learning technology. Empirical evidence typically indicates that talent, or learning ability, is determined by social as well as genetic factors (cf Behrman et al [1981]). Yet, allowing social capital or family environment to influence talent in this framework would merely imply a smaller share of poor but talented individuals and thus as weakening of the efficiency case for income redistribution. Nevertheless, in combination with a more complex learning technology, an alternative definition of talent would possibly be more useful. In particular, it would be interesting to examine the efficiency and equitableness of redistributive tax policy in the case where talent depends on both social and genetic endowments and where the achievements of the most successful individuals trickle down on their peers through local or aggregate human capital spillovers. However, these are topics for future research.

References

Behrman, J., Z. Hrubec, P. Taubman and T. Wales [1980]. *Socioeconomic Success - A Study of the Effects of Genetic Endowments, Family Environment and Schooling*, Amsterdam, North-Holland.

Bénabou, R. [1996]. "Inequality and Growth" in Bernanke, B. and J. Rotemberg (ed.), *NBER Macroeconomics Annual*, 11–74, Cambridge and London, MIT Press.

Bénabou, R. [2001]. "Tax and Education Policy in a Heterogeneous Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?", mimeo.

Card, D. [1998]. "The Causal Effect of Education on Earnings", mimeo.

Durlauf, S. [1996]. "A Theory of Persistent Income Inequality", *Journal of Economic Growth*, 1(1), 75–93.

Fernández, R. and R. Rogerson [1998]. "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform", *American*

Economic Review, 88(4), 813–33.

Fershtman, C., K. Murphy and Y. Weiss [1996]. "Social Status, Education and Growth", *Journal of Political Economy*, 104(1), 108–32.

Galor, O. and J. Zeira [1993]. "Income Distribution and Macroeconomics", *Review of Economic Studies*, 60(1), 35–52.

Glomm, G. and B. Ravikumar [1992]. "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality", *Journal of Political Economy*, 100(4), 813–34.

Hassler, J. and J.V. Rodríguez-Mora [2000]. "Intelligence, Social Mobility and Growth", *American Economic Review*, 90(4), 888–908.

Loury, G. [1981]. "Intergenerational Transfers and the Distribution of Earnings", *Econometrica*, 49, 843–67.

Murphy, K., A. Shleifer and R. Vishny [1991]. "The Allocation of Talent: Implications for Growth", *Quarterly Journal of Economics*, 106(2), 503–30.

Perotti, R. [1993]. "Political Equilibrium, Income Distribution and Growth", *Review of Economic Studies*, 60, 755–76.

Saint-Paul, G. and T. Verdier [1993]. "Education, Democracy and Growth", *Journal of Development Economics*, 42(2), 399–407.

Appendix A.

The transition matrices of individuals in each of the four equilibria are depicted below.

Equilibrium 1

	α	β	φ	γ
α	ρ	0	$1-\rho$	0
β	$\pi\rho$	$(1-\pi)\rho$	$\pi(1-\rho)$	$(1-\pi)(1-\rho)$
φ	$\mu(1-\rho)$	$(1-\mu)(1-\rho)$	$\mu\rho$	$(1-\mu)\rho$
γ	0	$1-\rho$	0	ρ

Equilibrium 2

	α	β	φ	γ
α	ρ	0	$1-\rho$	0
β	ρ	0	$1-\rho$	0
φ	$\mu(1-\rho)$	$(1-\mu)(1-\rho)$	$\mu\rho$	$(1-\mu)\rho$
γ	$\mu(1-\rho)$	$(1-\mu)(1-\rho)$	$\mu\rho$	$(1-\mu)\rho$

Equilibrium 3

	α	β	φ	γ
α	ρ	0	$1-\rho$	0
β	ρ	0	$1-\rho$	0
φ	0	$1-\rho$	0	ρ
γ	0	$1-\rho$	0	ρ

Equilibrium 4

	α	β	φ	γ
α	$\pi\rho$	$(1-\pi)\rho$	$\pi(1-\rho)$	$(1-\pi)(1-\rho)$
β	$\pi\rho$	$(1-\pi)\rho$	$\pi(1-\rho)$	$(1-\pi)(1-\rho)$
φ	0	$1-\rho$	0	ρ
γ	0	$1-\rho$	0	ρ

Furthermore, the stationary distributions of agents in equilibrium 1, 2, 3 and 4 are summarized in the table below.

	1	2	3	4
α	$\frac{\pi(\mu+\rho-2\mu\rho)}{2(1-\mu+\pi\mu-\rho+\mu\rho+\rho\pi-2\rho\mu\pi)}$	$\frac{\rho-\mu\rho+\mu}{2}$	$\frac{\rho}{2}$	$\frac{\rho\pi}{2}$
β	$\frac{(1-\rho)(1-\mu)}{2(1-\mu+\pi\mu-\rho+\mu\rho+\rho\pi-2\rho\mu\pi)}$	$\frac{(1-\mu)(1-\rho)}{2}$	$\frac{1-\rho}{2}$	$\frac{1-\rho\pi}{2}$
φ	$\frac{\pi(1-\rho)}{2(1-\mu+\pi\mu-\rho+\mu\rho+\rho\pi-2\rho\mu\pi)}$	$\frac{1-\rho+\mu\rho}{2}$	$\frac{1-\rho}{2}$	$\frac{\pi(1-\rho)}{2}$
γ	$\frac{(1-\mu)(1+2\rho\pi-\pi-\rho)}{2(1-\mu+\pi\mu-\rho+\mu\rho+\rho\pi-2\rho\mu\pi)}$	$\frac{\rho(1-\mu)}{2}$	$\frac{\rho}{2}$	$\frac{1+\rho\pi-\pi}{2}$

Since the shares of individual types sum up to 1, per capita income is equivalent to aggregate income. Thus, expected per capita income in each of the equilibria is calculated as

$$
\begin{aligned}
E(Y_1) &= \alpha y + \beta\left(\pi y + (1-\pi)x\right) + \varphi\left(\mu y + (1-\mu)x\right) + \gamma x \\
E(Y_2) &= (\alpha+\beta)\,y + (\varphi+\gamma)\left(\mu y + (1-\mu)x\right) \\
E(Y_3) &= (\alpha+\beta)\,y + (\varphi+\gamma)\,x \\
E(Y_4) &= (\alpha+\beta)\left(\pi y + (1-\pi)x\right) + (\varphi+\gamma)\,x
\end{aligned}
$$

Finally, realize that the expected degree of income inequality in equilibrium j, $E(\sigma_j)$, is defined by the standard deviation of expected after-tax income. This implies that the degree of income inequality in each of the equilibria is calculated as

$$\begin{aligned}
E(\sigma_1) &= (1-\tau)\sqrt{(\alpha+\beta\pi+\varphi\mu)(y-E(Y_1))^2+(\beta(1-\pi)+\varphi(1-\mu)+\gamma)(x-E(Y_1))^2}\\
E(\sigma_2) &= (1-\tau)\sqrt{(\alpha+\beta+(\varphi+\gamma)\mu)(y-E(Y_2))^2+(\varphi+\gamma)(1-\mu)(x-E(Y_2))^2}\\
E(\sigma_3) &= (1-\tau)\sqrt{(\alpha+\beta)(y-E(Y_3))^2+(\varphi+\gamma)(x-E(Y_3))^2}\\
E(\sigma_4) &= (1-\tau)\sqrt{(\alpha+\beta)\pi(y-E(Y_4))^2+((\alpha+\beta)(1-\pi)+\varphi+\gamma)(x-E(Y_4))^2}
\end{aligned}$$

Appendix B.

Definition 4. *Define the constant* $\Phi \equiv \frac{1}{2}\frac{\pi(1-\rho\mu)}{(1-\mu)(1-\rho)+\rho\pi(1-\mu\rho)}$. ■

In this appendix, z_j denotes the tax base in equilibrium j, where $j = 1, 2, 3, 4$. The proposition below ensures the existence of equilibrium 4.

Proposition 3. *The tax rate at which talented agents do not find education worthwhile,* $\frac{\lambda\Theta(1-\pi)(y-x)-q}{\lambda\Theta(1-\pi)(y-x)}$, *falls short of the tax rate at which education becomes unaffordable to rich agents,* $\frac{(1-\theta)y-q}{(1-\theta)(y-z_3)}$. ■

Proof. The equilibrium condition $\frac{\lambda\Theta(1-\pi)(y-x)-q}{\lambda\Theta(1-\pi)(y-x)} < \frac{(1-\theta)y-q}{(1-\theta)(y-z_3)}$, or $\frac{\Lambda(1-\pi)(y-x)-\frac{1}{1-\theta}q}{\Lambda(1-\pi)(y-x)} < \frac{y-\frac{1}{1-\theta}q}{y-z_3}$, is satisfied if and only if

$$q < \frac{2\Lambda(1-\theta)(1-\pi)}{2\Lambda(1-\pi)-1}\left(\tfrac{1}{2}y+\tfrac{1}{2}x\right) \tag{5.1}$$

Clearly, (5.1)holds by transitivity if its RHS exceeds $(1-\theta)(\Phi y+(1-\Phi)x)$, that is if $\frac{2\Lambda(1-\pi)}{2\Lambda(1-\pi)-1}\left(\frac{1}{2}y+\frac{1}{2}x\right) > \Phi y+(1-\Phi)x$. We know that $\Phi < \frac{1}{2}$, hence the condition is satisfied. This verifies the proposition. ■

The existence of equilibrium 2 is ensured by the second part of Assumption 2. Finally, note that Assumption 1 implies that $\frac{q-(1-\theta)x}{(1-\theta)(z_1-x)} > 0$ and that the condition that the expected value of education be higher to talented than to non-talented agents implies that $\frac{\lambda\Theta(1-\pi)(y-x)-q}{\lambda\Theta(1-\pi)(y-x)} > \frac{\lambda\Theta\mu(y-x)-q}{\lambda\Theta\mu(y-x)}$, and thus that equilibrium 1 and equilibrium

3 exist. In other words, the cut-off tax rates of the model may be ordered according to Figure B1.

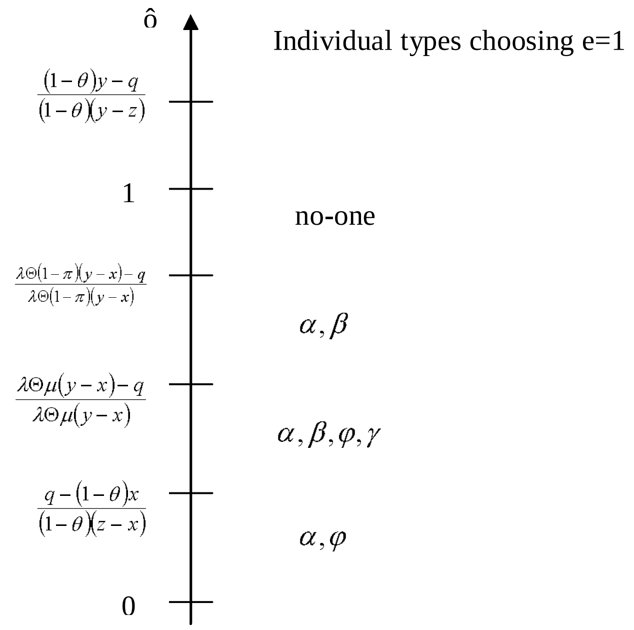

Figure B1.

Appendix C.

Proof of Proposition 1. Consider first the case where $\tau_0 < \tau^*$, that is the case where the economy is initially in equilibrium 1. In the case where $\tau_0 < \tau^*$ and $\tau' < \tau^{**}$, that is the case where the economy jumps from equilibrium 1 to equilibrium 2, the change in per capita income is positive if and only if

$$\tfrac{1}{2}\left(1-\mu\right)\tfrac{(2\pi\mu-\mu+\pi-1)\rho+(1-\pi)(1+\mu)}{(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)}\left(y-x\right)>0 \tag{5.2}$$

Indeed, (5.2) is satisfied for all parameter values if its LHS is positive at its minimum, that is if $\rho < \frac{(1-\pi)(1+\mu)}{1+\mu-\pi-2\pi\mu}$. In turn, this inequality holds by transitivity if its RHS exceeds 1, that is if $\frac{\pi\mu}{1+\mu-\pi-2\pi\mu} > 0$, which is clearly true. Thus $Y_2 - Y_1 > 0$. Further, in the case where $\tau_0 < \tau^*$ and $\tau^{**} < \tau' < \tau^{***}$, that is the case where the economy jumps from equilibrium 1 to equilibrium 3, the change in per capita income is positive if and only if

$$\tfrac{(1-\rho)(1-\mu-\pi)}{2(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)}\left(y-x\right)>0$$

which is clearly true for any set of parameter values. Hence, it must hold that $Y_3 - Y_1 > 0$.

Finally, in the case where $\tau_0 < \tau^*$ and $\tau^{***} < \tau'$, that is the case where the economy jumps from equilibrium 1 to equilibrium 4, the change in per capita income is negative if and only if

$$\tfrac{1}{2}\pi \tfrac{(3\mu-1+\pi-2\pi\mu)\rho-\mu(2-\pi)}{(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)}\,(y-x) < 0 \tag{5.3}$$

Clearly, (5.3) is satisfied for all sets of parameter values if its LHS is negative at its maximum, that is if $\rho < \frac{\mu(2-\pi)}{(3\mu-1+\pi-2\pi\mu)}$. In turn, this condition holds by transitivity if its RHS is in excess of 1, that is if $\frac{(1-\pi)(1-\mu)}{3\mu-1+\pi-2\pi\mu} > 0$, which is obviously true. It follows that $Y_4 - Y_1 < 0$.

Consider now the case where $\tau_0 > \tau^*$, that is the case where the economy is initially in equilibrium 2 or 3. In the case where $\tau^* < \tau_0 < \tau^{**}$ and $\tau' < \tau^{***}$, that is the case where the economy jumps from equilibrium 2 to equilibrium 3, the change in per capita income is negative if and only if $-\frac{1}{2}\mu\,(y-x) < 0$, which is clearly true. Hence, $Y_3 - Y_2 < 0$. Furthermore, in the case where $\tau^{**} < \tau_0 < \tau^{***}$ and $\tau^{***} < \tau'$, that is the case where the economy jumps from equilibrium 3 to equilibrium 4, the change in per capita income is negative if and only if $-\frac{1}{2}(1-\pi)(y-x) < 0$, which is indeed true, thus $Y_4 - Y_3 < 0$. Note that our proofs of $Y_2 > Y_3$ and $Y_3 > Y_4$ imply by transitivity that as the economy jumps from equilibrium 2 to equilibrium 4, per capita income decreases, thus $Y_4 - Y_2 < 0$. This verifies the proposition. ■

Proof of Proposition 2. The change in income inequality between equilibrium j and k, where j and k denote the initial and the current equilibrium, respectively, is positive for all feasible τ by transitivity if $\sigma_k^{\min} - \sigma_j^{\max} > 0$, and negative if $\sigma_k^{\max} - \sigma_j^{\min} < 0$. Consider first the case where $\tau' < \tau^{**}$, that is the case where the economy jumps from equilibrium 1 to equilibrium 2. In this case, the change in inequality is negative for all τ if $\sigma_2^{\max} - \sigma_1^{\min} < 0$, that is if $\sigma_2[\tau^*] - \sigma_1[\tau^*] < 0$. Thus, $\sigma_2 - \sigma_1 < 0$ if and only if

$$\tfrac{1}{2}(1-\tau^*)\,\frac{\sqrt{(1-\mu)(1+\mu)}(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)-\sqrt{\pi(1-\mu)(1-2\rho\mu+\mu)(2\rho\pi-2\rho+2-\pi)}}{(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)}\,(y-x) < 0$$

which, in turn, is satisfied if and only if

$$\sqrt{\pi(1-2\rho\mu+\mu)(2\rho\pi-2\rho+2-\pi)} > (1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)\sqrt{1+\mu} \tag{5.4}$$

Note that the LHS and RHS of (5.4) increases and decreases, respectively, in ρ. This implies that a sufficient condition for (5.4) to be satisfied is that it holds for the lower bound of ρ, which is equal to $\frac{1}{2}$. Replacing $\frac{1}{2}$ in (5.4) yields

$$2\sqrt{\pi} > (1-\mu+\pi)\sqrt{1+\mu} \tag{5.5}$$

the RHS of which clearly decreases in μ. Thus, a sufficient condition for (5.5) to be satisfied is that it holds for the upper bound of μ, which is equal to $1-\pi$. In turn, inserting $1-\pi$ in (5.5) yields $\sqrt{\pi} > \pi\sqrt{2-\pi}$, which is evidently true. It follows that $\sigma_2 - \sigma_1 < 0$.

Consider now the case where $\tau^{**} < \tau' < \tau^{***}$, that is the case where the economy jumps from equilibrium 1 or 2 to equilibrium 3. In the former case, the change in inequality is positive for all τ if $\sigma_3^{\min} - \sigma_1^{\max} < 0$, that is if $\sigma_3[\tau^{***}] - \sigma_1[0] > 0$. Thus, $\sigma_3 - \sigma_1 > 0$ if and only if

$$\frac{(1-\tau^{***})(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)-\sqrt{\pi(1-\mu)(1-2\rho\mu+\mu)(2\rho\pi-2\rho+2-\pi)}}{2(1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu)}(y-x) > 0$$

that is if

$$q > \lambda\Theta\frac{(1-\pi)\sqrt{\pi(1-\mu)(1-2\rho\mu+\mu)(2\rho\pi-2\rho+2-\pi)}}{1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu}(y-x) \tag{5.6}$$

Indeed, it follows from Assumption 3 that (5.6) is satisfied by transitivity if its RHS falls below $\lambda\Theta x$, that is if

$$\frac{1-\mu-\rho+\rho\mu+\rho\pi+\pi\mu-2\rho\pi\mu}{1-\pi} > \frac{\sqrt{\pi(1-\mu)(1-2\rho\mu+\mu)(2\rho\pi-2\rho+2-\pi)}(y-x)}{x} \tag{5.7}$$

Note that the LHS and RHS of (5.7) decreases and increases, respectively, in ρ. Note also that the RHS of (5.7) is an increasing function of y. This implies that the lower is ρ and the lower is $\frac{y}{x}$, the more likely is a non-marginal tax increase to enhance income inequality. Thus, (5.7) is satisfied for any parameter values if it holds for the upper bound of ρ and $\frac{y}{x}$, respectively. Consider the case where $\rho < \frac{2}{3}$ and $\frac{y}{x} < \frac{7}{3}$. Substituting $\frac{2}{3}$ for ρ and $\frac{7}{3}$ for $\frac{y}{x}$ in (5.7) yields

$$4(1-\pi)\sqrt{\pi(1-\mu)(3-\mu)(\pi+2)} < 3(1-\mu+2\pi-\pi\mu) \tag{5.8}$$

Clearly, the LHS and RHS of (5.8) decreases and increases, respectively, in μ. This implies that a sufficient condition for (5.8) to be satisfied is that it holds for the lower bound of μ, which is equal to π. Replacing π in (5.8) yields

$$3 + 3\pi - 3\pi^2 - 4(1-\pi)\sqrt{\pi(1-\pi)(3-\pi)(\pi+2)} > 0 \tag{5.9}$$

Plotting the LHS of (5.9) yields

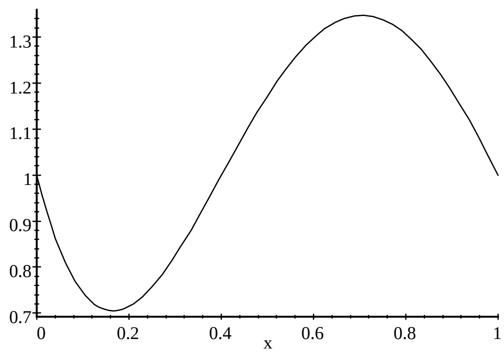

Figure 5.

It follows from the plot that the LHS of (5.9) exceeds zero for all feasible values of π. Hence, it must hold that $\sigma_3 - \sigma_1 > 0$ if $y < \frac{7}{3}x$ and $\rho < \frac{2}{3}$, and that $\sigma_3 - \sigma_1 \lesseqgtr 0$ otherwise.

Consider now the latter of the two cases where $\tau^{**} < \tau' < \tau^{***}$, that is the case where the economy jumps from equilibrium 2 to equilibrium 3. In this case, the change in inequality is positive for all τ if $\sigma_3^{\min} - \sigma_2^{\max} > 0$, that is if $\sigma_3[\tau^{***}] - \sigma_2[\tau^{**}] > 0$. But since σ is a decreasing function of τ, it follows by transitivity from our proofs of $\sigma_3[\tau^{***}] - \sigma_1[0] > 0$ and $\sigma_1[\tau^*] > \sigma_2[\tau^*]$ that $\sigma_3[\tau^{***}] > \sigma_2[\tau^{**}]$ or, in other words, that $\sigma_3 - \sigma_2 > 0$.

Consider finally the case where $\tau' > \tau^{***}$, that is the case where the economy jumps from equilibrium 1, 2 or 3 to equilibrium 4. In the second of these cases, that is the case where the economy is initially in equilibrium 2, the change in inequality is negative for all τ if $\sigma_4^{\max} - \sigma_2^{\min} < 0$, that is if $\sigma_4[\tau^{***}] - \sigma_2[\tau^{**}] < 0$. Thus, $\sigma_4 - \sigma_2 < 0$ if and only if

$$\frac{(1-\tau^{***})\sqrt{\pi(2-\pi)} - (1-\tau^{**})\sqrt{(1-\mu)(1+\mu)}}{2}(y-x) < 0 \tag{5.10}$$

But since $\tau^{**} < \tau^{***}$, (5.10) must hold by transitivity if it is satisfied in the case where $\tau^{**} = \tau^{***}$, that is if $\sqrt{\pi(2-\pi)} < \sqrt{(1-\mu)(1+\mu)}$. Clearly, this condition is satisfied if $(1-\mu-\pi)(1+\mu-\pi) > 0$, which indeed holds. It follows that $\sigma_4 - \sigma_2 < 0$.

Note finally that in the case where the economy jumps from equilibrium 1 to equilibrium 4, it follows by transitivity and our proofs of $\sigma_1 > \sigma_2$ and $\sigma_2 > \sigma_4$ that $\sigma_4 - \sigma_1 < 0$. Likewise, in the case where the economy jumps from equilibrium 3 to equilibrium 4, it is implied by transitivity and our proofs of $\sigma_3 > \sigma_1$ and $\sigma_1 > \sigma_4$ that $\sigma_4 - \sigma_3 < 0$. This verifies the proposition. ■