Interregional Inequality and Robin Hood Politics

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Abstract

This paper studies the implications of interregional redistributive taxation on interregional and interpersonal inequality and on social welfare. We introduce a model of two regions, where individuals are differentiated by their ability and opportunity, the former being determined by heritage and the latter by their residence. Moreover, agents are immobile and respond to interregional transfers by adjusting their labour supply, rather than by re-locating. The analysis shows, firstly, that increases in the rate of interregional redistribution need not generate neither reduced interregional inequality nor higher social welfare, and secondly, that their effects are highly dependent on the initial state of the economy. In particular, interregional redistribution seems most likely to be beneficial in terms of interregional and interpersonal equity as well as social welfare in low-tax economies, where the degree of income dispersion is high between, but not within regions.

1. Introduction

A subject of contention on today's political agenda is to what extent rich regions should share their affluence with less fortunate communities. Due to the interregional differences in average factor income that prevail within as well as between European countries, a number of redistributive tax programs have been introduced during the last decades, with the intention to equalize regional income.

In Sweden, for example, the system for reducing interregional fiscal disparities has four components, income equalization, cost equalization, transitional regulation and general government grants.[1] The latter type of grant includes compensation for adverse structural circumstances, such as long distances, cold temperature, demographic

[1]In addition to these explicit systems of interregional redistribution, factor income is also transferred endogenously from rich to poor regions via for example the unemployment insurance system

differences or sparse population.[2] In 2002, nine out of 280 municipalities and one out of 20 county councils made net contributions to the equalization system. The largest and smallest net grant per inhabitant amounted to SEK 23 194 and SEK -11 675, and was received by Dorotea (in the county of Lapland) and Danderyd (in the county of Stockholm), respectively. These net grants accounted for 89 and 26 per cent, respectively, of the regional tax revenue per capita in Dorotea and Danderyd in the same year. Finally, while the income and cost equalizing part of the system was in principle self-financing in 2002, the remaining parts accounted for xx per cent of total transfers in Sweden. In particular, general government grants to municipalities and county councils amounted to SEK 51,8 billion and SEK 17,7 billion, respectively.

Another example is the European Union, where four so-called structural funds have been established with the intention of supporting weak member regions. For instance, the funds encourage the restructuring and modernization of rural areas and low-income regions, and support the training of the unemployed, particularly those who are young or have been out of work for a long time. In 2002, the expenditures of the structural funds accounted for more than a third of the union's budget.

Interregional income redistribution also takes place to various extents within most European countries, which has spurred a large empirical literature investigating the efficiency and equitableness of fiscal equalization between regions. For instance, Decressin [1999] examines the degree of income redistribution and risk sharing among Italian regions, and its implications for public policy, while Berthold et al [2001] study the German system of fiscal federalism and its effects on growth. Further, Garcia-Mila and McGuire [2001] evaluate the efficiency of EU grants as well as interregional transfers among the regional governments of Spain.

In addition to being empirically explored, the topics of fiscal federalism and interregional income equalization have also been subject to a substantial amount of theoretical

("accidental redistribution"). Also, to the extent that income-based tax schedules and transfer systems are progressive, income is equalized not only among individuals, but also across regions.

[2] For a detailed description of the Swedish system for fiscal equalization, see for instance SCB, Statistiska meddelanden OE SM 0201.

work. The primary concern of these studies has typically been to study the optimal design of (redistributive) tax policy. In particular, one strand of this literature focuses on the first-best design of interregional grants when tax bases are mobile or when there are externalities in the provision of public goods and services (e.g., Boadway and Flatters [1982], Brown and Oates [1987], Myers [1990], Wildasin [1991] and Caplan et al. [2000]), while another concentrates on the most efficient redistributive policy under incomplete information (e.g., Cornes and Silva [2002] and Bordignon et al. [2001]). The efficiency of centralized vs decentralized tax policy is discussed in Inman and Rubinfeld [1996], while the equity and efficiency arguments for interregional income equalization are reviewed and commented in Oakland [1994].

Contrary to previous work within the theoretical field, I abstract in this paper from the optimal design of interregional transfer systems, and focus instead on their implications in terms of efficiency and equity. Specifically, the paper addresses the important questions whether interregional redistribution unambiguously equalizes regional income, and under which conditions interregional transfers are most likely to enhance interregional and interpersonal equity as well as society's welfare. The novelty of the analysis is twofold. Firstly, I consider an alternative externality of interregional transfers to what has been common in past work; in this context, individuals are assumed to be immobile and to respond to interregional redistribution by changing their labour supply, rather than their residence. Secondly, I assume that heterogeneity is two-dimensional, thus agents are differentiated by individual- as well as region-specific characteristics.

The paper presents a simple model of an economy consisting of two regions, which differ from each other with respect to their employment opportunities, and are populated by talented and non-talented individuals, respectively. Labour income depends on talent and job opportunities, thus individual and regional disparities imply that average income differs between the two neighbourhoods. In each period, income is taxed and redistributed from the rich to the poor neighbourhood, the sole aim of redistribution being to reduce regional income differentials. However, in addition to equalizing income, interregional redistributive taxation affects individuals' incentives to work, which in turn alters the pre-tax distribution of income. Consequently, the effects of increasing the rate of interregional redistribution are uncertain on interregional and interpersonal inequality as well as on society's welfare.

Two main conclusions emerge from the analysis. The first is that increases in the rate of interregional redistribution need not give rise to neither declining interregional inequality nor increasing social welfare. The second is that the initial rate of interregional redistributive taxation as well as the degree of pre-tax income dispersion and the degree of intra-regional income heterogeneity seem to be highly critical to the effects of an increase in redistributive taxation. In particular, interregional redistributive tax schedules seem most likely to reduce interregional and interpersonal inequality as well as to increase social welfare in low-tax economies where the pre-tax dispersion of income is wide between, but narrow within regions.

The paper is organized as follows. The next two sections outline the theoretical model and its equilibrium properties. In Section 4, the effects of income redistribution on interregional and interpersonal inequality, and on society's welfare are analyzed. Finally, Section 5 concludes.

2. The Model

Consider an economy with missing asset markets that consists of two spatially separated neighbourhoods, $\aleph_{rich}$ and $\aleph_{poor}$. The economy is populated by a unit mass continuum of infinitely-lived individuals, who are differentiated by their residence, $\aleph$, and their talent, ξ, the latter reflecting their aptitude for perceiving new concepts and dealing with advanced tasks. Talent can take on two values, $\underline{\xi}$ and $\overline{\xi}$, where $\underline{\xi} < \overline{\xi}$, and is assigned randomly to each individual. It follows that there are four types of agents, which are characterized by their talent on the one hand, and their residence on the other. Thus, type α is described by $\left\{\overline{\xi}, \aleph_{rich}\right\}$, type β by $\left\{\underline{\xi}, \aleph_{rich}\right\}$, type φ by $\left\{\overline{\xi}, \aleph_{poor}\right\}$ and type γ by $\left\{\underline{\xi}, \aleph_{poor}\right\}$, where $\alpha + \beta + \varphi + \gamma = 1$.

In each period, agents allocate their time between working, l, and leisure, $1-l$, where $l \in \{0, \ell\}$. All agents have identical preferences, which are defined over consumption,

c, and leisure, and are formally represented as[3]

$$u = c^{\theta} (1 - l)^{1-\theta} \tag{2.1}$$

There is no interregional mobility, and agents supply their labour in their home neighbourhood. Moreover, the two neighbourhoods are differentiated by their employment opportunities, but are otherwise identical. In $\aleph_{rich}$, there are high-paying as well as low-paying jobs, while in $\aleph_{poor}$, there are only low-paying jobs.[4] An agent's labour income, w, depends on her talent as well as on the job opportunities of her neighbourhood, thus $w = f(\xi, \aleph)$.[5] There are two levels of labour income, $w \in \{\underline{w}, \overline{w}\}$, where $\overline{w} = f\left(\overline{\xi}, \aleph_{rich}\right)$, $\underline{w} = f\left(\underline{\xi}, \aleph_{rich}\right) = f(\xi, \aleph_{poor})$ and $\overline{w} > \underline{w}$. For simplicity, we define $y \equiv \ell\overline{w}$ and $x \equiv \ell\underline{w}$. Non-working individuals who receive no non-labour income collect fruit and berries costlessly in the backyard, thereby earning a non-pecuniary subsistence wage, q.

Finally, there is a government, that taxes income proportionally at the rate τ in the rich neighbourhood and uses the proceeds of taxation to finance transfers τz to

[3]Due to the simplicity of the setting, particularly the absence of asset markets and intertemporal considerations, we may assume that individuals' time horizon be finite without loss of generality.

[4]An interpretation of this assumption is that "rich" and "poor" regions represent metropolitan and rural areas, respectively, rather than for example neighbour cities or suburbs. This is, firstly, because labour markets are typically geographically separated in the former case, and common in the latter, and secondly, because the supply of high-paying jobs is naturally higher in urban than in rural regions.

Another explanation of regional disparities in employment opportunities is provided by Rosen [200x], who argues that to the extent that goods and services are locally produced as well as consumed, interregional inequality in income as well as job opportunities may be self-reinforcing. The reason is that high-income earners tend to demand a larger variety of goods and services, thus, a wider range of jobs, including more sophisticated ones, are created in high-income regions.

[5]For a discussion of the concept of individual talent and empirical evidence in favour of a positive relationship between earnings and intellectual ability, see for example Behrman et al. [1981]. An alternative way to model income differentials within a neighbourhood is to assume individual disparities in factor endowments, as in for instance Wildasin [1991], where poor agents own only (low-skill) labour, while the rich own a combination of production factors, such as high-skill labour and a fixed factor, typically some natural resource.

the poor neighbourhood, the aim of redistribution being to equalize average regional income. The government's budget constraint is given by

$$\tau z = \begin{cases} \frac{1}{\varphi+\gamma}\tau\left(\alpha y+\beta x\right) & \text{if } l_\alpha = l_\beta = \ell \\ \frac{1}{\varphi+\gamma}\tau\alpha y & \text{if } l_\alpha = \ell,\ l_\beta = 0 \\ 0 & \text{if } l_\alpha = l_\beta = 0 \end{cases} \tag{2.2}$$

where z is the tax base, τz is the transfer per head from the rich to the poor region and l_α and l_β are the labour supply choices of high-paid and low-paid residents of $\aleph_{rich}$, respectively. The transfer may be interpreted as a subsidy to the public provision of private goods and services, such as social security benefits, health care and education. This interpretation implies that to residents of $\aleph_{poor}$, the transfer is a close substitute to labour income. Hence, in addition to equalizing regional income, redistribution from $\aleph_{rich}$ to $\aleph_{poor}$ worsens the incentives to work for residents of not only the providing, but also the recipient region.

In each period, each individual maximizes her instantaneous utility, as given by (2.1), subject to the conditions below. Note that in the case where residents of $\aleph_{rich}$ as well as $\aleph_{poor}$ supply zero labour units, the tax base is zero and thus all agents in the economy subsist by collecting fruit and berries, q.

$$\begin{aligned} l &\in \{0,\ell\} \\ c &= \begin{cases} (1-\tau)\,y & \text{if } \aleph = \aleph_{rich},\ \xi = \overline{\xi},\ l = \ell \\ (1-\tau)\,x & \text{if } \aleph = \aleph_{rich},\ \xi = \underline{\xi},\ l = \ell \\ q & \text{if } \aleph = \aleph_{rich},\ l = 0 \\ x+\tau z & \text{if } \aleph = \aleph_{poor},\ l = \ell \\ \tau z & \text{if } \aleph = \aleph_{poor},\ l = 0,\ z > 0 \\ q & \text{if } \aleph = \aleph_{poor},\ l = 0,\ z = 0 \end{cases} \end{aligned}$$

In order not to overburden the exposition, it is convenient to define the following constants.

Definition 1. *Define the constants* $\lambda \equiv (1-\ell)^{\frac{1-\theta}{\theta}}$ *and* $\widetilde{\ell} \equiv 1 - \left(\frac{2}{2^\theta+4^\theta}\right)^{\frac{1}{1-\theta}}$. ■

Solving the maximization problem yields two decision rules. The first is that high-paid residents of $\aleph_{rich}$, that is agents of type α, devote time to labour as well as to leisure if $u\left[(1-\tau)\,y, 1-\ell\right] \geq u\left[q,1\right]$, that is if the tax rate, τ, falls below $\frac{\lambda y - q}{\lambda y}$. Likewise, low-paid residents of $\aleph_{rich}$, that is agents of type β, find it worthwhile to work if

$u\left[(1-\tau)\,x, 1-\ell\right] \geq u\left[q, 1\right]$, that is if τ is less than $\frac{\lambda x-q}{\lambda x}$. The second rule is that residents of $\aleph_{poor}$, that is agents of type φ and type γ, can afford cutting down their labour supply and enjoying more leisure if $u\left[\tau z, 1\right] \geq u\left[x+\tau z, 1-\ell\right]$, that is if $\tau > \frac{\lambda x}{(1-\lambda)z}$ or, in other words, if the transfer level exceeds $\frac{\lambda x}{1-\lambda}$. Note that the non-negativity condition on $\frac{\lambda x-q}{\lambda x}$ implies that the transfer at which residents of $\aleph_{poor}$ decide to quit working, $\frac{\lambda x}{1-\lambda}$, is clearly higher than the subsistence level of consumption, q.

3. Equilibrium

The model generates four stationary equilibria, in each of which the respective shares of each individual type are equi-proportionate and equal to $\frac{1}{4}$.[6] Thus, by (2.2) the transfer, τz, equals $\frac{1}{2}\tau\,(x+y)$ if all residents of $\aleph_{rich}$ devote ℓ units each to working, and $\frac{1}{2}\tau y$ if only high-paid agents find it worthwhile to do so.

Definition 2. *The degree of interregional inequality in equilibrium k, Σ_k, is measured by the difference in average after-tax income of the residents in $\aleph_{rich}$ and $\aleph_{poor}$, respectively. Further, the degree of interpersonal inequality in equilibrium k, σ_k, is measured by the variance in personal income net of taxes. Finally, society's welfare in equilibrium k, W_k, is measured by the sum of individuals' utilities.* ■[7]

3.1. Equilibrium 1

In the first equilibrium, taxes are not high enough to impose any distortions on any agent's decisions. Thus, all individuals supply ℓ units each of labour and devote the rest of their time to leisure. The sufficient and necessary condition for this equilibrium

[6]Since agents are immobile and the population is evenly spread across $\aleph_{rich}$ and $\aleph_{poor}$, and since the allocation of talent is random, the shares of talented and non-talented agents are equal within as well as across regions in any stationary equilibrium.

[7]It can be shown that the results of the next section are the same in the case where interregional and interpersonal inequality are defined in terms of consumption, rather than income. In other words, fruit and berries may be included in the inequality measures without loss of generality.

is $\tau < \frac{2\lambda x}{(1-\lambda)(y+x)}$, while the degrees of interregional and interpersonal inequality are[8]

$$\Sigma_1 = \tfrac{1}{2}\left((1-2\tau)(y+x)-2x\right) \quad (3.1)$$

$$\sigma_1 = \tfrac{1}{4}\left((1-\tau)y-\widetilde{w}\right)^2+\tfrac{1}{4}\left((1-\tau)x-\widetilde{w}\right)^2+\tfrac{1}{2}\left(x+\tau\tfrac{1}{2}(x+y)-\widetilde{w}\right)^2 \quad (3.2)$$

where $\widetilde{w} \equiv \frac{1}{4}(y+3x)$. Moreover, society's welfare is given by

$$W_1 = \tfrac{1}{4}\left[\left((1-\tau)y\right)^\theta+\left((1-\tau)x\right)^\theta+2\left(x+\tfrac{1}{2}\tau(x+y)\right)^\theta\right](1-\ell)^{1-\theta} \quad (3.3)$$

3.2. Equilibrium 2

In the second equilibrium, the tax rate and thus the interregional transfer is high enough to induce residents of the poor region to work less and enjoy more leisure. The sufficient and necessary condition for this equilibrium is $\frac{2\lambda x}{(1-\lambda)(y+x)} < \tau < \frac{\lambda x - q}{\lambda x}$, while the degrees of interregional and interpersonal inequality are

$$\Sigma_2 = \tfrac{1}{2}(1-2\tau)(y+x) \quad (3.4)$$

$$\sigma_2 = \tfrac{1}{4}\left((1-\tau)y-\widetilde{w}\right)^2+\tfrac{1}{4}\left((1-\tau)x-\widetilde{w}\right)^2+\tfrac{1}{2}\left(\tfrac{1}{2}\tau(y+x)-\widetilde{w}\right)^2 \quad (3.5)$$

where $\widetilde{w} \equiv \frac{1}{4}(y+x)$. Further, social welfare is given by

$$W_2 = \tfrac{1}{4}\left[\left((1-\tau)y\right)^\theta+\left((1-\tau)x\right)^\theta\right](1-\ell)^{1-\theta}+\tfrac{1}{2}\left(\tfrac{1}{2}\tau(x+y)\right)^\theta \quad (3.6)$$

3.3. Equilibrium 3

In the third equilibrium, the tax rate is high enough to induce not only residents of $\aleph_{poor}$, but also low-paid residents of $\aleph_{rich,}$ to re-allocate time from labour to leisure, thus only high-paid agents find it worthwhile to devote time to working. The sufficient and necessary condition for this equilibrium is $\frac{\lambda x - q}{\lambda x} < \tau < \frac{\lambda y - q}{\lambda y}$, while the degrees of interregional and interpersonal inequality are

$$\Sigma_3 = \tfrac{1}{2}(1-2\tau)y \quad (3.7)$$

$$\sigma_3 = \tfrac{1}{4}\left((1-\tau)y-\widetilde{w}\right)^2+\tfrac{1}{4}\left(-\widetilde{w}\right)^2+\tfrac{1}{2}\left(\tfrac{1}{2}\tau y-\widetilde{w}\right)^2 \quad (3.8)$$

[8] The existence of equilibrium 1 is proved in Appendix A.

where $\widetilde{w} \equiv \frac{1}{4}y$. In turn, social welfare is given by

$$W_3 = \frac{1}{4}\left(\left(1-\tau\right)y\right)^{\theta}\left(1-\ell\right)^{1-\theta} + \frac{1}{4}q^{\theta} + \frac{1}{2}\left(\frac{1}{2}\tau y\right)^{\theta} \tag{3.9}$$

The assumption below ensures that the average after-tax income is at least as high in $\aleph_{rich}$ as in $\aleph_{poor}$ at all feasible tax rates.

Assumption 1. Assume that $\frac{y-x}{2(y+x)} > \frac{2\lambda x}{(1-\lambda)(y+x)}$ and that $\frac{1}{2} > \frac{\lambda y - q}{\lambda y}$. ∎

Note that the second part of Assumption 1 implies, together with the non-negativity condition on the tax rate $\frac{\lambda x - q}{\lambda x}$, that the upper bound of y is $2x$.

3.4. Equilibrium 4

Finally, in the fourth equilibrium, the tax rate is high enough to discourage all agents from working. Hence, residents of $\aleph_{rich}$ as well as $\aleph_{poor}$ supply zero labour, subsisting instead by collecting fruit and berries. The sufficient and necessary condition for this equilibrium is $\tau > \frac{\lambda y - q}{\lambda y}$. Moreover, the degrees of interregional and interpersonal inequality are $\Sigma_4 = \sigma_4 = 0$, while society's welfare is given by $W_4 = q^{\theta}$.

4. The Effects of Redistributive Income Taxation

In this section, we consider the effects of increasing the rate of income redistribution from the rich to the poor neighbourhood. Henceforth, τ_0 and τ' refer to initial and current tax rates, respectively, while $\sigma[\tau]$, $\Sigma[\tau]$ and $W[\tau]$ refer to the degrees of interpersonal and interregional inequality and the level of social welfare at the tax rate τ. According to the first of the definitions below, tax increases that induce agents to alter their labour supply decisions and thus cause the economy to jump from one equilibrium to another, are referred to as *non-marginal*, while tax increases that leave individual allocations unaffected are referred to as *marginal*.

Moreover, according to the second definition, an economy in which the rate of redistributive taxation is not high enough to affect individuals' labour supply allocations is referred to as a *low-tax society*, while an economy where taxes are distortionary is referred to as a *high-tax society*. In terms of this definition, an economy that fulfils the sufficient and necessary conditions for equilibrium 1 may be thought of as a low-tax society, while an economy that meets the conditions for equilibrium 2, 3 or 4 may rather be described as a high-tax society.

Finally, according to the third definition, the degree of pre-tax income dispersion is measured as the ratio of the high and the low income realization, respectively.

Definition 3. *If* $\tau_0 < \frac{2\lambda x}{(1-\lambda)(y+x)}$ *and* $\frac{2\lambda x}{(1-\lambda)(y+x)} < \tau'$, *or if* $\tau_0 < \frac{\lambda x - q}{\lambda x}$ *and* $\frac{\lambda x - q}{\lambda x} < \tau'$, *then* $\tau' - \tau_0$ *is a non-marginal tax increase, while if* τ_0 *and* τ' *both belong to one of the intervals* $\left[0, \frac{2\lambda x}{(1-\lambda)(y+x)}\right]$, $\left[\frac{2\lambda x}{(1-\lambda)(y+x)}, \frac{\lambda x - q}{\lambda x}\right]$, $\left[\frac{\lambda x - q}{\lambda x}, \frac{\lambda y - q}{\lambda y}\right]$ *or* $\left[\frac{\lambda y - q}{\lambda y}, 1\right]$, *then* $\tau' - \tau_0$ *is a marginal tax increase.* ■

Definition 4. *Define the following tax rates;* $\tau^* \equiv \frac{2\lambda x}{(1-\lambda)(y+x)}$, $\tau^{**} \equiv \frac{\lambda x - q}{\lambda x}$ *and* $\tau^{***} \equiv \frac{\lambda y - q}{\lambda y}$. *Also, define a low-tax society as an economy in which the tax rate falls short of* τ^*, *and a high-tax society as an economy where the tax rate exceeds* τ^*. ■

Definition 5. *Define the degree of income dispersion,* δ, *as* $\frac{y}{x}$. ■

Note that any marginal increase in redistributive taxation causes interregional as well as interpersonal inequality to decrease, and social welfare to increase.[9] Note also that any non-marginal increase in the tax rate that is large enough to discourage all agents in the economy from working unambiguously reduces interregional and interpersonal inequality as well as social welfare. In what follows, the implications of any other non-marginal tax increases are analyzed. The proofs of all propositions in this section are gathered in Appendix B.

The propositions below establish that the effects of interregional redistributive taxation on interregional and interpersonal inequality depend on the degree of income dispersion in the economy as well as on the initial tax level and the magnitude of the tax increase. Particularly, in a low-tax economy where the dispersion of pre-tax income is narrow enough, any non-marginal increase in the rate of interregional redistribution

[9]This result follows from Assumption 1, the convexity of σ_k, and the concavity of W_k, where $k = 1, 2, 3$.

produces an increase in interregional and interpersonal inequality, while in a high-tax economy, increases in the rate of redistributive taxation unambiguously cause interregional as well as interpersonal inequality to decline.

Proposition 1. *The more narrow is the dispersion of pre-tax regional income, the more likely is a non-marginal increase in the rate of interregional redistributive taxation to enhance interregional inequality. Moreover, the likelihood of a non-marginal tax increase to enhance interpersonal income inequality increases in the degree of income dispersion,* δ*, if* $\delta < \widetilde{\delta}(\tau_0, \tau^*)$*, where* $\frac{\partial \widetilde{\delta}}{\partial \tau_0} > 0$ *and* $\frac{\partial \widetilde{\delta}}{\partial \tau^*} < 0$*, and decreases in* δ *otherwise.*

Proposition 2. *If* $q > \frac{2}{3}\lambda y$*, then a non-marginal increase in interregional redistributive taxation generates a rise in interregional income inequality if* $\tau_0 < \tau^*$*, and a decline otherwise. However, if* $q < \frac{2}{3}\lambda y$*, then the effect of a non-marginal tax increase is ambiguous.*

Proposition 3. *If* $q > \frac{2}{3}\lambda y$*, then any non-marginal increase in interregional redistributive taxation generates a rise in interpersonal income inequality. However, if* $q < \frac{2}{3}\lambda y$*, then the effect of a non-marginal tax increase is ambiguous.*

An interesting implication of Proposition 2, 3 and 4 is that increases in the rate of interregional income redistribution need not, contrary to their purpose, generate a more equal distribution of income between neighbourhoods. The reason is that increases in interregional redistributive taxation give rise to two effects, which may be counter-acting, on interregional (as well as interpersonal) inequality. The first is the *equalization effect*, which refers to the reduction of after-tax inequality that is brought about by an increase in redistributive taxation. The second is the *allocation effect*, which refers to the increase or decrease in pre-tax inequality that arises to the extent that the tax increase induces individuals in either the providing or the recipient region to alter their labour supply decisions. If the allocation effect is positive and in excess of the equalization effect, interregional inequality rises in response to non-marginal tax increases, while in any other case, a non-marginal increase in the tax rate generates a decline in interregional inequality.

It turns out from Proposition 3 that increases in the rate of interregional redistribution seem to unambiguously generate lower interregional and interpersonal inequality only to the extent that individuals' incentives to work are unaffected by the tax increase, or if the initial tax rate is already moderate or high. However, while in the former case, the decrease in inequality is caused by the equalization effect, in the latter case inequality declines as a result of a negative allocation effect, that is declining labour supply in the providing region. Furthermore, as indicated by Proposition 2, increases in the rate of interregional redistributive taxation are more likely to produce lower inequality if the dispersion of pre-tax income, δ, is high between regions. The reason is that the larger is the interregional income differential, the larger is the transfer from $\aleph_{rich}$ to $\aleph_{poor}$ for a given tax rate, and the more likely is thus the equalization effect to offset the allocation effect of interregional redistribution.

Another implication of the propositions above is that depending on the degree of income dispersion before taxation, a rise in interregional transfers may produce either a more or a less even personal income distribution. In particular, the effect of a given tax increase on interpersonal inequality depicts a Laffer curve with respect to the dispersion of income, δ. The reason for this result is that in addition to the allocation and equalization effects described above, non-marginal tax increases also generate a de-equalizing effect on interpersonal inequality, arising from the presence of intra-regional income differentials. As the providing region, $\aleph_{rich}$, is populated by low- as well as high-income earners, individual residents of $\aleph_{rich}$ are not necessarily richer than residents of the recipient region, $\aleph_{poor}$. Hence, income redistribution from the rich to the poor region involves transferring income to middle-income earners not only from high-paid, but also from low-paid individuals. This implies that residents of the recipient region are typically made better off at the expense of the well-being of low-income earners in the providing region, and thus that the equalizing effect of interregional redistributive taxation is weakened. Clearly, the lower is the degree of income heterogeneity within regions, the more likely are non-marginal tax increases to equalize, rather than de-equalize after-tax income.

Finally, the condition $q > \frac{2}{3}\lambda y$ is most likely to be satisfied if λ is low, given the levels of q and y. The parameter λ takes on a low value if ℓ is high and θ is low, that is if individuals devote a larger share of their time to working, rather than leisure, and if their marginal valuation of leisure is higher than their marginal valuation of consumption. Since an average working week amounts to at least 40 hours for most people, the assumption about time devoted to working seems reasonable. Further, estimates of θ typically fall below 0.5 in most empirical studies. Thus, the restriction on q does not seem to be too controversial.

The next proposition establishes, firstly, that in a low-tax economy, small and moderate non-marginal increases in the rate of interregional redistribution give rise to higher social welfare, while large non-marginal tax increases generate the opposite effect, and secondly, that in a high-tax economy, non-marginal tax increases unambiguously cause society's welfare to decline. In other words, non-marginal tax increases are beneficial to society to the extent that they are not large enough to discourage any agents in the rich neighbourhood from working, and detrimental otherwise.

Proposition 4. *If $\ell < \widetilde{\ell}$, then a non-marginal increase in interregional redistributive taxation yields a rise in social welfare if $\tau' < \tau^{**}$, and a decline otherwise. However, if $\ell > \widetilde{\ell}$, then the effect of a non-marginal tax increase is ambiguous.*

An important implication of Proposition 5 is that non-marginal increases in the rate of interregional redistribution are not necessarily beneficial in terms of social welfare. This is because redistributive taxation may give rise to two opposite effects on society's welfare. The first is the *income effect*, that an increase in interregional redistribution imposes on the utility of agents in the recipient region. The income effect refers to the decrease in labour supply, and the corresponding increase in leisure consumption, that residents of $\aleph_{poor}$ can afford to undertake as soon as the lump-sum transfer exceeds $\tau^* z$. The second is the *substitution effect*, that an increase in the tax rate imposes on the utility of agents in the providing region, that is the re-allocation of time from labour to leisure, that residents of $\aleph_{rich}$ need to undertake as the rate of income taxation exceeds $\frac{\lambda x - q}{\lambda x}$ and $\frac{\lambda y - q}{\lambda y}$, respectively. To the extent that the income effect on recipients' utility is greater than the substitution effect on taxpayers' utility, society's welfare increases in response to non-marginal increases in the rate of interregional redistribution. However, if the substitution effect is stronger than the income effect, non-marginal increases in interregional redistribution rather cause social welfare to decline.

The condition $\ell < \widetilde{\ell}$ is typically satisfied if ℓ is low and θ is high. Although the present framework does not require any formal restrictions on these parameters, it would be empirically reasonable to assume that $\ell > 1-\ell$ and $\theta < 1-\theta$ (see references). Figure 1 depicts $\widetilde{\ell}$ as a function of θ. Obviously, given that the restrictions $\ell > 1-\ell$ and $\theta < 1-\theta$ be satisfied, the range within which ℓ falls below $\widetilde{\ell}$ is relatively small, thus $\ell < \widetilde{\ell}$ seems to be a strong parameter restriction. However, it appears (see Appendix B) that this condition is needed only to prove the most extreme case of the proposition. In other words, although the restriction on ℓ and θ may be strong, it is critical to the results only to a limited extent.

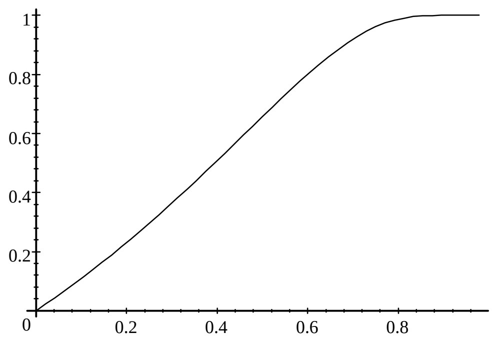

Figure 1.

Table 1.

	Income effect	Substitution effect
Allocation effect	$W\uparrow$, $\Sigma\uparrow$	$W\downarrow$, $\Sigma\uparrow$
Equalization effect	$W\uparrow$, $\Sigma\downarrow$	$W\downarrow$, $\Sigma\downarrow$

Obviously, interregional income equalization is not necessarily equitable. Further, there is no unambiguous relationship between interregional inequality, social welfare and interpersonal inequality. In particular, there need not be a negative relationship between interregional inequality and social welfare. The reason is, for one thing, that the implications of increasing the rate of interregional redistribution are determined by the relationships between the equalization and allocation effects on the one hand, and the income and substitution effects, on the other, and for another, that the relative strength of these effects seems to be highly variable with respect to the rate of taxation. In what follows, we consider the special case where $q > \frac{2}{3}\lambda y$ and $x > \frac{2}{3}y$, that is the case described in Proposition 3. Table 1 shows how interregional inequality and social welfare respond to non-marginal increases in the rate of redistribution, depending on which of these effects are in dominance. Furthermore, Figure 2 depicts the consequences of non-marginal increases in the rate of redistributive taxation in terms of welfare and interregional equity, depending on the initial tax rate and the size of the tax increase.

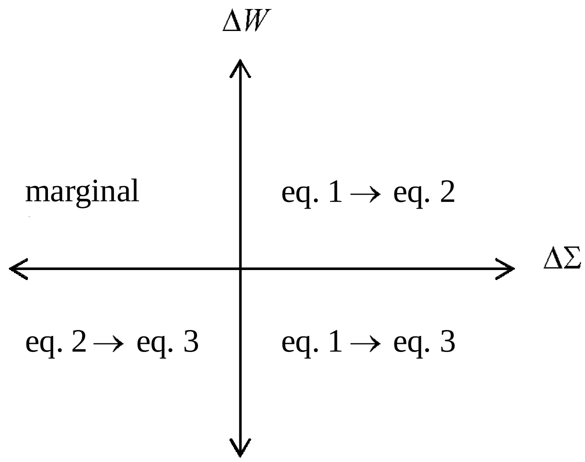

Figure 2.

Clearly, *Figure 1* indicates that the effects of an increase in the rate of interregional redistribution on interregional inequality and social welfare are highly dependent on the initial rate of taxation as well as on the magnitude of the tax increase. In the case of a low-tax society, a small or moderate non-marginal increase in the rate of taxation causes interregional as well as interpersonal inequality to rise, rather than to decline. However, at the same time the tax increase implies a rise in social welfare, albeit at the cost of lower labour market participation (cf the upper right corner of *Figure 1*). In comparison, a large non-marginal increase in redistributive taxation not only causes interregional and interpersonal inequality to rise, but also generates a decline in social welfare (cf the lower right corner of *Figure 1*). In the case of a high-tax society, on the other hand, non-marginal increases in redistributive taxation typically reduce interregional inequality, although at the expense of increased interpersonal inequality, reduced social welfare and lower labour market participation (cf the lower left corner of *Figure 1*).

The effects of increasing the rate of interregional redistribution also seem to be sensitive to the degree of pre-tax income dispersion as well as the degree of income heterogeneity within the neighbourhoods. In particular, the degrees of interregional as well as interpersonal income inequality are less likely to rise in response to non-marginal tax increases if the degree of pre-tax income dispersion is high between, but not within regions. The reason is that the greater is the extent to which interregional redistribution coincides with redistribution from the rich to the poor, the less likely is the tax increase to generate adverse effects on interregional and interpersonal inequality. It follows that interregional redistribution seems most likely to be beneficial in terms of inequality as well as social welfare in low-tax societies, where the degree of interregional, but not the intra-regional income dispersion is high. The reason is that while marginal or small marginal tax increases in a low-tax economy always cause society's welfare to increase, they are less likely to give rise to increases in interregional and interpersonal inequality if the dispersion of income is wide between, but narrow within regions.

In summary, the analysis indicates that due to its uncertain implications, interregional redistributive taxation seems to be a fairly inefficient policy tool. For one thing, although increases in the rate of interregional redistribution are beneficial to society to some extent, they are also highly likely to generate not only rising interregional and interpersonal inequality, but also declining aggregate working hours and destroyed incentives for residents of the recipient region. For another, the fact that providing and recipient regions need not be populated exclusively by rich and poor individuals, respectively, inevitably addresses the question whether resident- rather than source-based redistribution is equitable. The earliest proponent of so-called horizontal equity, that is the idea that individuals or groups with the same income should not be subject to different rates of taxation, was James Buchanan. In a seminal paper (Buchanan

[1950]), he questioned the ethics of inter-governmental grants, arguing that redistribution schemes should be targeted at individuals only, not taking into account their residence. Although not undisputed, this idea seems to have been generally accepted in the literature (see Mieszkowski and Musgrave [1999] for a discussion of Buchanan's argument and an overview of subsequent contributions in the field). For instance, Yinger [1986] advocates the principle of "fair compensation", that is the idea that an individual's tax burden should be independent of her residence.

However, to the extent that the political goal of redistribution is equality in economic opportunity, rather than in income or public consumption, there might be a case for interregional equalization. For example, as suggested by Oakland [1994], interregional transfers may be used to correct for regional differentials in the cost of providing public goods and services, supplies of natural resources or local opportunities in terms of education or employment.[10] In a model where individuals are mobile between regions, the target of equalization is likely to be even more critical to the implications of interregional redistribution.[11] Particularly, a redistributive policy that successfully equalizes educational or professional opportunity, rather than regional income, might possibly prevent de-population of less developed regions.

In this framework, introducing mobility would imply that talented agents who were born in $\aleph_{poor}$, that is type φ agents, improve their job opportunities by moving to $\aleph_{rich}$, while untalented agents who were born in $\aleph_{rich}$, that is type β agents, increase their after-tax income by re-locating to $\aleph_{poor}$. Consequently, depending on the rate of taxation and the cost of moving, the analysis would give rise to a large number of short-term equilibria with varying population distributions, and two long-run equilibria. In those short-run equilibria where the tax rate is sufficiently low, φ agents, but not β agents would find it worthwhile to re-locate. Thus, all talented and some untalented agents would concentrate in $\aleph_{rich}$, while those who were born untalented in $\aleph_{poor}$ would

[10] A discussion of the efficiency of educational equalization in the US is provided by Reschovsky [1994].

[11] The assumption of mobility opens up for further arguments in favour of residential taxation; see for example Inman and Rubinfeld [1996] and Oakland [1994].

choose to stay there. Moreover, in short-run equilibria where the tax rate is higher, all untalented and some talented agents would be concentrated in $\aleph_{rich}$, while those who were born talented in $\aleph_{rich}$ would remain there. Accordingly, in the two long-run equilibria, all agents would reside either in the rich or in the poor neighbourhood. This result is similar to that of Wildasin [1991], where unequal transfer levels across regions give rise to migration and thus to a socially inefficient allocation of labour.

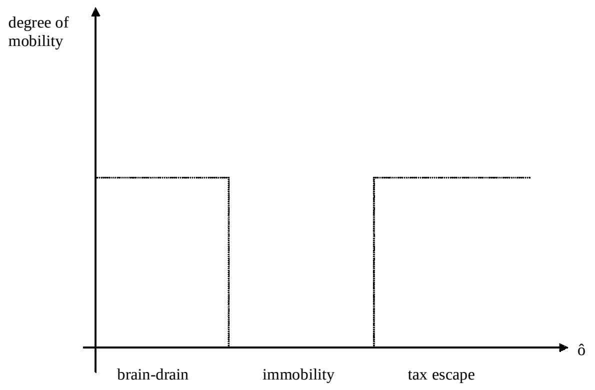

Figure 3.

However, alternating the framework so that the aim of equalization is to improve the economic opportunity of residents of the poor region, does not necessarily yield the same implications. Rather, to the extent that grants to $\aleph_{poor}$ are successfully used to extend the variety of job opportunities in the region, thus giving type φ agents incentives to stay there, and that the tax burden of residents of $\aleph_{rich}$ is not excessive, interregional transfers may possibly enhance efficiency as well as equity, without generating adverse side effects on labour supply (as in the present framework). Nevertheless, as improvement of regional employment opportunities in $\aleph_{poor}$ most likely requires a minimum level of investment, low levels of interregional transfers may not be sufficient to prevent migration of talented agents, or brain-drain, from $\aleph_{poor}$ to $\aleph_{rich}$. Likewise, if taxation becomes too burdensome, type β agents, and eventually type α agents, will typically migrate from $\aleph_{rich}$ to $\aleph_{poor}$ in order to escape taxes. Figure 3 depicts the degree of mobility with respect to the rate of interregional redistributive taxation.

5. Concluding Remarks

I have shown that increases in the rate of interregional redistribution need not generate neither reduced interregional inequality nor higher social welfare, and that their effects are highly dependent on the initial state of the economy. In particular, interregional redistribution seems most likely to be beneficial in low-tax societies, where the degree of income dispersion is high between, but not within regions.

Clearly, a variety of extensions of the current model remain to be analyzed. Among these are, for instance, the assumption of geographical mobility. In the present framework, introducing mobility would merely give rise to new non-interior equilibria, that is equilibria with congestion or depopulation. However, in combination with more complex preferences, or a more advanced accumulation technology, a framework where workers are mobile could possibly yield new results. In the former case, a possible extension would be to define preferences over residence as well as consumption and leisure. This approach would typically imply that redistributive taxation be even more harmful to efficiency and equity than in the present framework, but not necessarily to society's welfare. In the latter case, an interesting approach would be to study the welfare implications of interregional income redistribution in the presence of local or global human capital spillovers. Particularly, it would be interesting to analyze the optimal distribution of individuals across regions in the case of local spillovers, or so-called neighbourhood effects. Nevertheless, these are topics for future papers.

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Appendix A

The proposition below ensures the existence of an equilibrium where the tax rate is high enough to affect the incentives of residents of the poor, but not the rich neighbourhood.

Proposition 5. *The tax rate at which individuals in $\aleph_{poor}$ decide to cut their labour supply falls below the tax rate at which individuals in $\aleph_{rich}$ choose to do so, thus* $\frac{2\lambda x}{(1-\lambda)(y+x)} < \frac{\lambda x - q}{\lambda x}$. ■

Proof. Suppose that the tax rate at which poor residents of $\aleph_{rich}$ decide to quit working falls below the tax rate at which residents of $\aleph_{poor}$ choose to do so, thus $\frac{\lambda x - q}{\lambda x} < \frac{2\lambda x}{(1-\lambda)y}$. Then, in order for (the first part of) Assumption 1 to be satisfied, that is the assumption that the average regional after-tax income be at least as high in $\aleph_{rich}$ as in $\aleph_{poor}$ at any tax rate, it must hold that $\frac{y-2x}{2y} > \frac{2\lambda x}{(1-\lambda)y}$. In turn, this implies that $\lambda < \frac{y-2x}{2x+y}$. However, this inequality clearly violates the condition $y < 2x$. It follows that $\frac{\lambda x - q}{\lambda x} < \frac{2\lambda x}{(1-\lambda)y}$ cannot be satisfied. Consequently, it must hold by contradiction that $\frac{2\lambda x}{(1-\lambda)(y+x)} < \frac{\lambda x - q}{\lambda x}$. ■

Appendix B

Proof of Proposition 2. Consider first the implications of non-marginal tax increases on interregional inequality. The net change in interregional inequality as the economy jumps from equilibrium 1 to 2, from equilibrium 1 to 3 and from equilibrium 2 to 3, respectively, is given by

$$\begin{aligned}
\Sigma_2 - \Sigma_1 &= -\tfrac{1}{2}\left(\tau' - \tau_0\right) y + \tfrac{1}{2}\left(1 - \tau' + \tau_0\right) x \\
\Sigma_3 - \Sigma_1 &= -\tfrac{1}{2}\left(\tau' - \tfrac{1}{2}\tau_0\right) y + \tfrac{1}{2}\left(\tfrac{1}{2} + \tau_0\right) x \\
\Sigma_3 - \Sigma_2 &= -\tfrac{1}{2}\left(\tau' - \tau_0\right) y - \tfrac{1}{2}\left(\tfrac{1}{2} - \tau_0\right) x
\end{aligned}$$

Clearly, all these expressions are decreasing functions of y, and hence by $\frac{y}{x}$. Consider now the effects of non-marginal tax increases on interpersonal inequality. The net change in interpersonal inequality as the economy jumps from equilibrium 1 to 2, from equilibrium 1 to 3 and from equilibrium 2 to 3, respectively, is given by

$$\begin{aligned}
\sigma_2 - \sigma_1 &= \tfrac{1}{8}\left(3\tau' - 4 + 3\tau_0\right)\left(\tau' - \tau_0\right) y^2 + \tfrac{1}{4}\left(1 - \tau_0^2 - 2\tau_0 + \tau'^2\right) xy - \tfrac{1}{8}\left(4\tau' - 3\tau'^2 + 3\tau_0^2\right) x^2 \\
\sigma_3 - \sigma_1 &= \tfrac{1}{8}\left(3\tau' - 4 + 3\tau_0\right)\left(\tau' - \tau_0\right) y^2 + \tfrac{1}{8}\left(3 - 2\tau_0^2 - 4\tau_0\right) xy - \tfrac{3}{16}\left(1 - 2\tau_0^2\right) x^2 \\
\sigma_3 - \sigma_2 &= \frac{1}{8}\left(3\tau' - 4 + 3\tau_0\right)\left(\tau' - \tau_0\right) y^2 + \tfrac{1}{8}\left(1 - 2\tau_0^2\right) xy + \tfrac{1}{16}\left(8\tau_0 - 3 - 6\tau_0^2\right) x^2
\end{aligned}$$

Note first that by Assumption 1, it must hold that $4 - 3\tau' - 3\tau_0 > 0$. Then, the first and second order derivatives of $\sigma_2 - \sigma_1$, $\sigma_3 - \sigma_1$ and $\sigma_3 - \sigma_2$ indicate that $\frac{\partial(\sigma_k - \sigma_j)}{\partial\left(\frac{y}{x}\right)} > 0$, where j and k denote the initial and current equilibrium, if $\frac{y}{x}$ falls below $\frac{1-\tau_0^2-2\tau_0+(\tau')^2}{(4-3\tau'-3\tau_0)(\tau'-\tau_0)}$, $\frac{1}{2}\frac{3-2\tau_0^2-4\tau_0}{(4-3\tau'-3\tau_0)(\tau'-\tau_0)}$ and $\frac{1}{2}\frac{1-2\tau_0^2}{(4-3\tau'-3\tau_0)(\tau'-\tau_0)}$, respectively, and that $\frac{\partial(\sigma_k - \sigma_j)}{\partial\left(\frac{y}{x}\right)} < 0$ otherwise. Moreover, it is easy to see that the critical values of $\frac{y}{x}$ are also decreasing in τ', and increasing in τ_0. This verifies the Proposition. ■

Proof of Proposition 3. The difference in interregional income inequality, Σ, between equilibrium j and k, is positive for all feasible τ by transitivity if $\Sigma_k^{\min} - \Sigma_j^{\max} > 0$, and negative if $\Sigma_k^{\max} - \Sigma_j^{\min} < 0$, where j and k denote the initial and current equilibrium, respectively. Consider first the cases where $\tau_0 < \tau^*$, that is the cases where the economy jumps from equilibrium 1 to equilibrium 2 and 3, respectively. In the former case, the difference in Σ is positive if $\Sigma_2^{\min} - \Sigma_1^{\max} > 0$, that is if $\Sigma_2\left[\tau^{**}\right] - \Sigma_1\left[0\right] > 0$, where Σ_1 and Σ_2 are given by (3.1) and (3.4). Hence, $\Sigma_2 - \Sigma_1 > 0$ if and only if

$$q > \lambda \tfrac{yx}{y+x} \tag{5.1}$$

Equation (5.1) is satisfied by the second part of Assumption 1 and transitivity if $\frac{\lambda y}{2} > \frac{\lambda yx}{y+x}$, that is if $\frac{1}{2}\lambda y\frac{y-x}{y+x} > 0$, which is obviously true. Thus, it holds that $\Sigma_2 - \Sigma_1 > 0$.

In the latter case, the difference in Σ is positive if $\Sigma_3^{\min} - \Sigma_1^{\max} > 0$, that is if $\Sigma_3[\tau^{***}] - \Sigma_1[0] > 0$, where Σ_1 and Σ_3 are given by (3.1) and (3.7). Thus, $\Sigma_3 - \Sigma_1 > 0$ if and only if

$$q > \lambda\frac{2y-x}{2} \tag{5.2}$$

Given that $q > \frac{2}{3}\lambda y$, equation (5.2) is satisfied by transitivity if $\frac{2}{3}\lambda y > \lambda\frac{2y-x}{2}$, that is if $\frac{3}{2}x > y$. In turn, this inequality is satisfied by the conditions $q > \frac{2}{3}\lambda y$ and $q < \lambda x$. Hence, it must hold that $\Sigma_3 - \Sigma_1 > 0$.

Consider finally the case where $\tau_0 > \tau^*$, that is the case where the economy jumps from equilibrium 2 to equilibrium 3. In this case, the difference in Σ is negative if $\Sigma_3^{\max} - \Sigma_2^{\min} < 0$, that is if $\Sigma_3[\tau^{**}] - \Sigma_2[\tau^{**}] < 0$, where Σ_2 and Σ_3 are given by (3.4) and (3.7). Hence, $\Sigma_3 - \Sigma_2 < 0$ if and only if $q > \frac{1}{2}\lambda x$, which is true by the second part of Assumption 1 and transitivity. It follows that $\Sigma_3 - \Sigma_2 < 0$. This verifies the Proposition. ■

Proof of Proposition 4. The change in interpersonal inequality, σ, between equilibrium j and k, is positive for all feasible τ by transitivity if $\sigma_k^{\min} - \sigma_j^{\max} > 0$, and negative if $\sigma_k^{\max} - \sigma_j^{\min} < 0$, where j and k denote the initial and current equilibrium, respectively. It follows that in the case where the economy jumps from equilibrium 1 to equilibrium 2, the change in interpersonal inequality is positive for all τ if $\sigma_2^{\min} - \sigma_1^{\max} > 0$, that is if $\sigma_2[\tau^{**}] - \sigma_1[0] > 0$, where σ_1 and σ_2 are given by (3.2) and (3.5). Thus, $\sigma_2 - \sigma_1 > 0$ if and only if

$$q > \frac{(y+x)^2 + \sqrt{4y^2+2yx+4x^2}(y-x)}{3y^2+3x^2+2yx}\lambda x \tag{5.3}$$

Clearly, (5.3) is satisfied by transitivity and the second part of Assumption 1 if the RHS falls below $\frac{\lambda y}{2}$. In turn, this condition is satisfied if and only if $y > x$, which is definitely true. Hence, it must hold that $\sigma_2 - \sigma_1 > 0$.

Further, in the case where the economy jumps from equilibrium 1 to equilibrium 3, the change in σ is positive for all τ if $\sigma_3^{\min} - \sigma_1^{\max} > 0$, that is if $\sigma_3[\tau^{***}] - \sigma_1[0] > 0$, where σ_3 and σ_1 are given by (3.8) and (3.2). Hence, $\sigma_3 - \sigma_1 > 0$ if

$$q > \frac{2y+\sqrt{16y^2-36yx+18x^2}}{6}\lambda \tag{5.4}$$

Clearly, it must hold that if $q > \frac{2}{3}\lambda y$, (5.4) is satisfied by transitivity if the RHS falls short of $\frac{2}{3}\lambda y$. In other words, (5.4) holds if $2y - \sqrt{2}\sqrt{(4y-3x)(2y-3x)}$, which is clearly satisfied for all $y < 2x$. Consequently, it must hold that $\sigma_3 - \sigma_1 > 0$.

Finally, in the case where the economy jumps from equilibrium 2 to equilibrium 3, the change in σ is positive for all τ if and only if $\sigma_3^{\min} - \sigma_2^{\max} > 0$, that is if $\sigma_3[\tau^{***}] - \sigma_2[\tau^*] > 0$, where σ_2 and σ_3 are given by (3.5) and (3.8), respectively. Thus, $\sigma_3 - \sigma_2 > 0$ if

$$q > \frac{2y(1-\lambda)(y+x) - \sqrt{(16\lambda^2y^4+116\lambda^2y^3x-32y^4\lambda-136y^3\lambda x+250\lambda^2y^2x^2-116y^2\lambda x^2+16y^4+20y^3x+10x^2y^2+24yx^3-144yx^3\lambda+216y\lambda^2 x}}{6(y+x)(1-\lambda)} \tag{5.5}$$

Equation (5.5) is satisfied by the second part of Assumption 1 and transitivity if $\frac{\lambda y}{2}$ is in excess of the RHS of (5.5), that is if

$$\frac{y(1-\lambda)(y+x)+\sqrt{(16\lambda^2y^4+116\lambda^2y^3x-32y^4\lambda-136y^3\lambda x+250\lambda^2y^2x^2-116y^2\lambda x^2+16y^4+20y^3x+10x^2y^2+24yx^3-144yx^3\lambda+216y\lambda^2x^3+258}}{6(y+x)(1-\lambda)}$$

which is clearly true for all non-negative y and x. It follows that $\sigma_3 - \sigma_2 > 0$. This verifies the Proposition. ■

Proof of Proposition 5. The change in social welfare, W, between equilibrium j and k, is positive or non-decreasing for all feasible τ by transitivity if $W_k^{\min} - W_j^{\max} \geq 0$, and negative if $W_k^{\max} - W_j^{\min} < 0$, where j and k denote the initial and current equilibrium, respectively. Consider first the case where $\tau' < \tau^{**}$, that is the case where the economy jumps from equilibrium 1 to equilibrium 2. The change in social welfare is positive for all τ if $W_2^{\min} - W_1^{\max} > 0$, that is if $W_2[\tau^*] - W_1[\tau^*] \geq 0$, where W_1 and W_2 are given by (3.3) and (3.6). Thus, $W_2 - W_1 \geq 0$ if and only if

$$\left(\tfrac{\lambda}{1-\lambda}x\right)^{\theta} \geq \left(x + \tfrac{\lambda}{1-\lambda}x\right)^{\theta}(1-\ell)^{1-\theta} \tag{5.6}$$

By Definition 1, (5.6) simplifies to $\frac{\lambda}{1-\lambda}x \geq \left(x + \frac{\lambda}{1-\lambda}x\right)\lambda$, which is clearly satisfied. It follows that $W_2 - W_1 \geq 0$.

Consider now the cases where $\tau' > \tau^{**}$. In the case where the economy jumps from equilibrium 1 to equilibrium 3, the change in W is negative for all τ if $W_3^{\max} - W_1^{\min} < 0$, that is if $W_3[\tau^{***}] - W_1[0] < 0$, where W_3 and W_1 are given by (3.9) and (3.3). Thus, $W_3 - W_1 < 0$ if and only if

$$(1-\ell)^{1-\theta}\left((2q)^{\theta} - \tfrac{1}{2}(2\lambda y)^{\theta} - \tfrac{3}{2}(2\lambda x)^{\theta}\right) + (\lambda y - q)^{\theta} < 0 \tag{5.7}$$

It can be shown that the LHS of (5.7) decreases in q as well as in x. Hence, (5.7) holds for all q by transitivity if it is satisfied for the lower bounds of q and x, which are equal to $\frac{\lambda y}{2}$ and $\frac{y}{2}$, respectively, by the second part of Assumption 1. Replacing $\frac{\lambda y}{2}$ and $\frac{y}{2}$ in (5.7) and rearranging yields

$$1 - \left(\frac{2\left(\frac{1}{2}\right)^{\theta}}{1+2^{\theta}}\right)^{\frac{1}{1-\theta}} > \ell \tag{5.8}$$

Note that the LHS of (5.8) is equivalent to $\widetilde{\ell}$. Hence, given that ℓ is assumed to fall below $\widetilde{\ell}$, (5.8) is satisfied. In other words, it must hold that $W_3 - W_1 < 0$.

Finally, in the case where the economy jumps from equilibrium 2 to equilibrium 3, the change in social welfare is negative for all τ if $W_3^{\max} - W_2^{\min} < 0$, that is if $W_3[\tau^{***}] - W_2[\tau^*] < 0$. Recall that it was shown above that $W_3[\tau^{***}] < W_1[0]$ and that $W_2[\tau^*] > W_1[\tau^*]$. Since W is an increasing function of τ, it must hold that $W_3[0] < W_1[\tau^*]$. Hence, it is implied by transitivity that $W_3[\tau^{***}] < W_2[\tau^*]$ and, consequently, that $W_3 - W_2 < 0$. This verifies the Proposition. ■