PRICE COMPETITION AND CONVEX COSTS

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Abstract. In the original model of pure price competition, Bertrand (1883), firms have linear cost functions. For any number $n \geq 2$ of identical such price-setting firms, the unique equilibrium price equal the firms' (constant) marginal cost. This paper provides a generalization of Bertrand's model from linear to convex cost functions. I analyze pure price competition both in a static setting - where the firms interact once and for all - and in dynamic setting - where they interact repeatedly over an indefinite future. Sufficient conditions are given for the existence of Nash equilibrium in the static setting and for subgame perfect equilibrium in the dynamic setting, and the equilibrium sets are characterized. It is shown that there typically exists a whole interval of equilibrium prices both in the static and dynamic setting. Firms may earn sizable profits and equilibrium profits may be increasing in their production costs.

Keyword: Bertrand competition.
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1. Introduction

In the original model due to Joseph Bertrand (1883), firms have linear cost functions. For any number of identical such price-setting firms, the unique equilibrium price equal the firms' (constant and common) marginal cost. Hence, under pure competition, already two competitors is enough to obtain the perfectly competitive outcome, despite the fact that each firm has a lot of market power; the slightest unilateral price cut will rob all competitors of their entire demand. Francis Edgeworth (1925) pointed out that, except in the case of linear costs, there are serious existence

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problems in Bertrand's model of pure price competition if marginal costs are not constant. Edgeworth proposed, in particular, a stark modification of Bertrand's model in which firms have zero marginal cost and a fixed capacity. He showed that, unless demand is highly elastic, pure-strategy equilibrium may then fail to exist. Eric Maskin (1986) and Beth Allen and Martin Hellwig (1986) showed that *mixed* strategy equilibria may nevertheless exist. However, mixed equilibria in price competition do not appear convincing as models of many real-life market interactions. For although price-setting is a common practice, most firms do not appear to use randomization devices when setting their prices.[1] Given the prevalence of price setting, models of pure-strategy equilibrium in price competition are called for.

This paper provides a generalization of Bertrand's model from linear to convex cost functions. More exactly, I here analyze price competition in a homogeneous product market among a fixed number of price-setting firms. Each firm is characterized by its (continuous, non-decreasing and convex) cost function. Two distinct settings are analyzed. The first is a one-shot interaction in which all firms simultaneously set their prices and where each firm is committed to serve the demand it faces. Such commitment is mandated in some regulated industries, such as electricity and telephone, and is sometimes supported by consumer protection laws. Consumers observe all prices and buy only from those sellers who ask the lowest price. In the second setting studied here, this interaction is repeated an infinite number of times, with demand regenerated anew each period.

The main results are as follows. First, sufficient conditions are given for the existence of Nash equilibrium in the first setting and for subgame perfect equilibrium in the second. Second, it is shown that, unlike in the classical case of linear cost functions, the equilibrium outcome is not necessarily competitive. While marginal-cost pricing indeed may be a Nash equilibrium, typically there exists a whole interval of Nash equilibrium prices, both above and below the marginal-cost price. The intuition is that with strictly convex costs, price undercutting is less profitable than with linear costs—since serving the whole market is more than proportionately costly than serving a fraction thereof. Hence, also prices above marginal cost are possible in equilibrium. Thirdly, by way of comparative static analysis in two cases, it is shown that firms' equilibrium profits in the one-shot interaction may *increase* when their production costs go up. This is true even if they price at marginal cost. The intuition is again convexity: the marginal cost may rise more than average cost.

[1] For an analysis of so-called Edgeworth cycles as perfect Markov equilibria involving randomization, see Maskin and Tiole (1988). Noel (2004) reports empirical support for such phenomena.

Hence, producers' and consumers may disagree as to the desirability of technological progress. Fourthly, the set of subgame perfect equilibrium prices is characterized and it is shown how the subgame perfection condition is nested in a simple way with the condition for Nash equilibrium in the one-shot game. Thus, insights from the static Nash equilibrium analysis can readily be carried over to the dynamic case.

This is not the first study of pure-strategy equilibrium among price-setting firms with convex costs. Grossman (1981) and Hart (1985) developed so-called supply function equilibria, that is competition between firms that simultaneously commit to whole supply schedules, that is, functions that assign a price to each possible quantity supplied. Such models have a plethora of equilibria and many of these rely on non-credible threats. Klemperer and Meyer (1989) generalize this set-up to stochastic demand. They model a one-shot interaction between two identical firms, and provide sufficient conditions for the existence and uniqueness of Nash equilibria. Dastidar (1995) analyzes one-shot interaction among price-setting firms, just as here, and provides conditions for the existence of Nash equilibria. See also Vives (1999), who briefly discuss one-shot interaction among identical price-setting firms. However, I have found no reference to pure-strategy equilibria in repeated interaction between price-setting firms with convex costs. This paper is intentionally short and restricted to certain relatively simple questions within the given framework. The purpose is to stimulate more research and debate about the nature of price competition, both from a positive and normative perspective. More complicated questions and many relevant extensions have to be left for future work.

The paper is organized as follows. Section 2 provides the model, section 3 gives an analysis of the one-shot interaction and section 4 of the infinitely repeated interaction. Welfare aspects are studied in section 5, section 6 elaborates a parametric specification of the model, and section 7 suggests directions for future research.

2. The model

Suppose there are n firms in a market for a homogenous good. The market operates over an infinite sequence of time periods, $t = 0, 1, 2, ...$ Aggregate demand in each period is given by a continuous and non-increasing demand function $D : \mathbb{R}_+ \to \mathbb{R}_+$ with $\lim_{p \to +\infty} D(p) = 0$.

All firms simultaneously set their prices at the beginning of each period and are committed to provide the quantity demanded at that price during the period. Let $p_{it} \geq 0$ be firm $i's$ price in period t. All consumers observe all posted prices and buy from the firm(s) with the lowest price. The lowest price in any period will be called the *market price* in that period,

$$p_t = \min\{p_{1t}, ..., p_{nt}\}. \tag{1}$$

If more than one firm posts the market price, then sales are split equally between these. Firm i produces the good at cost $C_i(q_i)$, where q_i is its output quantity, and where $C_i : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, non-decreasing and convex. All firms are risk neutral and discount future profits by the same *discount factor* $\delta \in (0,1)$ between successive market periods. Resale is not possible. The good is non-storable.

For each firm i, let its *profit function* $\pi_i : \mathbb{R}_+ \to \mathbb{R}$ be defined by

$$\pi_i(p) = \frac{1}{n} pD(p) - C_i\left[\frac{1}{n}D(p)\right]. \tag{2}$$

This is the profit that the firm will make in a period when all firms post the same price p. The *industry profit function* is defined as the sum of all firms' profits when they all post the same price in a period:

$$\Pi(p) = \sum_{i=1}^{n} \pi_i(p) = pD(p) - \sum_{i=1}^{n} C_i\left[\frac{1}{n}D(p)\right]. \tag{3}$$

For each firm i, let its *monopolistic profit function* $\hat{\pi}_i : \mathbb{R}_+ \to \mathbb{R}$ be defined by

$$\hat{\pi}_i(p) = pD(p) - C_i[D(p)].$$

This is the profit that the firm would make in a period were it to post the price p while all other firms post higher prices. By Jensen's inequality,

$$C_i\left[\frac{1}{n}D(p)\right] \leq \frac{1}{n}C_i[D(p)] + \left(1 - \frac{1}{n}\right)C_i(0).$$

Hence,

$$\hat{\pi}_i(p) \leq n\pi_i(p) + (n-1)C_i(0)$$

for each firm i and any price p. In particular, $\hat{\pi}_i(p) \leq n\pi_i(p)$ in the absence of fixed costs, and $\hat{\pi}_i(p) = n\pi_i(p)$ in the classical case of constant average and marginal cost (C_i linear).

Throughout this study, I make two assumptions with regard to the functions π_i and $\hat{\pi}_i$. The first assumption is that each function $\hat{\pi}_i$ is quasi-concave and obtains its maximum at a finite price:

[C1] $\forall i$: $\hat{\pi}_i$ is quasi-concave with $\hat{P}_i = \arg\max_{p \geq 0} \hat{\pi}_i(p)$ non-empty.

In other words, each firm's profit function is unimodal and its *monopoly price* finite. Let $\hat{p}$ be the minimal such monopoly price across all firms: $\hat{p} = \min \cup_i \hat{P}_i$.[2] Heuristically, $\hat{p}$ is the monopoly price of the "largest" firm. It follows from [C1] that all functions $\hat{\pi}_i$ are non-decreasing on $[0, \hat{p}]$, a property that turns out to be analytically convenient. The second assumption is that there exist prices, not exceeding $\hat{p}$, at which the industry profit is positive:

[C2] $\Pi(p) > 0$ for some $p \in [0, \hat{p}]$.

This set-up contains the classical Bertrand model as a special case and has the Bertrand-Edgeworth model as a limiting case, with the important *proviso* that here firms are, by assumption, committed to serve the demand they face. The classical Bertrand model—constant marginal and average costs—here corresponds to all cost functions being linear: $C_i(q) \equiv c_i q$ for some $c_i \geq 0$. In order to obtain Edgeworth's model—zero marginal cost up to a certain capacity and thereafter infinite cost—first let the cost function of each producer i be

$$C_i(q) \equiv k_i + c \cdot \max\{0, q - K_i\}, \tag{4}$$

for some fixed costs $k_i \geq 0$ and capacities $K_i > 0$. Edgeworth's model is reached in the limit as $c \to +\infty$.

3. One-shot interaction

Suppose that the interaction takes place only once. Suppose also that each firm has the option of leaving the market. Let the value of this outside option to each firm be normalized to zero. A *pure strategy* for a firm is a pair $(s_i, p_i) \in \{0,1\} \times \mathbb{R}_+$, where $s_i = 1$ means that firm i stays in the market. The total number of firms that participate in the market is thus $m = \sum s_i$. We here investigate Nash equilibria in which all firms participate in the market and they all set the same price.

The set of such "symmetric" equilibria is characterized by two conditions. First, each firm's profit should not be below its' outside option, and, secondly, unilateral price under-cutting should be unprofitable:

$$\pi_i(p) \geq \max\{0, v_i(p)\} \qquad \forall i, \tag{5}$$

[2] Each of the constituent sets is closed and bounded from below. Being finitely many, their union is also closed and bounded from below and hence contains its infimum.

where

$$v_i(p) = \sup_{p'<p} \hat{\pi}_i(p') = \max_{p' \in [0,p]} \hat{\pi}_i(p') \tag{6}$$

defines a continuous function $v_i : \mathbb{R}_+ \to \mathbb{R}$.[3] A price p satisfying condition (5) will be called a (*symmetric Nash) equilibrium price*, and the (possibly empty) set of such prices will be denoted $P^{NE} \subset \mathbb{R}_+$.

Equilibrium prices above the minimal individualistic monopoly price are impossible, so the focus of the equilibrium analysis is, without loss of generality, on prices in the interval $[0, \hat{p}]$. Formally:

Lemma 1. $P^{NE} \cap (\hat{p}, +\infty) = \varnothing$.

Proof: Let firm i be such that $\hat{p}_i = \hat{p}$ and suppose that all firms price at some price $p > \hat{p}$. Then firm i produces and sells a quantity $q_i < D(p)$. By assumption, D is continuous with $\lim_{p\to+\infty} D(p) = 0$. Hence, there exists a price $p^* > p$ such that $D(p^*) = q_i$. It follows that

$$\hat{\pi}_i(p^*) = p^* q_i - C_i(q_i) > pq_i - C_i(q_i).$$

However, by definition, $\hat{\pi}_i(\hat{p}) \geq \hat{\pi}_i(p^*)$. Moreover, $\hat{p} < p$, so firm i can make a profitable unilateral deviation to $\hat{p} < p$. Hence, $p \notin P^{NE}$. **End of proof.**

It follows from [C1] that a price $p \in [0, \hat{p}]$ is an equilibrium price if and only if it delivers a profit to each firm that weakly exceeds the profit that the firm would have earned had it served the whole market at that price:[4]

Proposition 1. $p \in P^{NE}$ *if and only if* $\pi_i(p) \geq \max\{0, \hat{\pi}_i(p)\} \quad \forall i$.

Re-arranging the terms in the inequality, we obtain the following equivalent condition:

$$nC_i\left[\frac{1}{n}D(p)\right] \leq pD(p) \leq \frac{n}{n-1}\left(C_i[D(p)] - C_i\left[\frac{1}{n}D(p)\right]\right) \qquad \forall i. \tag{7}$$

The following result establishes existence of equilibrium in the special case of identical firms without fixed costs. Indeed, it goes beyond that claim and establishes the existence of a zero-profit equilibrium in that case.

[3]The second equality in this equation follows by continuity of $\hat{\pi}_i$, and the claimed continuity of the value function v_i follows from Berge's maximum theorem.

[4]By [C1], each function $\hat{\pi}_i$ is non-decreasing on $[0, \hat{p}]$, so $v_i(p) = \hat{\pi}_i(p)$ for all $p \leq \hat{p}$.

Proposition 2. *If firms are identical with $C(0) = 0$, then $\exists p^0 \in P^{NE}$ with $\pi(p^0) = 0$.*

Proof: According to [C2], $\Pi(p) > 0$ for some $p \in [0, \hat{p}]$. Since $\Pi(0) = -nC(0) \leq 0$ and Π is continuous, there exists a $p^0 \in [0, p)$ such that $\Pi(p^0) = -(n-1)C(0)$. But then $p^0 D(p^0) = nC[D(p^0)/n] - (n-1)C(0) \leq C[D(p^0)]$ by Jensen's inequality applied to C. Hence, $\hat{\pi}(p^0) \leq 0$ and thus $p^0 \in P^{NE}$ if $C(0) = 0$. **End of proof.**

In the classical Bertrand model, the condition for symmetric Nash equilibrium boils down to $p = c_i$ for all i. Such equilibria thus exist if and only if firms are identical, and the unique symmetric equilibrium price then equals the common constant marginal cost. In order to relate to the Bertrand-Edgeworth model, consider a duopoly with identical firms with cost functions as in equation (4). It is easily verified from the above analysis that any price p such that $K < D(p) \leq 2K$ then is a symmetric Nash equilibrium price, granted k is low enough and c high enough:

$$\pi \geq \hat{\pi} \Leftrightarrow c \geq \frac{pD(p)}{2(D(p) - K)},$$

and $\pi \geq 0$ if and only if $pD(p) \geq 2k$. In the limit as $c \to +\infty$, the inequality $\pi \geq \hat{\pi}$ is met. Hence, then any price p such that $K < D(p) \leq 2K$ and $pD(p) \geq 2k$ is a symmetric Nash equilibrium price.

4. Infinitely repeated interaction

Suppose that the oligopoly faces the same demand function D in each period. We here consider the possibility for prices $p \in [0, \hat{p}]$ above the one-shot Nash equilibrium prices to be sustainable in subgame perfect equilibrium in the infinitely repeated game by trigger strategies of the following sort: all firms initially ask the price p and continue to do so as long as all firms quote this price. In the wake of any unilateral price deviation, all firms switch to a price $p^* < p$ that is a symmetric Nash equilibrium price in the stage game. Such a trigger-strategy profile constitutes a subgame perfect equilibrium if and only if for all i:

$$\pi_i(p) \geq 0 \quad \text{and} \quad v_i(p) + \frac{\delta}{1-\delta}\pi_i(p^*) \leq \frac{\pi_i(p)}{1-\delta}, \tag{8}$$

where $p^* \in P^{NE}$, v_i is defined in equation (6), and where $\pi_i(p^*) \leq \pi_i(p)$ for all i since $p^* < p \leq \hat{p}$. The quantity on the left-hand side is the present value of the profit to a firm that undercuts the collusive price optimally. Let the set of prices $p \in [p^*, \hat{p}]$ satisfying (8), for some $p^* \in P^{NE}$ be denoted $P^{SPE}(\delta)$. In force of assumption [C1], $v_i(p) = \hat{\pi}(p)$ and hence

Proposition 3. *Suppose $\delta \in (0,1)$, $p^* \in P^{NE}$ and $p \in [p^*, \hat{p}]$. Then $p \in P^{SPE}(\delta)$ if and only if*

$$\pi_i(p) \geq \max\{0, (1-\delta)\hat{\pi}_i(p) + \delta\pi_i(p^*)\} \qquad \forall i. \tag{9}$$

As is the case more generally, all Nash equilibrium prices in the one-shot game are subgame perfect, for all δ $(p^* = p)$. It follows from assumption [C2] that if firms are identical with no fixed costs, then p^* can be taken to result in zero profits to all firms and this is also the "harshest" trigger strategy possible, so

Proposition 4. *Suppose firms are identical, $C(0) = 0$, $\delta \in (0,1)$ and $p \in [0, \hat{p}]$. Then $p \in P^{SPE}(\delta)$ if and only if*

$$\pi(p) \geq (1-\delta)\max\{0, \hat{\pi}(p)\}. \tag{10}$$

Proof: By proposition 2, there exists a price $p^0 \in P^{NE}$ with $\pi(p^0) = 0$. Setting $p^* = p^0$ in (9) shows that (10) is sufficient for $p \in P^{SPE}(\delta)$. Secondly, assume that $p \in P^{SPE}(\delta)$. Then (9) holds for some $p^* \in P^{NE}$ such that $\pi_i(p^*) \leq \pi_i(p)$ for all i. But then it also hold for $p^0 \in P^{NE}$ since $\pi(p^0) = 0 \leq \pi_i(p)$ for all $p \in P^{NE}$. **End of proof.**

We note the similarity between the condition (**??**) for Nash equilibrium in the one-shot game and the condition (10) in the infinitely repeated game, in the case of identical firms: the only difference being the factor $1-\delta \in (0,1)$.

We also note that a necessary condition for (10) to hold is:

$$\delta \geq 1 - \frac{\pi(p)}{\hat{\pi}(p)}.$$

In the absence of fixed costs, convexity of C gives

$$\frac{\pi(p)}{\hat{\pi}(p)} = \frac{pD(p) - nC[D(p)/n]}{npD(p) - nC[D(p)]} \geq \frac{pD(p) - C[D(p)]}{npD(p) - nC[D(p)]} = \frac{1}{n}.$$

Hence, collusion is then "easier" than in the classical case of linear costs in the sense that the range of discount factors δ satisfying the subgame perfection condition, *ceteris paribus*, is a superset of that in the linear case. This is intuitively evident: the deviating firm serves the whole market and hence has a more than proportionally increased cost to deliver the demanded quantity if its cost function is strictly convex.

5. Welfare

Having studied conditions for a market price to be a symmetric Nash equilibrium in a one-shot interaction as well as in the infinitely repeated game with discounting, I turn to the question of efficiency. For this purpose, define *social welfare,* at any price p quoted by all firms, as the sum of consumer surplus and firms' profits. Thus, $W(p) = S(p) + \Pi(p)$, where Π is defined in equation (3) and

$$S(p) = \int_p^{+\infty} D(x)\,dx.$$

It should be noted that this definition is restrictive and applies best in situations when firms are quite similar. For it is focused on situations in which all firms are active in the market, whether or not some firms make losses and whether or not it would be socially desirable to shut some firms.

However, staying with the function W as our social welfare function, it follows that, if the cost functions are differentiable, then a necessary condition for a common price $p > 0$ to be *socially optimal* is the first-order condition $W'(p) = 0$, where

$$W'(p) = \left(p - \frac{1}{n} \sum_{i=1}^{n} C_i' \left[D(p)/n \right] \right) D'(p)$$

Hence, granted $D'(p) \neq 0$, a necessary condition for efficiency is pricing at the *average marginal cost*:

$$p = \frac{1}{n} \sum_{i=1}^{n} C_i' \left[D(p)/n \right]. \tag{11}$$

This is a fixed-point equation in p. For convex cost functions, the right-hand side is non-increasing in p. Hence, there then exists at most one solution. If, moreover, C' and D are continuous functions, then existence is guaranteed. When a solution to equation (11) exists, it will be denoted p^{mc}.

6. A parametric specification

Suppose that demand is "linear" (more precisely, piece-wise affine):

$$D(p) = \max\{0, 1-p\},$$

and that firms have cost functions of the polynomial form

$$C_i(q) = k_i + c_i q + \gamma_i q^2/2,$$

where $k_i, \gamma_i \geq 0$ and $0 \leq c_i < 1$ for all i. Let $\bar{c} = \sum_i c_i/n$ and $\bar{\gamma} = \sum_i \gamma_i/n$. Hence, for each producer i, k_i is its fixed cost and $c_i + \gamma_i q$ its marginal cost when operating at any output level $q \geq 0$.

The monopoly price for firm i (were it alone in the market) is

$$\hat{p}_i = \frac{c_i + \gamma_i + 1}{\gamma_i + 2}.$$

It follows that $\hat{p}_i \geq 1/2$ and $\hat{p}_i > c_i$[5] Moreover,

$$\hat{p} = \min_i \frac{c_i + \gamma_i + 1}{\gamma_i + 2}, \tag{12}$$

with $\hat{p} \geq 1/2$ and $\hat{p} > \min_i c_i$. The condition (11) for marginal cost pricing gives

$$p^{mc} = \frac{n\bar{c} + \bar{\gamma}}{n + \bar{\gamma}}, \tag{13}$$

and the condition (7) for symmetric Nash equilibrium prices p becomes

$$p \in \cap_{i=1}^{n} \left[\frac{2n^2 k_i}{(2n + \gamma_i)(1-p)} + \frac{2nc_i + \gamma_i}{2n + \gamma_i}, \frac{2nc_i + (n+1)\gamma_i}{2n + (n+1)\gamma_i} \right]. \tag{14}$$

It follows by continuity that this set is non-empty if firms are sufficiently similar and have strictly convex cost functions with sufficiently low fixed costs:

Proposition 5. *There exists $\delta > 0$ such that $P^{NE} \neq \varnothing$ if $\max_i k_i < \delta$, $\min_i \gamma_i > \delta$, $\max_{i \neq j} |c_i - c_j| < \delta$ and $\max_{i \neq j} |\gamma_i - \gamma_j| < \delta$.*

Proof: Let n be fixed. The set on the right-hand side of (14) is a closed interval of positive length in the special case of identical firms with zero fixed costs and strict convexity in costs:

$$\begin{aligned} f(c, \gamma, k, p) &= \frac{2n^2 k}{(2n+\gamma)(1-p)} + \frac{2nc + \gamma}{2n + \gamma} \\ &< \frac{2nc + (n+1)\gamma}{2n + (n+1)\gamma} = g(c, \gamma) \end{aligned}$$

for $k = 0$, any $c \in [0, 1)$, $\gamma > 0$ and $p < 1$. Moreover, we then also have

$$f(c, \gamma, 0, p) < \frac{c + \gamma + 1}{\gamma + 2} = \hat{p}.$$

[5] To see this, note that $\hat{p}_i$ is increasing in γ_i, and note also that for $\gamma_i = 0$, $\hat{p}_i$ is the mid-point between c_i and 1.

any $c \in [0,1)$, $\gamma > 0$ and $p < 1$. Let

$$p^{+} = \min\left\{\hat{p}, g\left(c, \gamma\right)\right\}.$$

Clearly $p^{+} < 1$ and $P^{NE} \cap [0, \hat{p}] = \left[f\left(c, \gamma, k, p\right), p^{+}\right]$. By continuity, $f\left(c, \gamma, k, p^{+}\right) < p^{+}$ for any $c \in [0,1)$ and $\gamma > 0$, granted $k > 0$ is sufficiently small. The claim in the proposition now follows by continuity of f, g and $\hat{p}$, viewed as functions of parameters k_i, c_i and γ_i. **End of proof.**

In the special case of linear costs ($k_i = \gamma_i = 0$), the equilibrium condition (14) boils down to the requirement that $p = c_i$ for all i. Hence symmetric equilibrium then occurs only if all firms have the same marginal cost, and the equilibrium price equals this marginal cost. By equation (13), price competition then results in a socially efficient outcome.

However, this is in general not the case. To see this, suppose that fixed costs are absent and all firms are identical with strictly convex cost functions. Then the marginal cost price, p^{mc}, belongs to the interior of the interval of symmetric equilibrium prices:

$$\frac{n\bar{c} + \bar{\gamma}}{n + \bar{\gamma}} \in \left(\frac{2n\bar{c} + \bar{\gamma}}{2n + \bar{\gamma}}, \frac{2n\bar{c} + (n+1)\,\bar{\gamma}}{2n + (n+1)\,\bar{\gamma}}\right).$$

In other words, there exists a whole continuum of equilibrium prices, out of which the marginal cost price is but one.

This algebraic analysis can be illustrated geometrically as follows. Figure 1 below illustrates condition (10), for a firm i with $k_i = c_i = 0$ and $\gamma_i = 0.2$, when $n = 3$ and $\delta = 0$ (one-shot interaction). The thick curve is the graph of π_i, and the thin curve the graph of $\hat{\pi}_i$. The thick curve intersects the x-axis at $p_i^0 \approx 0.05$ and the thin curve peaks at $\hat{p}_i \approx 0.58$. The set of equilibrium prices for this firm is the small interval where the thick curve exceeds both the thin curve and the horizontal axis, roughly $0.05 \leq p_i \leq 0.22$. If all three firms are identical, then this is also the set P^{NE} of equilibrium prices, and in this case the socially optimal price is $p^{mc} = 0.2/1.7 \approx 0.12$.

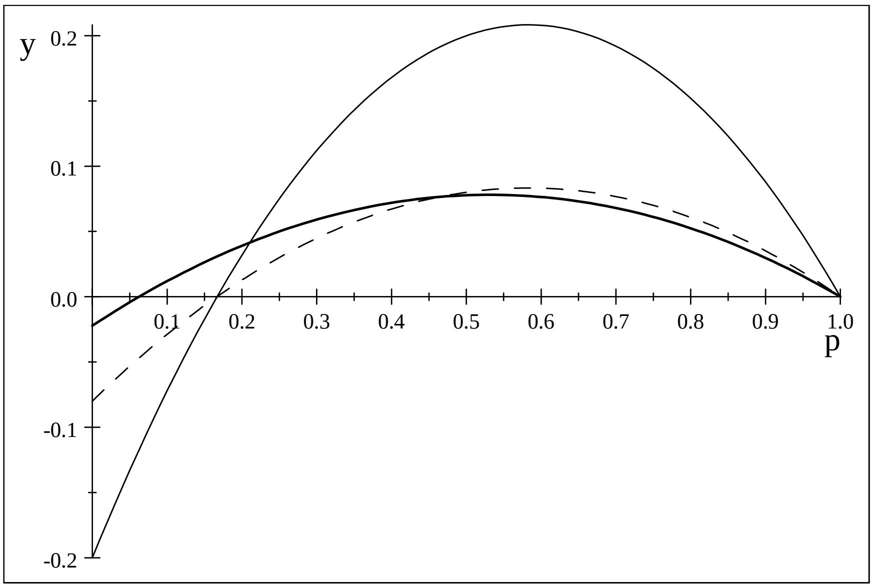

Figure 1: The graphs of the two profit functions in (10).

For positive δ, the thin curve is closer to the x-axis, and the diagram then shows, likewise, the set of symmetric subgame perfect equilibrium prices in the infinitely repeated game. For δ sufficiently large, the thin curve lies below the thick curve for all $p \in [p^0, \hat{p}]$, in which case P^{SPE} consists of all such prices p. The dashed thin curve is the graph of $(1-\delta)\,\hat{\pi}$, drawn for $\delta = 0.6$, in which case the interval of subgame perfect equilibrium prices is approximately $[0.05, 0.47]$.

6.1. Comparative statics in costs. Suppose that firms are identical and price at marginal cost. The profit to each firm is then

$$\pi\left(p^{mc}\right) = \frac{\gamma}{2}\left(\frac{1-c}{n+\gamma}\right)^2 - k.$$

It is noteworthy that this profit is non-monotonic in γ, see Figure 3 below, showing how the profit depends on the cost parameter γ, for k, c and n fixed. (Here $k = 0$, $c = 0.3$ and $n = 2$.)

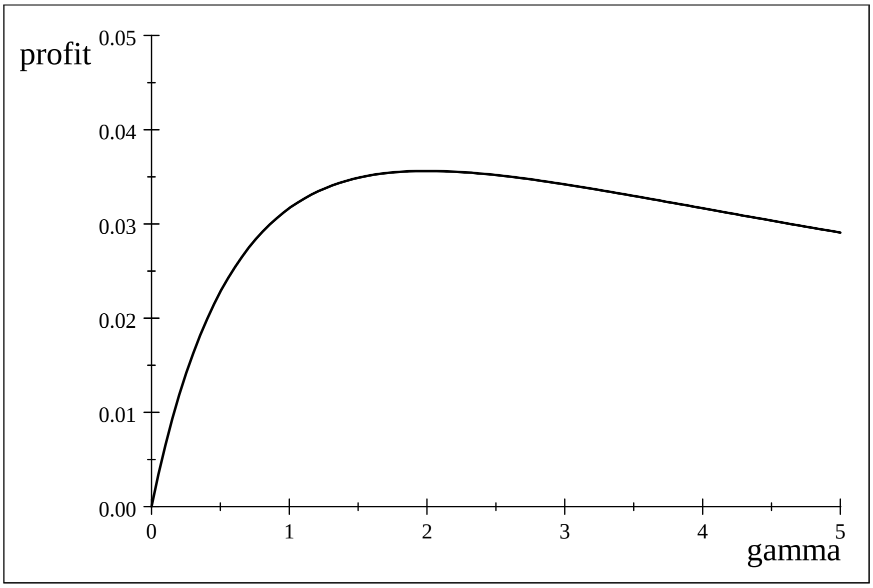

Figure 2: Equilibrium profit as a function of the cost parameter γ.

The diagram shows how the profit increases with γ, for all $\gamma < 2$. Hence, a general reduction in production costs, for all firms (for example due to technological progress) may be against the firms' profit interest. The reason is that a cost reduction may reduce the equilibrium price so much that profits fall. Hence, for certain cost parameters, there is a conflict of interest between consumers and the firms concerning technological progress in the production technology of firms. It may be noted that the property is robust in the sense that in the absence of fixed costs, equilibrium profits are zero when costs are linear ($\gamma = 0$) but positive when costs are slightly convex ($\gamma > 0$ and small).

6.2. Asymmetric duopoly. Consider the case of a firm with a positive fixed cost and a positive and increasing marginal cost competing with a firm à la Bertrand, that is, with constant marginal and average costs. Suppose, moreover, that the second firm's constant marginal cost is relatively high in comparison with the first: $k_1 \geq k_2 = 0$, $c_1 < c_2 < 1$ and $\gamma_1 > \gamma_2 = 0$. Condition (14) then pins down the symmetric Nash equilibrium price in the one-shot game: $p = c_2$. In symmetric equilibrium, the profit to the first firm is

$$\pi_1 (c_2) = \left[\frac{1}{2} (c_2 - c_1) - \frac{\gamma_1}{4} (1 - c_2) \right] (1 - c_2) - k_1 \tag{15}$$

Suppose that the cost parameters are such that $\pi_1(c_2) \geq 0$. A necessary and sufficient condition for $p = c_2$ to constitute a symmetric Nash equilibrium is then that $\pi_1(c_2) \geq \hat{\pi}_1(c_2)$, or, equivalently, that

$$2(c_2 - c_1) \leq 3\gamma_1(1 - c_2). \tag{16}$$

Do such parameter combinations exist? For $k_1 = c_1 = 0$, the requirement on γ_1 and c_2 can be re-written as

$$\frac{2c_2}{3(1-c_2)} \leq \gamma_1 \leq \frac{2c_2}{1-c_2}.$$

In other words, the convexity of the first firm's cost function should be moderate. This condition is met, for instance, by $\gamma_1 = 1$ and $c_2 = 1/2$. It follows, by continuity, that $p = c_2$ constitutes a symmetric Nash equilibrium also for positive but sufficiently low values of k_1 and c_1. For instance, for $\gamma_1 = 1$ and $c_2 = 1/2$, the requirement on k_1 and c_1 is $k_1 \leq 1/16 - c_1/4$.

Thus, symmetric Nash equilibria do exist for certain parameter combinations. Moreover, for certain parameter combinations there also exists an asymmetric Nash equilibrium, in which firm 1 sets $p_1 = c_2$ and firm 2 opts out ($s_2 = 0$). Then firm 1 serves the whole market and earns profits $\hat{\pi}_1(c_2)$. This situation constitutes a Nash equilibrium in the one-shot game if $\hat{\pi}_1(c_2) \geq 0$. Note that if condition (16) is met, then firm 1 does not earn a higher payoff in this asymmetric equilibrium—where it is a monopolist under the threat of a competitor's entry—than in the symmetric duopolistic equilibrium.

Secondly, consider the effect of a change in firm 2's marginal cost, c_2, upon the two firms' profits in the symmetric equilibrium. While firm 2's profit is unaffected by such a change—its profit remains at zero—the equilibrium profit to firm 1 rises, granted c_2 is below firm 1's monopoly price:

$$\frac{\partial \pi_1}{\partial c_2} > 0 \Leftrightarrow c_2 < \hat{p}_1 = \frac{c_1 + \gamma_1 + 1}{\gamma_1 + 2}. \tag{17}$$

Hence, for such cost parameters, a technological innovation that decreases c_2, without affecting firm 1's cost function, would not be welcomed by firm 1, since it would result in a lower market price and hence lower profits. This is not so surprising. More surprising, perhaps, is the fact that this would remain true for a range of cost parameters, even if the technological innovation were to benefit firm 1 to the same extent (that is, by way of reducing c_1 by the same amount):

$$\frac{\partial \pi_1}{\partial c_1} + \frac{\partial \pi_1}{\partial c_2} > 0 \Leftrightarrow c_2 < \frac{c_1 + \gamma_1 + 1/2}{\gamma_1 + 3/2}, \tag{18}$$

where the upper bound on c_2 is (somewhat) lower than in (17). The condition in (18) is met, for example, when $\gamma_1 = 1$, $c_1 = 0$ and $c_2 = 1/2$; the parameter combination discussed above.

7. Directions for further research

The above analysis is restrictive in many dimensions. Maybe the most apparent is its focus on symmetric equilibria, that is, equilibria in which all firms are active in the market. Asymmetric equilibria, where some firms opt out, as well as entry decisions by firms, are important phenomena that should be analyzed. Of particular interest for many applications is the case when one firm is much larger than the others. A second restriction of the present analysis is that it does not include investment decisions. What are the incentives for firms to investment in cost-reducing equipment and technologies? The present comparative-statics analysis suggests that these incentives may be weak or even negative. A relevant extension of the present one-shot interaction would thus be to consider a two-stage interaction, where firms invest in stage one. For the sake of realism one should probably also consider non-convex (but still continuous and non-increasing) cost functions. Such non-convexities are said to arise in the production of electricity; before a spare generator is turned on, marginal cost goes up and after the generator has started spinning, marginal cost again goes down. A forth relevant extension would be to consider periodic and/or stochastic fluctuations in demand. A fifth extension would be to include the possibility of incomplete information, that is, the realistic possibility that producers often lack precise information about each others' costs. A sixth extension would be to consider markets where sellers can choose not to serve all the demand they face. This possibility can turn otherwise unprofitable price deviations profitable. In order to analyze this possibility, some form of rationing scheme has to be defined. A seventh, and final, extension of great relevance for some markets—and which goes somewhat in the same direction as the preceding extension—would be to consider situations in which sellers can pre-commit to whole supply schedules, that is, rules that prescribe how much they are willing to supply at different prices (see Grossman (1981), Hart (1985) and Klemperer and Meyer (1989)).

It is well-known that all Nash equilibria in the classic case of identical firms with linear costs are weakly dominated; in equilibrium at least two firms set their prices equal to marginal cost, and while that is a best reply, it is weakly dominated by any higher price at which demand is positive. By contrast, with strictly convex costs, Nash equilibria are generically strict; a unilateral deviation causes a profit loss.

Moreover, based on arguments from the evolutionary game theory literature one may conjecture that only the highest Nash equilibrium price is viable in the very long run. For the highest Nash equilibrium price both Pareto dominates (in terms of the players, here the producers) and risk dominates (again in terms of the players) any other symmetric Nash equilibrium price, and it is known that recurrent play of 2×2 coordination games of this type (in large populations with boundedly rational agents subject to perpetual strategy perturbations) leads to play of the Pareto dominant equilibrium. To substantiate this conjecture requires a detailed analysis (including a discretization of prices), which falls outside the scope of this short paper. If the conjecture is correct, then the conclusion is that in the very long run, firms should be expected to price at the maximal Nash equilibrium price, that is, well above the perfectly competitive price.

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