Will Privatization Reduce Costs?*

Erik Lindqvist†

SSE/EFI Working Paper Series in Economics and Finance No. 660

March 5, 2007

Abstract

I develop a model of public sector contracting based on the multitask framework by Holmström and Milgrom (1991). In this model, an agent can put effort into increasing the quality of a service or reducing costs. Being residual claimants, private owners have stronger incentives to cut costs than public employees. However, if quality cannot be perfectly measured, providing a private firm with incentives to improve quality forces the owner of the firm to bear risk. As a result, private firms will always be cheaper for low levels of quality but might be more expensive for high levels of quality. Extending the model to allow for differences in task attractiveness, I find that public firms shun unattractive tasks, whereas private firms undertake them if incentives are strong enough.

Keywords: Privatization, public sector contracting, incomplete contracts, contracting out.
JEL codes: H11, H40, L32, L33.

*This work originates from an unpublished paper co-authored by Jens Sandblom, to whom I am very grateful. I am also indebted to Tore Ellingsen. In addition, I would like to thank Martin Bech Holte, Daniel Ferreira, Guido Friebel, Bengt Holmström, Jonathan Levin, Paul Milgrom, Robert Östling, Per Sonnerby and seminar participants at the Econometric Society European Meeting 2006, Massachussetts Institute of Technology, Stockholm Institute of Transition Economics and the Research Institute of Industrial Economics (Stockholm) for valuable comments. Financial support from the Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged.

†Department of Economics, Stockholm School of Economics, P.O. Box 6501, SE-113 83 Stockholm, Sweden. E-mail: erik.lindqvist@hhs.se

1 Introduction

Governments often procure services, such as garbage collection or elderly care, from private firms. A central issue in the debate on the merits of privatization has been how private and public firms differ regarding quality and cost. The standard view of economic theory is that private firms are cheaper than public firms, but that they might shirk on quality provision (Schmidt 1996, Hart, Shleifer and Vishny 1997, Shleifer 1998). In this paper, I develop a model of privatization that combines the multitask framework by Holmström and Milgrom (1991) with incomplete contracts.[1] My main result is that private firms are cheaper for low levels of quality, but that they might be more expensive for high levels of quality when quality is measured imperfectly.

As shown by Sappington and Stiglitz (1987), ownership matters only if contracts are incomplete. If it were possible to write complete contracts, taking every contingency over the entire horizon of the firm into account, the government could provide a private and a public firm with exactly the same incentives. Many models of incomplete contracting build on Grossman and Hart (1986), who assume that ownership allocates residual control rights that influence bargaining power in later stages of a contractual relationship. Notably, Hart, Shleifer and Vishny (1997) assume that an owner of a private firm has a stronger bargaining position toward the government than a manager of a public firm. Therefore, the private provider has stronger incentives to implement innovations that improve quality or reduce costs. However, the private provider's incentives to cut costs might be too strong since he ignores any negative effects on quality that the cost reduction might entail. As a result, private provision will always be cheaper than public provision, but may be either superior or inferior in terms of quality.

A distinguishing feature of Hart, Shleifer and Vishny (1997) is that ex ante contracts are the same under private and public provision. This view contrasts with the extensive use of procurement agencies in developed economies. Whereas explicit contracts are scarce under public provision, privatization in service contracting seems to go hand in hand with extensive ex ante contracting. In this paper, I take the view that there is some scope for ex ante contracting on quality with private providers, but not with public providers. The basic difference between the view of Grossman and Hart (1986) and this

[1] The model could also be thought of as a general model of contracting out, applicable to the make-or-buy decision of private firms. Yet in this paper we will restrict our discussion to the case of privatization.

paper is that whereas they distinguish between the ownership of assets and contractual compensation, I assume that integration shrinks the set of feasible compensation contracts. If the government holds the residual control rights, contracts on profit sharing or rewards for the completion of tasks could be manipulated ex post. Hence, like Schmidt (1996), I view privatization as a commitment device of the government that resolves a time-inconsistency problem.

The agent in my model can invest effort in improving service quality or reducing costs. Because of the commitment problem, employees of public firms cannot be given strong incentives to engage in either of these tasks, implying that work effort is low but balanced between quality improvements and cost reductions. In contrast, owners of private firms always have strong incentives to cut costs and must be provided with financial incentives for quality provision in order to improve quality. As a result, when only weak incentives are feasible, the maximum level of quality is higher under public provision than under private provision. On the other hand, if strong incentives are feasible, the maximum level of quality is higher under private provision.

Although it is possible to contract on quality outcomes with a private provider, outcomes may only partly reflect effort. When this is the case, providing a private firm with incentives to improve quality will force the owner of the firm to bear risk, raising the price for a given level of quality. Public employees are insensitive to such measurement problems since they are paid a fixed wage regardless of the outcome. If owners of private firms are sufficiently risk averse, private firms are more expensive than public firms for high levels of quality, even though they operate more efficiently. I thus identify a potential non-monotonicity regarding the choice of ownership structure in government preference for quality: If strong incentives for quality are feasible, but effort on quality is hard to measure, private provision is preferable when quality is considered either unimportant or very important whereas public provision is optimal when quality is of intermediate importance. This result differs from the prediction of Hart, Shleifer and Vishny (1997) that private provision is always cheaper.

The model formalizes arguments that have previously been put forward in an informal way. For example, Donahue (1989, p. 87 and p. 105) and Sclar (2000, p. 109) both argue that the inability to accurately measure effort on quality may lead to an suboptimal allocation of risk under private provision.

I present the model in the next section, and the main results are laid out in Section

3. In Section 4, I discuss some extensions of the basic model. Allowing for contracting on cost, I find that although contracts are complete in the sense that a private firm can be provided with exactly the same incentives as a public firm, ownership still matters. If effort on quality and cost are sufficiently hard to measure, public provision is still cheaper than private provision for high levels of quality. Extending the model to allow for differences in task attractiveness, I find that public firms shun unattractive tasks, whereas private firms undertake them if incentives are strong enough. Section 5 concludes the paper.

2 The Model

The principal is a public agency that has to decide whether to procure a service from the market or produce it in-house. The agent is a service provider. When service provision is organized in-house, I refer to the service provider as the manager, with private provision, the service provider is called owner.

A service provider can put effort on to two different tasks. The first task (t_q) improves the quality of the service and the other (t_c) reduces its costs. Engagement in the two tasks entails a cost $C(t)$ for the service provider, where $t = t_q + t_c$. The cost function, $C(t)$, is strictly convex, twice continuously differentiable and minimized at $t^* > 0$, i.e., people are assumed to exert some effort even in the absence of financial incentives.

The outcome of the cost reducing task is certain, but effort on cost reductions is not observable to the public agency.[2] The cost reduction is given by a function $S(t_c)$ which is increasing, strictly concave and twice continuously differentiable in t_c. Total cost is thus decreasing in $S(t_c)$. In contrast to cost savings, the effort on quality is observable, albeit imperfectly. Formally, the public agency receives a signal of quality $q = t_q + \varepsilon_q$, where ε_q is a stochastic term with distribution $\varepsilon_q \sim N\left(0, \sigma_q^2\right)$. It is only possible to contract on outcomes (q) and not directly on effort (t_q).

The public agency's valuation of service quality is described by a function $B(t_q, \psi)$, which is increasing and strictly concave in t_q for all $\psi > 0$. ψ is a parameter that denotes the public agency's valuation of quality. I assume that $B(t_q, \psi)$ is twice continuously differentiable in both t_q and ψ, $\lim_{\psi \longrightarrow \infty} B(t_q, \psi) = \infty$, $B_{t_q}(t_q, 0) = 0$ for all t_q and $\lim_{\psi \longrightarrow \infty} B_{t_q}(t_q, \psi) = \infty$ for all t_q, where $B_{t_q}(t_q, \psi)$ denotes the partial derivative of

[2] I relax the assumption that effort on cost reductions is not contractible in Section 4.1.

$B(t_q, \psi)$ with respect to t_q. The service provider's utility is defined as $u(x) = -e^{-rx}$, where r is a measure of risk aversion and x is the financial return minus the cost of effort. I assume that $r > 0$. The public agency is risk neutral.

The service provider receives a pecuniary compensation $w(q) = \alpha q + \beta$ and a share of cost savings $\lambda \in [0, 1]$. The parameter α is thus a measure of the economic incentive to engage in the quality improving task and β is a fixed wage component. Similarly, λ denotes the incentive to reduce costs. We get $x = w(q) + \lambda S(t_c) - C(t)$ and the manager/owner's expected utility is thus

$$E[u(w(q) - C(t) + \lambda S(t_c))] = u(CE),$$

where CE is the certainty equivalent. Solving for CE, we get

$$CE = \alpha t_q + \beta - C(t) + \lambda S(t_c) - \frac{1}{2} r \sigma_q^2 \alpha^2,$$

where $\frac{1}{2} r \sigma_q^2 \alpha^2$ is the agent's risk premium. I refer to the product $r\sigma_q^2$ as the cost of risk.

The agent's outside option is normalized to zero, implying that the participation constraint is $CE \geq 0$. Assuming perfect competition, the fixed wage component β will be set such that the participation constraint always binds.

How do private and public firms differ in this model? The key assumption is that the public agency cannot credibly commit to high-powered incentives, e.g., payment depending on the completion of tasks or profit sharing, under public (in-house) provision. In terms of the model, this means that $\alpha = 0$ and $\lambda = 0$. The argument for this assumption is that ownership gives control over key contractual parameters, which enlarges the public agency's scope for opportunistic behavior. For example, if the manager has a share in the firm's profits, the owner could influence stated profits by changing accounting procedures. Alternatively, the public agency could override the manager's decisions in a way that inflicts on his chances of reaching performance clauses. Another fundamental commitment problem is the existence of the public firm itself. If the public agency cannot commit to not sell or close down the public firm in the future, it is hard to commit to incentives for outcomes that are observable only in the long run. This in turn affects the optimal level of incentives for tasks that are observable in the short run as strong incentives on such tasks will crowd out effort on tasks where outcomes are only observable in the long run.

Hence, the inability to commit to long-run incentives creates a problem of intertemporal incentive balance. In practice, there are also some less fundamental reasons for why it might be difficult to provide managers of public firms with strong incentives. In many countries, employment protection laws and union wage bargaining restrict the space of feasible employment contracts. For example, it might more difficult to fire an employee should he deviate from the contractual terms than cease business with a private firm.

Private firms have a stronger legal position toward the public agency in the sense that it is harder for the public agency to manipulate contracts ex post. In addition, contracts with private firms are not constrained by employment regulations. However, even though contractual compensation under private provision can be based on observable outcomes, not every contract is credible ex ante. For example, if bankruptcy of the private provider implies that service delivery is interrupted, a contract that entails bankruptcy for the private firm in some states of the world might not be renegotiation-proof. Moreover, public procurement procedures are often subject to regulations that aim to reduce the risk of collusion between bureaucrats and firms. As pointed out by Wilson (2000, p. x), such regulations make it hard for the government purchaser to take past experience of a firm into account, thereby limiting the scope for implicit contracting on quality. If high-powered incentives for quality provision are not credible ex ante, the set of possible contracts is limited by some upper bound on α, i.e. $\alpha \in [0, \overline{\alpha}]$.[3]

Since effort on the cost reducing task is unobservable to the public agency and the owner of the private firm has the residual control rights, $\lambda = 1$ in case of private provision. The assumption that cost savings are unobservable might seem stark and I will relax it in an extension below. However, the assumption makes more sense if we include activities that affect the firm's cost structure over the long term in the definition of cost savings. In this case, a wide range of activities that might have a negative immediate impact on profits would count as effort on cost reductions. Examples include many tasks which arguably are very hard to contract on directly, such as equipment maintenance, the development of administrative systems and human resource management.

[3] We can think of α as being the result of both explicit and implicit contracting (such as relational contracting or reputation effects), though the game played by principal and agent is not modelled explicitly.

2.1 Public firm

Since the public agency cannot commit to incentive payment schemes under public provision, α and λ are equal to zero. The manager's maximization problem is therefore[4]

$$\max_{t_q, t_c} \quad \beta - C(t). \tag{1}$$

Since the agent is indifferent between the two tasks he just minimizes his cost function with respect to total effort.[5] This gives the optimality condition $t_q + t_c = t^*$ where $C'(t^*) = 0$. Since the manager is indifferent between tasks I assume that he divides his effort according to the wishes of the public agency.[6] The public agency's maximization problem is then

$$\begin{aligned} &\max_{t_q, t_c} \quad B(t_q, \psi) + S(t_c) \\ &\text{s.t.} \qquad t_q + t_c = t^*. \end{aligned} \tag{2}$$

Let the superscript m denote the manager of the public firm. The solution to this problem is

$$\left(t_q^m, t_c^m\right) = \begin{cases} t_q = 0 \text{ and } t_c = t^* & \text{if } B_{t_q}(0, \psi) \leq S'(t^*); \\ \left\{ (t_q, t_c) \middle| B_{t_q}(t_q, \psi) = S'(t_c) \right\} & \text{if } B_{t_q}(0, \psi) > S'(t^*) \text{ and } B_{t_q}(t^*, \psi) < S'(0); \\ t_q = t^* \text{ and } t_c = 0 & \text{if } B_{t_q}(t^*, \psi) \geq S'(0). \end{cases}$$

Hence, the manager of the public firm sets the total level of effort and the public agency decides how this effort should be divided between the two tasks. The public agency orders the manager to set $t_q = 0$ if the marginal value of quality at $t_q = 0$ is weakly lower than the marginal benefit of cost savings at $t_c = t^*$. When the public agency's valuation of quality is higher, the manager is instructed to put effort on quality until the marginal value of quality equals the marginal value of cost reductions. The second corner solution occurs when the public agency's marginal valuation of quality at $t_q = t^*$ is weakly higher than the marginal returns to cost savings at $t_c = 0$. The division of effort between tasks is therefore optimal under public provision, whereas total work effort is suboptimally low.

[4] Note that since $u(CE)$ is a monotone transformation of CE, maximizing CE is equivalent to maximizing $u(CE) = E[u]$.

[5] The assumption that the manager is indifferent between tasks is discuseed in Section 4.2.

[6] I thus implicitly assume that the agent has lexicographic preferences where his first priority is to maximize his own utility and his second priority to maximize his principal's utility.

2.2 Private firm

Since cost savings are unobservable and the owner of the private firm is the residual claimant to the firm's profit, we have that $\lambda = 1$. The owner's maximization problem is

$$\max_{t_q, t_c} \quad \alpha t_q + \beta - C(t) + S(t_c) - \tfrac{1}{2} r \sigma_q^2 \alpha^2. \tag{3}$$

Let the superscript o denote the owner of the private firm. The solution to this problem is

$$\left(t_q^o, t_c^o\right) = \begin{cases} t_q = 0 \text{ and } t_c = \underline{t} \text{ s.t. } S'(\underline{t}) = C'(\underline{t}) & \text{if } 0 \leq \alpha \leq S'(\underline{t}); \\ \{(t_q, t_c) |\, \alpha = S'(t_c) = C'(t)\} & \text{if } S'(\underline{t}) < \alpha < S'(0); \\ t_q \text{ s.t. } \alpha = C'(t) \text{ and } t_c = 0 & \text{if } \alpha \geq S'(0). \end{cases}$$

In the interior solution, I get t^o, t_q^o and t_c^o as continuous functions of α with derivatives[7]

$$\frac{\partial t^o}{\partial \alpha} = \frac{1}{C''(t)} > 0; \frac{\partial t_c^o}{\partial \alpha} = \frac{1}{S''(t_c)} < 0 \text{ and } \frac{\partial t_q^o}{\partial \alpha} = \frac{1}{C''(t)} - \frac{1}{S''(t_c)} > 0.$$

In the first corner solution, t, t_q and t_c are unaffected by α. Accordingly, for $0 \leq \alpha \leq S'(\underline{t})$ we have that $t_q^o(\alpha) = 0$ and $t^o(\alpha) = t_c^o(\alpha) = \underline{t}$. Hence, if incentives for quality provision are weak, the agent only puts effort into reducing costs and total work effort is low, though still higher than under public provision. As the incentive for quality provision gets stronger, the owner increases total effort and reduces effort on cost reductions, implying that effort on quality increases. If incentives for quality provision are very strong, the owner of the private firm only puts effort into increasing quality. In addition to its effect on effort choice, α also affects the agent's risk-premium, given that there is uncertainty in the signal q (i.e. $\sigma_q^2 > 0$). The public agency's maximization problem is

$$\begin{aligned} \max_{\alpha, \beta} \quad & B\left(t_q^o(\alpha), \psi\right) - \left(\alpha t_q^o(\alpha) + \beta\right) \\ \text{s.t.} \quad & \alpha \leq \overline{\alpha}, \\ & CE \geq 0. \end{aligned}$$

Since the fixed wage component β is set to satisfy the owner's participation constraint,

[7] It is shown in the Appendix that t^o, t_q^o and t_c^o are continuous functions of α.

the public agency's maximization problem can be reformulated as[8]

$$\begin{array}{ll} \max\limits_{\alpha} & B\left(t_q^o(\alpha), \psi\right) - C\left(t^o(\alpha)\right) + S\left(t_c^o(\alpha)\right) - \frac{1}{2} r\sigma_q^2 \alpha^2 \\ \text{s.t.} & \alpha \leq \overline{\alpha}. \end{array} \tag{4}$$

In the interior solution, we get the first-order condition

$$\alpha^* = \frac{B_{t_q}(t_q, \psi)\left[1/C''(t) - 1/S''(t_c)\right]}{\left[1/C''(t) - 1/S''(t_c) + r\sigma_q^2\right]},$$

which can be rewritten as

$$\alpha^* = \frac{B_{t_q}(t_q, \psi)}{1 + r\sigma_q^2 / \frac{\partial t_q}{\partial \alpha}}. \tag{5}$$

Though condition (5) seems to imply that the higher is the cost of risk, the lower is the strength of incentives for quality provision, a positive cost of risk also implies that the public agency's maximization problem need not be concave in α, and that there might be more than one interior solution.[9] Let $W(\alpha)$ denote the public agency's valuation of a certain contract, i.e. $W(\alpha) = B\left(t_q^o(\alpha)\right) - C\left(t^o(\alpha)\right) + S\left(t_c^o(\alpha)\right) - \frac{1}{2} r\sigma_q^2 \alpha^2$. Further, let $\alpha^{**} = \arg\max_{\alpha \in A} W(\alpha)$ where A is the set of feasible interior solutions.[10] To find the optimal contract, we have to compare the surplus under the corner solutions $\alpha = 0$ and $\alpha = \overline{\alpha}$ with the feasible interior solutions.[11] The optimal feasible contract is then:

$$\alpha = \begin{cases} 0 & \text{if} \quad W(0) \geq \max\left\{W(\alpha^{**}), W(\overline{\alpha})\right\}; \\ \alpha^{**} & \text{if} \quad W(\alpha^{**}) \geq \max\left\{W(0), W(\overline{\alpha})\right\}; \\ \overline{\alpha} & \text{if} \quad W(\overline{\alpha}) \geq \max\left\{W(\alpha^{**}), W(0)\right\}. \end{cases}$$

[8] I show this in the Appendix.

[9] Non-concavity could be ruled out by assuming that $C'''(t) \geq 0$ and $S'''(t_c) \geq 0$. This is shown in the Appendix.

[10] Note that the set of feasible interior solutions might be empty.

[11] $W(\alpha)$ is a continuous function since all the arguments are continuous in α (as shown in the proof to Proposition 1). As the derivatives of both t_q and t_c are zero for $\alpha < S'(\underline{t})$, it follows that the derivative of $W(\alpha)$ is negative for all $\alpha < S'(\underline{t})$. Hence, whenever the public agency's optimal solution implies $t_q = 0$, we have $\alpha = 0$.

3 Results

The highest possible level of quality from a public firm is simply $t_q^m = t^*$. In that case, the public agency orders the manager of the public firm to spend all his efforts on increasing quality. How high quality can be obtained from a private firm depends on the set of feasible incentives. In the one extreme, if the owner of the private firm can only be provided with weak incentives to invest in quality ($\overline{\alpha} \leq S'(\underline{t})$), it is not possible to induce the private firm to invest in quality at all. Yet if strong incentives on quality provision are feasible (that is, if $\overline{\alpha}$ is high), the maximum feasible level of quality from a private firm exceeds that of a public firm.

Proposition 1 *If strong incentives are feasible, in the sense that the permissible incentive intensity $\overline{\alpha}$ exceeds some finite threshold level $\widehat{\alpha}$, a private firm can be induced to produce a higher quality than the public firm. If $\overline{\alpha}$ instead falls short of $\widehat{\alpha}$, a private firm cannot be induced to produce as high quality as a public firm.*

Proofs are provided in the Appendix.

The result that private firms can have either higher or lower quality depending on the economic environment is also present in Hart, Shleifer and Vishny (1997), but their argument is different from mine. In their paper, the difference in quality depends on the relative importance of the quality-enhancing innovation and the negative effect on quality of cost reductions, whereas in my case it depends on the set of feasible incentives. Moreover, the difference between private and public production in Proposition 1 only refers to the *maximum* level of quality; whether a potential difference is realized hinges on the α set by the public agency. Let us now focus on differences in cost.

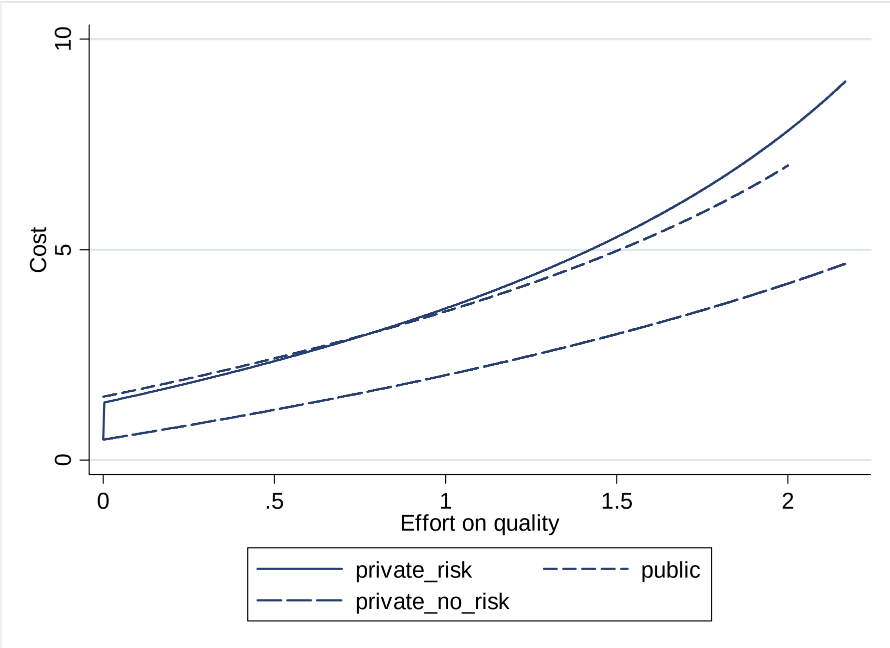
First, note that private provision is always cheaper than public provision for the lowest level of quality, $t_q = 0$. In that case, the manager of the public firm sets $t_c^m = t^*$ and total cost is $C(t^*) - S(t^*)$. Since $C'(t^*) = 0 < S'(t^*)$, this is clearly a suboptimally low level of effort. In contrast, the owner of the private firm is the residual claimant to cost savings and sets $t_c^o = \underline{t} > t^*$ which is the efficient level of effort. Moreover, the marginal cost for the public agency to increase quality is always lower under private provision than under public provision if the cost of risk $\left(r\sigma_q^2\right)$ is sufficiently small. To see this, note that for any given value of t_q we have that $t_c^o > t_c^m$ which by strict concavity of the cost-saving function implies that $S'(t_c^o) < S'(t_c^m)$. The cost of additional quality in terms of forgone cost savings is thus always larger for public than for private firms. However, the cost advantage of private provision vanishes for high levels of quality when the cost of risk is high. Since the public agency must increase α to induce the private firm to exert more effort on quality, the risk premium $\left(\frac{1}{2}r\sigma_q^2\alpha^2\right)$ rises with the level of quality. For high enough values of $r\sigma_q^2$, the marginal cost of quality is therefore higher under private provision than under public provision for any feasible level of quality. There is also a fixed cost of going from zero to positive levels of quality under private provision. For $0 < \alpha \leq S'(\underline{t})$, the owner of the private firm do not increase effort on t_q as α increases, but costs increase due to the higher the risk premium. Since private firms are always cheaper for zero quality, private provision is cheaper than public firms for low levels of quality, but more expensive for high levels of quality when $r\sigma_q^2$ is high.[12]

Proposition 2 *There is a cost of risk $\widehat{r\sigma_q^2}$ such that private firms are cheaper for no effort on quality, but more expensive for all levels of quality above a finite threshold $\widehat{t}_q$. If $\widehat{t}_q > 0$, private firms are cheaper for all quality levels below $\widehat{t}_q$.*

Figure 1 gives two parametric examples with different levels of cost of risk. In the first case, we set $r\sigma_q^2 = 0$ and private provision is cheaper for all levels of effort on quality and the difference is increasing in the level of quality. The cost for no effort on quality is unchanged when we assume a positive cost of risk $\left(r\sigma_q^2 = 0.45\right)$, but exhibits a discontinuous jump when quality is increased above zero. The reason for this is that the strength of incentives for quality provision must be increased from $\alpha = 0$ to $\alpha = S'(\underline{t}) + \epsilon$, where ϵ is some very small number larger than zero. In this particular case, costs are similar under public and private provision for medium levels of quality, but private provision is more expensive for high levels of quality. However, the highest levels of quality can only be obtained from a private firm. If we were to increase the cost of risk even higher, we would eventually reach the case where private provision is cheaper for zero effort on quality, but more expensive for all positive levels of quality.

Now, let us discuss the public agency's choice of ownership structure. Assuming that strong incentives are feasible, whether the service will be bought from a private or a public firm depends on the public agency's willingness to pay for quality, ψ, and the cost of risk, $r\sigma_q^2$. If either ψ or $r\sigma_q^2$ are low, private provision will be superior since it is cheapest.

[12]Note that, in the extreme case, "low levels of quality" refers to $t_q = 0$.



Conversely, if the public agency has a high valuation of quality and $r\sigma_q^2$ is high, a rational public agency prefers public provision since the risk premium makes private provision more expensive for high levels of quality. However, since it is not possible to extract quality in excess of t_q^* from public firms, private provision is better if the public agency's willingness to pay for quality is very high. There is thus a potential non-monotonicity regarding the choice of ownership structure in government preference for quality.

Proposition 3 *If the upper bound of contractibility on quality, $\overline{\alpha}$, is such that an effort on quality exceeding t_q^* can be extracted from private firms, private provision is superior if quality is either considered unimportant or very important, and if $r\sigma_q^2$ is sufficiently large there exists some intermediate level of the importance of quality such that public provision is superior.*

If strong incentives are not feasible, the only advantage of private provision is its lower cost for low levels of quality.

Proposition 4 *If the upper bound of contractibility on quality, $\overline{\alpha}$, is such that the maximum level of quality under private provision is below t_q^*, public provision is superior when quality is important.*

To summarize, we have two cases where public ownership may be superior to private ownership, namely: i) when weak feasible incentives for effort on quality make private provision above a certain quality level impossible, and ii) when the difficulty in measuring quality makes private firms more expensive for high levels of quality.

The model illustrates the importance of controlling for market selection in empirical studies of privatization. For services where quality is hard to measure, only public agencies with a low (or very high) valuation of quality will go for private provision. Hence, unless quality is adequately controlled for, empirical studies are likely to overstate the cost-saving effect of private provision.[13] In contrast, we would expect privatization to reduce costs without any deteriation of quality for services where quality is easy to measure.[14]

Another reason why privatization might not save costs is that agency problems within private firms make the incentives for top executives similar to those of public managers. If a firm owner that hires a manager to run his business faces the same commitment problem as the government, privatization does not change anything in the context of this model.[15] Hence, privatization might have its largest impact when private firms are relatively small and owner-led and economic models focusing on the impact of ownership (like this paper) might only be applicable to a subset of privatization cases.

Finally, let us briefly consider the differences between contracting out in the private and public sector. One might argue that the problem of providing managers with strong incentives is worse in the public than in the private sector, since the latter can rely on the information given by stock prices. This would imply that contracting out is particularly important in the public sector. However, there are also reasons for why contracting could be more difficult in the public sector. For example, procurement laws might make implicit contracting through repeated interaction harder and contracts with private providers are often in practice enforced by government bureaucrats with weak incentives to do a good

[13] Of course, the opposite case would hold if many public agency's have a very strong preference for quality and high-powered incentives for quality are feasible.

[14] There is indeed substantial evidence that privatization reduces costs without a detrimental effect on quality for services such as refuse collection and cleaning services, where measurability of quality can be assumed to be good. See, for example, Donahue (1989), Wilson (1989), Domberger and Jensen (1997) and the references citet in these works. Even a critic of privatization such as Elliot Sclar (2000) argues that "privatization of many blue-collar services certainly may be cost effective" (p. 57). However, Ohlsson (2003) argues that much of the literature on refuse collection has overstated the cost saving effect of privatization due to a failure to control for selection bias.

[15] Note that low incentives to reduce costs will imply less of an incentive balance problem, making it easier for the government to induce private firms not to shirk on quality provision.

job.

4 Extensions

4.1 Contracting on cost

I have so far analyzed the case where it is only possible to contract on quality. Here I relax this assumption and let the public agency and the private firm contract on cost.[16] As we will see, because of the residual risk, it is not possible to replicate the public firm even under full contractibility on quality and cost.

Under no cost contracting, the owner of the firm is the residual claimant to cost savings and $\lambda = 1$. Hence, cost contracting implies that the firm and the government sign a contract that gives the private firm *less* than the full share of cost savings, in effect a cost-plus contract where the firm gets compensation for costs incurred.[17] Accordingly, complete cost-plus contracting corresponds to $\lambda = 0$. The cost contracting problem is thus not to provide private firms with strong incentives to cut costs, but to mitigate those incentives, and the commitment problem is to make a high degree of compensation credible. In terms of our model, the *lower* is t_c, the *larger* is the compensation from the government to the private firm.

Let $\gamma = 1 - \lambda$ denote the share of costs for which the owner is compensated. I assume that $\gamma \in [0, 1]$, though in principle there might be some upper bound on contractibility on cost $\overline{\gamma} \in [0, 1]$ so that $\gamma \in [0, \overline{\gamma}]$. The principal receives a signal of costs $v = S(t_c) + \varepsilon_v$ where $\varepsilon_v \sim N(0, \sigma_v^2)$ and the agent's compensation is $w(q, v) = \alpha q + \beta - \gamma v$. I assume that the errors in q and v are independent, i.e., $Cov(\varepsilon_q, \varepsilon_v) = 0$. The owner's monetary payoff, x, is then normally distributed with $E(x) = \alpha t_q + \beta - C(t) + (1-\gamma) S(t_c)$ and $V(x) = \sigma_q^2 \alpha^2 + \sigma_v^2 \gamma^2$. This gives us the owner's maximization problem

$$\max_{t_q, t_c} \alpha t_q + \beta - C(t) + (1-\gamma) S(t_c) - \frac{1}{2} r \left(\sigma_q^2 \alpha^2 + \sigma_v^2 \gamma^2 \right). \tag{6}$$

[16] I keep the assumption that no contract on the completion of tasks is possible under public provision.

[17] I assume that it is not possible for the owner of the private firm to extract revenue by increasing costs, for example by increasing his own wage.

The solution to this problem is

$$\left(t_q^o, t_c^o\right) = \begin{cases} \left\{(t_q, t_c) \middle|\, t_q + t_c = t^*\right\} & \text{if } \alpha = 0 \text{ and } \gamma = 1; \\ t_q = 0 \text{ and } t_c = \underline{t} \text{ s.t. } (1-\gamma)\, S'(\underline{t}) = C'\left(\underline{t}\right) & \text{if } 0 \leq \alpha \leq (1-\gamma)\, S'(\underline{t}) \text{ and } \gamma < 1; \\ \left\{(t_q, t_c) \middle|\, \alpha = (1-\gamma)\, S'(t_c) = C'\left(t\right)\right\} & \text{if } (1-\gamma)\, S'(\underline{t}) < \alpha < (1-\gamma)\, S'(0); \\ \left\{(t_q, t_c) \middle|\, \alpha = C'\left(t\right), t_c = 0\right\} & \text{if } \alpha > (1-\gamma)\, S'(0). \end{cases}$$

In the interior solution (excluding the case when $\alpha = 0$ and $\gamma = 1$), we get t^o, t_q^o and t_c^o as continuous functions of α and γ with partial derivatives[18]

$$\begin{aligned} \frac{\partial t^o}{\partial \alpha} &= \frac{1}{C''\left(t\right)} > 0 \ ; \ \frac{\partial t^o}{\partial \gamma} = 0 \ ; \\ \frac{\partial t_c^o}{\partial \alpha} &= \frac{1}{(1-\gamma)\, S''\left(t_c\right)} < 0 \ ; \ \frac{\partial t_c^o}{\partial \gamma} = \frac{\left[S'\left(t_c\right)\right]^2}{\alpha S''\left(t_c\right)} < 0 \ ; \\ \frac{\partial t_q^o}{\partial \alpha} &= \frac{1}{C''\left(t\right)} - \frac{1}{(1-\gamma)\, S''\left(t_c\right)} > 0 \text{ and } \frac{\partial t_q^o}{\partial \gamma} = -\frac{\left[S'\left(t_c\right)\right]^2}{\alpha S''\left(t_c\right)} > 0. \end{aligned}$$

The public agency's maximization problem is therefore

$$\begin{aligned} \max_{\alpha,\gamma} \quad & B\left(t_q^o\left(\alpha,\gamma\right)\right) - C\left(t^o\left(\alpha\right)\right) + S\left(t_c^o\left(\alpha,\gamma\right)\right) - \tfrac{1}{2} r\left(\sigma_q^2 \alpha^2 + \sigma_v^2 \gamma^2\right) \\ \text{s.t.} \quad & \alpha \leq \overline{\alpha}, \\ & \gamma \in [0,1]\,. \end{aligned}$$

We get the first-order condition with respect to α,

$$\alpha^* = \frac{B_{t_q}\left(t_q, \psi\right)}{1 + r\sigma_q^2 / \frac{\partial t_q^o}{\partial \alpha}}, \tag{7}$$

and the first-order condition with respect to γ,

$$\gamma^* = \frac{B_{t_q}\left(t_q, \psi\right) - S'\left(t_c\right)}{r\sigma_v^2} \frac{\partial t_q^o}{\partial \gamma}. \tag{8}$$

As above, the maximization problem is not necessarily concave in α and γ.

The basic insight from this extension of the model is that the level of incentives for quality provision (α) needed to ensure a certain level of quality (t_q) is falling in

[18]This is shown in the Appendix.

the extent of cost-plus contracting (γ). By compensating the owner of the firm for his costs, incentives to invest in cost reductions are muted, implying that effort on the cost reducing task falls, which makes the cost of effort for engaging in quality provision lower on the margin.[19] Hence, by lowering the incentives for cost reductions, a higher level of quality can be exerted for any given level of incentives for quality provision. This has two potential advantages: First, it mitigates the problem of weak feasible incentives for quality provision. Second, for a given level of quality, incentives for quality can be less high-powered, thereby lowering the cost of risk.[20]

Note that setting $\alpha = 0$ and $\gamma = 1$ gives the owner of the private firm exactly the same incentives as the manager of the public firm. However, if $r\sigma_v^2 > 0$, the owner of the private firm have to bear risk, making private provision more expensive. Hence, ownership matters also when a private firm can be provided with exactly the same incentives as a public firm. Actually, even if contracts are complete (in the sense that $\overline{\alpha} = \infty$ and $\overline{\gamma} = 1$), public provision is still cheaper than private provision for high levels of quality if $r\sigma_q^2$ and $r\sigma_v^2$ are high enough.

Proposition 5 *For every positive level of quality there exists a nonempty set of vectors* $\left(\widehat{r\sigma}_q^2, \widehat{r\sigma}_v^2\right)$ *such that public firms are cheaper than private firms.*

4.2 Task attractiveness

A key result of the model is that private firms underinvest in tasks for which they have no financial incentives, whereas the drawback of public provision is the low level of total effort. This is in accordance with previous models of privatization like Schmidt (1996) and Hart, Shleifer and Vishny (1997). An implicit assumption in these models, as well as my model, is that tasks do not differ in terms of their inherent attractiveness. Yet if some tasks are inherently less attractive than others (for example because they are boring or difficult to undertake), the incentive balance problem might be reversed. The manager of a public firm, who lacks financial incentives for any particular task, underinvests in those tasks that he or she for some reason finds unattractive. The owner of the private firm

[19] As long as $\alpha > (1-\gamma) S'(\underline{t})$, total effort is determined by α, and γ does only determine the allocation across tasks – the higher is γ, the larger must $S'(t_c)$ be for a given α, implying that t_c is falling in γ. For $0 \leq \alpha \leq (1-\gamma) S'(\underline{t})$, total effort is determined solely by γ, but then there's no rationale for setting $\gamma > 0$, except for eliminating all incentives for cost reductions by setting $\gamma = 1$

[20] Conditions under which the principal will set $\gamma > 0$ are provided in the Appendix.

also dislikes the unattractive task, but this is at least partly offset by an explicit financial incentive.

Let t_u denote effort on the unattractive task. I assume that t_u, t_q and t_c are perfect substitutes in the cost of effort, i.e., $t = t_u + t_q + t_c$. Effort on t_u also comes at an additional cost of effort $D(t_u)$, which is an increasing and strictly convex function with $D'(0) > 0$. I assume that effort on t_u is perfectly observable and hence contractible in case of private provision. The owner of the private firm is compensated according to the linear wage contract

$$w(q) = \alpha q + \omega t_u + \beta.$$

The owner's maximization problem then becomes

$$\max_{t_q,\, t_c,\, t_u} \quad \alpha t_q + \omega t_u + \beta + S(t_c) - C(t) - D(t_u) - \tfrac{1}{2} r \sigma_q^2 \alpha^2. \tag{9}$$

Solving this problem, we find that the owner of the private firm will put effort on the unattractive task if $\omega > D'(0) + \max\{\alpha, S'(\underline{t})\}$. Note that effort on cost reductions and quality are both crowded out by strong incentives for the unattractive task.

Since the public agency cannot commit to any incentive scheme under public provision, we have $\alpha = 0$ and $\omega = 0$, implying that the public manager's maximization problem is

$$\max_{t_q, t_c, t_u} \quad \beta - C(t) - D(t_u). \tag{10}$$

Hence, under public provision, the manager sets $t_u = 0$ and $t_q + t_c = t^*$. Note that I implicitly assume that the government cannot restrict the work description of the public firm to only include t_u, in which case the task allocation problem might be overcome.[21] By the same argument, differences in task attractiveness also provides a rationale for limiting worker freedom within organizations.

Proposition 6 *Public firms shun unattractive tasks, but private firms undertake them if incentives are strong enough.*

[21] For the manager of the public firm to set $t_u > 0$, we must have that $|C'(0)| > D'(0)$. Note that if t_u is not an argument in $C(t)$ (that is, the cost of effort on t_q and t_c are independent of t_u), then the manager of the public firm will always set $t_u = 0$.

Instead of an additional unattractive task, we could consider the relative task attractiveness of t_q and t_c. For example, it is conceivable that improving quality is a more attractive task than reducing costs. If so, it would not be possible to induce a manager of a public firm to put effort on cost reductions. This is an important qualification to the assumption above that the manager of the public firm is indifferent between improving quality and reducing costs.

5 Concluding remarks

This paper has argued that differences in ex ante contracts is an important feature of privatization of service provision. Under private provision, procurement agencies write elaborate contracts with private firms, whereas managers under in-house provision seldom have explicit incentive contracts. This is a stark view and there are of course examples of less clear-cut cases. Private firms might in effect be run by employed managers with only a limited stake in the firm's profits. Public agencies might create independent firms that have to compete with private service providers for contracts. Such arrangements could reduce, or even eliminate, differences between private and public provision. However, it might still be fruitful to analyze such intermediate cases as examples of different ex ante contracts.

6 References

Domberger, S and Jensen, P. (1997), "Contracting Out by the Public Sector: Theory, Evidence, Prospects", *Oxford Review of Economic Policy*, Vol. 13, 4, 67-78.

Donahue, John D. (1989), *The privatization decision: public ends, private means*, Basic Books, New York.

Grossman, Sanford J. and Hart, Oliver D. (1986), "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration", *Journal of Political Economy*, 94, Issue 4, 691-719.

Hart, Oliver; Shleifer, Andrei and Vishny, Robert W. (1997), "The Proper Scope of Government: Theory and an Application to Prisons", *Quartely Journal of Economics*, November 1997, Vol. 112, 1127-1161.

Holmström, Bengt and Milgrom, Paul (1991), "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design", *Journal of Law, Economics and Organization*, Vol 7., 24-51.

Ohlsson, H. (2003), "Ownership and Production Costs: Choosing between Public Production and Contracting-Out in the Case of Swedish Refuse Collection", *Fiscal Studies*, Vol. 24, 4, 451-476.

Sappington, David E. and Stiglitz, Joseph E. (1987), "Privatization, Information and Incentives", *Journal of Policy Analysis and Management*, 6, 567-582.

Schmidt, Klaus M. (1996), "The Costs and Benefits of Privatization: An Incomplete Contracts Approach", *Journal of Law, Economics and Organization*, Vol. 12, No. 1, 1-24.

Sclar, Elliott D. (2000), *You don't always get what you pay for: the economics of privatization*, Ithaca, New York.

Shleifer, Andrei (1998), "State versus Private Ownership", *Journal of Economic Perspectives*, Fall 1998.

Wilson, James Q. (1989), *Bureaucracy: What Government Agencies Do and Why They Do It*, New York: Basic Books.

7 Appendix

7.1 Owner's effort choice

7.1.1 Without contracting on costs

Consider the owner's effort on cost reductions as a function of α. From the solution to (3), we know that $t_c^o(\alpha) = \underline{t} > t^*$ for $0 \leq \alpha \leq S'(\underline{t})$ and $t_c^o(\alpha) = 0$ for all $\alpha \geq S'(0)$. Since $S(t_c)$ is increasing and strictly concave, there exists an inverse function to $\alpha = S'(t_c)$ for $0 < t_c < \underline{t}$. As $S(t_c)$ is also twice continuously differentiable, the inverse function $t_c^o(\alpha)$ is continuously decreasing in α for $S'(\underline{t}) < \alpha < S'(0)$. Now consider the owner's total effort as a function of α. From the solution to (3), we know that $t_q^o(\alpha) = 0$ for $0 \leq \alpha \leq S'(\underline{t})$ and so $t^o(\alpha) = t_c^o(\alpha) = \underline{t}$ for $0 \leq \alpha \leq S'(\underline{t})$. Since $C(t)$ is strictly convex and increasing for $t > t^*$, $\alpha = C'(t)$ is invertible for $\underline{t} < t < \bar{t}$ where $\bar{t}$ is such that $\lim_{t \to \bar{t}} C'(t) = \infty$. As $C(t)$ is also twice continuously differentiable, the inverse function $t^o(\alpha)$ is continuously increasing in α for $\alpha > S'(\underline{t}) = C'(\underline{t})$ with $\lim_{\alpha \to \infty} t^o(\alpha) = \bar{t}$. The owner's effort on quality is given by $t_q^o(\alpha) = t^o(\alpha) - t_c^o(\alpha)$. It follows from above that $t_q^o(\alpha) = 0$ for $0 \leq \alpha \leq S'(\underline{t})$ and that $t_q^o(\alpha)$ is continuously increasing in α for all $\alpha > S'(\underline{t}) = C'(\underline{t})$ with $\lim_{\alpha \to \infty} t_q^o(\alpha) = \bar{t}$.

7.1.2 With contracting on costs

For $\alpha = 0$ and $\gamma = 1$, the owner of the firm is indifferent between t_q and t_c and all allocations of effort such that $t_q + t_c = t^*$ are feasible. However, as soon as $\alpha > 0$ and $\gamma = 1$, the owner will set $t_c = 0$ or, alternatively, when $\alpha = 0$ and $\gamma < 1$, the owner will set $t_q = 0$. Hence, $t_q^o(\alpha, \gamma)$ and $t_c^o(\alpha, \gamma)$ are not necessarily continuous in α and γ when $\alpha = 0$ and $\gamma = 1$.

From the solution to (6), it follows that for $\gamma < 1$, $t^o(\alpha, \gamma)$, $t_c^o(\alpha, \gamma)$ and $t_q^o(\alpha, \gamma)$ are continuous in α by the same argument as in the case without contracting on costs (to see this, just replace α by $\alpha/(1-\gamma)$ in the optimality conditions). For $\gamma = 1$, $t^o(\alpha, \gamma)$ and $t_q^o(\alpha, \gamma)$ are continuously increasing in α.

From the solution to (6), we see that $t_q = 0$ for $0 \leq \frac{\alpha}{1-\gamma} \leq S'(\underline{t})$, $t_c^o(\alpha, \gamma) = 0$ for $\frac{\alpha}{1-\gamma} \geq S'(0)$ and that $t^o(\alpha, \gamma)$ is independent of γ for $\frac{\alpha}{1-\gamma} > S'(\underline{t})$. Since $S(t_c)$ is increasing and strictly concave, there exists an inverse function to $\frac{\alpha}{1-\gamma} = S'(t_c)$ for $0 < t_c < \underline{t}$. As $S(t_c)$ is also twice continuously differentiable, the inverse function $t_c^o(\alpha, \gamma)$ is continuously decreasing in $\frac{\alpha}{1-\gamma}$ for $S'(\underline{t}) < \frac{\alpha}{1-\gamma} < S'(0)$ and hence continuously decreasing in γ for fixed α. It follows that $t_q^o(\alpha, \gamma) = t^o(\alpha, \gamma) - t_c^o(\alpha, \gamma)$ is continuously increasing in γ for $S'(\underline{t}) < \frac{\alpha}{1-\gamma} < S'(0)$ and independent of γ for $0 \leq \frac{\alpha}{1-\gamma} \leq S'(\underline{t})$ and $\frac{\alpha}{1-\gamma} \geq S'(0)$.

7.2 The public agency's maximization problem

The public agency's maximization problem without contracting on costs is

$$\begin{array}{ll} \max\limits_{\alpha,\beta} & E\left[B\left(t_q^o\left(\alpha\right),\psi\right)-w\left(q\right)\right] \\ \text{s.t.} & \alpha\leq\overline{\alpha}, \\ & CE\geq 0. \end{array}$$

Since the public agency is risk neutral, we get

$$\begin{array}{ll} \max\limits_{\alpha,\beta} & B\left(t_q^o\left(\alpha\right),\psi\right)-\left(\alpha t_q+\beta\right) \\ \text{s.t.} & \alpha\leq\overline{\alpha}, \\ & CE\geq 0. \end{array} \tag{11}$$

The fixed wage β is set just to let the participation constraint bind, i.e., so that $CE=0$. Reformulating the CE, we get

$$\beta=C\left(t^o\left(\alpha\right)\right)+\frac{1}{2}r\sigma_q^2\alpha^2-S\left(t_c^o\left(\alpha\right)\right)-\alpha t_q.$$

Substituting β into (11), we get

$$\begin{array}{ll} \max\limits_{\alpha} & B\left(t_q^o\left(\alpha\right),\psi\right)-\left(\alpha t_q+C\left(t^o\left(\alpha\right)\right)+\frac{1}{2}r\sigma_\varepsilon^2\alpha^2-S\left(t_c^o\left(\alpha\right)\right)-\alpha t_q\right) \\ \text{s.t.} & \alpha\leq\overline{\alpha}. \end{array}$$

This maximimization problem can be reformulated as

$$\begin{array}{ll} \max\limits_{\alpha} & B\left(t_q^o\left(\alpha\right),\psi\right)-C\left(t^o\left(\alpha\right)\right)+S\left(t_c^o\left(\alpha\right)\right)-\frac{1}{2}r\sigma_q^2\alpha^2 \\ \text{s.t.} & \alpha\leq\overline{\alpha}. \end{array}$$

7.2.1 First and second derivatives

Without contracting on cost The first derivative of the public agency's maximization problem with respect to α is

$$\begin{aligned} \frac{\partial W\left(\alpha\right)}{\partial\alpha} = {} & B_{t_q}\left(t_q^o\left(\alpha\right),\psi\right)\left[\frac{1}{C''\left(t\left(\alpha\right)\right)}\right]-B_{t_q}\left(t_q^o\left(\alpha\right),\psi\right)\left[\frac{1}{S''\left(t_c\left(\alpha\right)\right)}\right]-C'\left(t\left(\alpha\right)\right)\left[\frac{1}{C''\left(t\left(\alpha\right)\right)}\right] \\ & +S\left(t_c\left(\alpha\right)\right)\left[\frac{1}{S''\left(t_c\left(\alpha\right)\right)}\right]-r\sigma_q^2\alpha. \end{aligned}$$

The second derivative with respect to α becomes

$$\frac{\partial^2 W(\alpha)}{\partial \alpha^2} = \underbrace{B_{t_q,t_q}(t_q,\psi)\left[[1/C''(t) - 1/S''(t_c)]^2\right]}_{-} + \underbrace{\left[B_{t_q}(t_q,\psi) - S'(t_c)\right][S''(t_c)]^{-3} S'''(t_c)}_{\pm}$$
$$+\underbrace{\left[C'(t) - B_{t_q}(t_q,\psi)\right][C''(t)]^{-3} C'''(t)}_{\pm} + \underbrace{[1/S''(t_c)] - [1/C''(t)]}_{-} - \underbrace{r\sigma_q^2}_{+}$$

which can be either larger or smaller than zero. The second derivative is negative for certain if $B_{t_q}(t_q,\psi) = S'(t_c) = C'(t)$ (which is the case if $r\sigma_q^2 = 0$) or if $C'''(t) \geq 0$ and $S'''(t_c) \geq 0$.

With contracting on cost The first derivative of the public agency's maximization problem with respect to α is

$$\frac{\partial W(\alpha,\gamma)}{\partial \alpha} = B_{t_q}\left(t_q^o(\alpha,\gamma)\right)\left[\frac{1}{C''(t)}\right] - B_{t_q}\left(t_q^o(\alpha,\gamma)\right)\left[\frac{1}{(1-\gamma)S''(t_c)}\right] - C'(t^o(\alpha))\left[\frac{1}{C''(t)}\right]$$
$$+S'(t_c^o(\alpha,\gamma))\left[\frac{1}{(1-\gamma)S''(t_c)}\right] - r\sigma_q^2\alpha$$

which is not defined for $\gamma = 1$. The second derivative with respect to α is

$$\frac{\partial^2 W(\alpha,\gamma)}{\partial \alpha^2} = \underbrace{\left[B_{t_q}(t_q,\psi) - S'(t_c)\right][(1-\gamma)S''(t_c(\alpha))]^{-3}(1-\gamma)S'''(t_c)}_{\pm}$$
$$+\underbrace{\left[1/\left[(1-\gamma)^2 S''(t_c(\alpha))\right]\right] - [1/C''(t(\alpha))]}_{-}$$
$$\underbrace{B_{t_q,t_q}(t_q,\psi)\left[[1/C''(t(\alpha)) - 1/[(1-\gamma)S''(t_c(\alpha))]]^2\right]}_{-}$$
$$+\underbrace{\left[C'(t) - B_{t_q}(t_q,\psi)\right][C''(t(\alpha))]^{-3} C'''(t)}_{\pm} - \underbrace{r\sigma_q^2}_{+}$$

which is negative for certain if $B_{t_q}(t_q,\psi) = S'(t_c) = C'(t)$ or if $C'''(t) \geq 0$ and $S'''(t_c) \geq 0$.

The first derivative of the public agency's maximization problem with respect to γ is

$$\frac{\partial W(\alpha,\gamma)}{\partial \gamma} = -B_{t_q}(t_q,\psi)\left[\frac{[S'(t_c)]^2}{\alpha S''(t_c)}\right] + S'(t_c)\left[\frac{[S'(t_c)]^2}{\alpha S''(t_c)}\right] - \gamma r\sigma_v^2.$$

This derivative is positive for $\gamma = 0$ whenever $B_{t_q}(t_q, \psi) > S'(t_c)$.

For the public agency to set $\gamma > 0$ it must either be the case that the cost of risk is positive or that strong incentives for quality are infeasible. To see this, suppose that we restict γ to zero. The public agency's problem is then exactly the same as without contracting on cost. Further, suppose that quality is so important that the solution to the public agency's problem is such that $t_q > 0$ and that the incentive feasibility constraint $\overline{\alpha}$ does not bind. Then if $r\sigma_q^2 > 0$, we see from (7) that $\alpha = S'(t_c) < B_{t_q}(t_q, \psi)$ and so $\gamma = 0$ cannot be optimal. Similarly, γ will be strictly above zero if strong incentives for quality provision are not feasible, i.e., if $B_{t_q}(t_q(\overline{\alpha}, 0), \psi) > S'(t_c(\overline{\alpha}, 0))$, but quality is so important that $t_q(\overline{\alpha}, 0)$ is preferred over $t_q = 0$.

Before we consider the second derivative of the public agency's maximization problem with respect to γ, note that the derivative of

$$\left[\frac{[S'(t_c)]^2}{\alpha S''(t_c)}\right]$$

with respect to γ can be written as

$$\begin{aligned}
& \frac{1}{\alpha}\left[2[S'(t_c)][S''(t_c)]^{-1} S''(t_c)\frac{[S'(t_c)]^2}{\alpha S''(t_c)} + [S'(t_c)]^2(-1)[S''(t_c)]^{-2} S'''(t_c)\frac{[S'(t_c)]^2}{\alpha S''(t_c)}\right] \\
= & \frac{1}{\alpha^2}\left[2[S'(t_c)]^3[S''(t_c)]^{-2} S''(t_c) - [S'(t_c)]^4[S''(t_c)]^{-3} S'''(t_c)\right] \\
= & \frac{[S'(t_c)]^3}{\alpha^2 S''(t_c)}\left[2 - \frac{S'(t_c) S'''(t_c)}{[S''(t_c)]^2}\right] \gtrless 0.
\end{aligned}$$

Using this, we can write the second derivative of the owner's maximization problem with respect to γ as

$$\begin{aligned}
\frac{\partial^2 W(\alpha, \gamma)}{\partial \gamma^2} = & \underbrace{[S'(t_c) - B_{t_q}(t_q, \psi)]\left[\frac{[S'(t_c)]^3}{\alpha^2 S''(t_c)}\left[2 - \frac{S'(t_c) S'''(t_c)}{[S''(t_c)]^2}\right]\right]}_{\pm} \\
& \underbrace{+ [S''(t_c) + B_{t_q,t_q}(t_q, \psi)]\left[\frac{[S'(t_c)]^2}{\alpha S''(t_c)}\right]^2}_{-} - \underbrace{r\sigma_v^2}_{+}.
\end{aligned}$$

Note that it can never be optimal to set (α, γ) such that $S'(t_c) > B_{t_q}(t_q, \psi)$, since the public agency then can achieve both a better allocation of the agents effort and reduce the cost of risk by reducing α or γ. Since $B_{t_q}(t_q, \psi) \geq S'(t_c)$, a sufficient condition for the second derivative to be negative is that

$$\frac{S'(t_c)\, S'''(t_c)}{[S''(t_c)]^2} \geq 2.$$

Finally, the cross-partial derivative is

$$\frac{\partial^2 W(\alpha,\gamma)}{\partial\alpha\partial\gamma} = \frac{\partial^2 W(\alpha,\gamma)}{\partial\gamma\partial\alpha} = \underbrace{\left[B_{t_q}(t_q) - S'(t_c)\right] \left[\frac{[S'(t_c)]^2 \frac{S'''(t_c)}{S''(t_c)} - S'(t_c)\, S''(t_c)}{\alpha(1-\gamma)\,[S''(t_c)]^2}\right]}_{\pm}$$
$$+ \underbrace{\left[\frac{B_{t_q t_q}(t_q) + S''(t_c)}{S''(t_c)}\right] \frac{[S'(t_c)]^2}{\alpha(1-\gamma)\, S''(t_c)}}_{-} - \underbrace{\frac{B_{t_q t_q}(t_q)}{C''(t)} \frac{[S'(t_c)]^2}{\alpha S''(t_c)}}_{+}.$$

Since the cross-partial is not zero, it is a nontrivial exercise to derive conditions under which the Hessian matrix is negative definite. As it is not important for any of the results presented in the paper, I abstain from deriving these conditions here.

7.3 Proofs

7.3.1 Proof of Proposition 1

From (1), we know that the maximum level of quality under public provision is $t_q^m = t^*$. Since $\bar{t} > t^*$, there exists some $S'(\underline{t}) < \widehat{\alpha} < \infty$ by continuity of $t_q^o(\alpha)$ such that $t_q^o(\widehat{\alpha}) = t^*$, $t_q^o(\alpha) < t^*$ for all $\alpha < \widehat{\alpha}$ and $t_q^o(\alpha) > t^*$ for all $\alpha > \widehat{\alpha}$.

7.3.2 Proof of Proposition 2

Consider the function $t_q^o(\alpha)$ from the owner's effort choice above. As $t_q^o(\alpha)$ is continuously increasing in α for all $\alpha > S'(\underline{t})$, it has an inverse function $\alpha\left(t_q^o\right)$ for $t_q^o > 0$ which is continuously increasing in t_q^o. From the solution to (4) we also know that $\alpha(0) = 0$. Total costs for a given level of quality under private provision is then given by

$$K_o(\alpha(t_q)) = C(t^o(\alpha(t_q))) - S(t_c^o(\alpha(t_q))) + \frac{1}{2}\left[r\sigma_q^2(\alpha(t_q))^2\right]$$

for $0 \leq t_q \leq \bar{t}$. As $\alpha\left(t_q^o\right)$, $C(t)$ and $S(t_c)$ are continuous functions for $t_q > 0$, $K_o(\alpha(t_q))$ is a continuous function of t_q for $t_q > 0$.

Total costs under public provision is given by

$$K_m(t_q) = C(t^*) - S(t^* - t_q)$$

for $0 \leq t_q \leq t_q^*$, which is a continuous function by continuity of $S(t_c)$. First, consider the cost for no effort on quality. In case of private provision, we get

$$\begin{aligned} K_o(0) &= C(t^o(0)) - S(t_c^o(0)) \\ &= C(\underline{t}) - S(\underline{t}) \end{aligned}$$

which is the amount of effort that minimizes K. Since t^* is such that $C(t^*) = 0 < S(t^*)$, it follows that

$$K_o(0) < K_m(0).$$

Second, consider the cost for $t_q > 0$. In case of private provision, we get

$$K_o\left(\alpha\left(t_q^o\right)\right) = C\left(t^o\left(\alpha\left(t_q^o\right)\right)\right) - S\left(t_c^o\left(\alpha\left(t_q^o\right)\right)\right) + \frac{1}{2}\left[r\sigma_q^2(\alpha(t_q))^2\right].$$

Since $C(t^o(\alpha(t_q))) - S(t_c^o(\alpha(t_q)))$ is a finite number and $\alpha\left(t_q^o\right) > 0$ for $t_q^o > 0$, it follows that $\lim_{r\sigma_q^2 \to \infty} K_o\left(\alpha\left(t_q^o\right)\right) = \infty$ for all $t_q^o > 0$. In contrast, total costs under public provision is given by

$$K_m(t_q) = C(t^*) - S(t^* - t_q)$$

which is a finite number for all $t_q \leq t_q^*$. By continuity of $K_o(\alpha(t_q))$ and $K_m(t_q)$ for $t_q > 0$, there exists a finite value $\widehat{r\sigma}_q^2$ for all $0 < t_q \leq t_q^*$ such that, $K(t_q)_o > K(t_q)_m$ whenever $r\sigma_q^2 > \widehat{r\sigma}_q^2$.

Third, consider the marginal cost of quality under private provision for $t_q > 0$,

$$\frac{\partial K(\alpha(t_q))}{\partial t_q} = \frac{\partial K}{\partial \alpha}\frac{\partial \alpha}{\partial t_q} = \left(\frac{C'(t^o(\alpha(t_q)))}{C''(t(\alpha(t_q)))} - \frac{S'(t_c^o(\alpha(t_q)))}{S''(t_c(\alpha(t_q)))} + \alpha r\sigma_\varepsilon^2\right)\frac{\partial \alpha}{\partial t_q}.$$

Since

$$\frac{C'\left(t^{o}\left(\alpha\left(t_{q}\right)\right)\right)}{C''\left(t\left(\alpha\left(t_{q}\right)\right)\right)}-\frac{S'\left(t_{c}^{o}\left(\alpha\left(t_{q}\right)\right)\right)}{S''\left(t_{c}\left(\alpha\left(t_{q}\right)\right)\right)}$$

is positive, and

$$\frac{\partial\alpha}{\partial t_{q}}>0,$$

it follows that

$$\lim_{r\sigma_{q}^{2}\rightarrow\infty}\frac{\partial K_{o}\left(\alpha\left(t_{q}\right)\right)}{\partial t_{q}}=\infty.$$

Since

$$\frac{\partial K_{m}\left(t_{q}\right)}{\partial t_{q}}=S'\left(t^{*}-t_{q}\right)$$

is finite for all $t_q \leq t^*$, for $t_q > 0$ there exists a finite value $\widetilde{r\sigma}_{q}^{2}$ such that

$$\frac{\partial K_{o}\left(\alpha\left(t_{q}\right)\right)}{\partial t_{q}}>\frac{\partial K_{m}\left(t_{q}\right)}{\partial t_{q}}$$

if $r\sigma_q^2 > \widetilde{r\sigma}_{q}^{2}$.

It follows that there exists some $r\sigma_q^2 > \max\left\{\widehat{r\sigma}_{q}^{2},\widetilde{r\sigma}_{q}^{2}\right\}$ such that private provision is cheaper for zero effort on quality but more expensive for effort on quality above some finite threshold $t_q < t_q^*$. As marginal cost is higher under private provision for $t_q > 0$, costs can be the same for at most one level of effort on quality.

7.3.3 Proof of Proposition 3

I first show that public provision can be superior. As $K_m\left(t_q^m\right) - K_o\left(0\right)$ is a finite number and $B\left(t_q,\psi\right)$ is continuous in ψ with $\lim_{\psi\rightarrow\infty} B\left(t_q,\psi\right)=\infty$, there always exists some finite $\widetilde{\psi}$ such that $B\left(t_q^m,\psi\right) - K_m\left(t_q^m\right) > B\left(0,\psi\right) - K_o\left(0\right)$ for all $\psi > \widetilde{\psi}$. That is, if quality is sufficiently important, the optimal level of quality from public provision is superior to zero quality from a private firm. We know from Proposition 2 that $\lim_{r\sigma_q^2\rightarrow\infty} K_o\left(t_q\right)=\infty$ for all $t_q^o > 0$. It follows that the surplus under private provision approaches $B\left(0,\psi\right) - K_o\left(0\right)$ from above when $r\sigma_q^2$ goes to infinity. Hence, for all $\psi > \widetilde{\psi}$, there also exists a finite cost of risk $\widehat{r\sigma}_{q}^{2}\left(\psi\right)$ such that public provision is preferred to private provision for $r\sigma_q^2 > \widehat{r\sigma}_{q}^{2}\left(\psi\right)$.

Second, consider the case when quality is unimportant ($\psi = 0$). Since $B_{t_q}\left(t_q,0\right)=0$ for all t_q we get from the solutions to (2) and (4) that the optimal level of t_q is equal to zero both under private and public provision. From Proposition 2, we know that

$K_o(0) < K_m(0)$, and so private provision is superior.

Before we consider the case when quality is very important, we establish that there exists some finite ψ such that the public agency will set α so that $t_q^o(\alpha) > t^* \geq t_q^m$. As $\lim_{\psi\to\infty} B(t_q,\psi) = \infty$ and $K_o\left(t_q^o\right)$ is a finite number for any $t_q < \bar{t}$, there exists for all $0 < t_q < \bar{t}$ some finite value of ψ such that $B(t_q,\psi) - K_o(t_q) > B(0,\psi) - K_o(0)$. We can therefore rule out the corner solution $\alpha = 0$. Since $\lim_{\psi\to\infty} B_{t_q}(t_q,\psi) = \infty$, we get from (5) that $\lim_{\psi\to\infty} \alpha^* = \infty$. As $\lim_{\alpha\to\infty} t_q^o(\alpha) = \bar{t}$, we have that $\lim_{\psi\to\infty} t_q^o = \bar{t}$. To see that private provision is superior when quality is very important, note that since $K_o\left(t_q^o\right) - K_m\left(t_q^m\right)$ is a finite number, $\lim_{\psi\to\infty} B_{t_q}(t_q,\psi) = \infty$ for all $t_q > 0$ and $\lim_{\psi\to\infty} t_q^o = \bar{t} > t^* \geq t_q^m$, there always exists a $\widehat{\psi}\left(r\sigma_q^2\right)$ such that $B\left(t_q^o,\psi\right) - K_o\left(t_q^o\right) > B\left(t_q^m,\psi\right) - K_m\left(t_q^m\right)$ for all $\psi > \widehat{\psi}$.

Now, suppose that $r\sigma_q^2 > \widehat{r\sigma}_q^2(\psi)$ for some $\psi > \widetilde{\psi}$. We know from above that private provision is superior if $\psi = 0$ or $\psi > \widehat{\psi}\left(r\sigma_q^2\right)$ and that there exists some ψ above $\widetilde{\psi}$ but below $\widehat{\psi}\left(r\sigma_q^2\right)$ such that public provision is superior.

7.3.4 Proof of Proposition 4

We know from Proposition 1 that there exists an $\widehat{\alpha}$ such that $t_q^o(\overline{\alpha}) < t_q^*$ if $\overline{\alpha} < \widehat{\alpha}$. Since $\lim_{\psi\to\infty} B_{t_q}(t_q,\psi) = \infty$ and $t_q^m = t^*$ if $B_{t_q}(t_q,\psi) \geq S'(0)$, there exists a finite $\widetilde{\psi}$ such that $t_q^m > t_q^o$ if $\psi > \widetilde{\psi}$ and $\overline{\alpha} < \widehat{\alpha}$. Since $K_o\left(t_q^o\right) - K_m\left(t_q^m\right)$ is finite and $\lim_{\psi\to\infty} B_{t_q}(t_q,\psi) = \infty$ there exists a $\widehat{\psi}$ such that $B\left(t_q^m,\psi\right) - B\left(t_q^o,\psi\right) > K_m\left(\overline{t_q}\right) - K_o\left(\widetilde{t_q}\right)$ for all $\psi > \widehat{\psi}$.

7.3.5 Proof of Proposition 5

For any feasible level of effort on quality, there is some nonempty set of parameter values (α,γ) that induces the private firm to put down this particular level of effort (the particular parameter values that minimize costs for a certain level of quality will vary depending on $r\sigma_q^2$ and $r\sigma_v^2$). Let $\alpha(t_q)$ and $\gamma(t_q)$ denote the values of the incentive parameters that minimize total cost to the public agency for a certain level of t_q. Then, total cost for a given level of quality is

$$K(t_q) = C\left(t^o\left(\alpha(t_q),\gamma(t_q)\right)\right) - S\left(t_c^o\left(\alpha(t_q),\gamma(t_q)\right)\right) + \frac{1}{2}r\left(\sigma_q^2\left(\alpha(t_q)\right)^2 + \sigma_v^2\left(\gamma(t_q)\right)^2\right).$$

Note that $(\alpha, \gamma) = (0, 0)$ is the only optimal solution for $t_q = 0$. We therefore get the same result as in Proposition 2 regarding cost for zero effort on quality, i.e., $K_o(0, 0) = K(0)_o < K(0)_m$.

Second, for any $t_q > 0$, we must have that either α and γ or both are strictly larger than zero. Since $C(t^o(\alpha)) - S(t_c^o(\alpha, \gamma))$ is finite, costs approach infinity for the whole set of parameter values that give a certain t_q (including the set that minimize costs) when both $r\sigma_q^2$ and $r\sigma_v^2$ approach infinity. Hence, we have that $\lim_{r\sigma_q^2 \to \infty; r\sigma_v^2 \to \infty} K_o(t_q) = \infty$ for all feasible $t_q > 0$. Since costs are continuously increasing in $r\sigma_q^2$ and $r\sigma_v^2$ for all combinations of α and γ that give a certain level of quality, $K_o(t_q)$ is a continuous function of $r\sigma_q^2$ and $r\sigma_v^2$ and there always exists a pair of finite values of $r\sigma_q^2$ and $r\sigma_v^2$ such that $K_o(t_q) > K_m(t_q)$ for any $t_q \in (0, t^*]$.

7.3.6 Proof of Proposition 6

Since $D'(t_u) > 0$ for all $t_u > 0$, $t_u = 0$ and $t_q + t_c = t^*$ solves the manager's problem (10). In case of private provision (9), the first-order condition with respect to t_u is $\omega - D'(t_u) - C'(t)$. Since $C'(t) = \max\{\alpha, S'(\underline{t})\}$ the owner will set $t_u > 0$ if and only if $\omega > D'(0) + \max\{\alpha, S'(\underline{t})\}$.

7.4 Numerical example

Assume the functional forms

$$
\begin{aligned}
C(t) &= (t-2)^8, \\
S(t_c) &= 5 \log(t_c + 1),
\end{aligned}
$$

which give

$$
\begin{aligned}
C'(t) &= 8(t-2)^7, \\
t^* &= 2, \\
S'(t_c) &= \frac{5}{t_c + 1}.
\end{aligned}
$$

Also, set

$$
r\sigma_q^2 = 0.45.
$$

In the corner solution where $t_q = 0$, we get

$$\begin{aligned} S'(\underline{t}) &= C'(\underline{t}), \\ \frac{5}{\underline{t}+1} &= 8(\underline{t}-2)^7, \\ \underline{t} &= 2.7735. \end{aligned}$$

and so the highest level of α for which $t_q^o(\alpha) = 0$ is

$$\alpha = 8(2.7735 - 2)^7 = 1.3253.$$

In the interior solution, we get

$$\begin{aligned} \alpha &= S'(t_c) = C'(t), \\ \alpha &= \frac{5}{t_c+1} = 8(t-2)^7, \\ t^o(\alpha) &= \sqrt[7]{\frac{\alpha}{8}} + 2, \\ t_c^o(\alpha) &= \frac{5}{\alpha} - 1, \\ t_q^o(\alpha) &= \sqrt[7]{\frac{\alpha}{8}} + 3 - \frac{5}{\alpha}. \end{aligned}$$

For

$$\alpha \geq \max S'(t_c) = 5$$

we have $t_c = 0$, but this point is never reached in the cases I simulated.