Optimal Social Insurance with Endogenous Health*

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Abstract

This paper analyzes optimal insurance against unemployment and disability in a private information economy with endogenous health and search effort. Individuals can reduce the probability of becoming disabled by exerting, so-called, prevention effort, which is costly in terms of utility. A healthy, i.e., not disabled, individual either works or is unemployed. An unemployed individual can exert search effort in order to increase the probability of finding a new job. I show that the optimal sequence of consumption is increasing for a working individual and constant for a disabled individual. During unemployment, decreasing benefits are not necessarily optimal in this setting. The prevention constraint implies increasing benefits over time while the search constraint demands decreasing benefits while being unemployed. However, if individuals respond sufficiently much to search incentives, the latter effect dominates the former and the optimal consumption sequence is decreasing during unemployment.

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1 Introduction

The question of how a government should insure workers against unemployment and disability is a recurring and controversial theme in the public policy debate. In the last twenty years many developed countries have reformed their unemployment insurance programs in order to decrease costs for the government and make people return to employment. These changes have often included a decrease in benefits if the worker is unemployed longer than a certain amount of time. While these reforms obviously increase the incentives of unemployed workers to find new employment, they might also create situations in which the long-term unemployed would rather exit the labor force, and go on disability insurance, than continue trying to find new employment. Empirical studies, e.g. Autor and Duggan (2003), Larsson (2006) and Karlström, Palme, and Svensson (2008), have shown that substitution between social insurance programs is indeed a common phenomenon. In order to receive disability insurance benefits, an individual either has to falsely claim to be unable to work or actually become unemployable. The former is a well known problem and taken into account when designing unemployment and disability insurance systems. This issue is also addressed in this paper. The idea that strict unemployment insurance systems can create incentives for individuals to actually become disabled has, to my knowledge, not yet been formally investigated.

To study this issue, I combine unemployment and disability insurance in one framework and assume that the probability of becoming disabled is endogenous. Combining moral hazard and adverse selection, enables me to study the optimal design of unemployment insurance when individuals have the option to leave the labor force. In this framework I am also able to analyze how unemployment and disability benefits should depend on the employment history. In this sense, combining unemployment and disability insurance yields more than just the sum of its parts. For example, employment-history dependent disability benefits allow the planner to use disability benefits not only to provide incentives for preventing disability but also as an additional way of creating search incentives for the unemployed individual.

I assume that individuals, whether employed or unemployed, can exert so-called prevention effort. In so doing, they increase the probability of remaining healthy, and hence staying in the labor force, for one more period. Furthermore, an unemployed individual has the option of exercising search effort, which increases the probability of finding employment in the next period. Both the prevention and the search effort are costly in terms of utility. Disability is an absorbing state. The planner's goal is to minimize the expected discounted cost of providing the individual with a certain level of utility. I first consider the full information case in which the planner can observe effort levels and the individual's state of health and employment. I then relax those assumptions and assume that the planner can only observe whether an individual works or not, i.e., the planner cannot distinguish between unemployment and disability, and he cannot observe effort levels. I show that the optimal sequence of consumption and promised utilities of an employed worker is increasing with tenure. Once a worker is disabled, she will receive constant benefits and her utility will remain constant. During unemployment, decreasing benefits are not necessarily optimal anymore. The prevention constraint implies increasing benefits and promised utilities during unemployment while the search constraint has the opposite effect. However, if individuals respond sufficiently much to search incentives, the latter effect dominates the former and the optimal consumption sequence is decreasing during unemployment.

This result has important policy implications. While it is commonly accepted that decreasing unemployment benefits create incentives for individuals to find employment, I show in this paper that this is only optimal if individuals indeed react sufficiently much to those incentives and exert enough search effort. If this is not the case, making unemployment less attractive over time may not be optimal because it can create incentives for individuals to leave the labor force.

The literature on optimal unemployment insurance started with Shavell and Weiss (1979). They show that if individuals have no wealth, cannot borrow and can influence the probability of finding a job, monotonically decreasing benefits throughout unemployment are optimal.

Hopenhayn and Nicolini (1997) extend the model of Shavell and Weiss (1979) and apply the recursive solution methods for repeated games and dynamic principal-agent problems.[1] They show that unemployment benefits are monotonically decreasing with the length of unemployment. Furthermore, after the individual finds employment there is a wage tax which depends on the unemployment history. A tax which is increasing with the length of previous unemployment is optimal, because this reduces claims to consumption in all future states (employment and unemployment), and hence, gives a stronger incentive to search for a job. In numerical simulations the authors also show that, for certain parameter values, the wage tax is negative, i.e., a subsidy, for individuals who were unemployed for five weeks or less.

In a setting in which an individual must exert effort not only to find a job but also to remain employed, Hopenhayn and Nicolini (2009) show that monotonically decreasing unemployment benefits are optimal. In other words, the presence of moral hazard while working does not alter this basic finding. The authors assume that work causes a certain level of disutility and allow for the possibility of quits which are indistinguishable from

[1] For more on these solution methods, see for example Spear and Srivastava (1987), Thomas and Worrall (1990), Abreu, Pearce, and Stacchetti (1990), Atkeson and Lucas (1992) and Chang (1998).

lay-offs. For a high enough level of disutility, unemployed individuals who find a new job prefer to quit after one period of employment and return to unemployment with increased benefits. In order to prevent quits, the promised utilities have to increase with the duration of employment. This increasing utility profile is achieved by decreasing the wage tax (i.e., increasing consumption) with the length of employment.

Fredriksson and Holmlund (2001) analyze a model with two states of unemployment: insured and uninsured. Here, decreasing benefits are defined as a drop in consumption when going from insured to uninsured unemployment. The optimality of this drop cannot be established analytically. Higher benefits for insured individuals increase the search incentives of the uninsured because finding employment entitles the individual to those higher benefits in the future. However, more generous benefits in the first stage of unemployment reduce the search incentives for insured individuals by making employment comparatively less attractive. The authors numerically show that this wage pressure effect is dominated by the entitlement effect and that decreasing unemployment benefits are therefore optimal.

For a more thorough review on the literature on optimal unemployment insurance see Fredriksson and Holmlund (2006).[2] An issue not addressed in the literature is the possibility for workers to leave the labor force. If individuals only have a choice between being unemployed or working, providing them with incentives to search for a job is easier than when they also have the option to go into other social insurance systems, e.g. disability insurance. Given the importance of disability insurance in many countries, it is, therefore, crucial to take disability insurance into account when designing optimal unemployment insurance.

Optimal disability insurance was first investigated by Diamond and Mirrlees (1978). An example of the more recent literature on disability insurance is Golosov and Tsyvinski (2006). In those models, individuals either have full or no work capacity and disability arises exogenously. Furthermore, the government cannot distinguish between those who cannot work and those who choose not to work. Diamond and Mirrlees (1978) show that, in the optimum, individuals are indifferent between working and not working and that a tax on savings should be part of the optimal social insurance policy. Disability benefits are higher the longer the individual previously worked. Golosov and Tsyvinski (2006) characterize a second-best optimum in which an able individual's consumption is increasing in the duration of her work history. Once disability occurs, the individual's consumption drops and remains constant after that.

A common assumption in the literature on optimal disability insurance is that indi-

[2] A few examples include Baily (1978), Flemming (1978), Wang and Williamson (1996), Acemoglu and Shimer (1999), Boone, Fredriksson, Holmlund, and van Ours (2007) Pavoni and Violante (2007), Pavoni (2007), Hagedorn, Kaul, and Mennel (2010).

viduals can only be in two possible states: work or retirement. Introducing a third state, namely unemployment, implies that the age at which one stops working and the retirement age are not necessarily equivalent. The question, whether benefits are still increasing in the retirement age or only increasing in the time spent working, thus arises. Moreover, it is worth exploring how unemployment affects the level of disability benefits.

The work closest related to mine is Höglin (2008). He was the first to look at unemployment and disability insurance simultaneously. In the second chapter of his thesis he combines the models of Hopenhayn and Nicolini (1997) and Diamond and Mirrlees (1978). In his framework, there is an exogenous probability that employed individuals can become unemployed or disabled. The probability of an unemployed individual finding a job depends on her search effort. The probability of becoming disabled is exogenous and the same as for employed individuals. Disability is an absorbing state. The author shows that in the optimum employed and disabled workers have constant consumption, while unemployed individuals face decreasing benefits over time. These findings resemble the results of Hopenhayn and Nicolini (1997). Another result is that the length of employment is irrelevant for the level of benefits the individual receives when she becomes unemployed or disabled. This stands in stark contrast to the results of Diamond and Mirrlees (1978) who find that benefits are increasing in the length of employment. The decreasing unemployment benefits and the irrelevance of employment history both stem from the assumption that disability is an exogenous state. The individual does not need to be provided with an incentive to work because job loss and disability are exogenous. While unemployed, benefits can be decreasing in order to provide search incentives without running the risk of creating incentives to leave the labor force.

The key distinction between this paper and Höglin (2008) is that here the probability of becoming disabled is endogenous. This implies that when designing optimal unemployment insurance one faces a trade-off between providing incentives to search for a job while at the same time keeping the individual in the labor force. This additional constraint makes characterizing the solution more difficult but it is necessary in order to avoid creating unintended incentives.

The structure of the paper is as follows. In section 2, I characterize the environment of the model. Section 3 analyzes the autarky case, where there is no insurance provided by the planner. In section 4, I describe the planner's problem, and in section 5, I derive the optimal insurance for the full information case and for the case of asymmetric information. Section 6 provides a numerical illustration of the model, and section 7 concludes. All proofs can be found in the appendix.

2 The Environment

In this model an individual can be in one of three possible states; healthy and employed, healthy and unemployed, or disabled. Following Diamond and Mirrlees (1978), I assume that disability is an absorbing state. Disability should, in this context, be interpreted as a state of non-employability, i.e., a permanent loss of work capacity. The prevention effort is hence an effort to remain employable. Disability itself, however, has no direct utility effects and individuals do not necessarily want to avoid it at any cost.

An employed individual earns the constant wage $w > 0$ and maximizes her utility by choosing a level of prevention effort $a_t \geq 0$. An unemployed individual earns no income and chooses jointly the optimal levels of prevention and search effort: $a_t \geq 0$ and $e_t \geq 0$. A disabled individual also has no income and, since disability is an absorbing state, does not exert any search or prevention effort.

Hopenhayn and Nicolini (2009) make the simplifying assumption of discrete effort levels, i.e., $e_t = e > 0$ or $e_t = 0$. They argue that a continuum of effort levels[3] complicates the analysis significantly without providing additional insights. That is true for their case with only one incentive problem, that is, the search-incentive problem. However, making this assumption here, in the presence of two incentive problems (search and prevention), leads to multiple equilibria with different combinations of binding and non-binding constraints.

If a worker exerts prevention effort a_t, then the probability of remaining employable in the next period is $p(a_t)$. The function $p(\cdot)$ is strictly increasing, strictly concave and twice differentiable. Prevention effort can be thought of as typical prevention measures, such as exercising, a healthy diet and regular medical checkups, as well as seeking treatment for medical problems. Jönsson, Palme, and Svensson (2012) show that circulatory diseases and musculoskeletal diseases are common reasons for awarding disability benefits in Sweden. These are arguably areas in which the individual can influence the probability of becoming sick by exerting some kind of prevention effort.

The probability of remaining on the job when employed is exogenous and given by $s > 0$. The unemployed individual will remain employable in the next period with probability $p(a_t)$ and she will find a job with probability $q(e_t)$. The function $q(\cdot)$ is also strictly increasing, strictly concave and twice differentiable.

As in Hopenhayn and Nicolini (1997), effort is costly in terms of utility. An individual's expected lifetime utility is then given by

$$E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - a_t - e_t \right],$$

[3]As it is the case in this model or in Hopenhayn and Nicolini (1997).

where $0 < \beta < 1$ is the discount factor, c_t is consumption and $u(\cdot)$ is strictly increasing, strictly concave and twice differentiable. I further assume that $u(0)$ is well defined.

Following the literature on repeated moral hazard, individuals have no access to credit markets or storage technology. The planner can hence directly control their consumption. Obviously, this is a very strong assumption and relaxing it could have important effects on the results. However, this assumption makes it possible to derive analytical results and not solely rely on simulations.

The individual's only source of income is the transfer from the planner and her wage if she is working.[4] The planner has unlimited access to a perfect capital market (with a constant gross interest rate equal to $1/\beta$) while the individual can neither lend nor borrow.

The state of an individual is private information. The planner can only observe the individual's income and from this infer whether she is employed or not. He cannot distinguish between an unemployed and a disabled individual. The effort levels are also unobservable.

3 Autarky

In autarky there is no planner to provide insurance against unemployment and disability. An employed worker consumes her wage, w, and decides on how much prevention effort to exert. The autarky value of being employed is then

$$V^e_{aut} = \max_{a \geq 0} \left\{ u(w) - a + \beta p(a) \left[s V^e_{aut} + (1-s) V^u_{aut} \right] + \beta [1 - p(a)] V^d_{aut} \right\}, \tag{1}$$

where V^u_{aut} and V^d_{aut} are the autarky values of being unemployed and disabled respectively. Recall that, $p(a)$ is the endogenous probability of remaining employable while s is the exogenous probability of remaining employed. Since there are no state variables in this problem, there is a time-invariant optimal prevention effort and an associated value of being healthy and employed.

The healthy but unemployed individual has no income in autarky, and hence, zero consumption. She has to determine the optimal levels of prevention and search effort. Her problem is given by

$$V^u_{aut} = \max_{a,e \geq 0} \left\{ u(0) - a - e + \beta p(a) \left[q(e) V^e_{aut} + [1 - q(e)] V^u_{aut} \right] + \beta [1 - p(a)] V^d_{aut} \right\}, \tag{2}$$

where $q(e)$ is the endogenous probability of finding a new job.

[4] Since the planner can control the individual's consumption with the transfers, I use the terms transfer and consumption interchangeably throughout this paper.

The disabled worker has no consumption in autarky. Since disability is an absorbing state, there are no decisions about effort levels. The value of being in this state is

$$V^d_{aut} = u(0) + \beta V^d_{aut} \quad \Leftrightarrow \quad V^d_{aut} = \frac{u(0)}{1-\beta}. \tag{3}$$

Equations (1) – (3) together determine the autarky levels of utility. These utility levels provide a lower bound for the planner. If he were to promise less than those values, the individual would not participate in the insurance system.

4 The Planner's Problem

The aim of the planner is to minimize the expected discounted cost of providing the individual with a certain level of utility. The planner does so by choosing the individual's consumption in the current period, and by promising her a certain utility level in the next period in an incentive compatible way. As in Spear and Srivastava (1987), the problem can be defined recursively with the individual's promised utility acting as a state variable which summarizes the employment history.

4.1 Employed Worker

An employed worker faces the problem of determining the optimal prevention effort. She receives the transfer $c^e - w$ from the planner, and hence, has a consumption of c^e. Furthermore, she is promised a utility of $V^{e,e}$ if she remains employed in the next period. If she becomes unemployed, she is promised a utility of $V^{e,u}$. If she becomes disabled, her promised utility is $V^{e,d}$. Given these continuation values and consumption, the employed worker solves the following problem

$$\max_{a^e \geq 0} f^e(a^e) = u(c^e) - a^e + \beta p(a^e)\left[sV^{e,e} + (1-s)V^{e,u}\right] + \beta\left[1 - p(a^e)\right]V^{e,d}.$$

The first-order condition is given by

$$f^e_a = \beta p'(a^e)\left[sV^{e,e} + (1-s)V^{e,u} - V^{e,d}\right] - 1 \leq 0, \tag{4}$$

with equality for $a^e > 0$. The left hand side of this inequality represents the benefit from a marginal increase in prevention effort, which consists of an increase in the probability of remaining employable multiplied by the corresponding utility benefit. The latter is the difference between the utility of being healthy, that is, the probability weighted average of the utilities in case of employment and unemployment, and the utility of being disabled.

The optimal prevention effort, therefore, depends on the utility levels promised by the planner.

The second-order condition is fulfilled since $p(\cdot)$ is strictly concave and since the utility of being healthy is larger, in equilibrium, than the utility of being disabled

$$f_{aa}^{e} = \beta p''(a^{e}) \left[sV^{e,e} + (1-s)V^{e,u} - V^{e,d} \right] < 0.$$

Since this is a repeated game, the individual working in the current period was promised a certain utility level in the previous period. This utility promise has to be kept by providing the individual with a consumption level c^{e}, and in turn, promising utility levels for the next period. If the planner previously promised to provide the employed worker with a utility level of V^{e}, the promise-keeping constraint is given by[5]

$$V^{e} = u(c^{e}) - a^{e} + \beta p(a^{e}) \left[sV^{e,e} + (1-s)V^{e,u} \right] + \beta \left[1 - p(a^{e}) \right] V^{e,d}. \tag{5}$$

Since the planner cannot distinguish between an unemployed and a disabled individual, the unemployed worker has to be given an incentive to not falsely claim disability. This moral hazard can be prevented by making sure that an unemployed individual always enjoys a utility at least as high as that of a disabled individual. This truth-telling constraint is given by

$$V^{e,d} \leq V^{e,u}. \tag{6}$$

The problem of the planner is now to minimize the cost of providing consumption and continuation values to the employed worker such that the prevention-incentive constraint (4), the promise-keeping constraint (5) and the truth-telling constraint (6) are fulfilled.

$$C_{e}(V^{e}) = \min_{c^{e}, a^{e}, V^{e,e}, V^{e,u}, V^{e,d}} \Big\{ c^{e} - w + \beta p(a^{e}) \left[sC_{e}(V^{e,e}) + (1-s)C_{u}(V^{e,u}) \right] + \beta \left[1 - p(a^{e}) \right] C_{d}(V^{e,d}) \Big\}$$

subject to conditions (4) – (6). $C_{e}(\cdot)$, $C_{u}(\cdot)$ and $C_{d}(\cdot)$ are the minimized costs of providing utility to an employed, an unemployed and a disabled individual, respectively.

Assigning the Lagrange parameters γ^{e} to the promise-keeping constraint (5), δ^{e} to the prevention-incentive constraint (4) and $\beta\eta^{e}$ to the truth-telling constraint (6), yields the

[5] It is straightforward to show that the promise-keeping constraints are always binding in equilibrium.

following first-order conditions

$$\begin{aligned}
\gamma^e =& \frac{1}{u'(c^e)}, && (7)\\
C_e'(V^{e,e}) =& \gamma^e + \delta^e \frac{p'(a^e)}{p(a^e)}, && (8)\\
C_u'(V^{e,u}) =& \gamma^e + \delta^e \frac{p'(a^e)}{p(a^e)} + \eta^e \frac{1}{p(a^e)(1-s)}, && (9)\\
C_d'(V^{e,d}) =& \gamma^e - \delta^e \frac{p'(a^e)}{1-p(a^e)} - \eta^e \frac{1}{1-p(a^e)}. && (10)
\end{aligned}$$

The Envelope theorem further implies that

$$C_e'(V^e) = \gamma^e. \tag{11}$$

4.2 Unemployed Worker

The unemployed individual receives a transfer c^u from the planner. In addition, she is promised a utility of $V^{u,e}$ if she finds employment. If she remains employable but does not find a job, she is promised a utility level of $V^{u,u}$. If she becomes disabled, her promised utility is $V^{u,d}$. The individual takes these values as given and maximizes her utility with respect to the levels of prevention and search effort.

$$\begin{aligned}
\max_{a^u, e^u \geq 0} f^u(a^u, e^u) =& u(c^u) - a^u - e^u + \beta p(a^u)\left[q(e^u)V^{u,e} + [1-q(e^u)]\, V^{u,u}\right]\\
&+ \beta\left[1 - p(a^u)\right] V^{u,d}.
\end{aligned}$$

The first-order condition with respect to prevention effort is given by

$$f_a^u = \beta p'(a^u)\left[q(e^u)V^{u,e} + [1-q(e^u)]\, V^{u,u} - V^{u,d}\right] - 1 \leq 0, \tag{12}$$

which has a similar interpretation as condition (4). The first-order condition with respect to search effort reads

$$f_e^u = \beta p(a^u) q'(e^u)\left[V^{u,e} - V^{u,u}\right] - 1 \leq 0. \tag{13}$$

Both first-order conditions hold with equality for $a^u, e^u > 0$, respectively. A marginal increase in search effort yields an increase in the probability of finding employment. Multiplying this increase by the probability of remaining employable, $p(a^u)$, and the utility difference associated with finding a job, $V^{u,e} - V^{u,u}$, yields the benefit of a marginal increase in search effort.

The sufficient conditions for the unemployed individual's problem are given by

$$f_{aa}^{u} = \beta p''(a^{u}) \left[q(e^{u})V^{u,e} + [1 - q(e^{u})]\, V^{u,u} - V^{u,d} \right] < 0,$$
$$f_{ee}^{u} = \beta p(a^{u}) q''(e^{u}) \left[V^{u,e} - V^{u,u} \right] < 0,$$
$$f_{aa}^{u} f_{ee}^{u} - [f_{ae}^{u}]^{2} \geq 0.$$

The first two conditions are fulfilled since $p(\cdot)$ and $q(\cdot)$ are strictly concave and since the expressions in the square brackets are both strictly positive in equilibrium. Substituting in the first-order conditions and rearranging yield that the last condition is fulfilled if

$$\frac{q''(e^{u})}{q'(e^{u})} \leq \frac{[p'(a^{u})]^{3}}{[p(a^{u})]^{2}\, p''(a^{u})}.$$

An individual who was unemployed in the previous period was promised a utility of V^{u} in case of continued unemployment. The planner now has to fulfill this promise by providing the individual with consumption and by promising utility levels for all three possible future states. The promise-keeping constraint can be written as

$$V^{u} = u(c^{u}) - a^{u} - e^{u} + \beta p(a^{u}) \left[q(e^{u})V^{u,e} + (1 - q(e^{u}))V^{u,u} \right] + \beta \left[1 - p(a^{u}) \right] V^{u,d}. \quad (14)$$

Since the planner cannot distinguish unemployment from disability, the individual needs to be provided with an incentive to truthfully report her state of employability. The utility from being unemployed has to be at least as high as that from disability. The truth-telling constraint is given by

$$V^{u,d} \leq V^{u,u}. \quad (15)$$

The planner now solves the problem of providing consumption and promising utilities to an unemployed individual in a cost-minimizing and incentive compatible way

$$C_{u}(V^{u}) = \min_{c^{u}, a^{u}, e^{u}, V^{u,e}, V^{u,u}, V^{u,d}} \Big\{ c^{u} + \beta p(a^{u}) \left[q(e^{u}) C_{e}(V^{u,e}) + [1 - q(e^{u})]\, C_{u}(V^{u,u}) \right] + \beta \left[1 - p(a^{u}) \right] C_{d}(V^{u,d}) \Big\}$$

subject to the conditions (12) – (15). The first-order conditions are given by

$$\begin{aligned}
\gamma^u =& \frac{1}{u'(c^u)}, && (16)\\
C_e'(V^{u,e}) =& \gamma^u + \delta^u \frac{p'(a^u)}{p(a^u)} + \mu^u \frac{q'(e^u)}{q(e^u)}, && (17)\\
C_u'(V^{u,u}) =& \gamma^u + \delta^u \frac{p'(a^u)}{p(a^u)} - \mu^u \frac{q'(e^u)}{1-q(e^u)} + \eta^u \frac{1}{p(a^u)[1-q(e^u)]}, && (18)\\
C_d'(V^{u,d}) =& \gamma^u - \delta^u \frac{p'(a^u)}{1-p(a^u)} - \eta^u \frac{1}{1-p(a^u)}. && (19)
\end{aligned}$$

where γ^u is the Lagrange parameter for the promise-keeping constraint (14), δ^u is the multiplier for the prevention-incentive constraint (12), μ^u is the multiplier for the search-incentive constraint (13), and $\beta\eta^u$ is the parameter for the truth-telling constraint (15).

The Envelope conditions is

$$C_u'(V^u) = \gamma^u. \qquad (20)$$

4.3 Disabled Worker

Disability is an absorbing state, and the disabled individual cannot exert any search or prevention effort. Therefore, the planner does not have to consider any incentive constraints when providing consumption and continuation values to a disabled individual. The only constraint to consider is the promise-keeping constraint. For a disabled individual who was promised a utility of V^d, this constraint reads

$$V^d = u(c^d) + \beta V^{d,d}, \qquad (21)$$

where c^d is the transfer to the disabled individual and $V^{d,d}$ is the utility promised in the next period of disability.

The planner's problem is now given by

$$C_d(V^d) = \min_{c^d, V^{d,d}} \left\{ c^d + \beta C_d(V^{d,d}) \right\}$$

subject to the promise-keeping constraint (21).

Letting γ^d be the Lagrange parameter on the promise-keeping constraint, the first-order conditions are

$$\begin{aligned}
\gamma^d =& \frac{1}{u'(c^d)}, && (22)\\
C_d'(V^{d,d}) =& \gamma^d. && (23)
\end{aligned}$$

The Envelope condition reads

$$C_d'(V^d) = \gamma^d. \tag{24}$$

5 Optimal Insurance

Using the results from the previous section I am now able to derive the optimal insurance against unemployment and disability. In order to establish a benchmark case, I first assume that the planner has full information.

5.1 Full Information

Full information implies that the planner can distinguish between an unemployed and a disabled individual and that he is able to observe the individual's effort levels. The first assumption implies that the truth-telling constraint does not have to be considered, while the second assumption makes the prevention-incentive and the search-incentive constraints obsolete. The Lagrange parameters associated with those constraints can hence be set equal to zero in the first-order conditions.

5.1.1 Employed Worker

After setting the Lagrange parameters δ^e and η^e equal to zero, the first-order conditions with respect to the promised utilities together with the Envelope condition imply[6]

$$C_e'(V^e) = C_e'(V^{e,e}) = C_u'(V^{e,u}) = C_d'(V^{e,d}). \tag{25}$$

Because of the strict convexity of $C_e(\cdot)$, the first equality implies that the utility level of an employed worker remains constant while employed, i.e., $V^e = V^{e,e}$.

If an employed individual remains healthy and does not lose her job, she is guaranteed a utility of $V^{e,e}$ in the next period. Facing such an individual, the planner has first-order conditions similar to the ones presented above. In particular, the first-order condition with respect to consumption and the Envelope condition are given by

$$\gamma^{e,e} = \frac{1}{u'(c^{e,e})} \quad \text{and} \quad C_e'(V^{e,e}) = \gamma^{e,e}.$$

This implies

$$C_e'(V^{e,e}) = \frac{1}{u'(c^{e,e})}.$$

[6]See equations (8) – (11).

For individuals that are employed in this period and unemployed or disabled in the next, a similar result can be derived

$$C_u'(V^{e,u}) = \frac{1}{u'(c^{e,u})} \quad \text{and} \quad C_d'(V^{e,d}) = \frac{1}{u'(c^{e,d})}.$$

Plugging these conditions, together with the Envelope condition (11), into equations (25) and using the strict concavity of the utility function yields

$$\frac{1}{u'(c^e)} = \frac{1}{u'(c^{e,e})} = \frac{1}{u'(c^{e,u})} = \frac{1}{u'(c^{e,d})} \quad \Leftrightarrow \quad c^e = c^{e,e} = c^{e,u} = c^{e,d}.$$

As expected in a case without asymmetric information and moral hazard, this implies full insurance against unemployment and disability and a constant consumption while remaining employed.

5.1.2 Unemployed Worker

If the Lagrange parameters δ^u, μ^u and η^u are all equal to zero, the first-order conditions with respect to the promised utilities and the Envelope condition[7] can be combined to yield

$$C_u'(V^u) = C_e'(V^{u,e}) = C_u'(V^{u,u}) = C_d'(V^{u,d}). \tag{26}$$

The strict convexity of $C_u(\cdot)$ implies that the utility level of an unemployed worker remains constant during unemployment, i.e., $V^u = V^{u,u}$.

As in the previous section, combining the first-order condition with respect to consumption and the Envelope condition of an individual who has been unemployed for two periods yields

$$C_u'(V^{u,u}) = \frac{1}{u'(c^{u,u})}.$$

Similarly, we have the following conditions for an individual who has been unemployed and then found a job and an individual who has been unemployed and then became disabled that

$$C_e'(V^{u,e}) = \frac{1}{u'(c^{u,e})} \quad \text{and} \quad C_d'(V^{u,d}) = \frac{1}{u'(c^{u,d})}.$$

Substituting theses conditions, together with the Envelope condition (20), into equation (26) and applying the strict concavity of the utility function yields

$$\frac{1}{u'(c^u)} = \frac{1}{u'(c^{u,e})} = \frac{1}{u'(c^{u,u})} = \frac{1}{u'(c^{u,d})} \quad \Leftrightarrow \quad c^u = c^{u,e} = c^{u,u} = c^{u,d}.$$

[7] See equations (17) – (20).

In other words, the unemployed worker is fully insured against disability and her consumption remains constant during unemployment.

5.1.3 Disabled Worker

The first-order condition with respect to promised utility (23) and the Envelope condition (24) can be combined to yield

$$C_d'(V^d) = C_d'(V^{d,d}). \tag{27}$$

Due to the strict convexity of the cost function, this implies that $V^d = V^{d,d}$, i.e., constant utility during disability.

As before, the first-order condition with respect to consumption and the Envelope condition of an individual who has been disabled for two periods together imply

$$C_d'(V^{d,d}) = \frac{1}{u'(c^{d,d})}.$$

This can again be plugged into equation (27), together with the Envelope condition (24), to yield

$$\frac{1}{u'(c^d)} = \frac{1}{u'(c^{d,d})} \quad \Leftrightarrow \quad c^d = c^{d,d}.$$

Once a worker becomes disabled his consumption and utility levels remain constant.

5.2 Asymmetric Information

After having derived the optimal insurance against unemployment and disability in the full information setting, I turn now to the case of asymmetric information. While it was optimal to have perfect insurance against unemployment and disability in the full information case, this is no longer incentive compatible in the asymmetric information setting. Here, the planner is neither able to observe effort levels nor can he distinguish between an unemployed and a disabled individual. The planner merely observes whether the individual has income or not. The individual has to be provided with incentives to prevent disability, search for employment and truthfully report her state of health. When providing consumption and continuation values, the planner has, therefore, to consider the prevention-incentive, the search-incentive, the truth-telling as well as the promise-keeping constraint. Due to the continuous effort levels, the prevention-incentive constraints and the search-incentive constraint are always binding. Whether the truth-telling constraints are binding or slack cannot be determined analytically. We, therefore, know the following about the Lagrange parameters

$$\gamma^e, \delta^e, \gamma^u, \delta^u, \mu^u > 0 \quad \text{and} \quad \eta^e, \eta^u \geq 0.$$

5.2.1 Employed Worker

A planner facing an employed individual has the following first-order conditions with respect to the promised utilities[8]

$$\begin{aligned} C_e'(V^{e,e}) &= \gamma^e + \delta^e \frac{p'(a^e)}{p(a^e)}, \\ C_u'(V^{e,u}) &= \gamma^e + \delta^e \frac{p'(a^e)}{p(a^e)} + \eta^e \frac{1}{p(a^e)(1-s)}, \\ C_d'(V^{e,d}) &= \gamma^e - \delta^e \frac{p'(a^e)}{1-p(a^e)} - \eta^e \frac{1}{1-p(a^e)}. \end{aligned}$$

Since $\gamma^e, \delta^e > 0$ and $\eta^e \geq 0$ these conditions, together with the Envelope condition (11), imply that

$$C_u'(V^{e,u}) \geq C_e'(V^{e,e}) > C_e'(V^e) > C_d'(V^{e,d}). \tag{28}$$

As in the full-information case, these inequalities of marginal costs can be transformed to inequalities in transfers

$$c^{e,u} \geq c^{e,e} > c^e > c^{e,d}.$$

The consumption of an employed worker increases as long as she remains healthy and employed. Increasing consumption does not, however, necessarily imply a positive transfer from the planner. Since the worker earns a positive wage, increasing consumption can also be achieved by a decreasing wage tax, i.e., $c^e - w < c^{e,e} - w < 0$.

Moreover, consumption increases when the individual becomes unemployed and decreases at the time of disability. It can be seen below that the former does not necessarily imply a higher utility when becoming unemployed since the individual will have to exert search effort. In either case, there is no problem of moral hazard because job loss is exogenous. The drop in consumption upon disability, together with the increasing consumption while employed, creates an incentive to prevent disability.

In (28) it can further be seen that

$$C_e'(V^{e,e}) > C_e'(V^e) \quad \Leftrightarrow \quad V^{e,e} > V^e,$$

which implies that the continuation value of an employed worker increases with tenure. The truth-telling constraint further implies that $V^{e,u} \geq V^{e,d}$. Concerning the relationship

[8] See equations (8) – (10).

between the current level of utility V^e and the promised utility in case of disability, $V^{e,d}$, the following proposition holds.

Proposition 1: *The utility of a worker is decreasing when she becomes disabled, i.e.,*

$$V^e > V^{e,d}.$$

The results can be summarized as follows

$$V^{e,e} > V^e > V^{e,d} \quad \text{and} \quad V^{e,u} \geq V^{e,d}.$$

Consumption and utility of an employed individual are increasing with tenure. This stands in contrast to the results of Hopenhayn and Nicolini (1997) where consumption and utility are constant while working. However, it resembles the conclusions from models with disutility of work, e.g. Hopenhayn and Nicolini (2009). In that context, it is necessary to provide incentives to remain employed because work decreases utility, while here it is necessary because keeping up employability requires effort. If the individual becomes disabled, her consumption and her utility decrease. This decrease together with the increasing utility while healthy and employed gives her an incentive to prevent disability. In the case of unemployment, the individual's consumption increases. This increase compensates for the search effort the individual will have to exert. Whether the utility level increases when becoming unemployed is unclear. What is certain, is that the utility when unemployed is (weakly) larger than the utility when disabled.

5.2.2 Unemployed Worker

The planner's first-order conditions with respect to promised utilities in the case of an unemployed individual are given by[9]

$$\begin{aligned}
C_e'(V^{u,e}) &= \gamma^u + \delta^u \frac{p'(a^u)}{p(a^u)} + \mu^u \frac{q'(e^u)}{q(e^u)}, \\
C_u'(V^{u,u}) &= \gamma^u + \delta^u \frac{p'(a^u)}{p(a^u)} - \mu^u \frac{q'(e^u)}{1-q(e^u)} + \eta^u \frac{1}{p(a^u)[1-q(e^u)]}, \\
C_d'(V^{u,d}) &= \gamma^u - \delta^u \frac{p'(a^u)}{1-p(a^u)} - \eta^u \frac{1}{1-p(a^u)}.
\end{aligned}$$

The Envelope condition (20) and the fact that $\gamma^u, \delta^u, \mu^u > 0$ and $\eta^u \geq 0$ together imply

$$C_e'(V^{u,e}) > C_u'(V^u) > C_d'(V^{e,d}). \tag{29}$$

[9]See equations (17) – (19).

Applying the same logic as before yields

$$c^{u,e} > c^{u} > c^{u,d}.$$

An unemployed worker has higher consumption in the next period if she finds employment and remains healthy, giving her an incentive to both prevent disability and exert search effort. If she becomes disabled, her consumption level will be lower than under unemployment, which provides an additional incentive to exert prevention effort.

If the individual remains healthy but does not succeed in finding a job, the change in utility, and hence consumption, is ambiguous. In other words, it is not clear wether $V^{u,u}$ and $c^{u,u}$ are larger, smaller or equal to V^{u} and c^{u}, respectively. The intuition is that the incentive to search for a job has a negative effect on the promised utility while the incentive to prevent disability and the truth-telling constraint both have a positive one. Decreasing benefits and utilities as in Hopenhayn and Nicolini (1997) are therefore not necessarily optimal in this model. As an extreme example, consider a case where the probability of finding employment is independent of search effort, i.e., $q(e) = \bar{q}$. Then, the negative effect of the search constraint disappears and benefits as well as utility are increasing during unemployment.

While it may seem that this ambiguity can only be resolved numerically, it turns out that there is a condition under which this trade-off between providing incentives for search and providing incentives for prevention disappears.

A decrease in the promised utility in case of continued unemployment, $V^{u,u}$, increases the incentives to search for a job and thereby also search effort, e^{u}. This can be seen in the search-incentive constraint (13). In the prevention-incentive constraint (12), a decrease in $V^{u,u}$ has a direct and an indirect effect on the value of remaining employable, which is a weighted average of the value of being employed and the value of being unemployed

$$q(e^{u})V^{u,e} + [1 - q(e^{u})]\,V^{u,u}. \tag{30}$$

The direct effect is that the value of remaining employable decreases because $V^{u,u}$ decreases. This reduces the incentives to exert prevention effort. The indirect effect comes from the fact that a decrease in $V^{u,u}$ increases search effort, and therefore the probability of reemployment, $q(e^{u})$. This in turn increases the value of remaining healthy, because the value of becoming employed is larger than the value of remaining unemployed. The indirect and the direct effect are therefore of opposing signs. If the positive indirect effect is as least as large as the negative direct effect, then the value of remaining employable is not reduced when $V^{u,u}$ decreases.

In other words, if individuals react sufficiently to changed search incentives, the positive effect of decreasing utilities during unemployment on prevention effort outweighs the negative one and there is no trade-off between providing incentives for search and prevention. How much individuals react to changed search incentives depends on the concavity of $q(e)$. The less concave the function $q(e)$ is, the more individuals react to a given change in promised utility during unemployment. In order for individuals to react sufficiently, $q(e)$ can therefore not be too concave. This implies that the convex function $1-q(e)$ cannot be too convex, which is the case if $1-q(e)$ is log-concave, i.e., the log of the convex function is concave.[10] This condition is stated in the following proposition.

Proposition 2: *If the search technology satisfies the following condition*

$$-\frac{1-q(e)}{q'(e)} \geq \frac{q'(e)}{q''(e)}, \tag{31}$$

then decreasing utilities during unemployment lead to constant or increasing levels of prevention effort.

Examples of functions that satisfy this condition are the the logistical and the exponential distribution functions. The latter is commonly used in the literature as a search function.

Proposition 2 establishes that if individuals react sufficiently to search incentives, prevention effort does not decrease when the promised utility of continued unemployment decreases. If the search function $q(e)$ fulfills condition (31), the positive effect of the prevention-constraint in the first-order condition with respect to the promised utility in case of continued unemployment disappears or becomes negative. Since there is also the negative effect of the search-incentive constraint as well as the (possibly) positive effect of the truth-telling constraint, it remains unclear how consumption and utility change during unemployment. To shed further light on the analysis the following lemma can be established.

Lemma 1: *For promised utility and consumption to increase or remain constant during unemployment, the truth-telling constraint has to be binding.*

Increasing or constant utilities when remaining unemployed, together with the binding truth-telling constraint, imply that

$$V^u \leq V^{u,u} = V^{u,d} \quad \Leftrightarrow \quad V^u \leq V^{u,d}.$$

[10] A function $f(x)$ is log-concave if $f(x)f''(x) \leq [f'(x)]^2$. The function $1-q(e)$ is therefore log-concave if

$$[1-q(e)][-q''(e)] \leq [-q'(e)]^2 \quad \Leftrightarrow \quad -\frac{1-q(e)}{q'(e)} \geq \frac{q'(e)}{q''(e)}.$$

In other words, in order to have increasing or constant utilities during unemployment, the utility of an unemployed agent has to increase at the time of disability. However, the following proposition establishes that increasing utility upon disability cannot be optimal.

Proposition 3: *The utility of an unemployed agent decreases when she becomes disabled, i.e.,*

$$V^u > V^{u,d}.$$

Since an increase in utility when going from unemployment into disability is not optimal, consumption and promised utility can neither increase nor remain constant during unemployment. Therefore, Proposition 3 together with Lemma 1 implies that decreasing utilities and benefits during unemployment, as in Hopenhayn and Nicolini (1997), are optimal if individuals react sufficiently to changed search incentives.

Finally, the binding search-incentive constraint demands that the promised value of employment is strictly larger than the corresponding value for unemployment, $V^{u,e} > V^{u,u}$. The truth-telling constraint, on the other hand, implies that the promised value of unemployment has to be weakly larger than the value of disability, $V^{u,u} \geq V^{u,d}$.

The results can then be summarized as follows

$$V^{u,e} > V^{u,u} \geq V^{u,d} \quad \text{and} \quad V^u > V^{u,u} \geq V^{u,d}.$$

Finding a job guarantees higher utility and higher consumption compared to further unemployment. This utility promise, together with the decreasing utility and consumption during unemployment, provides an incentive to exert search effort. However, remaining unemployed yields a weakly higher utility than becoming disabled, which generates an incentive to be truthful about the health state. This, together with the higher utility when finding a job, creates an incentive to prevent disability and remain in the labor force.

5.2.3 Disabled Worker

The first-order condition associated with the continuation value of a disabled individual together with the corresponding Envelope condition implies[11]

$$C_d'(V^d) = C_d'(V^{d,d}) \quad \Leftrightarrow \quad c^d = c^{d,d} \quad \text{and} \quad V^{d,d} = V^d.$$

Since disability is an absorbing state and there are no incentive problems to consider, it is optimal to provide the disabled individual with a constant utility and a constant stream of consumption.

[11] See equations (23) and (24).

6 Numerical Simulations

In order to illustrate the results from the previous section and to answer the questions which cannot be solved analytically, I simulate the model numerically. Since the model is computationally very intensive, I focus my analysis on the problem of the unemployed individual and assume that employment is an absorbing state. The utility of finding employment is therefore given by $V^e = u(w)/(1-\beta)$. Hence, the planner only decides on the utility promises for unemployment and disability.

6.1 Functional Forms

The utility function exhibits constant relative risk aversion (CRRA) and has the form

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where σ is the coefficient of relative risk aversion. The hazard functions are given by the exponential distribution function which is commonly used in the literature. The probability of finding employment therefore depends on search effort, e, in the following way

$$q(e) = 1 - \exp(-\theta e).$$

This function fulfills condition (31). The probability of remaining healthy is

$$p(a) = 1 - \exp(-\phi a),$$

where a is prevention effort.

6.2 Calibration

A model period is one week. The discount factor β is equal to 0.999, which implies a yearly discount rate of roughly 0.95. The coefficient of relative risk aversion, σ, is set to 0.5.

The values of the parameters ϕ and θ are calibrated in the following way. Consider a stationary economy where workers pay a constant wage tax τ^e and unemployed (disabled) workers receive a constant benefit equal to a fraction τ^u (τ^d) of their previous wages. Considering the U.S. economy for this exercise, I set τ^e equal to 0.27, τ^u equal to 0.36

and τ^d equal to 0.57.[12] This allows me to solve the following problem

$$V^u_{stat} = \max_{a,e \geq 0} \left\{ u(\tau^u w) - a - e + \beta p(a) \left[q(e) V^e_{stat} + [1 - q(e)] V^u_{stat} \right] + \beta [1 - p(a)] V^d_{stat} \right\},$$

where

$$V^e_{stat} = \frac{u((1-\tau^e)w)}{1-\beta} \quad \text{and} \quad V^d_{stat} = \frac{u(\tau^d w)}{1-\beta}.$$

The solution to this problem yields a probability of remaining healthy for another week, $p(a)$, as well as a probability of finding employment until the next week, $q(e)$. Using these two probabilities, I can compute the probability of becoming disabled during the course of one year as well as the probability of finding employment within one month. These values can then be matched with their corresponding averages from the U.S. data.[13] Table 1 presents the parameter values used in the simulation.

Target	Data	Model
Monthly job finding rate	0.45	0.41
Yearly DI award rate	0.051	0.068
Calibrated Parameters	Value	Description
ϕ	1.0	Prevention parameter
θ	0.0008	Search parameter
Other Parameters	Value	Description
β	0.999	Discount factor
σ	0.5	Parameter of risk aversion
w	100	Wage

Table 1: Parameter Values

6.3 Results

Using the functional forms and parameter values described in the previous sections, I can now simulate the simplified version of the model. Specifically, I am looking at an individual who remains unemployed for one year and compute the optimal utility promises as well as the replacement rates and effort levels those promises imply. Table 2 and Figure 1 show the results of this simulation. All promised utilities as well as consumption are decreasing over time. I set the initial promised utility so that the initial unemployment

[12]Source: McDaniel (2007), OECD Employment Outlook (http://dx.doi.org/10.1787/182506528237) and U.S. Social Security Administration (http://www.ssa.gov/policy/docs/rsnotes/rsn2008-01.html), respectively.

[13]The monthly job finding rate is taken from Shimer (2005) and the yearly disability insurance award rate comes from the U.S. Social Security Administration, Annual Statistical Supplement, 2012 (http://www.ssa.gov/policy/docs/statcomps/supplement/2012/6c.html).

insurance replacement rate is roughly 100%. The unemployment insurance replacement rate decreases from 98.7% to 70.3% during the first year of unemployment. The promised disability insurance replacement rate decreases from 30.1% to 25.7% during one year of unemployment. Search effort is increasing during unemployment, increasing the probability of finding employment by 7.5%. Prevention effort also increases, reducing the probability of becoming disabled by 9.7%.

Weeks	UI Rep. Rate	DI Rep. Rate	Prob. Disab.	Prob. Empl.
1	98.67	30.12	12.88×10^{-3}	11.71
10	81.58	27.61	12.15×10^{-3}	12.22
20	74.58	26.45	11.83×10^{-3}	12.45
30	71.88	25.99	11.70×10^{-3}	12.54
40	70.78	25.79	11.65×10^{-3}	12.58
52	70.28	25.70	11.63×10^{-3}	12.60

Table 2: Simulation of an Unemployed Worker's Values (in %)

In summary, consumption and all promised utilities are decreasing during unemployment creating incentives to find employment. The promised disability benefits are decreasing in the length of the unemployment spell which provides the individual with an incentive to remain in the labor force despite the decreasing utility during unemployment. This resembles the results of Hopenhayn and Nicolini (1997) who show that it is optimal to reduce all claims to future consumption in order to create incentives to leave unemployment.

These results can now be compared to a policy in the spirit of Hopenhayn and Nicolini (1997) where the planner chooses only the utility during unemployment and the possibility of disability is not taken into account. I assume that individuals who become disabled in this setting are treated as unemployed by the planner. Figure 1 shows a simulation of the model presented in this paper (continuous line) and a model where disability is not taken into account (dashed line). Both policies start out with the same utility promise.

The upper left panels show the replacement rates of an unemployed individual under the two policies over 52 weeks. Both policies imply decreasing replacement rates during unemployment. As expected, the Hopenhayn-Nicolini policy is less generous. Since the planner's only problem in the Hopenhayn-Nicolini model is to provide search incentives, the replacement rate can be lower than in the model presented in this paper where the planner provides incentives for search as well as for preventing disability. In the upper right panel the replacement rates promised to the individual in case of disability are plotted over 52 weeks of unemployment. I assume that under the Hopenhayn-Nicolini policy individuals who become disabled receive the same treatment as unemployed indi-

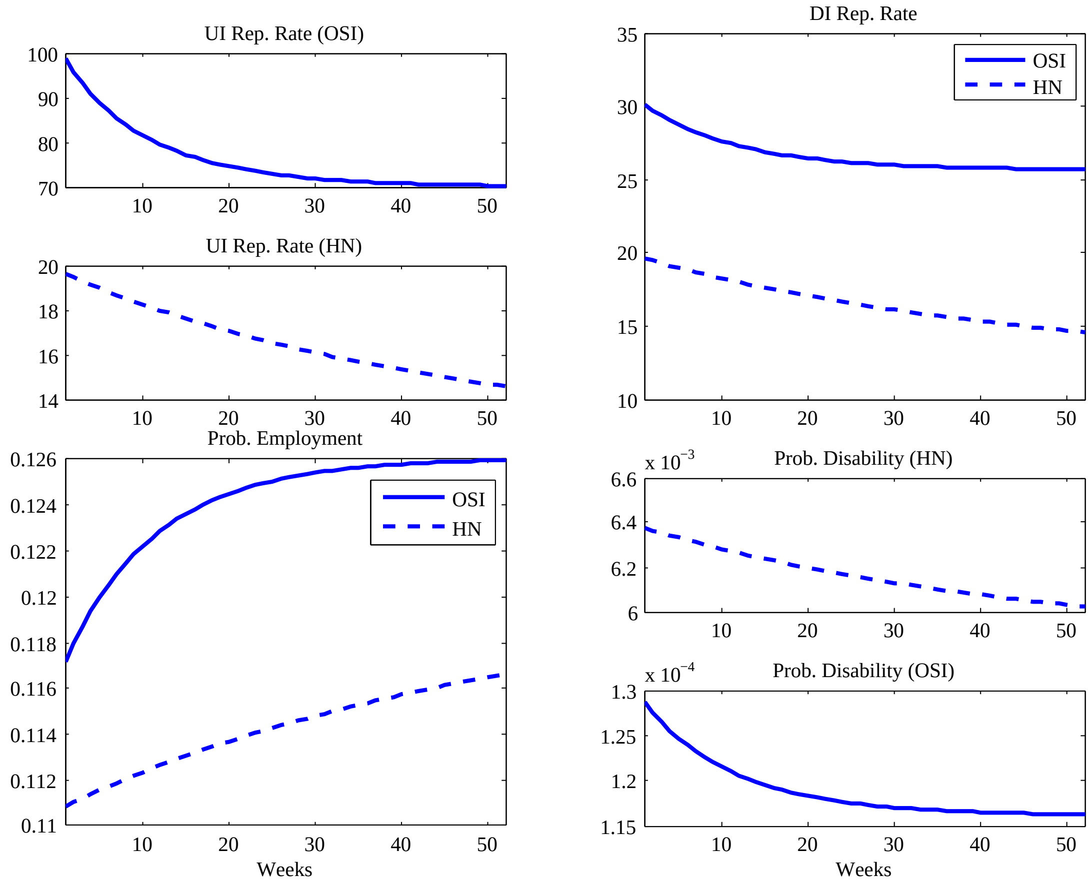

Figure 1: Simulation of an Unemployed Worker's Values. Optimal Insurance against Unemployment and Disability (OSI) vs Hopenhayn-Nicolini Insurance (HN).

viduals. Although, the disability benefits implied by the Hopenhayn-Nicolini policy are lower than the ones under the optimal insurance derived in this paper, the less generous unemployment insurance implies weaker incentives to remain in the labor force under the Hopenhayn-Nicolini policy compared to the optimal policy in this paper. The latter can be seen in the lower right panels where the probability of becoming disabled for the two policies is shown. Finally, in the lower left panel it can be seen that the probability of finding employment increases during unemployment under both policies, but the probabilities are lower under the Hopenhayn-Nicolini policy.

7 Conclusion

In this paper, I derive the optimal insurance against unemployment and disability in a private information economy with endogenous health and search effort. Introducing endogenous health makes the analysis significantly more difficult but it addresses a crucial shortcoming in the literature, namely the possibility of individuals to leave the labor force and go into other insurance systems such as disability insurance. If individuals only have a choice between being unemployed or working, providing them with incentives to search for a job is easier than when they also have the option to exit the labor force.

I demonstrate that the optimal sequence of consumption and promised utilities of an employed worker is increasing with tenure. Once a worker is disabled she will receive constant benefits and her utility will remain constant. For an unemployed worker I show that the prevention constraint implies increasing benefits and utility levels during unemployment while the search constraint has the opposite effect. Decreasing benefits and utilities as in Hopenhayn and Nicolini (1997) are therefore not necessarily optimal in this model. I show, however, that the search effect dominates the prevention effect if individuals respond sufficiently much to a change in search incentives. The sequence of consumption and utility levels is then decreasing during unemployment.

Moreover, I show numerically that the promised utility in case of disability is decreasing during unemployment. This means that disability benefits are no longer monotonically increasing in the retirement age as in Diamond and Mirrlees (1978), instead they are lower the longer the individual was previously unemployed. Comparing this policy to a policy in the spirit of Hopenhayn and Nicolini (1997) where the planner does not distinguish between unemployed and disabled individuals, I show that the policy presented in this paper provides better incentives for search and prevention leading to higher probabilities of finding employment and remaining in the labor force.

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Appendix

A1 Proof of Proposition 1

Since the incentive constraint (4) is fulfilled, the equilibrium effort level will give the individual the highest possible utility. This utility level is, therefore, as least as large as the utility implied by zero effort. The promise-keeping constraint (5), which is binding in equilibrium, can then be rearranged to yield

$$\begin{aligned} V^e &= u(c^e) - a^e + \beta p(a^e)\left[sV^{e,e} + (1-s)V^{e,u}\right] + \beta\left[1 - p(a^e)\right]V^{e,d} \\ &\geq u(c^e) - 0 + \beta p(0)\left[sV^{e,e} + (1-s)V^{e,u}\right] + \beta\left[1 - p(0)\right]V^{e,d} \\ &= u(c^e) + \beta V^{e,d}, \end{aligned} \tag{32}$$

where the last equality comes from the fact that $p(0) = 0$.

The first-order condition (28) implies that

$$C'_e(V^e) > C'_d(V^{e,d}),$$

which can be transformed to yield

$$\frac{1}{u'(c^e)} > \frac{1}{u'(c^{e,d})} \quad \Leftrightarrow \quad u(c^e) > u(c^{e,d}) \quad \Leftrightarrow \quad \frac{u(c^e)}{1-\beta} > \frac{u(c^{e,d})}{1-\beta}.$$

Since disability is an absorbing state, consumption will be constant at the optimum and the latter inequality can be rewritten as

$$\frac{u(c^e)}{1-\beta} > \frac{u(c^{e,d})}{1-\beta} = V^{e,d} \quad \Leftrightarrow \quad u(c^e) > (1-\beta)V^{e,d}. \tag{33}$$

(32) and (33) can then be combined to yield

$$V^e \geq u(c^e) + \beta V^{e,d} > (1-\beta)V^{e,d} + \beta V^{e,d} = V^{e,d} \quad \Leftrightarrow \quad V^e > V^{e,d} \quad \square$$

A2 Proof of Proposition 2

The value of being employable (30) is affected by the promised utility in case of continued unemployment, $V^{u,u}$, in two ways: directly through the promised utility and indirectly through search effort, e^u, which depends on $V^{u,u}$. The value of being employable can therefore be expressed as a function of $V^{u,u}$

$$F(V^{u,u}) \equiv q\left(e^u\left(V^{u,u}\right)\right)V^{u,e} + \left[1 - q\left(e^u\left(V^{u,u}\right)\right)\right]V^{u,u}.$$

If $V^{u,u}$ decreases, the direct effect reduces the value of being employable while the indirect effect increases the value. If the positive effect is as least as large as the negative one, the value of F does not increase with $V^{u,u}$, i.e.,

$$\frac{\mathrm{d}F(V^{u,u})}{\mathrm{d}V^{u,u}} = q'\left(e^u\left(V^{u,u}\right)\right)\frac{\partial e^u\left(V^{u,u}\right)}{\partial V^{u,u}}\left[V^{u,e} - V^{u,u}\right] + 1 - q\left(e^u\left(V^{u,u}\right)\right) \leq 0,$$

which implies for the change in effort

$$\frac{\partial e^u\left(V^{u,u}\right)}{\partial V^{u,u}} \leq -\frac{1 - q\left(e^u\left(V^{u,u}\right)\right)}{q'\left(e^u\left(V^{u,u}\right)\right)}\frac{1}{\left[V^{u,e} - V^{u,u}\right]}. \tag{34}$$

Equation (34) describes how equilibrium effort, e^u, should react to a change in the promised utility in case of continued unemployment, $V^{u,u}$, so that the value of being employable is not reduced when $V^{u,u}$ is decreasing.

How much equilibrium effort actually changes can be derived from the search-incentive constraint (13)

$$\beta p(a^u)q'(e^u)\left[V^{u,e}-V^{u,u}\right]=1 \quad \Leftrightarrow \quad q'(e^u)=\frac{1}{\beta p(a^u)\left[V^{u,e}-V^{u,u}\right]}\equiv G(V^{u,u}). \quad (35)$$

The equilibrium effort is then given by

$$e^u\left(V^{u,u}\right)=(q')^{-1}\left[G(V^{u,u})\right],$$

and the derivative with respect to $V^{u,u}$ is

$$\frac{\partial e^u\left(V^{u,u}\right)}{\partial V^{u,u}}=\frac{\partial (q')^{-1}\left[G(V^{u,u})\right]}{\partial G(V^{u,u})}\frac{\partial G(V^{u,u})}{\partial V^{u,u}}, \quad (36)$$

where $(q')^{-1}$ is the inverse of the first derivative of the search function $q(\cdot)$. The first part of this derivative can be transformed using the inverse function theorem[14]

$$\frac{\partial (q')^{-1}\left[G(V^{u,u})\right]}{\partial G(V^{u,u})}=\frac{1}{q''\left((q')^{-1}\left[G(V^{u,u})\right]\right)}=\frac{1}{q''\left(e^u(V^{u,u})\right)}. \quad (37)$$

The second part of the derivative in (36) is given by

$$\frac{\partial G(V^{u,u})}{\partial V^{u,u}}=\frac{\partial}{\partial V^{u,u}}\left(\frac{1}{\beta p(a^u)\left[V^{u,e}-V^{u,u}\right]}\right)=\frac{1}{\beta p(a^u)\left[V^{u,e}-V^{u,u}\right]^2}.$$

Substituting the second equality from (35) into this expression gives

$$\frac{\partial G(V^{u,u})}{\partial V^{u,u}}=q'\left(e^u(V^{u,u})\right)\frac{1}{\left[V^{u,e}-V^{u,u}\right]}. \quad (38)$$

Plugging in the two expressions from (37) and (38) into equation (36) yields

$$\frac{\partial e^u\left(V^{u,u}\right)}{\partial V^{u,u}}=\frac{q'\left(e^u(V^{u,u})\right)}{q''\left(e^u(V^{u,u})\right)}\frac{1}{\left[V^{u,e}-V^{u,u}\right]} \quad (39)$$

The expression in (39) shows how much the optimal search effort *will* react to a change in the promised utility of continued unemployment. Equation (34) demonstrates how much search effort *should* change in order for a decrease in promised utility of unemploy-

[14]The theorem states that

$$\left(f^{-1}\right)'(b)=\frac{1}{f'(f^{-1}(b))}=\frac{1}{f'(a)},$$

where $b=f(a)$.

ment not to reduce the value of remaining employable. Combining equations (34) and (39) yields

$$-\frac{1-q(e^u)}{q'(e^u)} \geq \frac{q'(e^u)}{q''(e^u)} \quad \square$$

A3 Proof of Lemma 1

If condition (31) is satisfied, the first-order condition with respect to $V^{u,u}$ reads

$$C_u'(V^{u,u}) = \gamma^u - \delta^u \frac{p'(a^u)}{p(a^u)} - \mu^u \frac{q'(e^u)}{1-q(e^u)} + \eta^u \frac{1}{p(a^u)[1-q(e^u)]}.$$

Since the Envelope condition is given by $C_u'(V^u) = \gamma^u$, increasing or constant utilities (and hence consumption) during unemployment are optimal if

$$\gamma^u - \delta^u \frac{p'(a^u)}{p(a^u)} - \mu^u \frac{q'(e^u)}{1-q(e^u)} + \eta^u \frac{1}{p(a^u)[1-q(e^u)]} \geq \gamma^u,$$

or

$$\eta^u \geq \delta^u p'(a^u)[1-q(e^u)] + \mu^u p(a^u) q'(e^u) > 0,$$

i.e., the Lagrange parameter associated with the truth-telling constraint has to be positive, and hence the truth-telling constraint has to be binding.

A4 Proof of Proposition 3

Since the incentive constraints (12) and (13) are fulfilled, the equilibrium effort levels will give the individual a weakly higher utility than zero search and prevention effort. The binding promise-keeping constraint (14) can then be rearranged to yield

$$\begin{aligned} V^u &= u(c^u) - a^u - e^u + \beta p(a^u)\left[q(e^u)V^{u,e} + (1-q(e^u))V^{u,u}\right] + \beta\left[1-p(a^u)\right]V^{u,d} \\ &\geq u(c^u) - 0 - 0 + \beta p(0)\left[q(0)V^{u,e} + (1-q(0))V^{u,u}\right] + \beta\left[1-p(0)\right]V^{u,d} \\ &= u(c^u) + \beta V^{u,d}, \end{aligned} \tag{40}$$

where the last equality comes from the fact that $p(0) = q(0) = 0$.

The first-order condition (29) implies that

$$C_u'(V^u) > C_d'(V^{u,d})$$

which can be transformed to yield

$$\frac{1}{u'(c^u)} > \frac{1}{u'(c^{u,d})} \quad \Leftrightarrow \quad u(c^u) > u(c^{u,d}) \quad \Leftrightarrow \quad \frac{u(c^u)}{1-\beta} > \frac{u(c^{u,d})}{1-\beta}.$$

Since consumption is constant once the individual is disabled, this condition can be rewritten as

$$\frac{u(c^u)}{1-\beta} > \frac{u(c^{u,d})}{1-\beta} = V^{u,d} \quad \Leftrightarrow \quad u(c^u) > (1-\beta)V^{u,d}. \tag{41}$$

(40) and (41) can then be combined to yield

$$V^u \geq u(c^u) + \beta V^{u,d} > (1-\beta)V^{u,d} + \beta V^{u,d} = V^{u,d} \quad \Leftrightarrow \quad V^u > V^{u,d} \quad \square$$