Accuracy in Contests: Players' Perspective

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Abstract

We propose a political theory for the slow adoption of technology in sports and other contests. We investigate players' preferences for new technology that improves contest accuracy. Modeling accuracy as the elasticity of "production" in a standard Tullock contest, we show that players may be against higher accuracy if heterogeneity among them is: (1) sufficiently low; (2) moderate but the initial accuracy is low; or (3) high but the initial accuracy is high. We apply our results to the recent adoption of goal-line technology by major European soccer leagues.

1 Introduction

Despite being a multi-billion dollar industry[1], European soccer has witnessed serious refereeing errors.[2] As technology has advanced, soccer fans have grown intolerant of

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[1]According to Deloitte Consulting, in the midst of economic pressures, the European soccer market reached $24.6 billion in revenue terms in 2012, which implies 11% growth in 2011/12. More details can be found at <http://www.deloitte.com/view/en_LB/ly/press/press-releases/4c11cefd2e23f310VgnVCM3000003456f70aRCRD.htm#>

[2]Perhaps, the most memorable ones are England's second goal against Germany in the 1966 World Cup Final, Argentina's first goal against England in the 1986 World Cup, and England's second goal against Germany in the 2010 World Cup.

these errors. Yet, European soccer leagues, and the international organizations such as UEFA and FIFA, seem reluctant to introduce any advanced technology that can minimize refereeing errors at a reasonable cost and with little disruption to the games. It is only in 2013 that several major soccer leagues, including the English and Dutch, have decided to adopt a goal-line technology while others including the Spanish and German have agreed to follow suit in the near future. In contrast to team-based European soccer, technology adoption appears timely in individualistic sports like tennis, athletics, horse-racing, etc. The objective of this paper is to offer a political theory for this discrepancy. Specifically, we examine players' incentives to support a new technology that improves contest accuracy. We show that these incentives may substantially differ from those of a contest designer because conceivably, players care more about winning than increasing the aggregate effort. Therefore, in contests where players retain a significant say in contest design, technology adoption may be delayed. Indeed, the English Premier League only recently adopted the new goal-line technology after votes from its twenty clubs.[3]

The same may be true for other contests where accuracy improvement is feasible. Today, some educational institutions use plus/minus grading instead of letter grading, where the former better differentiates students and is thus believed to enhance grading accuracy. One rationale for this is the contention that plus/minus grading is superior to letter grading, its less accurate counterpart, in motivating student achievement. While those not using it may have various reasons such as financial and administrative costs, one reason could be student resistance. In fact, an ad-hoc committee on plus/minus grading, established by Eastern Kentucky University in 2013, reported that more than half of the universities in Kentucky are still not using plus/minus grading for various reasons, one being student resistance.[4] Likewise, Dixon (2004) finds that the ratio of students choosing plus/minus grading over those choosing letter grading is 1 to 2, whenever they are given a choice.

Our model is a standard Tullock contest with heterogeneous players. We define contest *accuracy* as the elasticity of "production" in the Tullock contest success function since higher elasticity implies that winning depends more on the effort than

[3]For more information see <http://www.bbc.co.uk/sport/0/football/22107409>

[4]For more information see <http://www.eku.edu/academics/facultysenate/minutes/2003-04/11-03-03/plus_minus_report/final.pdf>

on “exogenous uncertainty”. In practice, accuracy can be improved through various mechanisms depending on the context: in sports by allowing referees to get access to a better technology, and in lobbying by providing interest groups with extra information about the preferences of decision-makers. We assume there are two types of players: those with a high marginal cost and those with a low marginal cost. Following Dixit (1987), we call the former type underdogs and the latter type favorites.

In the unique equilibrium, we find that while the underdog’s payoff is always decreasing in accuracy, the favorite’s payoff is ambiguous. In particular, when the initial accuracy is very low, the favorite prefers higher accuracy if the cost advantage is significant, and lower accuracy otherwise. This makes sense because when the cost advantage is small, players are essentially identical and therefore compete most fiercely with little change in their equilibrium probabilities of winning. The intuition for a significant cost advantage is similar. This result fits well with the adoption of technology in European soccer in general. The soccer leagues of England (resp. EPL) and the Netherlands (resp. Eredevisie) have recently decided to implement a goal-line technology while those of Germany (Bundesliga) and Spain (La Liga) have delayed their decision until 2015. In agreement with our prediction, EPL and Eredevisie are more heterogenous than Bundesliga and La Liga as evidenced by Figure 1.[5]

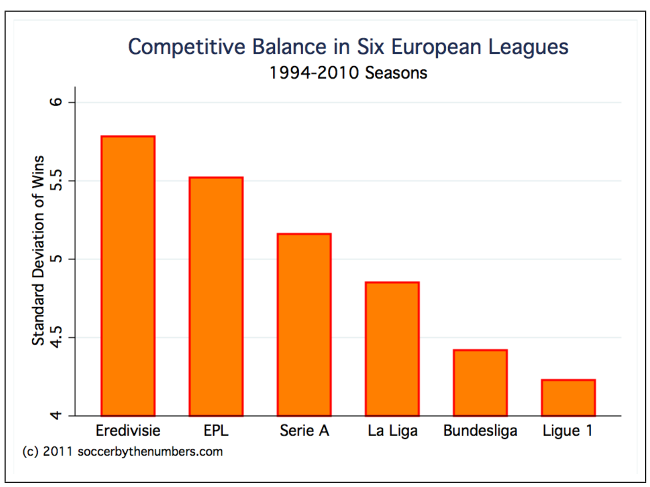

Figure 1. Heterogeneity of Six European Soccer Leagues

[5]In Figure 1, the competitive balance measures the dispersion of wins across teams. Formally, it is the standard deviation of wins. Thus, while 0 implies that the number of wins is the same for all teams, any non-zero number shows the degree of heterogeneity among teams’ number of wins with respect to the mean number of wins. More details can be found at <http://www.soccerbythenumbers.com/2011/06/comparing-competitiveness-of-european.html>

Related Literature:

Our paper falls into a large collection of literature on contest design. These include: the choice of prizes (Glazer and Hassin (1988), Moldovanu and Sela (2001)), the choice of contest success function (Dasgupta and Nti (1998), Nti (2004)), the number of contestants (Baye et al. (1993), Amegashi (1999)), the structure of multi-stage contests (Gradstein (1998), Gradstein and Konrad (1999), Amegashie (2000), Yildirim (2005)), and the structure of information (Wärneryd (2003,2012)).[6] This literature is mainly concerned about contest design aimed at maximizing total effort.[7] More importantly, the contest designer is treated as independent in the design process. That is, only the designer's preferences matter for the contest design. In contrast, our focus is on the players' preferences for the contest design.

In highlighting accuracy differences across contests, our paper relates to Alcalde and Dahm (2007), Che and Gale (1997), Dasgupta and Nti (1998), Micheals (1988), Nti (1999,2004), Wang (2010). In these models, the contest designer employs *accuracy*, the extent to which winning depends on effort rather than exogenous uncertainty, to adjust these incentives. Moreover, they model accuracy as elasticity of production in a standard Tullock contest as in our paper. To these, Alcalde and Dahm (2007), Che and Gale (1997), Dasgupta and Nti (1998), Nti (1999,2004), Wang (2010) introduce heterogeneity, while Dasgupta and Nti (1998), Micheals (1988) focus on homogeneous contests. None of these papers investigate players' preferences for accuracy.

The remainder of the paper is organized as follows. The basic model is presented in the next section, followed by the equilibrium characterization in Section 3. Sections 4 and 5 provide findings regarding players and the designer who is concerned about maximizing total effort respectively. Section 6 discusses the findings. Section 7 extends the model to a pairwise contest, and Section 7 concludes.

2 The Model

The model is a standard Tullock contest. A population of $n + m$ risk-neutral players simultaneously exert effort to win a prize $V > 0$. The cost of effort for player i is

[6] See Nitzan (1994) and Konrad (2009) for a detailed survey of contest literature.

[7] If efforts are interpreted as rent-seeking, then the design aims to minimize total error.

$C_i(x_i) = c_i x_i$ where $x_i \geq 0$ denotes his effort and $c_i \in \{c_L, c_H\}$ denotes his marginal cost, where $0 < c_L < c_H$. Let $\mathbf{x} = (x_1, ..., x_n, x_{n+1}, ..., x_{n+m})$ represent an effort profile such that $c_i = c_L$ for $i = 1, ..., n$. Given $\mathbf{x}$, player i's probability of winning or the contest success function (CSF) takes the Tullock form:

$$p_i(\mathbf{x}) = \begin{cases} \dfrac{x_i^r}{\sum_{j=1}^{n+m} x_j^r} & if \quad \mathbf{x} \neq \mathbf{0} \\ 1/(n+m) & if \quad \mathbf{x} = \mathbf{0}, \end{cases} \tag{1}$$

where $r \in (0, 1)$. Clearly, i's probability of winning is increasing with his own effort and decreasing with others', both at a decreasing rate. In particular,

$$\frac{\partial p_i(\mathbf{x})}{\partial x_i} = \frac{r}{x_i} p_i(\mathbf{x})[1 - p_i(\mathbf{x})] > 0 \text{ and } \frac{\partial p_i(\mathbf{x})}{\partial x_j} = -\frac{r}{x_j} p_i(\mathbf{x}) p_j(\mathbf{x}) < 0. \tag{2}$$

Note that the "production function" $f(x_i) = x_i^r$ has a constant elasticity, r. That is, as r increases, the probability of winning becomes more sensitive to efforts, and less sensitive to exogenous uncertainty. Depending on the context, the source of such effort sensitivity can be technological, political, or institutional. For instance, in sports r may be determined by the resolution of cameras used or by the ability of the referees in deciding close calls. In lobbying, r may be affected by the (unknown) preferences of decision-makers, and in organizations, it may be the result of the allocation of property rights. In general, we will call r the *accuracy* of the contest, and investigate players' preferences for this accuracy. We begin our analysis by equilibrium characterization.

3 Equilibrium Characterization

The expected payoff of player i can be written:

$$\pi_i(\mathbf{x}) \equiv p_i(\mathbf{x}) V - c_i x_i. \tag{3}$$

An effort profile $\mathbf{x}^*$ is a Nash equilibrium if and only if player i's effort is a best-reply to others'; namely

$$x_i^* = \arg\max_{x_i} \pi_i(x_1^*, ..., x_i, ..., x_{n+m}^*).$$

Lemma 1 *There is a unique* $\mathbf{x}^*$*.*

Proof. Directly follows from Szidarovski and Okuguchi (1997). ■

The argument for the equilibrium existence is routine. The uniqueness relies on the assumption that $r < 1$ so that there are diminishing marginal returns to effort. Since there are two sets of identical players in our contest, the uniqueness of equilibrium implies that players with equal marginal costs choose equal efforts in equilibrium. Let $x_i^* = x_L^*$ for $c_i = c_L$ and $x_i^* = x_H^*$ for $c_i = c_H$. The following proposition fully characterizes equilibrium efforts and players' payoffs.

Proposition 1 *Let* $c \equiv \dfrac{c_H}{c_L}$ *and* $\theta^* \equiv \dfrac{x_H^*}{x_L^*}$. *Then,* $\theta^* \in (0, \frac{1}{c}]$ *and*

(a) $p_L^* = \frac{1}{n+m\times(\theta^*)^r} > \frac{1}{m+n/(\theta^*)^r} = p_H^*$,

(b) $x_L^* = \frac{r}{c_L} p_L^*[1 - p_L^*]V > \frac{r}{c_H} p_H^*[1 - p_H^*]V = x_H^*$,

(c) $\pi_L^* = p_L^*(1 - r + rp_L^*)V > p_H^*(1 - r + rp_H^*)V = \pi_H^*$,

(d) θ^* *uniquely solves:*

$$g(\theta) \equiv (m-1)\theta^{2r-1} - mc\theta^r + n\theta^{r-1} - (n-1)c = 0. \tag{4}$$

Proof. Part (a) directly follows from (1). Next, differentiating (3) with respect to x_i and employing (2), player i's first-order condition can be stated as

$$\frac{\partial \pi_i}{\partial x_i} = \frac{r}{x_i^*} p_i(\mathbf{x}^*)[1 - p_i(\mathbf{x}^*)]V - c_i \leq 0 \ (= 0 \text{ if } x_i^* > 0). \tag{5}$$

Note first note that $x_L^* = x_H^* = 0$ cannot form an equilibrium because, given zero effort by others, a player can guarantee winning by an $\varepsilon > 0$ effort. In fact, because $r < 1$, it must be that $x_H^* > 0$ and $x_L^* > 0$; otherwise, (5) would be violated for the player with zero effort. Positive equilibrium efforts imply that (5) holds with equality for all i. In particular,

$$\begin{aligned} rp_H^*(1 - p_H^*)V &= c_H x_H^* \\ rp_L^*(1 - p_L^*)V &= c_L x_L^*. \end{aligned} \tag{6}$$

Together with the fact that $p_L^* > p_H^*$ from part (a), part (b) is obtained. Inserting (6) into (3) proves part (c). Finally, to show part (d), we divide both sides of (6) to obtain: $\frac{1-p_H^*}{1-p_L^*} = c \times (\theta^*)^{1-r}$. Inserting the expressions from part (a) and arranging terms yield the indifference condition, as desired. The observation that $\theta^* \leq \frac{1}{c}$ follows from part (b). ■

Proposition 1 is intuitive. It says that the low-cost players work harder and thus they are more likely to win the contest. In the terminology of Dixit (1987), we therefore call a low-cost player a *favorite* and a high-cost player an *underdog*. It also says that all players participate in the competition. This is due to the fact that, for small efforts, the marginal benefit is greater than the marginal cost. Formally, this relies on the assumption that $r < 1$. [8] Despite exerting more effort and incurring a higher cost, as $c_L x_L^* \geq c_H x_H^*$ by part (b), Proposition 1 reveals that the favorite is better off than the underdog.

Armed with the equilibrium characterization, we are ready to investigate how players' payoffs change with the accuracy, r.

4 Comparative Statics of the Accuracy, r

To establish a benchmark, we begin with the two-player case which is often adopted in the contest literature.

4.1 Benchmark: Two Players

Assume that there is one favorite and one underdog. The intuition suggests that the favorite should always prefer higher contest accuracy in order to make his effort – not the exogenous uncertainty – more decisive in winning, while the opposite should hold for the underdog.[9] The following proposition mostly confirms this intuition.

Proposition 2 *Suppose there is one favorite and one underdog, i.e.,* $n = m = 1$.

[8] For $r = 1$, it is possible that the high-cost players may drop out of the contest, especially when the number of the low-cost ones is sufficiently large. We ignore this corner case here both because our investigation is about the interaction between the two types of players and because one can take the limit $r \to 1$.

[9] As mentioned in the introduction, Wang(2010) also investigates the impact of contest accuracy in a two-player setting, but his focus is on total efforts.

Then, (a) $\theta^* = \frac{1}{c}$, *(b)* $\frac{d\pi_H^*}{dr} < 0$, *and (c)*

$$\frac{d\pi_L^*}{dr}\begin{cases} < 0 & \text{if} \quad c < \underline{c} \\ < 0 & \text{if} \quad \underline{c} \le c \le e \text{ and } r < \underline{r} \\ \ge 0 & \text{if} \quad \underline{c} \le c \le e \text{ and } r \ge \underline{r} \\ > 0 & \text{if} \quad c > e, \end{cases}$$

where $\underline{c} \approx 2.09$ *uniquely solves:* $2c\ln c - c - 1 = 0$, *and* $e \approx 2.71$ *is the natural number.*

Proof. Note that $p_L + p_H = 1$ for $n = m = 1$. Together with (6), this implies $c_H x_H^* = c_L x_L^*$. Dividing both sides by $c_H x_L^*$ proves part (a). To prove part (b), we differentiate π_H^* with respect to r

$$\frac{d\pi_H^*}{dr} = \frac{\partial \pi_H^*}{\partial r} + \frac{\partial \pi_H^*}{\partial \theta^*}\frac{\partial \theta^*}{\partial r}.$$

More simply,

$$\frac{d\pi_H^*}{dr} = \frac{\partial \pi_H^*}{\partial r},$$

since $\theta^* = \frac{1}{c}$ by part (a) implies $\frac{\partial \theta^*}{\partial r} = 0$. Now, calculating the derivative using Proposition 1c provides

$$\frac{d\pi_H^*}{dr} = \frac{\partial \pi_H^*}{\partial r} = -\frac{c^r V}{(c^r+1)^3}[1 + c^r + (1 + r + (1-r)c^r)\ln c],$$

which together with $c > 1$ yields $\frac{d\pi_H^*}{dr} < 0$ for all r, as desired.

Finally, to prove part (c), we differentiate π_L^* with respect to r to obtain

$$\frac{d\pi_L^*}{dr} = \frac{\partial \pi_L^*}{\partial r} = -\frac{c^r V}{(c^r+1)^3}\underbrace{[1 + c^r - (1 - r + (1+r)c^r)\ln c]}_{f(r,c)},$$

where

$$\frac{\partial f(r,c)}{\partial c} = -\frac{1}{c}[1 - r + c^r + (1+r)rc^r \ln c] < 0,$$

and

$$\frac{\partial^2 f(r,c)}{\partial r^2} = -c^r(\ln c)^2[1 + (1+r)\ln c] < 0.$$

Clearly, if $c \ge e$, then since $f(r,e) < 0$ and $\frac{\partial f(r,c)}{\partial c} < 0$ for any r, we have $f(r,c) < 0$ which, in turn, implies $\frac{d\pi_L^*}{dr} > 0$ for any r. If $c < e$, the second derivative together with the fact that $f(0,c) = 2(1 - \ln c) > 0$, and $f(1,c) = 1 + c - 2c\ln c$ gives us:

$f(r,c) > 0$ whenever $c < \underline{c}$, and $f(r,c) > 0$ whenever $\underline{c} < c < e$ and $r < \underline{r}$; and $f(r,c) < 0$ whenever $\underline{c} < c < e$ and $r > \underline{r}$. Combining our findings shows part (c). ■

Part (a) is well-known in the literature on Tullock contests (e.g., Nitzan (1994), Konrad (2011)). It says that equilibrium efforts are inversely proportional to marginal costs.[10] Part (b) indicates that the underdog's payoff decreases with accuracy because an increase in accuracy amplifies the underdog's cost disadvantage. The change in the favorite's payoff is, however, ambiguous. Part (c) reveals that as the accuracy improves so does the favorite's payoff as long as his cost advantage is sufficiently large. When the cost advantage is small (in the extreme, negligible), the favorite's payoff, like the underdog's, is also decreasing in the accuracy level. When the cost advantage is intermediate, part (c) shows that the change in the favorite's payoff depends on the initial level of accuracy. In particular, his payoff decreases if the initial level is low, and it increases if the initial level is high.

Although prominently used in the literature, the two-player case is restrictive in our investigation because discussion about the contest accuracy often involves more than two players, necessitating a more general analysis.

4.2 More than Two Players

Suppose that there are at least three players including one favorite and one underdog. The next proposition shows that the relative effort of the underdog in equilibrium, θ^*, is no longer independent of the accuracy level, r, even though its impact on the underdogs' payoff is qualitatively the same as in the two-player case.

Proposition 3 *Suppose there are n favorites, and m underdogs. Then, (a) $\frac{\partial \theta^*}{\partial r} < 0$ and (b) $\frac{d\pi_H^*}{dr} < 0$.*

Proof. Differentiating (4) with respect to r yields,

$$\frac{\partial \theta}{\partial r} = -\frac{g_r}{g_\theta}.$$

[10] Technically, since, with two players, $p_L + p_H = 1$, eq.(6) implies $c_L x_L^* = c_H x_H^*$, and thus $\theta^* = \frac{x_H^*}{x_L^*} = \frac{c_L}{c_H}$.

Routine algebra yields

$$\begin{aligned} g_r &= [n + 2(m-1)\theta^r - mc\theta]\theta^{r-1}(\ln\theta), \\ g_\theta &= -[(1-r)n + (1-2r)(m-1)\theta^r + rmc\theta]\theta^{r-2}. \end{aligned}$$

Now note from (4) that $c\theta = \dfrac{[n + (m-1)\theta^r]\theta^r}{[n + m\theta^r - 1]}$. Substituting this into the above expressions, we get:

$$\begin{aligned} g_r &= \frac{m(m-1)\theta^{2r} + 2(m-1)(n-1)\theta^r + n(n-1)}{n + m\theta^r - 1}\theta^{r-1}(\ln\theta), \\ g_\theta &= -\frac{(1-r)[m(m-1)\theta^{2r} + (m-1)(n-1)\theta^r + n(n-1)] + [mn - r(m-1)(n-1)]\theta^r}{n + m\theta^r - 1}\theta^{r-2}. \end{aligned}$$

Clearly, $g_r < 0$ and $g_\theta < 0$ since $\theta < 1$ by Proposition 1d, proving part (a). To prove part (b), we differentiate π_H^* with respect to r (as in the two-player case),

$$\frac{d\pi_H^*}{dr} = \frac{\partial \pi_H^*}{\partial r} + \frac{\partial \pi_H^*}{\partial \theta}\frac{\partial \theta^*}{\partial r}.$$

Employing Proposition 1c and 1a, we respectively get,

$$\frac{\partial \pi_H^*}{\partial \theta} = \frac{\partial p_H^*}{\partial \theta}(1 - r + rp_H^*) + rp_H^*\frac{\partial p_H^*}{\partial \theta},$$

and

$$\frac{\partial p_H^*}{\partial \theta} = rn\frac{\theta^{r-1}}{(n + m\theta^r)^2} > 0.$$

From here, it follows that $\frac{\partial \pi_H^*}{\partial \theta^*} > 0$, which implies $\frac{\partial \pi_H^*}{\partial \theta^*}\frac{\partial \theta^*}{\partial r} < 0$. Moreover,

$$\frac{\partial \pi_H^*}{\partial r} = \frac{\partial p_H^*}{\partial r}(1 - r + rp_H^*) + p_H^*(-1 + p_H^* + r\frac{\partial p_H^*}{\partial r}),$$

and

$$\frac{\partial p_H^*}{\partial r} = n\theta^r\frac{\ln\theta}{(n + m\theta^r)^2} < 0,$$

which, together implies, $\frac{\partial \pi_H^*}{\partial r} < 0$. As a result, we have $\frac{d\pi_H^*}{dr} < 0$. ■

To understand Proposition 3, note that in general, an increase in the accuracy level, r, introduces a direct "technological" effect and an indirect "competitive" effect measured by θ^*. Part (a) indicates that unlike in the two-player benchmark, superior accuracy creates a competitive advantage for the favorites by motivating them more than the underdogs. Part (b) shows that as the accuracy improves, the technological effect also works against the underdog because, all else equal, the winning becomes more sensitive to effort.

As with the two-player case, the impact of the accuracy on the favorite's payoff is ambiguous. This ambiguity is, however, qualitatively different depending on whether the underdogs or the favorites form the majority in the contest. To ease exposition, we present analytical results for the two extreme values of accuracy and show their robustness through numerical examples later.

Proposition 4 *Suppose there are n favorites and m underdogs. Then,*

(a) *as $r \to 0$,*

$$\frac{d\pi_L^*}{dr}\begin{cases} < 0 & \text{if} \quad c < \exp(1 + \dfrac{n-1}{m}) \\ > 0 & \text{if} \quad c > \exp(1 + \dfrac{n-1}{m}) \end{cases}.$$

(b) *as $r \to 1$,*

$$\frac{d\pi_L^*}{dr}\begin{cases} < 0 & \text{if} \quad c < \widetilde{c}(n,m) \text{ or } c > \dfrac{n}{n-1} \\ > 0 & \text{if} \quad \widetilde{c}(n,m) < c < \dfrac{n}{n-1} \end{cases},$$

where $\widetilde{c}(n,m) \in (1, \frac{n}{n-1})$ uniquely solves: $mc + n + 2mc\ln(\dfrac{n-(n-1)c}{mc-(m-1)}) = 0$. Moreover, $\widetilde{c}(n,m)$ is decreasing in n and m, each converging to 1.

Proof. Employing Proposition 1c to differentiate π_L^* with respect to r provides

$$\frac{d\pi_L^*}{dr} = \frac{dp_L^*}{dr}(1 - r + 2rp_L^*)V - p_L^*(1-p_L^*)V, \tag{7}$$

where

$$p_L^* = \frac{1}{n + m \times (\theta^*)^r} \text{ by Proposition 1a.} \tag{8}$$

Differentiating p_L^* with respect to r

$$\frac{dp_L^*}{dr} = \frac{\partial p_L^*}{\partial r} + \frac{\partial p_L^*}{\partial \theta^*}\frac{\partial \theta^*}{\partial r}.$$

Routine algebra yields

$$\frac{dp_L^*}{dr} = -(\frac{m \times (\theta^*)^r \ln \theta^*}{(m(\theta^*)^r + n)^2})(1 + r\frac{2(m-1)(\theta^*)^r - cm\theta^* + n}{(1-2r)(m-1)(\theta^*)^r + rcm\theta^* + (1-r)n}), \tag{9}$$

where we have also used (4) to obtain $\frac{\partial \theta^*}{\partial r}$. Using (4) to obtain the limit values of θ^*

$$\lim_{r \to 0} \theta^* = \frac{1}{c}. \tag{10}$$

$\lim\limits_{r \to 0} p_L^*$ and $\lim\limits_{r \to 0} \frac{dp_L^*}{dr}$ can be calculated after substituting (10) into (8) and (9)

$$\lim_{r \to 0} p_L^* = \frac{1}{m+n}, \tag{11}$$

$$\lim_{r \to 0} \frac{dp_L^*}{dr} = \frac{m}{(m+n)^2} \ln c > 0. \tag{12}$$

Likewise, $\lim\limits_{r \to 0} \dfrac{d\pi_L^*}{dr}$ can be calculated after substituting (11) and (12) into (7)

$$\lim_{r \to 0} \frac{d\pi_L^*}{dr} = \frac{m \ln c - (m+n-1)}{(m+n)^2} V,$$

which implies $\lim_{r \to 0} \dfrac{d\pi_L^*}{dr} < 0$ if and only if $c < \exp(1 + \dfrac{n-1}{m})$, proving part (a). To prove part (b), we follow the same steps as in part (a). So, we first refer to (4) to obtain

$$\lim_{r \to 1} \theta^* = \frac{n-(n-1)c}{m(c-1)+1}. \tag{13}$$

After substituting (13) into (8) and (9), $\lim\limits_{r \to 1} p_L^*$ and $\lim\limits_{r \to 1} \frac{dp_L^*}{dr}$ are found as

$$\lim_{r \to 1} p_L^* = \frac{m(c-1)+1}{n+mc}, \tag{14}$$

$$\lim_{r \to 1} \frac{dp_L^*}{dr} = -cm \frac{(m+n-1)}{(n+cm)^2} \ln\left(\frac{n-(n-1)c}{m(c-1)+1}\right) > 0. \tag{15}$$

Likewise, after substituting (14) and (15) into (7), $\lim\limits_{r \to 1} \dfrac{d\pi_L^*}{dr}$ is found as

$$\lim_{r \to 1} \frac{d\pi_L^*}{dr} = -\frac{(m+n-1)(m(c-1)+1)V}{(cm+n)^3} \left(cm + n + 2cm \ln\left(\frac{n-(n-1)c}{m(c-1)+1}\right)\right).$$

Letting,

$$f = mc + n + 2mc \ln\left(\frac{n-(n-1)c}{m(c-1)+1}\right),$$

it can be rewritten as

$$\lim_{r \to 1} \frac{d\pi_L^*}{dr} = \left(-\frac{(m+n-1)(m(c-1)+1)V}{(cm+n)^3}\right) * f.$$

Obviously, the term in parenthesis is always negative. Hence, we need to examine the sign of f. To this end, we differentiate f with respect to c to find

$$\frac{\partial f}{\partial c}=-m\frac{m\,(n-1)\,c^2-(2mn-3(m+n-1))\,c+n\,(m-1)}{(n-(n-1)c)\,(mc-(m-1))}+2m\ln(\frac{n-(n-1)c}{m(c-1)+1}).$$

$1\leq c<\frac{n}{n-1}$ assures

$$\frac{\partial f}{\partial c}<0,$$

it follows,

$$-\infty=f(\frac{n}{n-1})<f(c)<f(1)=m+n\ \text{ for } 1<c<\frac{n}{n-1},$$

which together with the continuity of $f(c)$ over $(1,\frac{n}{n-1})$ assures the existence of the solution by the intermediate value theorem. Moreover, the uniqueness comes from monotonicity. Denoting the unique solution by $\widetilde{c}(n,m)$ generates the conditions in part (b). Note that

$$\lim_{c\to 1}(\lim_{r\to 1}\frac{d\pi_L^*}{dr})=-\frac{m+n-1}{(m+n)^2}V,$$

which implies

$$\lim_{n\to\infty}\widetilde{c}=\lim_{m\to\infty}\widetilde{c}=1.$$

Finally, to show $\widetilde{c}(n,m)$ is decreasing in n and m, we utilize implicit differentiation.

$$\frac{\partial\widetilde{c}(n,m)}{\partial n}=-\frac{\frac{\partial f}{\partial n}}{\frac{\partial f}{\partial c}}|_{f=0}\ \text{and}\ \frac{\partial\widetilde{c}(n,m)}{\partial m}=-\frac{\frac{\partial f}{\partial m}}{\frac{\partial f}{\partial c}}|_{f=0}\ ,$$

where f is defined as above. Taking the derivatives and calculating them at $f=0$,

$$\frac{\partial\widetilde{c}(n,m)}{\partial n}=-\frac{[\widetilde{c}\,(m\widetilde{c}-(m-1))][2m\widetilde{c}(\widetilde{c}-1)+(n-1)\widetilde{c}-n]}{(2\widetilde{c}^2)\,m^2+((2n-1)\,n\widetilde{c}-\widetilde{c}^2\,(n-1)\,(n-2)-n^2)\,m+n\,(n-(n-1)\,\widetilde{c})}\ ,$$

and

$$\frac{\partial\widetilde{c}(n,m)}{\partial m}=-\frac{\widetilde{c}}{m}\frac{[(n-(n-1)\widetilde{c})][2m^2\widetilde{c}\,(\widetilde{c}-1)+(m\widetilde{c}-(m-1))\,n]}{(2\widetilde{c}^2)\,m^2+((2n-1)\,n\widetilde{c}-\widetilde{c}^2\,(n-1)\,(n-2)-n^2)\,m+n\,(n-(n-1)\,\widetilde{c})}\ .$$

Note that $1\leq\widetilde{c}<\frac{n}{n-1}$ implies

$$\frac{\partial\widetilde{c}}{\partial n}<0\ \text{and}\ \frac{\partial\widetilde{c}}{\partial m}<0,$$

showing that $\widetilde{c}(n, m)$ is decreasing in n and m. ■

Part (a) of Proposition 4 indicates that when the initial accuracy level is very low, the favorite prefers higher accuracy if his cost advantage is significant. This is in line with the benchmark case where, in a sufficiently uneven contest, higher accuracy discourages the underdog, though not to the extent of dropping out of the competition. More interestingly for the case of multiple players, the favorite is more likely to support higher accuracy as more underdogs and/or fewer favorites compete. This also makes sense because, in either situation, the balance of competition tips to the advantage of the favorite. Part (b) of Proposition 4 looks at the other extreme where the initial accuracy level is very high. Interestingly, in this case the favorite supports a further increase in accuracy only if his cost advantage is moderate. In particular, contrary to part (a), for high cost advantage, i.e., $c > \dfrac{n}{n-1}$, the favorite does not support a higher accuracy when r is close to 1. The reason is that if the contest technology is already very accurate, underdogs exert little effort, which means that the competition takes place mostly among favorites – in other words, among the identical players.

To understand how the favorite's payoff changes for intermediate values of the accuracy, r, we next present two numerical examples.

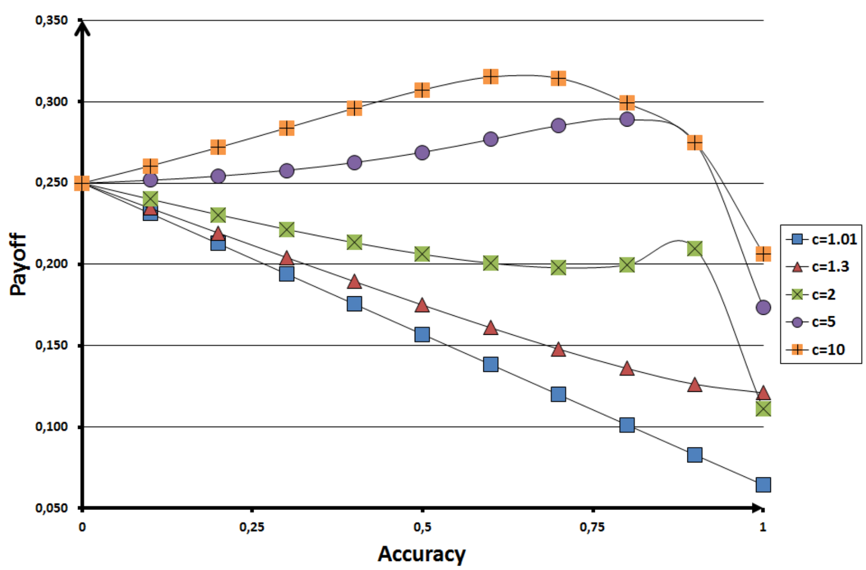

Figure 2. Favorite's Payoff vs Accuracy when n=m=2

Figure 2 illustrates the case in which there are 2 players from each type. It indicates that when the cost advantage is small enough, the favorite's payoff monotonically decreases with accuracy. An increase in the cost advantage, however, makes this monotonicity disappear. First, the pattern is "decrease-increase-decrease" followed by "increase-decrease" with further increases in the cost advantage.

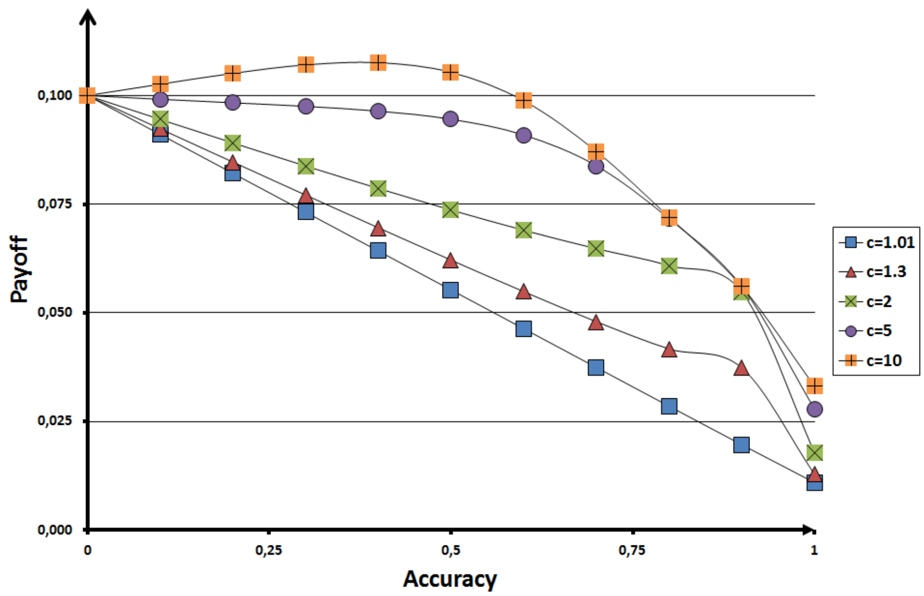

Figure 3. Favorite's Payoff when n=m=5

Figure 3 illustrates the case where there are 5 players from each type. It follows the same patterns as in Figure 2, and the only difference seems to be the smaller cutoff values. However, this is not true. Unlike in Figure 2 where the monotonicity is observed for small enough cost advantages, monotonicity is observed for some high cost advantages.

5 Total Effort and Accuracy

Up to now, we have analyzed players' willingness to support higher accuracy. We now turn our attention to the designer's incentives to adopt higher accuracy. Naturally, this requires making an assumption about his objective. While the designer's objective often varies across contests, total effort maximization has become the primary focus of many studies (see Related Literature).[11] So, in what follows, we assume that total effort maximization is the only concern of the designer.

The following proposition analyses the impact of higher accuracy on total effort (hereinafter, TE), the designer's only concern.

Proposition 5 *Suppose there are* n *favorites and* m *underdogs. If the initial accuracy is very low, then total effort increases with accuracy. However, if the initial accuracy is very high, then total effort increases with accuracy unless the cost advantage is moderate, and it decreases otherwise. Formally,*

(a) *as* $r \to 0$,

$$\frac{dTE}{dr} > 0 \quad .$$

(b) *as* $r \to 1$,

$$\begin{array}{ll} \frac{dTE}{dr} > 0 & \text{if} \quad c < \widehat{c}(n,m) \text{ or } c > \frac{n}{n-1} \\ \frac{dTE}{dr} < 0 & \text{if} \quad \widehat{c}(n,m) < c < \frac{n}{n-1} \end{array} ,$$

where $\widehat{c}(n,m) \in (1, \frac{n}{n-1})$ *uniquely solves:* $mc + n + mn\,(c-1)\ln(\frac{n-(n-1)c}{mc-(m-1)}) = 0$. *Moreover,* $\widehat{c}(n,m)$ *is decreasing in* n *and* m.

Proof. Total effort is given by

$$TE = nx_L^* + mx_H^* \ .$$

By taking the common parenthesis of x_H^* and remembering that $\theta^* = \frac{x_H^*}{x_L^*}$, it can be rewritten as

$$TE = (\frac{n}{\theta^*} + m)x_H^*. \tag{16}$$

Recall that x_H^* and p_H^* are given by

$$x_H^* = \frac{r}{c_H} p_H^*(1 - p_H^*)V, \tag{17}$$

[11] For instance, in sports, the designer is usually interested in total effort maximization since, all else equal, higher effort has a positive impact on viewers' willingness to pay for watching the game. In job interviews, however, he is mostly concerned about selecting the highest-ability candidate because such a candidate is more valuable for the firm.

and

$$p_H^* = \frac{\theta^r}{n + mx\theta^r}, \tag{18}$$

by Proposition 1a & 1b respectively. Substituting (17) and (18) into (16), it becomes

$$TE = \frac{V}{c_H} \frac{r\theta^{r-1}}{(m\theta^r + n)^2} (m\theta + n)\left((m-1)\theta^r + n\right). \tag{19}$$

Notice that θ can be rewritten as

$$\theta = \frac{\left((m-1)\theta^r + n\right)\theta^r}{(m\theta^r + n - 1)c}, \tag{20}$$

by (4). Substituting (20) into (19) and recalling that $c = \frac{c_H}{c_L}$, it becomes

$$TE = \frac{V}{c_L} \frac{r\,(m\theta + n)\,(m\theta^r + n - 1)}{(m\theta^r + n)^2}. \tag{21}$$

Letting

$$h = \frac{r\,(m\theta + n)\,(m\theta^r + n - 1)}{(m\theta^r + n)^2},$$

it becomes

$$TE = \frac{V}{c_L} h.$$

Differentiating TE with respect to r, we obtain

$$\frac{d(TE)}{dr} = \frac{V}{c_L}\left[\frac{\partial h}{\partial r} + \frac{\partial h}{\partial \theta}\frac{\partial \theta}{\partial r}\right], \tag{22}$$

where

$$\frac{\partial h}{\partial r} = (m\theta + n)\frac{(m\theta^r + n)\,(m\theta^r + n - 1) - rm\theta^r\,(m\theta^r + n - 2)\ln\theta}{(m\theta^r + n)^3}, \tag{23}$$

$$\frac{\partial h}{\partial \theta} = rm\frac{\theta\,(m\theta^r + n)\,(m\theta^r + n - 1) - r\theta^r(m\theta + n)(m\theta^r + n - 2)}{(m\theta^r + n)^3\,\theta},$$

$$\frac{\partial \theta}{\partial r} = \frac{n - mc\theta + 2(m-1)\theta^r}{(1-r)n + (1-2r)(m-1)\theta^r + rmc\theta}\theta\ln\theta.$$

Referring to (4) to obtain the limit values of θ^*, we get

$$\lim_{r\to 0}\theta^* = \frac{1}{c}, \tag{24}$$

and

$$\lim_{r\to 1}\theta^* = \frac{n-(n-1)c}{m(c-1)+1}. \tag{25}$$

Using (24), (23), and (22) together

$$\lim_{r\to 0}\frac{d(TE)}{dr} = \frac{V}{c_L}[\frac{(m+n-1)\,(m+cn)}{c\,(m+n)^2}],$$

which implies $\lim_{r\to 0}\frac{d(TE)}{dr} > 0$ for all c, showing part (a).
Similarly, using (25), (23), and (22) together

$$\lim_{r\to 1}\frac{d(TE)}{dr} = \frac{V}{c_L}[(m+n-1)\,\frac{mc+n+mn\,(c-1)\ln(\frac{n-(n-1)c}{mc-(m-1)})}{(mc+n)^2}].$$

Letting,

$$f = mc+n+mn\,(c-1)\ln(\frac{n-(n-1)c}{mc-(m-1)}),$$

it can be rewritten as

$$\lim_{r\to 1}\frac{d(TE)}{dr} = (\frac{V}{c_L}\frac{(m+n-1)}{(mc+n)^2}) * f.$$

As the term in parenthesis is always positive, we will just focus on f. Note that

$$\lim_{c\to 1} f = m \quad \text{and} \quad \lim_{c\to\frac{n}{n-1}} f = -\infty,$$

which together with the continuity of f over $c\in(1,\frac{n}{n-1})$ assures the existence of the solution by the intermediate value theorem. To show uniqueness, we first calculate the first and second derivatives of f,

$$\frac{\partial f}{\partial c} = -\frac{m\,(m\,(n-1)\,c^2-(m-n-1)\,(n-1)\,c-n^2)}{(n-(n-1)c)\,(mc-(m-1))} + mn\ln(\frac{n-(n-1)c}{mc-(m-1)}),$$

$$\frac{\partial^2 f}{\partial c^2} = -\frac{mn\,(m+n-1)\,((m-n+1)\,c+(n-m+1))}{(c+n-cn)^2\,(cm-m+1)^2} < 0.$$

Note that,

$$\frac{\partial f}{\partial c}|_{c=1} = m,\ f(c=1) = m \ \text{ and } \ \frac{\partial f}{\partial c}|_{c=\frac{n}{n-1}} = -\infty,\ \lim_{c\to\frac{n}{n-1}} f(c) = -\infty\ ,$$

which together with concavity assures the uniqueness of the solution. Letting $\widehat{c}(n,m)$ denote this solution generates the conditions in part (b). Finally, to show $\widehat{c}(n,m)$ is decreasing in n and m, we utilize implicit differentiation.

$$\frac{\partial \widehat{c}(n,m)}{\partial n} = -\frac{\frac{\partial f}{\partial n}}{\frac{\partial f}{\partial c}}|_{f=0} \text{ and } \frac{\partial \widehat{c}(n,m)}{\partial m} = -\frac{\frac{\partial f}{\partial m}}{\frac{\partial f}{\partial c}}|_{f=0}.$$

Routine algebra yields,

$$\frac{\partial \widehat{c}}{\partial n} = -\frac{m}{n}(\widehat{c}-1)\frac{(m\widehat{c}-(m-1))((n^2-n+1)\widehat{c}^2-(2n-1)n\widehat{c}+n^2)}{\widehat{c}(\widehat{c}-1)m^2+\widehat{c}m+n(n-(n-1)\widehat{c})},$$

and

$$\frac{\partial \widehat{c}}{\partial m} = -\frac{n}{m}(\widehat{c}-1)\frac{(n-(n-1)\widehat{c})(m^2\widehat{c}^2-(2m-1)m\widehat{c}+(m^2-m+1))}{\widehat{c}(\widehat{c}-1)m^2+\widehat{c}m+n(n-(n-1)\widehat{c})}.$$

$1 < c < \frac{n}{n-1}$ implies:

$$\frac{\partial \widehat{c}}{\partial n} < 0 \text{ and } \frac{\partial \widehat{c}}{\partial m} < 0,$$

which completes the proof. ■

Part (a) of Proposition 5 indicates that when the initial accuracy is very low, total effort increases with accuracy independent of the size of the cost advantage. This is in line with the intuition that a decrease in accuracy weakens players' incentives for effort because lower accuracy is associated with lower effort sensitivity of winning. In the extreme, when accuracy is zero, it becomes a pure lottery in which no players exert any effort.

Part (b) of Proposition 5 indicates that when the cost advantage is either small enough or big enough, total effort increases with accuracy for very high initial accuracy. The intuition behind it is simple. When the cost advantage is either small enough or high enough, the contest essentially takes place among identical players. Specifically, when it is small enough, all players are essentially identical. When it is big enough, however, the competition takes place among favorites for very high initial accuracy. This is because high initial accuracy intensifies the cost advantage, which is already significant. This discourages underdogs, leading to very low effort provision in the equilibrium by them. Evidently, when competition takes place among identical players, higher accuracy leads to greater total effort. Thus, higher accuracy leads to greater total effort when the initial accuracy is very high and the cost advantage is not moderate.

Up to now, our analysis has yielded conditions under which the players and the designer would want to adopt new technology that improves contest accuracy. Equipped with these conditions, we are ready to highlight the cases in which the conflict of interest arises between the players and the designer.

Proposition 6 *Suppose there are* n *favorites,* m *underdogs, and let* $\widetilde{c}(n,m)$ *and* $\widehat{c}(n,m)$ *be defined as above. Then, regardless of the asymmetry between types, conflict over an accuracy improvement arises. The type of conflict, however, depends on initial accuracy, asymmetry, and number of players from each type. Formally,*

(a) *as* $r \to 0$,

$$\begin{array}{ll} \frac{dTE}{dr} > 0 \text{ and } \frac{d\pi_L^*}{dr}, \frac{d\pi_H^*}{dr} < 0 & \text{if} \quad 1 \leq c < \exp(1 + \frac{n-1}{m}) \\ \frac{dTE}{dr}, \frac{d\pi_L^*}{dr} > 0 \text{ and } \frac{d\pi_H^*}{dr} < 0 & \text{if} \quad c > \exp(1 + \frac{n-1}{m}) \end{array}.$$

(b) *as* $r \to 1$,

$$\begin{array}{ll} \frac{dTE}{dr} > 0 \text{ and } \frac{d\pi_L^*}{dr}, \frac{d\pi_H^*}{dr} < 0 & \text{if} \quad 1 \leq c < \widetilde{c} \text{ or } c > \frac{n}{n-1} \\ \frac{dTE}{dr}, \frac{d\pi_L^*}{dr} > 0 \text{ and } \frac{d\pi_H^*}{dr} < 0 & \text{if} \quad \widetilde{c} \leq c < \widehat{c} \\ \frac{d\pi_L^*}{dr} > 0 \text{ and } \frac{dTE}{dr}, \frac{d\pi_H^*}{dr} < 0 & \text{if} \quad \widehat{c} \leq c < \frac{n}{n-1} \end{array}.$$

Proof. Directly follows from Propositions 4 & 5. ■

Part (a) of Proposition 6 highlights conflicts when the initial accuracy is very low. As mentioned earlier, the designer, unlike the underdog who never prefers higher accuracy under any circumstances, always prefers higher accuracy whenever the initial accuracy is very low. Thus, while the designer is always in conflict with the underdogs, he may be in agreement with the favorites. This suggests that there may be two conflict types where the conflict is either between the designer and both types of player or between the designer and underdogs. Part (a) of Proposition 6, indeed, points to these two conflict types. Specifically, it suggests that the designer vs. players conflict occurs when the cost asymmetry is below a certain threshold, and that the designer vs. underdog conflict occurs when the cost asymmetry is above a certain threshold. Moreover, the threshold increases either as favorites are introduced or as underdogs are removed. Clearly, a greater threshold narrows the no-conflict interval, the length of interval over which the designer and the favorites are in agreement on higher accuracy. Accordingly, adding a favorite and/or removing an underdog makes the consensus between the designer and the favorites more likely.

Part (b) of Proposition 6 highlights conflicts where the initial accuracy is very high. In addition to the two conflict types above, part (b) of proposition 6 points to an extra conflict type which is observed when the cost asymmetry is intermediate. In this type, the conflict arises between the designer who does not prefer higher accuracy and the favorites who do prefer. In the remaining two types, the designer always prefers higher accuracy as in part (a). However, the designer and the favorites reach a consensus on higher accuracy over an interval $[\widetilde{c}(n,m), \widehat{c}(n,m)]$ where $\widetilde{c}(n,m), \widehat{c}(n,m) < \dfrac{n}{n-1}$. This is in stark contrast to part (a) where the favorites and the designer reach a consensus on accuracy improvement for significantly high cost asymmetry (meaning that $c > \exp(1 + \frac{n-1}{m})$). However, it is not entirely clear, particularly compared to part (a), how the addition of either more favorites or more underdogs reduces the upper and lower bounds of the no-conflict interval, which reflects the possibility of conflict since $\Delta c = \widehat{c}(n,m) - \widetilde{c}(n,m)$. For example, consider the interval $[a, b]$. If both a and b decrease with the introduction of favorites and/or underdogs, then it is unclear exactly how the length of the $b - a$ interval changes, since it would depend on whether it is b or a that decreases at higher rate. In other words, both $\widehat{c}(n,m)$ and $\widetilde{c}(n,m)$ decrease with the addition of new players, making the change ambiguous. In order to make it clear, we present the following numerical examples.[12]

[12] Our attempt to show it analytically was unsuccessful.

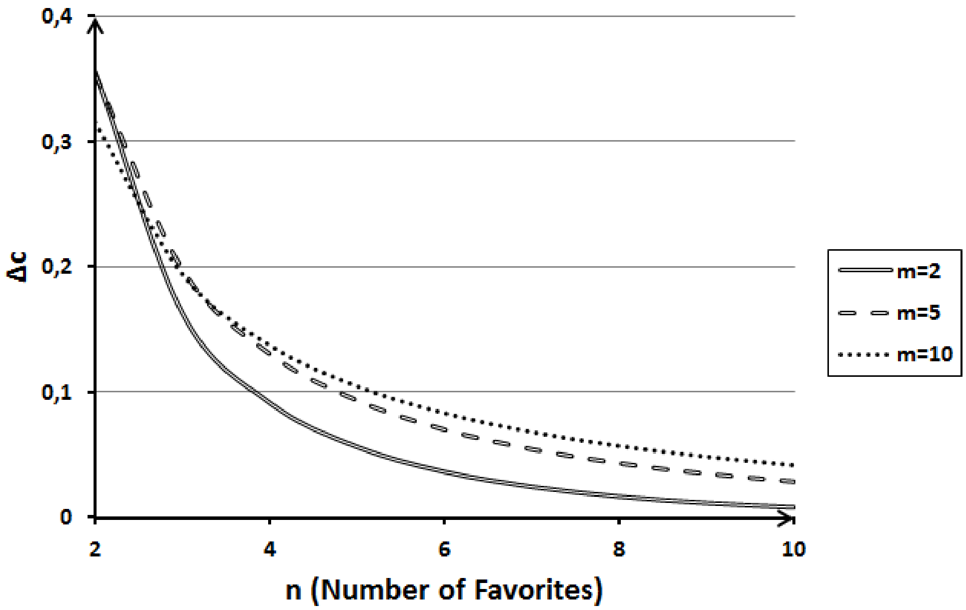

Figure 4. Length of No-Conflict Interval vs Number of Favorites

Figure 4 portrays, fixing the number of underdogs, how the length of no-conflict interval changes with an increase in the number of favorites. It clearly suggests that the no-conflict interval narrows as the number of favorites increases. Simply put, all else equal, this amounts to saying that adding more favorites to the contest, weakens the changes to improve accuracy. This makes sense in that higher accuracy is less appealing to the favorites whenever there are more favorites in competition.

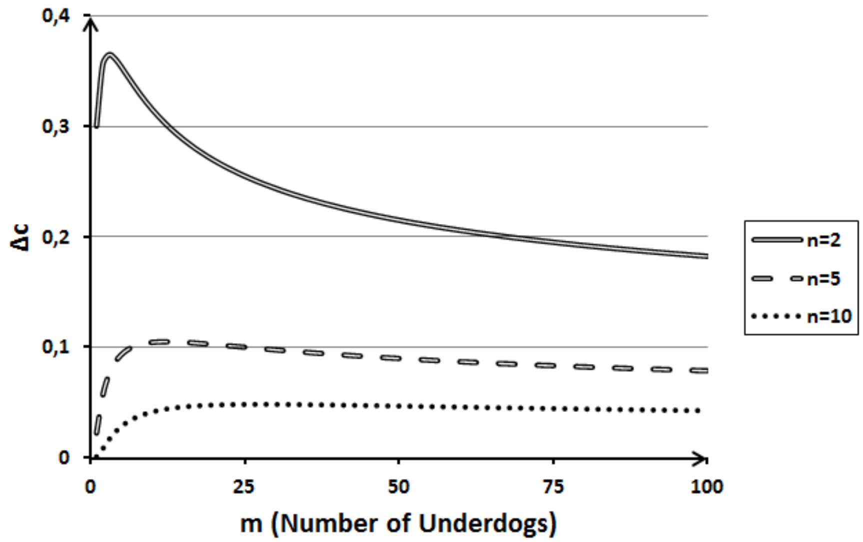

Figure 5: Length of No-Conflict Interval vs Number of Underdogs

Figure 5 portrays, fixing the number of favorites, how the length of no-conflict interval changes with an increase in the number of underdogs. In contrast to Figure 4, it points to non-monotonicity. Specifically, the no-conflict interval first expands, then it shrinks with an addition of underdogs. Said differently, while more underdogs in the contest make the adoption of higher accuracy more likely, this likelihood diminishes over time. To grasp the intuition, consider the extreme case where there are no underdogs, that is, the competition takes place among the favorites, i.e. homogeneous contest. As explained earlier, in such a case, higher accuracy is always preferred by the designer only not the favorites. In other words, the length of no-conflict interval is 0. Addition of an underdog to the contest, however, leads to a non-zero interval length. Consider now the other extreme where there are infinitely many underdogs. Clearly, this contest can be considered as homogeneous as long as the number of favorites is finite. Consequently, no-conflict interval eventually vanishes.

6 Discussion

Our analysis features discrepancies in the preferences of players and designer for higher accuracy. While these discrepancies may not be important for contests where players' preferences do not matter for the design process, they are clearly crucial for others where they do matter. According to Dixon (2004), most students choose letter grading over plus/minus grading despite the fact that the latter leads to greater grading accuracy than the former. Accordingly, we expect to see letter grading instead of plus/minus grading in schools whenever the players' preferences matter for the designer. The report of the Ad-Hoc Committee of Eastern Kentucky University revealed that those not using plus/minus grading cited student resistance as one factor. We conjecture that players' preferences are more likely to matter in settings where there are large and long-term players. The rationale behind our conjecture is multi-faceted. Firstly, the design process can be democratic in that any proposal by the designer needs to be agreed upon by players. Secondly, when the contest designer is selected by the players, he may value their preferences to increase his re-election prospects. Finally, large players may influence the designer's decision by various means such as lobbying or by pressuring him. In team sports such as soccer or baseball, for instance, competition takes place among long-term teams where some are disproportionately powerful. Yet, in individualistic sports such as tennis, golf, or athletics, the players are short-term and can not hold too much power. In line with this conjecture, our analysis predicts that team sports such as soccer or baseball are associated with less accuracy than individualistic sports. This prediction appears consistent with current practices in sports. In addition to the previous prediction that seems to correctly address the accuracy disparities across contests that differ in the significance of players' preferences on design process, our analysis has another prediction about accuracy disparities across contests that differ in the level of heterogeneity. Specifically, our analysis predicts that greater heterogeneity is closely linked with higher accuracy whenever players' preferences matter for the designer. The evidence from European soccer leagues presented in the Introduction, seems to support this prediction. More specifically, the evidence shows that the soccer leagues of England and the Netherlands, the most heterogeneous ones, have recently decided to implement a goal-line technology while those of Germany and Spain delayed their decision until 2015. Though somewhat extreme, our limit results in Proposition 4, which states that the favorites support higher accuracy only under certain cases, seems to explain why top tennis players such as Roger Federer and Novak Djokovic have expressed their opinion against the use of technology in tennis.[13]

Apart from the above predictions, our analysis underlines several distinctions between two-player contests and contests with more than two players. First, while a change in accuracy alters players' efforts through direct effect only in two-player contests, this happens with indirect effect in addition to direct effect in contests with more than two players. Second, in a more general case, the effort-maximizing designer does not necessarily decrease the contest accuracy with an increase in heterogeneity. More precisely, our analysis indicates that when there is more than one favorite, it is still optimal for the effort-maximizing designer to increase contest accuracy for significant heterogeneity levels.

7 Pairwise Contests and Accuracy

Up to now, we have assumed that all players compete simultaneously. However, in certain contests, competitions involve only two players. That is, while at the time of accuracy choice, players may not know their rivals. They know that there will be only one opponent. In this section, we demonstrate that our results above continue to hold.

Suppose there are once again, n favorites and m underdogs. Conditional on being a favorite, the opponent is a favorite with a probability $\frac{n-1}{m+n-1}$, and an underdog with a probability $\frac{m}{m+n-1}$.

Proposition 7 *Suppose each contest involves only two players and opponents are drawn randomly. Moreover, the accuracy, r, is decided before there are n favorites and m underdogs where a given player will compete with his opponent which is to be drawn randomly. Then,*

(a) The underdog's payoff is decreasing in the accuracy, i.e. $\frac{dE[\pi_H^*]}{dr} < 0$.

(b) The favorite's payoff is ambiguous in the accuracy. Specifically,

- as $r \to 0$,

$$\frac{dE[\pi_L^*]}{dr} \begin{cases} > 0 & \text{if} \quad c > \exp(1 + \frac{n-1}{m}) \\ < 0 & \text{if} \quad c < \exp(1 + \frac{n-1}{m}) \end{cases} .$$

[13] For more information, see: <http://www.independent.co.uk/sport/tennis/djokovic-resists-hawkeye-calls-1990418.html.>

- as $r \to 1$,

$$\frac{dE[\pi_L^*]}{dr} \begin{cases} > 0 & \text{if} \quad \frac{n-1}{m} < k \text{ and } \underline{c}(m,n) < c < \overline{c}(m,n) \\ < 0 & \text{if} \quad \text{otherwise} \end{cases} .$$

where $k \approx 1.06$ uniquely solves: $\max_{\{c\in[1,\infty)\}} \dfrac{4c(2c\ln c - c - 1)}{(c+1)^3}$.

Proof. Let π_{ij}^* denote the payoff of type i in a pairwise contest when his opponent is of type j. Also let $E[\pi_i^*]$ denote the expected payoff of type i. Since the opponent is chosen randomly,

$$E[\pi_H^*] = \frac{m-1}{m+n-1}\pi_{HH}^* + \frac{n}{m+n-1}\pi_{HL}^*.$$

Differentiating with respect to r,

$$\frac{dE[\pi_H^*]}{dr} = \frac{m-1}{m+n-1}\frac{d\pi_{HH}^*}{dr} + \frac{n}{m+n-1}\frac{d\pi_{HL}^*}{dr}$$

From the proof of Proposition 2, $\frac{d\pi_{HL}^*}{dr} < 0$, and we have $p_{HH}^* = \frac{1}{2}$. Substituting it into Proposition 1c which leads to $\pi_{HH}^* = \frac{2-r}{4}V$. Note that $\frac{d\pi_{HH}^*}{dr} = -\frac{V}{4} < 0$ and $\frac{d\pi_{HL}^*}{dr} < 0$ by Proposition 2, which proves part (a). In a similar vein, we have $E[\pi_L^*] = \frac{n-1}{m+n-1}\pi_{LL}^* + \frac{m}{m+n-1}\pi_{LH}^*$. Differentiating both sides with respect to r,

$$\frac{dE[\pi_L^*]}{dr} = \frac{n-1}{m+n-1}\frac{d\pi_{LL}^*}{dr} + \frac{m}{m+n-1}\frac{d\pi_{LH}^*}{dr}$$

where $\pi_{LL}^* = \dfrac{2-r}{4}V$ and $\pi_{LH}^* = \dfrac{c^r(c^r - r + 1)}{(c^r+1)^2}V$. Routine algebra yields,

$$\frac{dE[\pi_L^*]}{dr} = \frac{n-1}{m+n-1}(-\frac{V}{4}) + \frac{m}{m+n-1}\frac{c^r((1+c^r-r+rc^r)\ln c - (1+c^r))}{(1+c^r)^3}V$$

Taking the limit as $r \to 0$ we have,

$$\lim_{r\to 0}\frac{dE[\pi_L^*]}{\partial r} = \frac{m\ln c - (m+n-1)}{4(m+n-1)}V$$

it follows $\dfrac{d\pi_L^*}{dr} < 0$ if and only if $c < \exp(1 + \dfrac{n-1}{m})$, proving part (a).
Taking the limit as $r \to 1$ we have,

$$\lim_{r\to 1}\frac{dE[\pi_L^*]}{\partial r} = m(c+1)^3(\frac{4c(2c\ln c - c - 1)}{(c+1)^3} - \frac{n-1}{m})V$$

Similarly, $\lim_{r\to 1} \dfrac{\partial(E_P[\pi_L^*])}{\partial r} > 0$ if and only if $1+\overline{k}\times m \geq n$ and $\underline{c}(m,n) < c < \overline{c}(m,n)$, where $\overline{k} \approx 1.06$ uniquely solves $\overline{k} \in \max\{\dfrac{4c(2c\ln c - c - 1)}{(c+1)^3}\}$, and $\underline{c}$, $\overline{c}$ solve $\dfrac{4c(2c\ln c - c - 1)}{(c+1)^3} - \dfrac{n-1}{m} = 0$. ■

Proposition 6 confirms that the accuracy choices of the players in pairwise contests are qualitatively the same as in simultaneous contests with many players. Regardless of the initial accuracy, the underdogs never prefer higher accuracy in both settings. On the other hand, the favorites prefer higher accuracy if and only if either the initial accuracy is very low and the cost advantage is significant, or the initial accuracy is very high and the cost advantage is moderate.

8 Concluding remarks

Contest accuracy, or the extent to which winning depends on the effort rather than exogenous uncertainty, often varies across contests. While this variation may be expected among contests with different design objectives, it is rather a puzzle for those with exactly the same and /or relatively similar design objectives. Today, for instance, while some schools switched from letter grading to plus/minus grading to enhance grading accuracy, others continue to use letter grading. When asked, the latter points to student resistance among other factors. On the other hand, all sports are often designed with the common purpose of providing players with appropriate incentives to perform well. Yet, today only certain sports do use technology. A closer look reveals that they are mostly individualistic sports such as tennis, athletics, or horse-racing rather than team sports such as soccer or baseball. One notable distinction between them is that while the set of players often changes in individualistic sports, it remains the same in team sports. More importantly, the players hold less power in individualistic sports than in team sports. Our conjecture is that the contest designer takes the players' preferences more seriously in team sports compared individualistic sports. In the light of this conjecture, our analysis offers one possible explanation for this puzzle by emphasizing the discrepancies in the preferences of players and the contest designer. Our analysis also offers an explanation for the accuracy differences across the same contests differing only with regard to composition of the players. Specifically, our analysis predicts higher accuracy in less heterogeneous contests, which seems to be in line with the evidence in European soccer. One interesting finding is that the favorites support higher accuracy only under certain cases. This seems to explain why top tennis players such as Roger Federer and Novak Djokovic have expressed their opinions against the use of technology in tennis.[14]

Our analysis highlights several distinctions between two-player contests and more general contests that may be useful for future research. First, a change in accuracy alters players' efforts through an indirect effect, or competitive effect, which is absent in the two-player case. Second, in contrast to the two-player case, effort maximization does not necessitate lowering contest accuracy for excessive cost asymmetry between types.

[14] For more information, see: <http://www.independent.co.uk/sport/tennis/djokovic-resists-hawkeye-calls-1990418.html.>

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