

# Are the Nordic Stock Markets Mean Reverting ?

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## Abstract

In this paper we test for mean reversion in the Nordic stock markets using monthly nominal data 1947-1998. By simply account for the heteroscedasticity of the data with a regime-switching model of normal distributions and taking estimation bias into account via a Bayesian approach we can find no support of mean reversion. This is a contradiction to some previous result from Denmark and Sweden. Our findings suggest that the Nordic stock markets can be characterized by two regimes and within the regimes the stock markets is random. This finding of randomness is in line with recent evidence in literature.

# 1 Introduction

This paper addresses the question of whether or not the Nordic stock markets are subject to mean reversion. Previous research by Frennberg and Hansson (1993) on the Swedish stock market and by Risager (1998) on the Danish stock market has found stock returns to exhibit mean reversion. The evidence of mean reversion via variance ratio, VR, is controversial because as the null hypothesis of random walk is only valid under the assumption of constant expected return especially as asset returns are well known to exhibit time varying volatility. Hence, mean reversion might be explained by the time-variation in volatility and taking this heteroscedacity into consideration the market might be efficient. Kim, Nelson and Startz (1991, 1998a) questions the significant mean reversion some times found when using VR statistic and argue that it might in fact be explained by time varying volatility. They conclude that returns are white noise. Nielsen and Overgaard-Olesen (1999) find weak support of mean reversion when they employ Hidden Markov models and compute variance ratio test on annual Danish stock market data. Malliaropulos and Priestly (1999) utilize a bootstrap approach to test for mean reversion in international stock market data. Graflund (2000) can not find any support of mean reversion in Swedish excess returns when accounting for the time varying volatility with a regime-switching model.<sup>1</sup>

This study differs from previous studies on the Nordic stock markets in that we employ Bayesian approach to test for mean reversion on standardized returns as suggested by Kim et al (1998a). The idea is to capture the time variation in the variance by a regime-switching model of Gaussian mixtures. Goldfelt and Quandt (1973) introduced the regime-switching models but its application in economics and finance came a decade ago when Hamilton (1989) employed a two-state hidden Markov model (HMM) on GDP data. The drawback with the HMM is the multi modality of the likelihood function and ordinary optimization techniques are likely to fail.<sup>2</sup> Albert and Chib (1993) suggest a Bayesian approach and estimate the marginal distributions of the parameters in a two-state HMM suggested by Hamilton (1989) with Gibbs sampling.<sup>3</sup> Geman and Geman

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<sup>1</sup>This paper is an independent extended version of Graflund (2000) "A Bayesian approach to testing mean reversion in the Swedish stock markets".

<sup>2</sup>Ordinary optimisation algorithms often fail to estimate the true HMM correct. Another approach is to employ the simulated annealing, SA, algorithm. This is also a MCMC approach and thus, computer intensive.

<sup>3</sup>Kim et al (1998a, 1998b) extended Albert and Chib's model to a three-state HMM. See the papers by Lunginbuhl and De Vos (1999) and Dueker (1999) for other applications of the gibbs sampling framework

(1984)'s Bayesian framework of Gibbs sampling is very advantageous. First, we can use prior information in the estimation of the conditional distribution of the parameters, without estimation of a likelihood function. This is an appealing approach as the likelihood function of hidden Markov models can be cumbersome to estimate. Second, all inferences in Gibbs sampling are made from joint distributions of the variates and the unknown parameters of the model. Thus, we are able to account for the parameter uncertainty of the underlying parameters in the model.

The Nordic countries have many common attributions; first they are all small open economies.<sup>4</sup> Second, analysts often tend to view them as a group, and this is also our approach. Our data consists of monthly nominal stock market returns calculated as the logarithmic differences of stock market indexes from Finland (HEX), Norway (BOX), Denmark (KFX) and Sweden (AGI).<sup>5</sup> Our post-war sample period is December 1947 to December 1998 a total of 612 monthly observations.

In our analysis we find no support of mean reversion in the any of the Nordic stock markets. Our regime switching models of normal distributions suggests that mean reversion if found in the Nordic stock markets can be explained by time variation in the volatility and within the regimes the stock markets is random.

The outline of the paper is as follows: In section 2 we describe the underlying assumptions of the variance ratio test. The methodology is presented in section 3. Section 3.1 describes the regime-switching model. A brief presentation of Bayesian statistics is given in section 3.2. The Gibbs sampler and the prior distributions are specified in section 3.3. The Bayesian re-sampled variance ratio tests are presented in section 3.4. Section 4 presents the results and section 5 concludes the paper.

## 2 Variance ratio

The variance ratio test, VR, of Cochrane (1988) have been frequently used as a test of mean reversion. The advantage of the test is that it allows us to study if returns follows a random walk and if this property changes with the investment horizon  $q$ . We compute the  $q$  period return  $R_t^q$  as the  $q$  period difference between the log of the monthly index values of the portfolio  $I_t$  and  $I_{t-q}$ . In our case this is done for each one of the stock market

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of Albert and Chib (1993).

<sup>4</sup>The Nordic countries include Iceland. Due to lack of available data Iceland is excluded from the study.

<sup>5</sup><http://www.globalfindata.com>

portfolios from Sweden, Norway, Finland and Denmark.

$$R_t^q = I_t - I_{t-q} \quad (1)$$

Let  $r_q$  be the monthly return of a stock market portfolio. Compounded returns,  $I_t$ , are assumed to be a random walk. This implies the arithmetic return being a drift  $\mu$  plus a white noise term  $\varepsilon_t$ . In this context the  $q$ -month arithmetic return is:

$$R_t^q = q\mu + \varepsilon_t + \dots + \varepsilon_{t+q} \quad (2)$$

$$R_t^q = \mu + r_{q-1} + \varepsilon_{t+q} \quad (3)$$

The expected  $q$  period return is equal to the monthly mean return times the holding period  $q$  and the variance of the  $q$  period return is  $q$  times the variance of monthly returns.

$$E[R_t^q] = q\mu, \quad Var[R_t^q] = q\sigma^2 \quad (4)$$

The variance ratio statistic, VR, is defined as:

$$VR(q) = \frac{Var[R_t^q]}{q \cdot Var[R_t^1]} \quad (5)$$

The VR-test equals unity under the null hypothesis of random walk. In our investigation we have chosen the investment horizon  $q$  to range from two to twelve months and yearly up to ten years. This enables us to study the random walk hypothesis both in the short-run and the long-run.

## 3 Methodology

### 3.1 The Regime Switching Model

Let the monthly de-measured excess stock returns  $y_t$  be described as a  $k$ -state regime-switching model of Gaussian mixtures.<sup>6</sup> Where  $S_t$  is an unobserved state variable following a Markov process.

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<sup>6</sup>We have also done estimations using three-state hidden Markov model. The results suggest that a two-state hidden Markov model being more appropriate. The results of the estimations are available on request.

$$y_t \sim N(0, \sigma_i^2) \quad (6)$$

$$\sigma_t^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} \quad (7)$$

subject to the restriction:

$$\sigma_1 < \sigma_2 < \dots < \sigma_k \quad (8)$$

$$\Pr[S_t = j \mid S_{t-1} = i] = p_{ij}, \quad i, j = 1, \dots, k$$

$$\sum_{j=1}^k p_{ij} = 1, \quad i = 1, \dots, k$$

The above model is a standard Markov switching model that can be estimated with maximum likelihood (see Hamilton (1994)). In our case we have chosen two regimes ( $k=2$ ).<sup>7</sup>

### 3.2 Bayesian statistic

The fundamental idea behind Bayesian statistic is to condition on the observed data,  $Y$ , and regard the parameters,  $\theta$ , as random variables. Suppose that  $p(\theta)$  is a probability distribution of the parameter  $\theta$ .

$$p(Y \mid \theta) p(\theta) = p(Y, \theta) = p(\theta \mid Y) p(Y)$$

The probability distribution of  $\theta$  conditional on the observed data is expressed by Bayes theorem:

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<sup>7</sup>We have also done estimations using a three-state hidden Markov model. The results suggest that a two-state hidden Markov model being more appropriate. The results of the estimations are available on request.

$$p(\theta | Y) = \frac{p(Y | \theta) p(\theta)}{p(Y)} \quad (9)$$

where  $p(\theta)$  is the *prior probability density function* and describes the information in  $\theta$  without any knowledge about the data  $Y$ .  $p(\theta | Y)$  is the *posterior probability density function* and gives a description of what is known about  $\theta$  given the data,  $Y$ . Given the data,  $Y$ , the *conditional probability distribution*  $p(Y | \theta)$  can be seen as a function of the parameters  $\theta$ . This function is in fact proportional to the likelihood function of  $\theta$ ,  $L(Y | \theta)$ . Let us consider  $p(Y)$  as being constant, then we can write the above as

$$p(\theta | Y) \propto p(Y | \theta) p(\theta) \quad (10)$$

and

$$p(\theta | Y) \propto L(Y | \theta) p(\theta) \quad (11)$$

Hence, posterior probability density function is proportional to the likelihood function times the prior probability density function and we do not need to specify the likelihood function in order to sample from the marginal distributions of the parameters of interest.

### 3.3 The Gibbs sampler

Introduced in 1984 by Geman and Geman the breakthrough of Gibbs sampling, a special case of the Metropolis-Hastings algorithm (see Metropolis et al (1953) and Hastings (1970)), came with the seminal papers of Gefland and Smith (1990) and Gefland et al (1990). It is easiest explained, as a technique for numerical integration and the difference between Gibbs sampling and M-H algorithm is that all generated candidates are accepted. The Gibbs sampler provides the means to sample from the marginal distribution of the parameters of interest. The idea behind the algorithm is to sample from the conditional distribution of the parameter space  $\{\theta_1, \theta_2, \dots, \theta_k\}$ .

Step 1: Specify arbitrary initial values,  $(\theta_1^{(0)}, \theta_2^{(0)}, \dots, \theta_k^{(0)})$ , and set  $n = 1$ .

Step 2: Cycle through the full conditionals by drawing:

$$(1) \quad \theta_1^{(n)} \text{ from } \left[ \theta_1 | \theta_2^{(n-1)}, \dots, \theta_k^{(n-1)} \right]$$

$$\begin{aligned}
(2) \quad & \theta_2^{(n)} \text{ from } \left[ \theta_2 \mid \theta_1^{(n)}, \theta_3^{(n-1)}, \dots, \theta_k^{(n-1)} \right] \\
& \vdots \\
(k) \quad & \theta_k^{(n)} \text{ from } \left[ \theta_k \mid \theta_1^{(n)}, \dots, \theta_{k-1}^{(n)} \right]
\end{aligned}$$

Step 3: set  $n = n + 1$ , and go to step 2.

This cycle is then repeated  $N$  times and we obtain the sample values  $(\theta_1^{(N)}, \theta_2^{(N)}, \dots, \theta_k^{(N)})$ . Where  $N$  is set to a large number, in our case  $N$  is set to 20.000 iterations.<sup>8</sup> The first  $M$  iterations when the chains have not converged are discarded leaving us with a sample of  $m$  useful iterations. For a large number  $m$  the simulated values  $(\theta_1^{(M)}, \theta_2^{(M)}, \dots, \theta_k^{(M)}) \dots (\theta_1^{(N)}, \theta_2^{(N)}, \dots, \theta_k^{(N)})$  can be treated as an approximate sample from  $[\theta_1, \theta_2, \dots, \theta_k]$  (see Tierney (1994)).

### 3.3.1 Priors and prior distributions

We use conjugate prior distributions and the specification of the prior parameters and their distributions follows from Albert and Chib (1993), Tanner (1996), Kim et al (1998a), Robert and Casella (1999) and Roberts (Gilks et al (1996), Ch 24, pages 441-464).<sup>9</sup>

The probabilities for the Markov process to move from one state  $i$  at time  $t - 1$  to state  $j$  at time  $t$  are called transition probabilities,  $p_{ij} = p(S_t = j \mid S_{t-1} = i)$ . The transition probabilities  $p_{ij}$  are collected in the transition matrix  $\mathbf{P}$ , which forms the nucleus of the Markov model. Each row of the transition probability matrix  $\mathbf{P}$  is generated as random draws from a Dirichlet distribution.<sup>10</sup>

$$\mathbf{P}(i) \sim D(u_{i1} + n_{i1}, u_{i2} + n_{i2}), \quad i = 1, 2 \quad (12)$$

where  $n_{ik}$ , are the number of transitions from state  $i$  to state  $k$ . We consider  $u_{ik}$ ,  $i = 1, 2$ ,  $k = 1, 2$ , as non-informative priors and set them equal to 1.

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<sup>8</sup>This is a computer intensive simulation. All simulations are done in MATLAB. The estimation time is approximately 6 hours per country on a standard Intel PII 450 MHz CPU.

<sup>9</sup>See Gilks et al (1996) "Markov Chain Monte Carlo in Practice" and Tanner (1996) "Tools for Statistical Inference: Methods for the Exploration of Posterior Distributions and Likelihood Functions"

<sup>10</sup>The Dirichlet density function has the property that it can assume a large number of various shapes in the sample space  $[0, 1]$ . Another property of the multivariate Dirichlet distribution is that the sampled probabilities sum to unity. This makes the Dirichlet distribution family very suitable in representing any experiments on multivariate continuous random variables in the  $[0, 1]$  space.

We generate volatilities,  $\sigma_1^2$  and  $\sigma_2^2$ , from the inverse-gamma,  $IG$ , distribution family.<sup>11</sup> In order to satisfy the constraint,  $\sigma_1^2 < \sigma_2^2$ , we need to first generate  $\sigma_1^2$  and re-define  $\sigma_2^2$  conditional on  $\sigma_1^2$ .

$$\sigma_2^2 = \sigma_1^2 (1 + h) \quad (13)$$

where  $h > 0$ . Where  $\sigma_1^2$  and  $\bar{h} = (1 + h)$  are random draws from the inverse-gamma,  $IG$ , distribution family.

$$Y_{1t} = \frac{y_t}{\sqrt{\sigma_1^2 (1 + S_{2t}h)}} \quad (14)$$

$$[\sigma_1^2 \mid \tilde{Y}_{1T}, \tilde{S}_T, \tilde{\theta}_{j \neq \sigma_1^2}] \sim IG \left( \frac{v_1 + T}{2}, \frac{\delta_1 + \sum_{t=1}^T Y_{1t}^2}{2} \right), \quad (15)$$

$$Y_{2t} = \frac{y_t}{\sqrt{\sigma_1^2}} \quad (16)$$

We define  $N_2$  as the number of times state 2 occurs  $N_2 = \{t : S_t = 2\}$  and  $T_2$  is the sum of the elements in  $N_2$ .

$$[\bar{h} \mid \tilde{Y}_{2T}, \tilde{S}_T, \tilde{\theta}_{j \neq \bar{h}}] \sim IG \left( \frac{v_2 + T_2}{2}, \frac{\delta_2 + \sum_{t=1}^{N_2} Y_{2t}^2}{2} \right) I_{[\bar{h} > 1]}, \quad (17)$$

We use non-informative priors and set  $v_1$ ,  $v_2$ ,  $\delta_1$ , and  $\delta_2$  equal 1.

### 3.3.2 States

We regard the states as missing data. Thus, we cannot observe the states. However, we can compute the probability of a given observation  $y_t$  belongs to state  $i$ ,  $i = 1, \dots, k$ , and from this information construct forecast probabilities of which state  $i$ ,  $i = 1, \dots, k$ , observation  $y_{t+1}$  belongs to. The probabilities are computed for all observations  $y_t$ ,  $t = 1 \dots T$ , via the

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<sup>11</sup>A random sample from the inverse gamma is the reciprocal of a draw of a random number from the gamma distribution. The reason to employ the (inverse) gamma density function as a prior distribution is that it enables the researcher to sample nonnegative real numbers. See also Mittelhammer (1995).



local updating algorithm of Robert (1993).<sup>12</sup> This is repeated for every Gibbs run. The local updating algorithm is an forward algorithm in which each state is simulated from the full conditional ( $1 \leq i \leq k$ ). Thus we have utilized the fact that this is a first order Markov chain as the distribution only depends on the value of two neighboring states.

$$p(S_1 = i \mid S_2, \dots, \mathbf{P}) \propto \rho_i p_{iS_2} f(x_1 \mid 0, \sigma)$$

$$p(S_t = i \mid \dots, S_{t-1}, S_{t+1}, \dots, \mathbf{P}) \propto p_{S_{t-1}i} p_{iS_{t+1}} f(x_j \mid 0, \sigma), \quad (1 < t < T)$$

$$p(S_T = i \mid \dots, S_{T-1}, \mathbf{P}) \propto p_{S_{T-1}i} f(x_T \mid 0, \sigma)$$

Where  $(\rho_i, \dots, \rho_k)$  is the stationary distribution of the transition matrix  $\mathbf{P}$  and  $f(\cdot \mid 0, \sigma)$  denotes the density of the normal distribution. Thus, the  $\rho_i$ 's are computed from the transition matrix at each iteration of the Gibbs sampler. Using the probabilities from the local updating algorithm we generate the states  $S = 1, \dots, K$ , from a k-point distribution.  $q_{ijt}$  are the normalized probabilities from the local updating algorithm.

$$q_{ijt} = \frac{p_{jt}}{\sum_{j=1}^k}, \quad (18)$$

$$\sum_{j=1}^k q_{ijt} = 1 \quad (19)$$

Given the regime  $S_i$  the move to regime  $S_j$  is determined by comparing a random draw,  $\mu_t$ , with the cumulative probability distribution.

$$\mu_t = U(0, 1) \quad (20)$$

$$S_t := \min \left[ j : \sum_{i=1}^j q_{ijt} \geq \mu_t \right]. \quad (21)$$

This is then repeated for all observations  $t = 1 \dots T$  at each Gibbs sweep.

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<sup>12</sup>We have also made runs using the backward-forward algorithm. See pages 690-693 in Hamilton (1994) "*Time Series Econometrics*".

### 3.4 A Bayesian approach to variance ratio test

Remember our basic assumption that  $y_t$  is heteroscedastic de-meaned return with volatility  $\sigma_t(\theta)$  which can be described by a mixture of two normal distributions  $y_t \sim N(0, \sigma_t^2)$ .  $\theta = \{\sigma_1^2, \sigma_2^2, S_t, p_{11}, p_{22}\}$  is a parameter vector describing the dynamics of  $\sigma_t^2(\theta)$ . The following two re-sampled based variance ratio tests have been suggested by Kim et al (1998a). At the end of each run of the Gibbs-sampling algorithm the following procedure is computed:

Step 1: We divide the monthly returns  $y_t$  by the standard deviation  $\sigma_t$  in order to get the standardized returns  $y_t^*$ .

Step 2: Scramble the standardized returns  $y_t^*$  to yield a new randomized vector  $y_t^{r*}$ .

Step 3: Create a new series of de-standardized randomized monthly returns  $y_t^r$  by scaling the randomized-standardized returns  $y_t^{r*}$  by the standard deviation  $\sigma_t$ .

Thus we compute four return series, *original returns*  $y_t$ , *standardized original returns*  $y_t^*$ , *randomized standardized returns*  $y_t^{r*}$  and finally *randomized de-standardized returns*  $y_t^r$ . Then we calculate the  $q$ -month variance ratio for the four return series. The significance level of the null hypothesis is the fraction of the variance ratios for the artificial returns that fall below the variance ratios of the original historical returns. In this setting we compute a test on original returns and a test based on standardized returns for every  $q$ -month horizon. At the end of the 20.000 iterations we will have 20.000 realizations of each of the two tests for each of the 20  $q$ -month test horizons. An advantage with our Bayesian approach is that we are able to account for the parameter uncertainty in  $\theta$  as well as the effect of the randomization.

## 4 Empirical Results

### 4.1 Bayesian inferences on parameter estimates

The convergence of the Gibbs sampler or burn in time is determined via monitoring techniques. The convergence of the Gibbs sampler or burn in time has been determined by running several Gibbs sequences and by using different values of the priors. This is done in order to reveal possible slow mixing of the Markov chain. We monitor all parameters of the Gibbs sequence and the burn in time based on the worst scenario, the parameter with the slowest mixing. The mixing, being based on the average value versus

the number of iterations, the transition probability  $p_{11}$  for Sweden, can be seen in figure 1. The variance parameters converge quickly, but the transition probabilities exhibits slow convergence. Thus the burn in time is based on the latter and  $m$  is set to 8.000 iterations, leaving 12.000 Gibbs sequences from which to make statistical inference.

[Figure 1]

The stability of the states for different countries is quite clear from the graphs in figure 2- figure 5. These are called *assignment maps* and plot the assignments of the states as gray levels against the iterations (black for state 1 and white for state 2) (Robert and Mengersen (1999)). Random state vectors gives a blurred assignment map. If the Gibbs sampling algorithm have problems identifying the states the assignment map will have horizontal stripes. However, if the Gibbs sampler at each sweep assigns the same state to the same observation the assignment map will have vertical bars.

Our Gibbs sampler is able to find stable allocations for each one of our four data sets. Thus, we have quite clear allocations for Finland, Sweden and Norway while Denmark presents the lowest persistence of the allocations.

[Figure 2]

[Figure 3]

[Figure 4]

[Figure 5]

The mean, median and the 2.5 upper and lower percentiles of the posterior distribution of the transition probabilities are presented in table 1. Given that we are in regime  $S$  we can compute the duration of the regime by  $1/(1 - p_{ij})$  conditional on  $i = j$ <sup>13</sup>. The last column in table 1 shows the persistence or duration of a regime. The expected duration of the regimes varies between 1.5 up to 3 months. Finland has the most persistent regimes with duration of 1.67 months for and 18 months for regime 1 and regime 2. Thus, we seem to catch the heteroscedasticity by frequently switching between regimes with different volatility.

[Table 1]

The the mean, median and the 2.5% upper and lower percentiles of the conditional distributions of the estimated variance parameters are displayed in *table 2* and *figure 6-figure9*. There is a significant difference in the variance between the two regimes and this

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<sup>13</sup>For proof see Kim and Nelson (1999) pages 71-72

result is robust for all four countries. Denmark has the highest volatility for both regimes with 10.1% and 50.5% for regime 1 and regime 2. In the case of Norway, Sweden and Finland the volatility is around 7.8% to 8.1% for regime 1 and around 33.9% to 37.3%.

[Figure 6]

[Figure 7]

[Figure 8]

[Figure 9]

[Table 2]

## 4.2 Variance Ratios

Figure 10 shows the distributions of the variance ratio test computed on the Swedish data for the five-year horizon on the randomized standardized returns and randomized de-standardized returns and the standardized original returns. Table 3 and table 4 presents the tests based upon original returns and the variance ratio tests computed from the standardized randomized returns are presented in table 5 and table 6. The mean, median and the 2.5 upper and lower percentiles of the variance ratios for all twenty horizons are presented in the tables.

[Figure 10]

The probability values of the VR decrease as the horizon  $q$  increase. This is expected as the randomization of the returns leads to flatter posterior distributions of the VR as the horizon increases. Sweden presents us with the lowest variation in the VR from original returns whereas Finland and Denmark have a very high variation in the VR computed on the original returns. The maximum and minimum values are from Finland with  $(VR(q)=2.5896)$  at 24 months and  $(VR(q)=0.8950)$  at 120 months. This is an unexpected result especially as the high VR occur at 12, 24, 36 and 48 months and decay with  $q$ . This might also explain the evidence on mean reversion on annual horizons from the Danish stock market by Risager (1998) and Nielsen and Ovegaard-Olesen (1999). Thus, it justifies our approach of utilizing computations of monthly VR with short-run horizons of 2-12 months and long-run horizons of 1 to 10 years. The p-values from the standardized returns are lower then the p-values computed from the VR test of the original returns. Our lowest p-value is 0.4489 at 120 months horizon for standardized returns from Norway. Our highest p-values are all from the short run horizons. We cannot reject the null hypothesis of random walk for any of horizon  $q$  and any of the countries. This result is robust to VR computed on standardized or de-standardized returns.

[Table 3]

[Table 4]

[Table 5]

[Table 6]

Risager (1998) concludes that the Danish stock market is subject to mean reversion and this is verified by Nielsen and Overgaard-Olesen (1999) who find weak support of mean reversion when they employ regime switching models and compute variance ratio test on annual Danish stock market data. Frennberg and Hansson (1993) find support of mean reversion in the Swedish stock market and the mean reversion to increase with the length of the investment horizon. This they conclude indicates that the risk in the Swedish stock market decrease with the holding period. In our analysis we use monthly nominal stock market return and find no support of the above result. When we take the heteroscedasticity of the monthly data the estimation bias into account we find no support of mean reversion. The results suggest that two regime models with different volatility captures the heteroscedacity in nominal stock market returns in Finland, Sweden Norway and Denmark, and given the regimes the stock markets is random. Our result is in line with what Kim et al (1998a) finds for the U.S. stock market 1926-1986, Malliaropulos and Priestly (1999) results on international data and Graflund (2000) study on Swedish stock market excess returns 1918-1998. Thus, accounting for estimation bias and time-variation in volatility improves the variance ratio test.

## 5 Conclusion

This paper addresses the question if the Nordic Stock markets, i.e. Finland, Sweden, Denmark and Norway are subject to mean reversion. Previous studies find support of mean reversion in the Danish and the Swedish stock market and the mean reversion to increase with the length of the investment horizon. However the result of these studies are controversial as they ignore the assumption of constant expected return. Resent research has found that heteroscedasticity seriously affects the probability of the variance ratio test to reject the null hypothesis of random walk.

We model the well-known heteroscedasticity of the stock markets returns with a two regime hidden Markov model of normal mixtures. The model is estimated with Bayesian approach of Gibbs sampling, a computer intensive Markov chain Monte Carlo method. Our two regime hidden Markov model is clearly specified along with the priors and prior distributions employed in the Gibbs sampler. Further we use the information at each

run of Gibbs sampler to compute variance ratios test on standardized as well as de-standardized returns.

Our analysis finds no support for mean reversion in the any of the Nordic stock markets. We cannot reject the null hypothesis of random walk for any of the countries. This result is robust to VR computed on standardized or de-standardized returns. Our two regime models of normal distributions captures the variance and suggests that mean reversion if found in the Nordic stock markets can be explained by time variation in the volatility. Within the regimes the market is random.

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Table 1: Bayesian inferences on parameter estimates from two-state HMM's on monthly Nordic stock markets returns, 1919-1998.

| Parameter  | Posterior                  |        |          |
|--|----------------------------|--------|----------|
|  | mean                       | median | duration |
| Sweden   |                            |        |          |
| $p_{11}$   | 0.5836<br>[0.5835, 0.5837] | 0.5836 |          |
| $p_{22}$   | 0.4353<br>[0.4351, 0.4354] | 0.4353 |          |
| Denmark  |                            |        |          |
| $p_{11}$   | 0.5430<br>[0.5428, 0.5431] | 0.5429 |          |
| $p_{22}$   | 0.4747<br>[0.4746, 0.4748] | 0.4748 |          |
| Norway   |                            |        |          |
| $p_{11}$   | 0.6000<br>[0.5999, 0.6002] | 0.6001 |          |
| $p_{22}$   | 0.4323<br>[0.4322, 0.4325] | 0.4323 |          |
| Finland  |                            |        |          |
| $p_{11}$   | 0.6237<br>[0.6235, 0.6239] | 0.6237 |          |
| $p_{22}$   | 0.4428<br>[0.4428, 0.4430] | 0.4428 |          |
| Comment: Parameters estimated using diffuse priors |                            |        |          |



Table 2: Bayesian inferences on parameter estimates from two-state HMM's on monthly Nordic stock markets returns, 1947-1998.

| Parameter  | Posterior                  |        |
|------------|----------------------------|--------|
|            | mean                       | median |
| Sweden     |                            |        |
| $\sigma_1$ | 0.1108<br>[0.1000, 0.1230] | 0.1106 |
| $\sigma_2$ | 0.5100<br>[0.4602, 0.5664] | 0.5090 |
| Denmark    |                            |        |
| $\sigma_1$ | 0.1558<br>[0.1375, 0.1783] | 0.1551 |
| $\sigma_2$ | 0.7172<br>[0.6331, 0.8210] | 0.7142 |
| Norway     |                            |        |
| $\sigma_1$ | 0.0998<br>[0.0904, 0.1102] | 0.0996 |
| $\sigma_2$ | 0.4594<br>[0.4162, 0.5075] | 0.4586 |
| Finland    |                            |        |
| $\sigma_1$ | 0.1065<br>[0.0958, 0.1189] | 0.1062 |
| $\sigma_2$ | 0.4903<br>[0.4411, 0.5475] | 0.4889 |

Comment: Parameters estimated using diffuse priors

Table 3: Variance ratios test based on original returns for Sweden and Denmark

| Horizon | Sweden   |                            |        | Denmark  |                            |        |
|---------|----------|----------------------------|--------|----------|----------------------------|--------|
| $q$     | Original | Scrambled                  | Prob.  | Original | Scrambled                  | Prob   |
| 2       | 1.1603   | 0.9996<br>[0.9210, 1.0822] | 0.9988 | 1.2050   | 0.9991<br>[0.9192, 1.0820] | 1.0000 |
| 3       | 1.2063   | 0.9987<br>[0.8833, 1.1210] | 0.9959 | 1.3739   | 0.9980<br>[0.8816, 1.1230] | 1.0000 |
| 4       | 1.2476   | 0.9982<br>[0.8548, 1.1548] | 0.9873 | 1.5521   | 0.9967<br>[0.8526, 1.1577] | 1.0000 |
| 5       | 1.2793   | 0.9976<br>[0.8319, 1.1827] | 0.9777 | 1.7063   | 0.9955<br>[0.8274, 1.1838] | 1.0000 |
| 6       | 1.2942   | 0.9972<br>[0.8109, 1.2068] | 0.9619 | 1.8204   | 0.9944<br>[0.8070, 1.2079] | 1.0000 |
| 7       | 1.3211   | 0.9968<br>[0.7908, 1.2278] | 0.9518 | 1.9339   | 0.9930<br>[0.7854, 1.2301] | 1.0000 |
| 8       | 1.3370   | 0.9958<br>[0.7734, 1.2501] | 0.9418 | 2.0379   | 0.9916<br>[0.7694, 1.2494] | 1.0000 |
| 9       | 1.3485   | 0.9946<br>[0.7595, 1.2661] | 0.9315 | 2.1286   | 0.9902<br>[0.7547, 1.2668] | 1.0000 |
| 10      | 1.3666   | 0.9934<br>[0.7442, 1.2819] | 0.9288 | 2.1923   | 0.9889<br>[0.7396, 1.2850] | 1.0000 |
| 11      | 1.3934   | 0.9923<br>[0.7323, 1.2994] | 0.9343 | 2.2355   | 0.9875<br>[0.7260, 1.3000] | 1.0000 |
| 12      | 1.4258   | 0.9909<br>[0.7188, 1.3181] | 0.9418 | 2.2592   | 0.9861<br>[0.7106, 1.3155] | 1.0000 |
| 24      | 1.3581   | 0.9701<br>[0.5979, 1.4545] | 0.8452 | 1.7996   | 0.9682<br>[0.5893, 1.4672] | 0.9984 |
| 36      | 1.2454   | 0.9452<br>[0.5104, 1.5476] | 0.7327 | 1.5384   | 0.9454<br>[0.5049, 1.5710] | 0.9695 |
| 48      | 1.2734   | 0.9185<br>[0.4422, 1.6206] | 0.7294 | 1.3433   | 0.9230<br>[0.4389, 1.6405] | 0.9039 |
| 60      | 1.3103   | 0.8932<br>[0.3884, 1.6782] | 0.7505 | 1.2422   | 0.9012<br>[0.3888, 1.7162] | 0.8532 |
| 72      | 1.3294   | 0.8693<br>[0.3443, 1.7186] | 0.7411 | 1.0935   | 0.8789<br>[0.3422, 1.7825] | 0.7650 |
| 84      | 1.2927   | 0.8440<br>[0.3034, 1.7602] | 0.7050 | 0.9958   | 0.8563<br>[0.3081, 1.8425] | 0.7082 |
| 96      | 1.3103   | 0.8192<br>[0.2687, 1.8002] | 0.6929 | 0.8967   | 0.8333<br>[0.2746, 1.8824] | 0.6435 |
| 102     | 1.3650   | 0.7948<br>[0.2420, 1.8369] | 0.7019 | 0.9906   | 0.8090<br>[0.2457, 1.9087] | 0.7341 |
| 120     | 1.3561   | 0.7703<br>[0.2158, 1.8665] | 0.6634 | 0.9639   | 0.7842<br>[0.2204, 1.9435] | 0.7340 |

Comment: 2.5 and 97.5 percentiles within brackets

Table 4: Variance ratios test based on original returns for Norway and Finland

| Horizon | Norway   |                            |        | Finland  |                            |        |
|---------|----------|----------------------------|--------|----------|----------------------------|--------|
| $q$     | Original | Scrambled                  | Prob.  | Original | Scrambled                  | Prob   |
| 2       | 1.1612   | 0.9995<br>[0.9203, 1.0828] | 0.9991 | 1.2612   | 0.9988<br>[0.9157, 1.0845] | 1.0000 |
| 3       | 1.2133   | 0.9991<br>[0.8830, 1.1243] | 0.9986 | 1.3897   | 0.9977<br>[0.8771, 1.1253] | 1.0000 |
| 4       | 1.2817   | 0.9981<br>[0.8533, 1.1567] | 0.9990 | 1.4770   | 0.9967<br>[0.8467, 1.1554] | 1.0000 |
| 5       | 1.3144   | 0.9970<br>[0.8300, 1.1855] | 0.9983 | 1.5473   | 0.9958<br>[0.8233, 1.1826] | 1.0000 |
| 6       | 1.3207   | 0.9959<br>[0.8073, 1.2062] | 0.9967 | 1.6273   | 0.9954<br>[0.8038, 1.2075] | 1.0000 |
| 7       | 1.3420   | 0.9951<br>[0.7893, 1.2318] | 0.9958 | 1.7255   | 0.9944<br>[0.7843, 1.2310] | 1.0000 |
| 8       | 1.3588   | 0.9943<br>[0.7731, 1.2555] | 0.9952 | 1.8083   | 0.9932<br>[0.7675, 1.2514] | 1.0000 |
| 9       | 1.3521   | 0.9933<br>[0.7577, 1.2728] | 0.9919 | 1.8858   | 0.9923<br>[0.7524, 1.2679] | 1.0000 |
| 10      | 1.3672   | 0.9925<br>[0.7435, 1.2908] | 0.9908 | 1.9777   | 0.9913<br>[0.7398, 1.2881] | 1.0000 |
| 11      | 1.3939   | 0.9916<br>[0.7307, 1.3063] | 0.9913 | 2.0778   | 0.9903<br>[0.7240, 1.3041] | 1.0000 |
| 12      | 1.4215   | 0.9905<br>[0.7167, 1.3198] | 0.9915 | 2.1705   | 0.9894<br>[0.7119, 1.3209] | 1.0000 |
| 24      | 1.2604   | 0.9737<br>[0.6041, 1.4735] | 0.8967 | 2.5896   | 0.9729<br>[0.5941, 1.4807] | 1.0000 |
| 36      | 1.1141   | 0.9482<br>[0.5205, 1.5497] | 0.7603 | 2.4142   | 0.9472<br>[0.5090, 1.5762] | 0.9999 |
| 48      | 1.1027   | 0.9213<br>[0.4492, 1.6145] | 0.7559 | 2.1741   | 0.9197<br>[0.4411, 1.6465] | 0.9985 |
| 60      | 1.1307   | 0.8972<br>[0.3970, 1.6760] | 0.7850 | 2.0683   | 0.8967<br>[0.3875, 1.6972] | 0.9956 |
| 72      | 1.0996   | 0.8743<br>[0.3541, 1.7271] | 0.7712 | 1.9762   | 0.8740<br>[0.3429, 1.7630] | 0.9882 |
| 84      | 1.0936   | 0.8494<br>[0.3159, 1.7594] | 0.7784 | 1.7430   | 0.8493<br>[0.3030, 1.8247] | 0.9687 |
| 96      | 1.0592   | 0.8238<br>[0.2810, 1.8026] | 0.7685 | 1.4736   | 0.8235<br>[0.2705, 1.8645] | 0.9278 |
| 102     | 1.0146   | 0.7984<br>[0.2496, 1.8373] | 0.7530 | 1.1371   | 0.7965<br>[0.2439, 1.8764] | 0.8226 |
| 120     | 0.9544   | 0.7733<br>[0.2235, 1.8599] | 0.7314 | 0.8950   | 0.7706<br>[0.2174, 1.9060] | 0.6987 |

Comment: 2.5 and 97.5 percentiles within brackets

Table 5: Variance ratios test based on standardized returns for Sweden and Denmark

| Horizon | Sweden                     |                            |        | Denmark                    |                            |        |
|---------|----------------------------|----------------------------|--------|----------------------------|----------------------------|--------|
| $q$     | Scrambled                  | Standardized               | Prob.  | Standardized               | Scrambled                  | Prob   |
| 2       | 1.1570<br>[1.1061, 1.2088] | 0.9984<br>[0.9186, 1.0783] | 0.9998 | 1.1526<br>[1.0802, 1.2229] | 0.9986<br>[0.9202, 1.0774] | 0.9978 |
| 3       | 1.2017<br>[1.1140, 1.2939] | 0.9970<br>[0.8820, 1.1172] | 0.9998 | 1.2644<br>[1.1480, 1.3797] | 0.9975<br>[0.8845, 1.1173] | 0.9987 |
| 4       | 1.2278<br>[1.1013, 1.3624] | 0.9958<br>[0.8513, 1.1481] | 0.9982 | 1.3652<br>[1.2158, 1.5142] | 0.9963<br>[0.8554, 1.1459] | 0.9993 |
| 5       | 1.2454<br>[1.0873, 1.4145] | 0.9945<br>[0.8284, 1.1787] | 0.9972 | 1.4534<br>[1.2794, 1.6318] | 0.9947<br>[0.8325, 1.1758] | 0.9997 |
| 6       | 1.2440<br>[1.0611, 1.4403] | 0.9933<br>[0.8086, 1.2005] | 0.9958 | 1.5026<br>[1.3011, 1.7146] | 0.9930<br>[0.8106, 1.1991] | 0.9993 |
| 7       | 1.2528<br>[1.0470, 1.4738] | 0.9920<br>[0.7892, 1.2206] | 0.9951 | 1.5514<br>[1.3225, 1.7947] | 0.9913<br>[0.7918, 1.2219] | 0.9993 |
| 8       | 1.2612<br>[1.0353, 1.5038] | 0.9906<br>[0.7749, 1.2402] | 0.9941 | 1.5971<br>[1.3452, 1.8663] | 0.9896<br>[0.7740, 1.2388] | 0.9993 |
| 9       | 1.2627<br>[1.0204, 1.5235] | 0.9892<br>[0.7598, 1.2601] | 0.9928 | 1.6373<br>[1.3656, 1.9250] | 0.9879<br>[0.7586, 1.2526] | 0.9993 |
| 10      | 1.2736<br>[1.0173, 1.5513] | 0.9879<br>[0.7446, 1.2774] | 0.9926 | 1.6610<br>[1.3750, 1.9676] | 0.9860<br>[0.7425, 1.2660] | 0.9993 |
| 11      | 1.2960<br>[1.0277, 1.5869] | 0.9865<br>[0.7323, 1.2934] | 0.9934 | 1.6733<br>[1.3767, 1.9909] | 0.9842<br>[0.7287, 1.2811] | 0.9991 |
| 12      | 1.3214<br>[1.0435, 1.6217] | 0.9851<br>[0.7211, 1.3076] | 0.9939 | 1.6721<br>[1.3689, 1.9972] | 0.9825<br>[0.7144, 1.2948] | 0.9988 |
| 24      | 1.2575<br>[0.9061, 1.6586] | 0.9646<br>[0.5968, 1.4584] | 0.9512 | 1.2772<br>[0.9681, 1.6230] | 0.9618<br>[0.5956, 1.4385] | 0.8748 |
| 36      | 1.1552<br>[0.7330, 1.6531] | 0.9452<br>[0.5099, 1.5783] | 0.8698 | 1.1364<br>[0.7992, 1.5493] | 0.9412<br>[0.5158, 1.5521] | 0.7417 |
| 48      | 1.1612<br>[0.6792, 1.7589] | 0.9246<br>[0.4426, 1.6624] | 0.8798 | 1.0229<br>[0.6713, 1.4832] | 0.9199<br>[0.4455, 1.6316] | 0.6399 |
| 60      | 1.1886<br>[0.6789, 1.8559] | 0.9026<br>[0.3902, 1.7423] | 0.8908 | 0.9783<br>[0.6121, 1.4883] | 0.8985<br>[0.3930, 1.7098] | 0.6116 |
| 72      | 1.1655<br>[0.6379, 1.8725] | 0.8796<br>[0.3432, 1.8015] | 0.8916 | 0.9247<br>[0.5179, 1.5018] | 0.8762<br>[0.3460, 1.7634] | 0.5794 |
| 84      | 1.0953<br>[0.5471, 1.8297] | 0.8558<br>[0.3044, 1.8451] | 0.8764 | 0.9366<br>[0.4877, 1.5777] | 0.8536<br>[0.3050, 1.8265] | 0.6075 |
| 96      | 1.0531<br>[0.5159, 1.7881] | 0.8313<br>[0.2716, 1.8728] | 0.8873 | 0.9136<br>[0.4271, 1.6129] | 0.8304<br>[0.2677, 1.8630] | 0.6044 |
| 102     | 1.0323<br>[0.5157, 1.7401] | 0.8063<br>[0.2411, 1.8859] | 0.9029 | 0.9721<br>[0.4731, 1.7057] | 0.8059<br>[0.2432, 1.9015] | 0.6688 |
| 120     | 0.9354<br>[0.4544, 1.5965] | 0.7809<br>[0.2168, 1.9156] | 0.9023 | 0.9269<br>[0.4226, 1.7041] | 0.7810<br>[0.2180, 1.9301] | 0.6524 |

Comment: 2.5 and 97.5 percentiles within brackets

Table 6: Variance ratios test based on standardized returns for Norway and Finland

| Horizon | Norway                     |                            |        | Finland                    |                            |        |
|---------|----------------------------|----------------------------|--------|----------------------------|----------------------------|--------|
| $q$     | Standardized               | Scrambled                  | Prob.  | Standardized               | Scrambled                  | Prob   |
| 2       | 1.1418<br>[1.0833, 1.1970] | 0.9980<br>[0.9216, 1.0778] | 0.9965 | 1.1786<br>[1.1198, 1.2345] | 0.9986<br>[0.9187, 1.0774] | 0.9997 |
| 3       | 1.1947<br>[1.0903, 1.2922] | 0.9965<br>[0.8838, 1.1185] | 0.9916 | 1.2673<br>[1.1736, 1.3614] | 0.9972<br>[0.8815, 1.1184] | 0.9996 |
| 4       | 1.2585<br>[1.1210, 1.3855] | 0.9947<br>[0.8566, 1.1455] | 0.9929 | 1.3202<br>[1.1920, 1.4532] | 0.9956<br>[0.8532, 1.1498] | 0.9988 |
| 5       | 1.2963<br>[1.1360, 1.4473] | 0.9931<br>[0.8313, 1.1705] | 0.9932 | 1.3646<br>[1.2025, 1.5335] | 0.9942<br>[0.8283, 1.1795] | 0.9982 |
| 6       | 1.2937<br>[1.1197, 1.4647] | 0.9913<br>[0.8091, 1.1929] | 0.9869 | 1.4189<br>[1.2287, 1.6175] | 0.9929<br>[0.8071, 1.2020] | 0.9983 |
| 7       | 1.3080<br>[1.1209, 1.4928] | 0.9897<br>[0.7891, 1.2140] | 0.9831 | 1.4871<br>[1.2681, 1.7143] | 0.9916<br>[0.7858, 1.2251] | 0.9988 |
| 8       | 1.3225<br>[1.1243, 1.5186] | 0.9881<br>[0.7719, 1.2338] | 0.9811 | 1.5512<br>[1.3017, 1.8104] | 0.9899<br>[0.7694, 1.2443] | 0.9990 |
| 9       | 1.3162<br>[1.1075, 1.5221] | 0.9866<br>[0.7560, 1.2527] | 0.9738 | 1.6064<br>[1.3296, 1.8931] | 0.9882<br>[0.7541, 1.2597] | 0.9990 |
| 10      | 1.3277<br>[1.1091, 1.5451] | 0.9852<br>[0.7422, 1.2682] | 0.9730 | 1.6706<br>[1.3708, 1.9818] | 0.9865<br>[0.7393, 1.2749] | 0.9993 |
| 11      | 1.3511<br>[1.1247, 1.5774] | 0.9838<br>[0.7285, 1.2835] | 0.9748 | 1.7412<br>[1.4208, 2.0730] | 0.9848<br>[0.7256, 1.2879] | 0.9995 |
| 12      | 1.3739<br>[1.1398, 1.6078] | 0.9823<br>[0.7151, 1.3004] | 0.9758 | 1.8139<br>[1.4749, 2.1666] | 0.9831<br>[0.7143, 1.3031] | 0.9996 |
| 24      | 1.2846<br>[1.0065, 1.5820] | 0.9635<br>[0.5990, 1.4479] | 0.8878 | 2.2664<br>[1.7479, 2.8259] | 0.9633<br>[0.5969, 1.4447] | 0.9999 |
| 36      | 1.2194<br>[0.8671, 1.6145] | 0.9429<br>[0.5148, 1.5526] | 0.8117 | 2.3463<br>[1.7147, 3.0782] | 0.9420<br>[0.5129, 1.5455] | 0.9988 |
| 48      | 1.2440<br>[0.8304, 1.7307] | 0.9214<br>[0.4498, 1.6433] | 0.8106 | 2.3486<br>[1.5926, 3.2693] | 0.9196<br>[0.4514, 1.6182] | 0.9963 |
| 60      | 1.3044<br>[0.8408, 1.8566] | 0.8992<br>[0.3916, 1.7157] | 0.8366 | 2.3216<br>[1.4830, 3.3700] | 0.8961<br>[0.3944, 1.6944] | 0.9920 |
| 72      | 1.2764<br>[0.7804, 1.8759] | 0.8763<br>[0.3432, 1.7759] | 0.8203 | 2.2769<br>[1.3724, 3.4229] | 0.8721<br>[0.3453, 1.7484] | 0.9858 |
| 84      | 1.2674<br>[0.7536, 1.9022] | 0.8528<br>[0.3057, 1.8249] | 0.8173 | 2.1914<br>[1.2457, 3.3981] | 0.8472<br>[0.3063, 1.7853] | 0.9780 |
| 96      | 1.2143<br>[0.6964, 1.8730] | 0.8287<br>[0.2733, 1.8508] | 0.7977 | 2.0259<br>[1.0987, 3.2149] | 0.8222<br>[0.2737, 1.8167] | 0.9646 |
| 102     | 1.1167<br>[0.6159, 1.7775] | 0.8042<br>[0.2432, 1.8914] | 0.7553 | 1.7522<br>[0.9147, 2.8546] | 0.7967<br>[0.2450, 1.8464] | 0.9307 |
| 120     | 0.9890<br>[0.5099, 1.6531] | 0.7791<br>[0.2184, 1.9179] | 0.6980 | 1.4294<br>[0.7135, 2.4099] | 0.7707<br>[0.2215, 1.8751] | 0.8716 |

Comment: 2.5 and 97.5 percentiles within brackets

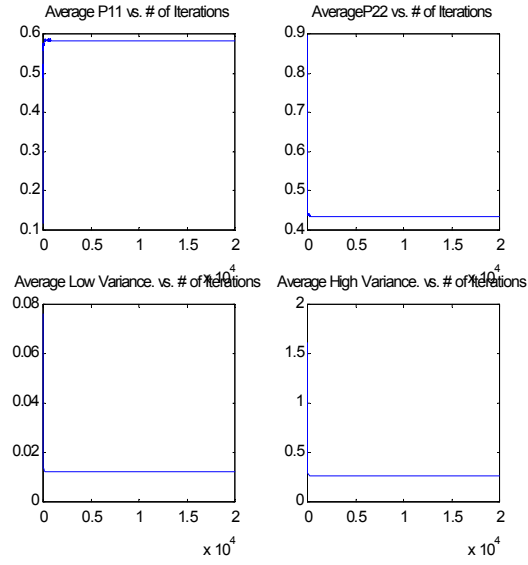


Figure1: Ergodic average of estimated parameters vs. # iterations

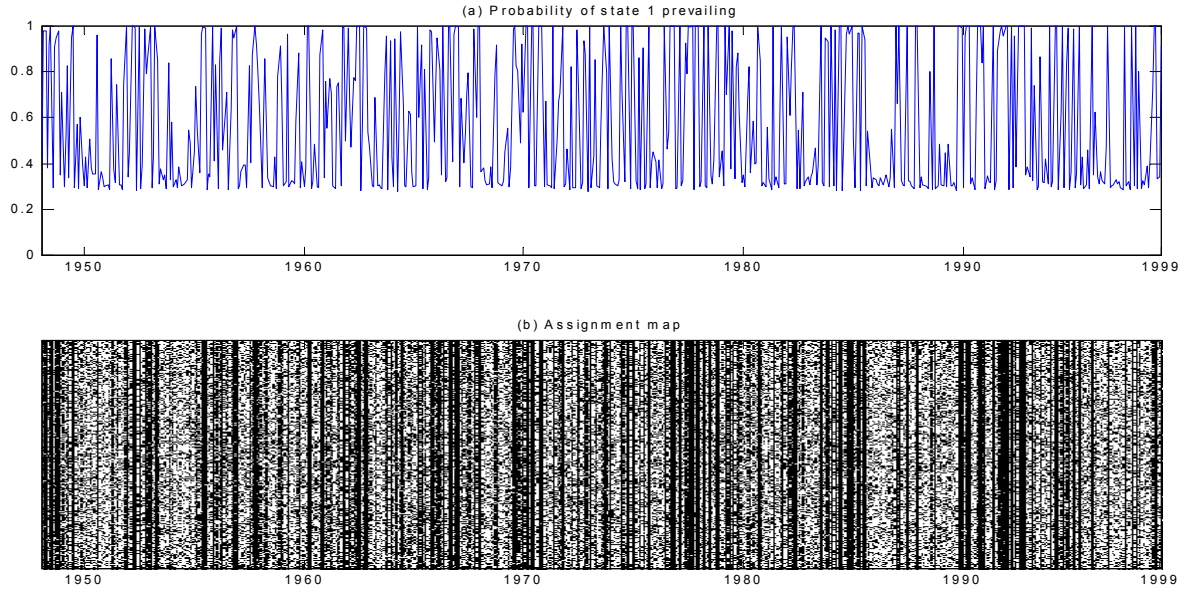


Figure 2: Assignment of state 1 for Sweden

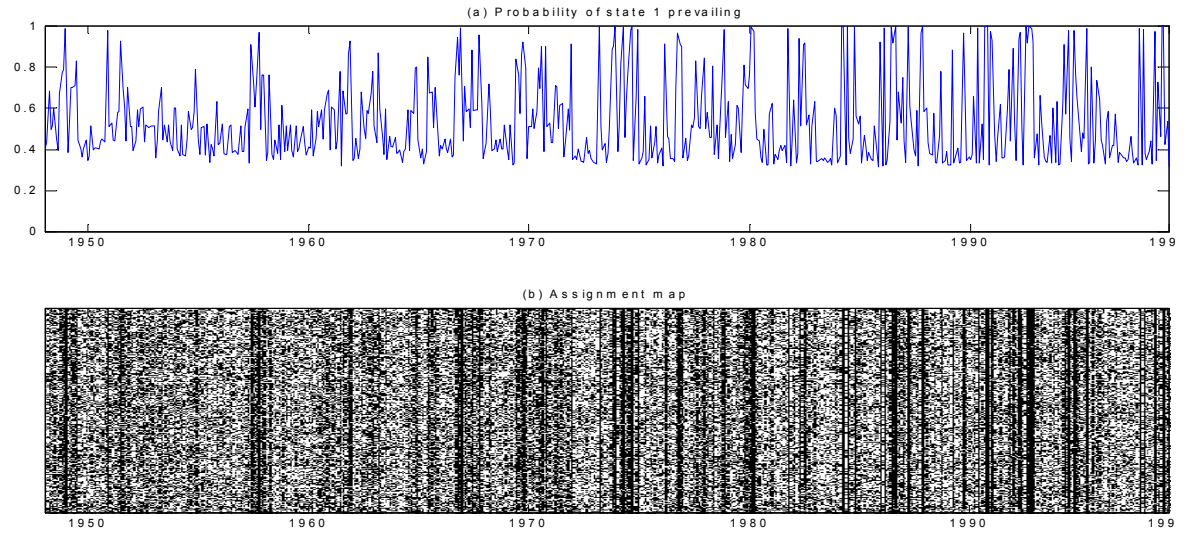


Figure 3: Assignment of state 1 for Denmark

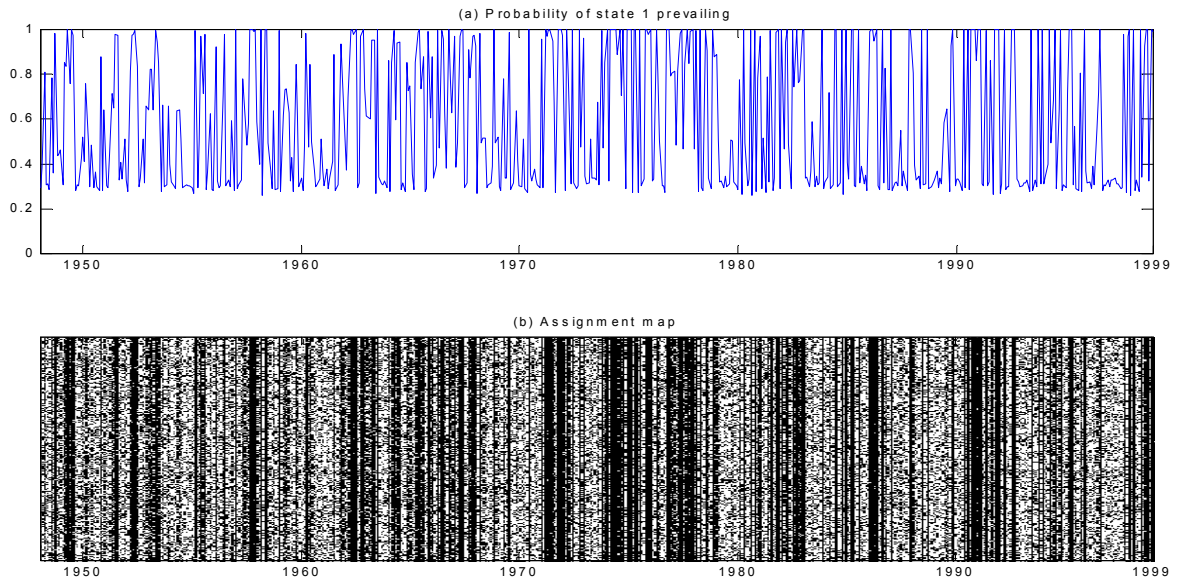


Figure 4: Assignment of state 1 for Norway

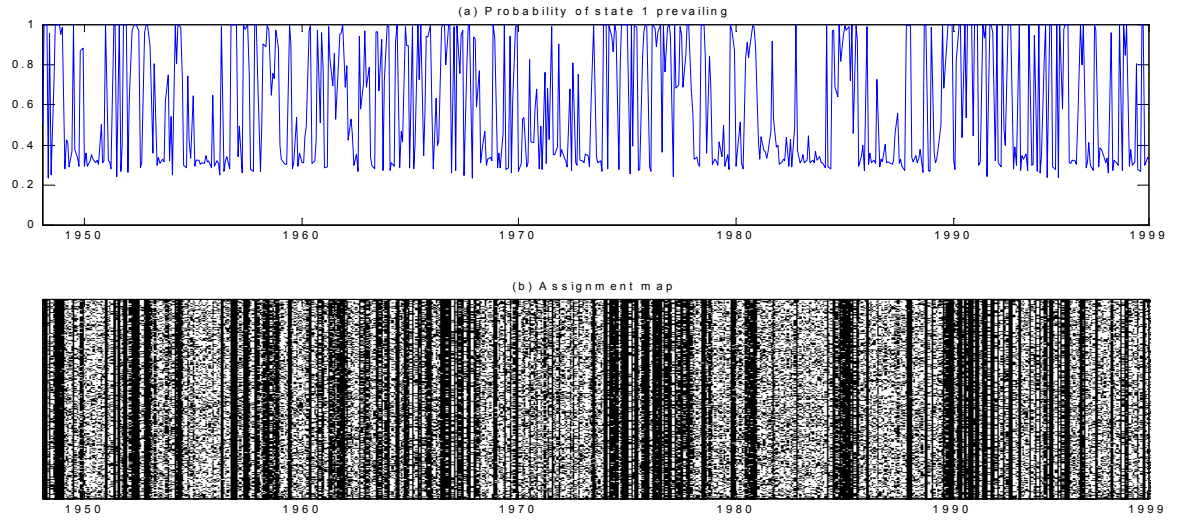


Figure 5: Assignment of state 1 for Finland

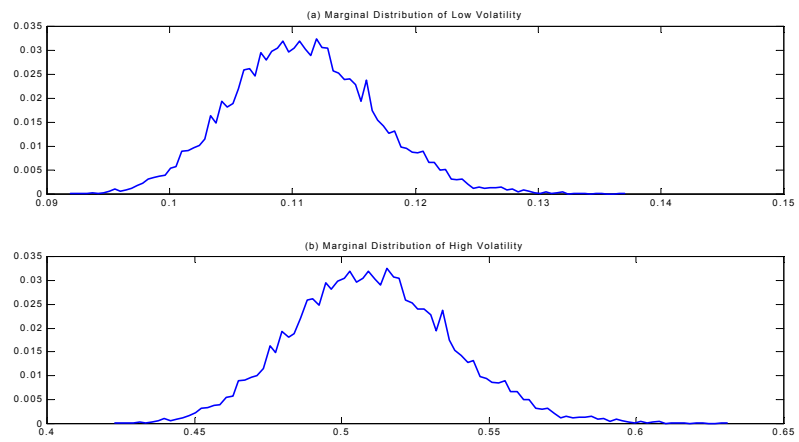


Figure 6: Posterior distribution of low and high volatility for Sweden



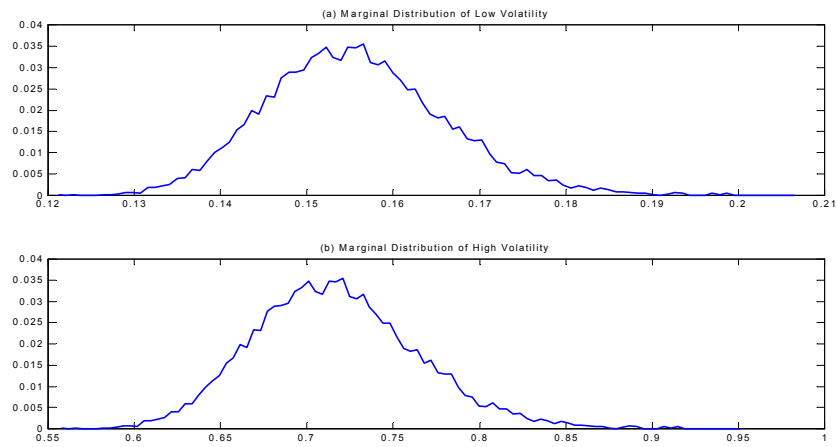


Figure 7: Posterior distribution of low and high volatility for Denmark

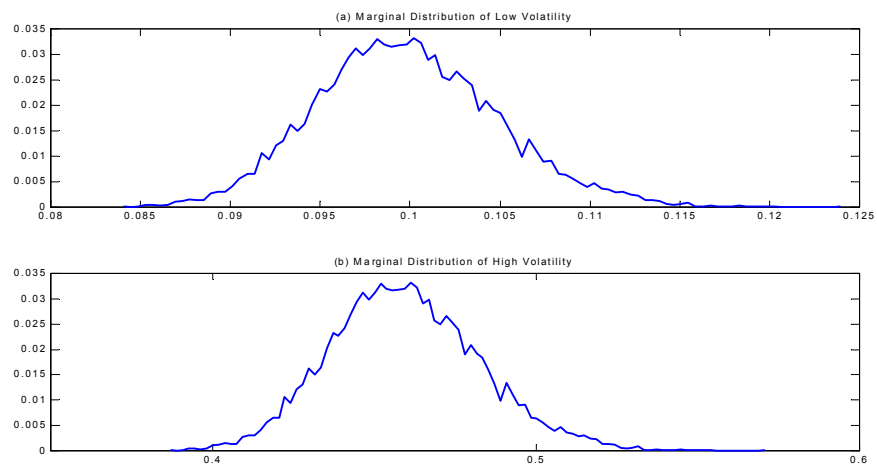


Figure 8: Posterior distribution of low and high volatility for Norway

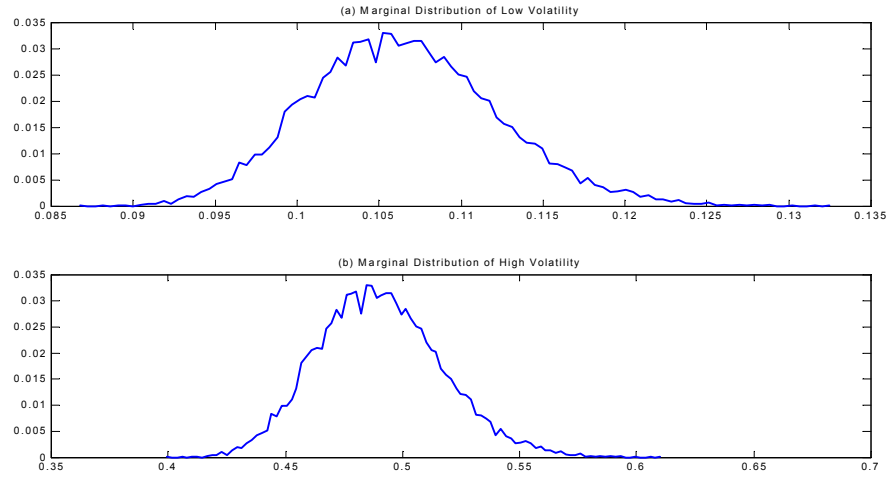


Figure 9: Posterior distribution of low and high volatility for Finland

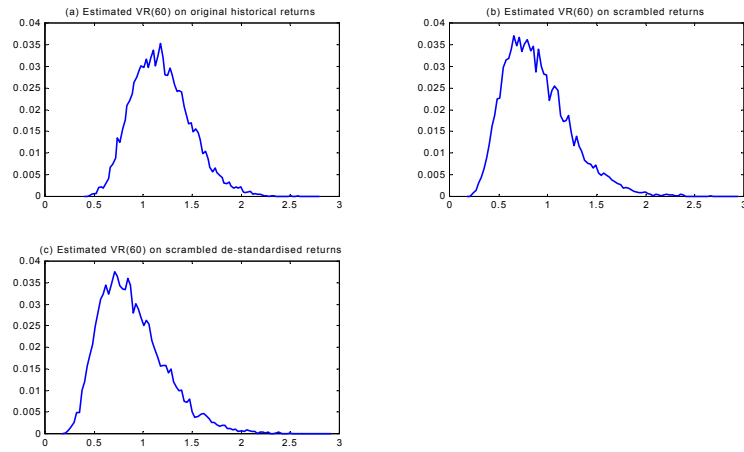


Figure 10: Conditional distribution of 5-year VR for Sweden