Expected Real Returns from Swedish Stocks and Bonds

Andreas Graflund

Department of Economics

Lund University

P.O. Box 7082

S-220 07 LUND, Sweden

Fax: +46 (0) 46 222 41 18

Email: andreas.graflund@nek.lu.se

Phone: +46 (0) 46 222 79 19

JEL: C11 C15 G10 G11

Keywords: *Empirical distribution, stock returns, bond returns, markovian bootstrap, MCMC.*

September 2001

Abstract

This paper introduces a new non-parametric approach to examine the predictability of the real return for different investment horizons for five portfolios of Swedish stocks and bonds. In our setting the problem reduces to generating new generalizations from a known empirical Markov chain. We find that the stocks yield a real return of about 7.5% and bonds about 3.0%. Our results suggest that an investor ought to avoid bonds in the long run. Finally if the investors goal is to minimize the risk of capital destruction the preferable long-run passive portfolio is a mix of bonds and stocks.

1 Introduction

The most important issue when investing is the concern of what real value growth one is likely to receive. Or as important, what is the risk of capital erosion when investing? Today eight out of ten Swedes invests in the stock market.[1] The number of shareholders and investments in mutual funds has increased dramatically during the last decade. In 2000 the Swedish government floated a new public pension system, *the premium pension*, in which the individual decides how 10 per cent of her retirement funds should be invested in a selection of about 450 mutual funds.[2] Altogether this underlines the importance of what real return an investor can expect to receive when investing in Swedish stocks and bonds.

The contribution of this paper is twofold. First, we utilize a new non-parametric approach to examine the predictability of the real return for different investment horizons for five mixed portfolios of Swedish stocks and bonds. The non-parametric approach enables us to find the empirical probability density functions of the annual real return for each investment horizon. Hence we are able to find the expected real return for Swedish stocks and bonds. Further we are able to find the probability of actually receiving a specific return, which make our approach applicable in risk management. Second we introduce a new Markov chain Monte Carlo method based on the empirical properties of the data for generating resamples. Our contribution is that we introduce a new general decision rule that governs the transition probabilities in the Markov chain (see also Carlstein, Edward. Do, Kim-Anh., Hall, Peter., Hesterberg, Tim and Hans R. Kunsch (1998), Paparoditis and Politis [2001a, 2001b]). In the empirical part of the paper the volatility governs the probability of moving between the states, this is analogous to the well-known time varying volatility of financial time series. The time varying volatility of Swedish stocks have been documented by Hanson and Hördahl (1997).

We find that the most likely real return from stocks varies between 7.5 per cent to 8.3 per cent and the real bond returns varies between 2.4 per cent and 3.1 per cent depending on the investment horizon. With our definition of risk as the probability that the investment yields a negative real return we find that the best solution avoiding capital erosion for a passive long-term investor is to diversify into mixed stock-bond portfolios.

[1] According to Aktiespararnas förening.

[2] The Swedish public pension funds have today an inflow of about 18.5 per cent of the individuals annual taxable income. 16 per cent is managed by the authorities in income retirement funds, *allmäna pensionsfonderna*. The remaining 2.5 per cent are invested according to the individuals choice of a selection of about 450 mutual funds. See also `www.PPM.nu`.

Notable is that for a long-run investment horizon the bond portfolio has about 10 per cent risk of reducing in real value. This is a significantly higher risk in comparison to the other portfolios in the study.

The outline of the paper is as follows. Section 2 specifies the framework utilized to investigate the predictability of real returns from Swedish stocks and bonds. The methodology and data are thoroughly described in section 3 along with the markovian bootstrap framework. The results and the empirical evidence are presented in section 4. Section 5 concludes the paper. The resampling methodology is presented in the appendix.

2 Predicted return

The purpose of this paper is to study the empirical distributions of mean annual real return from Swedish stock and bond portfolios for different investment horizons. The common approach is to focus on the mean and standard deviation of the return under the assumption that returns are normally distributed. However empirical research have found that the real world is more complex as financial assets often exhibit fat tail distributions and skewness (see Sweden: Frennberg and Hansson [1993] and US: Ibbotson and Singuefield [1976]). Jones and Wilson (1999) study the returns for different horizons by fitting lognormal distributions on for different U.S. stock and bond portfolio returns. In this approach they make the assumption returns actually are lognormal distributed and discard the fact that asset returns often exhibit heavy tail distributions.

The issue in this paper is to find the distribution of the annual mean real return, $F(x)$, for different investment horizons. However the distribution $F(x)$ is generally unknown. In this paper we introduce a new Markovian moving block bootstrap methodology that enables us to replace the unknown distribution $F(x)$ by its empirical distribution $F_n(x)$. This approach has several advantages as it enables us to capture possible fat tails and skewness of the returns. The framework of bootstrap original data a large number of times to capture the empirical distributions was first introduced by Efron (1979).

3 Methodology

3.1 Description of data

The data consists of monthly consumer price index and nominal returns from Swedish bonds and the Swedish stock market portfolio, including dividends, all from the Frennberg and Hansson database (Frennberg and Hansson [1992]). Our sample covers the period January 1919 – December 1999. A total of 80 years of monthly observations. Some descriptive statistics of the monthly real return is presented in table 1. The null hypothesis of normal distribution is rejected and the explanation is the very high kurtosis in the data. This also verifies that fitting a normal distribution to the data is not the correct approach.

[Table 1]

The real return from Swedish stocks and bonds are roughly the same as the returns reported by Jones and Wilson (1999) for the US were stocks yielded an annualized monthly real return of approximately 7.49% and bonds 2.68%.

3.2 Returns

From the data we construct the following five portfolios; all equity, all bonds, sixty per cent equity forty per cent bonds, fifty-fifty equities and bonds, and forty per cent equity sixty per cent bonds. The three latter portfolios are commonly held by institutions. The monthly real return is computed of the above portfolio as:

$$x_t = \frac{P_t}{P_{t-1}} - \frac{CPI_t}{CPI_{t-1}} \tag{1}$$

[Figure 1]

The real returns of the portfolios are computed for eight different investment horizons, q, of one, two, three, five, ten, fifteen, twenty, and twenty-five years. We will compute non-overlapping returns, as overlapping returns will exhibit a strong autocorrelation with increasing investment horizon. This will produce $n = \frac{T}{q}$ numbers of non-overlapping returns for investment horizon q. Where n is rounded to the nearest floor integer. The drawback is obvious as q, the investment horizon in months, increases the number of observed returns, n, decreases and for long investment horizons we will have to few returns to make any statistical inference.

The solution is to resample the original data, $\mathbf{x}$, to generate new vectors of resampled return $\mathbf{r}$, and from these construct new asset price paths, $\mathbf{I}$, of the portfolios as:

$$I_t = I_{t-1}(1 + r_t) = \prod_{i=1}^{t}(1 + r_t) \tag{2}$$

Now we can compute a new set of non-overlapping annual mean q-month returns for each new asset price path, $\mathbf{I}$, as:

$$r_{mq} = \left(\left(\frac{I_{mq} - I_{mq-q}}{I_{mq-q}} + 1 \right)^{\frac{12}{q}} - 1 \right) \cdot 100 \quad \text{for} \quad m = 1, ..., n \tag{3}$$

The returns will converge to the empirical distribution if this is repeated N number of times, where N is a large number. Hence we are able to compute number $N \times n$ of returns for investment horizon q.

3.3 Markov Chain Monte Carlo Methodology

Resampling time series with serial dependence raises the question of how to keep these properties, as ordinary wild bootstrap would destroy this dependence.[3] One solution is to bootstrap blocks of the original data (Carlstein (1986)) or utilize a moving block bootstrap suggested by and Kunsch (1989) and resample overlapping blocks of data.[4] However both approaches have been criticized by Carlstein et al (1996) as the dependence between the generated blocks is ignored. Here we suggest an improvement in line with the ideas suggested by Carlstein et al (1996) and construct an empirical Markov chain whose transition probabilities depend on the data. This is achieved by introducing a decision rule that can match states with similar information. In this sense we rule out the probabilities of moving between the extreme states.

3.3.1 States

We define a state S_i as a set of b number of observations $S_i = \{r_i, ..., r_{i+b}\}$. Thus, we construct k number of states, where $k = T - b$. This is a first order Markov Chain as a random state, S_i, conditional upon all of the past events only depend on the previous state S_{i-1}.[5]

[3] An excellent description of resampling techniques can be found in Hjort (1994) "Computer Intensive Statistical Methods"; Shao Jun and Dongsheng Tu (1995) "The Jackknife and Bootstrap ," and Davidson A.C. and D.V. Hinkley (1998) "Bootstrap Methods and Their Application".

[4] See also Davidson and Hinkley (1998) Ch. 8.2.3.

[5] This is referred to as the Markov property.

3.3.2 Block length

The idea behind matching blocks is that importance of the dependence between blocks increases with short blocks, as there is more blocks, and decrease with long blocks. Note that one should be cautious of using block lengths exceeding the shortest investment horizon as this introduces a discretization of the non-overlapping returns and that the problem becomes more severe as the block length increase, a proof of this statement is given in appendix 2.[6] Therefore we utilize a block length of six months as this is long enough to preserve some of the serial dependence and yet shorter than the shortest investment horizon.

3.3.3 Transition kernel

The transition probability is determined by a ratio of distances between information in the previous state and the information in the next state (see also Carlstein et al [1996] and Paparoditis and Politis [2001a, 2001b]) Our transition kernel has two advantages. First, the information set $\mathbf{y}$, twhich governs the transition probability, can easily be altered to account for dependence between the states, such as volatility. The further the distance between the information in the states y_i and y_j the lower the probability of moving from state S_i to state S_j. Second, the strength of our beliefs in the information, $\mathbf{y}$,is determined by a distance raised to the power, z. If we question the non-equal dependence between the states, then z is set to zero and all the transition probabilities of the Markov chain is equal. This is a special case of the decision rule and is commonly referred to as a moving block bootstrap in the literature (see Kunch (1989)). Further, our transition kernel is that of an ordinary bootstrap if we set z to zero and the number of observation in each state to one.

The transition kernel describes the probability to move from a given state S_i to a given state S_j given an information criteria $\mathbf{y}$ in the states. v_{ij}, denotes the probability of moving to a given state S_j conditional upon the previous state S_i.

$$v_{ij} = \left(1 - \frac{|y_{i,b} - y_{j,1}|}{|y_{\max} - y_{\min}|} \right)^z \tag{4}$$

Where $\mathbf{y}$ is an information vector from the data of interest such as return or volatility. The suffix i, b denotes the last observation for of the data set in state i, and $j, 1$ denotes the

[6] We have also done estimations with block lengths ranging up to 120 months. The results are available upon request.

first observation of the information set in state j. The factor z determines the probability of moving to states with a different value of $\mathbf{y}$. As one can see the decision rule, v_{ij}, can easily be modified to account for time variations in volatility and variance, where $\sum_{l=1}^{b} \frac{r_{il}^2}{b}$ can be employed to proxy variance, σ_i^2, at state, S_i, of block length b. Each state is then associated with a single value of volatility and hence the subscript then denotes the state as we will have one observation of volatility within the block.

$$v_{ij} = \left(1 - \frac{|\sigma_i - \sigma_j|}{|\sigma_{\max} - \sigma_{\min}|}\right)^z \tag{5}$$

In the paper blocks with similar volatility are the most likely to be simulated as this captures the clustering of volatility. Thus we assume that the real returns are martingale differences.

3.3.4 Transition probabilities

Given the fact that each realization of a bootstrap is a state in a Markov chain and v_{ij}, denotes the strength of moving to a given state j conditional upon the previous state i, we can compute the transition probabilities, p_{ij}, in a k-state Markov chain as:

$$p_{ij} = \frac{v_{ij}}{\sum_{j=1}^{k} v_{ij}}, \qquad i, j = 1, ...k \tag{6}$$

$$\sum_{j=1}^{k} p_{ij} = 1 \tag{7}$$

The transition probabilities p_{ij}, i.e. the probability of moving from one state S_i, to another state S_j, is gathered in a $k \times k$ transition matrix $\mathbf{P}$. Where $k = T - b$ and T is the number of observations and b is the block length. The transition probabilities in a row vector of the transition matrix $\mathbf{P}$ always sum to unity. Each state produces a unique row vector of transition probabilities for the Markov chain to move to another state. Note that this is a well-behaved Markov chain with no absorbing states as all transition probabilities are less than unity, $p_{ij} < 1$.

3.4 Generation via accept-rejection method

According to Casella et al (1999) an ideal accept/reject density $g(x)$, is a density such that $p(x) \leq Mg(x)$ for all x and for which the ratio $p(x)/Mg(x)$ is relatively constant over the range of x where $p(x)$ has most of its mass, where M is a scaling factor. In or case $g(x)$ is a uniform density, $U(01)$, $M = 1$ and $p(x)$ is density describing the probability, v_{ij}, of moving from a given state, S_i to a given state, S_j. Note that each and every, v_{ij}, is bounded by the [01] space.[7]

Let $\mathbf{r}$ denote the vector of the originally 1-month return series

Step 1: Determine the block length b and compute the number of states, i.e. blocks, $n = \frac{T}{b}$, that will fit in the original return series, where n is rounded to the nearest lowest integer.

Step 2: Create a vector $\mathbf{V}$ of increasing numbers ranging from 1 to $k = T - b$.

Step 3: Set $N = 1$ and randomize $\mathbf{V}$ to get a scrambled vector of numbers .

Step 4: Set $w = 1$, $a = 1$ and use the wth observation of the scrambled vector to pick the wth block and create a vector $\mathbf{r}_i^*$ of returns.

We use an accept-rejection kernel in order accept a proposed new state j given the previous state i.

$v_{ij} = \left(1 - \frac{|\sigma_i - \sigma_j|}{|\sigma_{\max} - \sigma_{\min}|}\right)$, $c \sim U(0,1)$.

Set $w = w + 1$.

Accept if; $c < v_{ij}$,

and set $a = a + 1$,

else generate a new state, j.

Repeat step 4 until $a = n$.

Step 5: Compute a price series from the resampled $\mathbf{r}_i^*$.

Step 6: Compute the non-overlapping returns $\mathbf{r}_q^*$ for all q investment horizons and set $N = N + 1$;

Step 7: Repeat step 3 to step 6 until say, N=20.000.

[7] The accept-rejection methodology have been criticized as being computationally inefficient as samples are being rejected and does not contribute with information. In our case this is a minor problem (see table 1).

The non-overlapping q-horizon returns will converge to their empirical distributions if this is repeated N number of times, where N is a large number. In our case N is set to 20.000. Hence we are able to compute $N \times n$ number of returns for each investment horizon q.

3.5 Risk

We will refer to risk as the risk of capital erosion or the probability that the investment yields a negative real return. If the investor expects the investment to yield a certain real return, target return, then risk can also be stated as the probability of receiving a below target return. This is referred to as downside risk in the literature (see Fishburn [1977]). The empirical cumulative distributions of the returns enable us to compute the probability of receiving an at least specified return level t, $P(r \geq \tau)$. Which is computed as:

$$P(r \geq \tau) = \frac{\#\{r_i \geq \tau\}}{n} \tag{8}$$

4 Results

4.1 Rejections from the Accept-Reject method

We generate a total of 20.000 monthly return series from which we construct 20.000 asset price paths. From these asset price paths we calculate non-overlapping returns for each of the eight investment horizons. This procedure is then repeated for each one of the five portfolios.

The accept-rejection algorithm rejects between 1.8 per cent and 4.1 per cent of the proposed blocks. These are tolerable numbers as the are quite low. Hence the algorithm is efficient. The bond portfolio has the highest number of accepted blocks and the stock portfolio the highest number of rejected blocks. The rejections are presented in table 2.

[Table 2]

4.2 The empirical distributions of the real return

For each of the investment horizons we compute the empirical probability density functions, PDF, and the empirical cumulative probability density functions, CDF. The empirical PDF's for stocks and bonds are presented in figure 2 and figure 3.

[Figure 2]
[Figure 3]

The mean return along with the 2.5 upper and lower percentiles of the real return for the portfolios and the investment horizon is presented in table 2. This presents us with some interesting results. Depending on the investment horizon stocks yield the highest average real return, between 7.46 per cent and 8.27 per cent, and bonds the lowest, between 2.50 per cent and 3.06 per cent.

[Table 3]

Stocks have the highest probability of rendering a high return for all investment horizons in comparison to the other investigated portfolios. Stocks also stand the highest probability of capital erosion at investment horizons up to 10 years. However at the long investment horizons 15, 20 and 25 years, bonds is the riskiest asset in this aspect. The latter is a somewhat surprising result. Moreover from table 3 to table 7 the diversification effect between stocks and bonds is evident as the mixed 40-60 stock/bond portfolio dominates the bond portfolio even at short horizons. As the investment horizon increases all stock/bond portfolios dominate the bond portfolio and at long-run investment horizons also the stock portfolio dominates the bond portfolio. It is interesting to note that at a 25 year investment horizon the bond portfolio just stand a 91.4 per cent chance of yielding any annual value growth whereas stocks has a 97.5 per cent probability of growth.[8]

Overall we find that a mixed bond/stock portfolio stand least risk of reducing the real value of the investment. Further these portfolios also have a higher chance of capital gain in comparison to the bond portfolio. However the results from this type of analysis are time specific. In comparison to the US stock market

[Table 4]
[Table 5]
[Table 6]
[Table 7]
[Table 8]

Our results indicates that a passive investor should clearly avoid bonds in the long run and ought to avoid them even at moderate investments horizons. The overall conclusion is that when investing in a long-run passive portfolio a mix of bonds and stocks minimizes the risk of capital destruction.

[8] Some explanation to this result might be the deflationary period in Sweden in the early 1930s during which bonds had a negative real return.

5 Conclusion

This paper is examines the empirical distributions of the real return from a number of portfolios of Swedish stocks and bonds. The contribution of this paper is twofold. First, we are able to answer the question of what real return an investor can expect to receive if investing in Swedish stocks and bonds. Further we are able to find the probability of actually receiving a specific return, which make our approach applicable in risk management. Second we propose an improvement to the moving block bootstrap methodology by introducing a new Markov chain Monte Carlo method based on the empirical properties of the data for generating samples.

We find that the most likely real return from stocks to vary between 7.5 per cent to 8.3 per cent depending on the investment horizon and the real bond returns vary between 2.4 per cent and 3.1 per cent. However the results from this type of analysis are time specific.

It is interesting to note that at a 25-year investment horizon the bond portfolio stand a 91.4 per cent chance of yielding any annual value growth whereas stocks has a 97.5 per cent probability. The effect of diversification is evident as our 60-40, 50-50 and 40-60 stock/bond portfolios have a higher probability of yielding a positive value growth and at less risk of receiving a negative annual mean real return than a bond portfolio. The overall conclusion is that when investing in a long-run passive portfolio a mix of bonds and stocks is important if ones goal is to minimize the risk of capital destruction.

Acknowledgements

I am grateful to Professor Tobias Rydén and Birger Nilsson for valuable comments on the paper. Financial support from *Jan Wallander & Tom Hedelius' Stiftelse*, *Stiftelsen för främjande av ekonomisk forskning vid Lunds Universitet*, *Crafoordska Stiftelsen* and *The Royal Swedish Academy of Science* is gratefully acknowledged.

References

Casella, George and Robert, P. Christian. "*Monte Carlo Statistical Methods.*" Springer-Verlag, New York, Inc. 1st Ed, (1999).

Carlstein, Edward., Do, Kim-Anh., Hall, Peter., Hesterberg, Tim and Hans R. Kunsch. (1996) "Matched-Block Bootstrap for Dependent Data" *Research Report No .74 Eidgenössische Technische Hochschule, Zurich, Switerland.*

Davidson A.C. and D.V. Hinkley. "*Bootstrap Methods and Their Application*", Cambridge University Press 2nd Ed 1998.

Efron, B. (1979). "Bootstrap Methods Another Look at the Jackknife." *Ann. Statist. 7, 1-26.*

Fishburn, C.P. (1977). "Mean-Risk Analysis with Risk Associated Below-Target Returns." *American Economic Review*, 67(5) 116-26.

Fitzenberger Bernd, (1997) "The Moving Block Bootstrap and Robust Inference for Linear Least Squares and Quantile Regressions", *Journal of Econometrics 82*, 235-287.

Frennberg, Per and Björn Hansson. (1992) "Computation of a Monthly Index for Swedish Stock Returns 1919-1990." *Scandinavian Economic History Review*, 40(2) 3-27.

Frennberg, Per and Hansson, Björn, (1993) Some Distributional Properties of Monthly Stock Returns in Sweden 1919-1990, *Finnish Economic Papers*, 6(2), p. 108-122.

Hansson, Björn and Hördahl, Peter, (1997) "Changing Risk Premia: Evidence from a Small Open Economy." *Scandinavian Journal of Economics*, **99** 335-350.

Hjort J. S. "*Computer Intensive Statistical Methods*", Chapman & Hall, 1994

Ibbotson, Roger G; Singuefield Rex A, (1976) Stocks, Bonds, Bills, and Inflation: Year-by-Year Historical Returns (1926 - 1974), *The Journal of Business*, 2, 11 - 48.

Jones, C. P. and J. W. Wilson, (1999) Expectations about Real Returns, *Journal of Portfolio Management*, Winter 1999, 45-53.

Kunsch, H. R. (1989) "The Jackknife and Bootstrap for General Stationary Observations" *Annals of Statistics 17*, 1217-1241.

Paparoditis Efstathios. and Dimitris N. Politis (2001) "A Markovian Local Resampling Scheme for Nonparametric Estimators in Time Series Analysis " forthcoming in *Econometric Theory* in 2001.

Paparoditis Efstathios. and Dimitris N. Politis (2001) "The Local Bootstrap for Markov Processes" forthcoming in *J. Statist. Plan. Infer.*, in 2001.

Shao Jun and Dongsheng Tu "*The Jackknife and Bootstrap* " Springer-Verlag, New York, Inc. 1st Ed, (1995).

Appendix

A1 Generation via random numbers

Another approach to generate samples from a Markov Chain is to first construct the transition matrix $\mathbf{P}$. Given the state S_i the move to state S_j is determined by comparing a random draw against the cumulative probability distribution of the ith row in the transition matrix such that:

$$u \sim U(0,1)$$

$$S_j := \min\left\{ j : \sum_{i=1}^{j} p_{ij} \geq u \right\} \tag{A1}$$

Note that this approach has the advantage that all generations are accepted. Hence this is a less computer intensive approach.

A2 Block length

The idea behind block bootstrap is to keep the (time) serial properties of the original sample in the generated series. A long block length assures that a large fraction of the serial properties is kept within the generated series. However, long block lengths keep the serial properties at a cost of less possible outcomes of the generated series. One can think of the extreme case application of a block length equal to the original sample. It case easily be seen that this would render the same sample. Thus, the total number of possible outcomes, O, from a moving block bootstrap is a function of the total number of observations, T, and the block length, b.

$$O = (T - bl)^{\frac{T}{bl}} \tag{A2}$$

In the approach of this paper the bootstrapped monthly return series is utilized to construct price paths. And compute the non-overlapping annual return series from these trajectories. In this setting the approach of long block lengths actually contributes to parameter uncertainty, see figure 4.

[Figure 4]

This discretization occurs for investment horizons shorter that the block length. The number of possible non-overlapping q month returns, o, for block lengths, b, longer or equal to the investment horizon, q, can be expressed by:

$$O = \frac{T}{I}(T - bl) \quad \text{for} \quad bl \geq 1 \tag{A3}$$

The solution to avoid this discretization of the generated series is to utilize shorter block lengths than the investment horizon as this increases the number of possible outcomes, o:

$$O = (T - bl)^{\frac{I}{bl}} \quad \text{for} \quad bl \leq 1 \tag{A4}$$

Let us compare the possible outcomes for a long block, $b = 60$, versus short block, $b = 6$.

Example 1: If we have 600 observations and a block length of 60 months the number of possible trajectories are $O = 540^{10} = 2.10 \cdot 10^{27}$. The number of possible 60-month returns are 540 and the number of possible 12 month return are $5 \cdot 540 = 2700$.

Example 2: If we have 600 observation and a shorter block length of 6 months the number of possible trajectories are $O = 594^{100} = 2.39 \cdot 10^{277}$. The number of possible 60-month returns increases to $o = 594^{10} = 5.46 \cdot 10^{27}$ and 12-months returns increases to $o = 594^{2} = 352836$.

Table 1: Descriptive statistics of monthly real return from Swedish stocks and bonds 1920-1999
Note: p-values in brackets.

Asset	Mean	Median	Standard-deviation	Skewness	Kurtosis	Doornik-Hansen test	Sign. level of DH-test
Bonds	0.28	0.12	2.20	0.45	13.40	875.82	0.00
Stocks	0.72	0.73	4.71	0.02	6.89	283.77	0.00

Table 2: Rejected blocks in number and per cent
Note: In our case the number of accepted blocks is always 3.200.000.

Asset mix	Stocks	60-40	50-50	40-60	Bonds
# of rejections	175078	127100	118281	111353	109254
Rejections in %	5.40	3.92	3.65	3.44	3.42

Table 3: Rejected blocks in number and per cent
Note: 2.5 and 97.5 percentiles within brackets

	Stocks	60/40 Stocks/Bonds	50/50 Stocks/Bonds	40/60 Stocks/Bonds	Bonds
1-Year	8.17 [−27.07, 51.75]	6.25 [−19.35, 36.79]	5.68 [−17.80, 33.95]	5.04 [−16.66, 31.73]	2.48 [−17.15, 31.17]
2-Year	7.79 [−17.61, 37.44]	6.14 [−12.21, 26.68]	5.66 [−11.36, 24.53]	5.16 [−10.90, 22.84]	2.86 [−12.53, 20.92]
3-Year	7.62 [−13.29, 31.29]	6.09 [−9.00, 22.52]	5.66 [−8.36, 20.67]	5.17 [−8.03, 19.24]	2.96 [−9.73, 17.19]
5-Year	7.51 [−8.89, 25.59]	6.04 [−5.70, 18.57]	5.61 [−5.30, 17.11]	5.16 [−5.12, 15.90]	3.03 [−6.79, 13.65]
10-Year	7.44 [−4.28, 20.03]	6.00 [−2.34, 14.76]	5.59 [−2.17, 13.59]	5.15 [−2.12, 12.62]	3.04 [−3.94, 10.41]
15-Year	7.42 [−2.20, 17.70]	5.99 [−0.83, 13.06]	5.59 [−0.77,12.03]	5.14 [−0.78, 11.21]	3.04 [−2.64, 8.99]
20-Year	7.39 [−0.93, 16.23]	5.98 [0.02, 12.13]	5.58 [0.07, 11.19]	5.12 [−0.06, 10.38]	3.04 [−1.88, 8.14]
25-Year	7.40 [−0.12, 15.27]	5.98 [0.68, 11.51]	5.58 [0.68, 10.57]	5.12 [0.50, 9.82]	3.04 [−1.35, 7.55]

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0.001	0	0	0	0	0	0	0
≥ 95	0.002	0	0	0	0	0	0	0
≥ 90	0.002	0	0	0	0	0	0	0
≥ 85	0.003	0	0	0	0	0	0	0
≥ 80	0.004	0	0	0	0	0	0	0
≥ 75	0.006	0	0	0	0	0	0	0
≥ 70	0.008	0	0	0	0	0	0	0
≥ 65	0.010	0.001	0	0	0	0	0	0
≥ 60	0.014	0.002	0	0	0	0	0	0
≥ 55	0.020	0.003	0	0	0	0	0	0
≥ 50	0.028	0.005	0.001	0	0	0	0	0
≥ 45	0.041	0.010	0.002	0	0	0	0	0
≥ 40	0.059	0.018	0.006	0.001	0	0	0	0
≥ 35	0.086	0.033	0.014	0.003	0	0	0	0
≥ 30	0.124	0.060	0.032	0.009	0.001	0	0	0
≥ 25	0.180	0.108	0.068	0.029	0.004	0.001	0	0
≥ 20	0.254	0.183	0.137	0.082	0.025	0.008	0.003	0.001
≥ 19	0.271	0.202	0.156	0.099	0.035	0.014	0.006	0.002
≥ 18	0.290	0.223	0.177	0.119	0.048	0.021	0.010	0.004
≥ 17	0.309	0.244	0.200	0.142	0.066	0.032	0.016	0.009
≥ 16	0.328	0.268	0.225	0.167	0.087	0.048	0.028	0.017
≥ 15	0.349	0.293	0.252	0.196	0.114	0.070	0.045	0.029
≥ 14	0.370	0.319	0.281	0.229	0.147	0.099	0.068	0.049
≥ 13	0.391	0.346	0.312	0.264	0.186	0.137	0.103	0.079
≥ 12	0.413	0.374	0.345	0.302	0.231	0.183	0.149	0.120
≥ 11	0.436	0.403	0.379	0.343	0.282	0.240	0.208	0.179
≥ 10	0.458	0.432	0.414	0.386	0.339	0.306	0.279	0.253
≥ 9	0.481	0.463	0.450	0.431	0.400	0.378	0.358	0.341
≥ 8	0.504	0.493	0.486	0.478	0.464	0.455	0.444	0.439
≥ 7	0.527	0.524	0.523	0.524	0.529	0.533	0.536	0.542
≥ 6	0.550	0.555	0.559	0.570	0.594	0.611	0.626	0.641
≥ 5	0.573	0.585	0.596	0.616	0.656	0.685	0.709	0.732
≥ 4	0.596	0.615	0.631	0.660	0.715	0.752	0.784	0.808
≥ 3	0.618	0.644	0.666	0.703	0.768	0.812	0.845	0.870
≥ 2	0.640	0.672	0.699	0.742	0.816	0.861	0.894	0.918
≥ 1	0.661	0.699	0.730	0.779	0.857	0.901	0.931	0.950
≥ 0	**0.682**	**0.726**	**0.760**	**0.813**	**0.891**	**0.933**	**0.957**	**0.972**
≥ -1	0.702	0.751	0.788	0.843	0.919	0.955	0.975	0.986
≥ -2	0.722	0.775	0.814	0.870	0.941	0.971	0.986	0.993
≥ -3	0.741	0.797	0.838	0.894	0.958	0.983	0.993	0.997
≥ -4	0.759	0.818	0.860	0.914	0.971	0.990	0.996	0.999
≥ -5	0.776	0.837	0.880	0.931	0.980	0.994	0.998	0.999
≥ -6	0.793	0.855	0.898	0.946	0.987	0.997	0.999	1
≥ -7	0.809	0.872	0.914	0.958	0.992	0.998	1	1
≥ -8	0.824	0.887	0.928	0.968	0.995	0.999	1	1
≥ -9	0.838	0.901	0.940	0.975	0.997	1	1	1
≥ -10	0.851	0.914	0.950	0.981	0.998	1	1	1
≥ -11	0.863	0.925	0.959	0.986	0.999	1	1	1
≥ -12	0.875	0.936	0.967	0.990	0.999	1	1	1
≥ -13	0.886	0.945	0.973	0.993	1	1	1	1
≥ -14	0.896	0.953	0.978	0.995	1	1	1	1
≥ -15	0.905	0.960	0.983	0.996	1	1	1	1
≥ -16	0.914	0.967	0.987	0.998	1	1	1	1
≥ -17	0.922	0.972	0.990	0.998	1	1	1	1
≥ -18	0.929	0.977	0.992	0.999	1	1	1	1
≥ -19	0.936	0.981	0.994	0.999	1	1	1	1
≥ -20	0.942	0.984	0.995	1	1	1	1	1
≥ -25	0.967	0.995	0.999	1	1	1	1	1
≥ -30	0.983	0.999	1	1	1	1	1	1
≥ -35	0.993	1	1	1	1	1	1	1
≥ -40	0.997	1	1	1	1	1	1	1
≥ -45	0.999	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 4: Probability of achieving at least Specified Stock Market Return.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0	0	0	0	0	0	0	0
≥ 95	0	0	0	0	0	0	0	0
≥ 90	0	0	0	0	0	0	0	0
≥ 85	0	0	0	0	0	0	0	0
≥ 80	0	0	0	0	0	0	0	0
≥ 75	0	0	0	0	0	0	0	0
≥ 70	0	0	0	0	0	0	0	0
≥ 65	0	0	0	0	0	0	0	0
≥ 60	0.001	0	0	0	0	0	0	0
≥ 55	0.001	0	0	0	0	0	0	0
≥ 50	0.002	0	0	0	0	0	0	0
≥ 45	0.005	0	0	0	0	0	0	0
≥ 40	0.009	0	0	0	0	0	0	0
≥ 35	0.016	0.001	0	0	0	0	0	0
≥ 30	0.028	0.004	0.001	0	0	0	0	0
≥ 25	0.044	0.011	0.003	0	0	0	0	0
≥ 20	0.073	0.030	0.012	0.002	0	0	0	0
≥ 19	0.082	0.036	0.015	0.003	0	0	0	0
≥ 18	0.091	0.044	0.020	0.005	0	0	0	0
≥ 17	0.101	0.053	0.026	0.007	0	0	0	0
≥ 16	0.112	0.063	0.034	0.010	0	0	0	0
≥ 15	0.126	0.075	0.043	0.015	0.001	0	0	0
≥ 14	0.142	0.089	0.055	0.021	0.002	0	0	0
≥ 13	0.159	0.106	0.070	0.031	0.005	0.001	0	0
≥ 12	0.179	0.126	0.088	0.045	0.009	0.002	0	0
≥ 11	0.201	0.149	0.111	0.063	0.017	0.005	0.002	0.001
≥ 10	0.226	0.176	0.139	0.088	0.031	0.011	0.004	0.002
≥ 9	0.254	0.207	0.172	0.119	0.053	0.024	0.011	0.006
≥ 8	0.284	0.242	0.211	0.160	0.085	0.048	0.027	0.016
≥ 7	0.317	0.283	0.256	0.209	0.134	0.089	0.060	0.042
≥ 6	0.352	0.329	0.308	0.270	0.202	0.155	0.122	0.096
≥ 5	0.391	0.379	0.366	0.340	0.288	0.248	0.220	0.192
≥ 4	0.432	0.434	0.430	0.419	0.392	0.369	0.351	0.334
≥ 3	0.475	0.492	0.497	0.502	0.505	0.505	0.506	0.506
≥ 2	0.523	0.552	0.566	0.588	0.619	0.641	0.663	0.680
≥ 1	0.571	0.612	0.636	0.670	0.724	0.765	0.795	0.819
≥ 0	**0.621**	**0.670**	**0.701**	**0.743**	**0.811**	**0.859**	**0.889**	**0.912**
≥ -1	0.669	0.725	0.759	0.807	0.879	0.922	0.947	0.964
≥ -2	0.716	0.775	0.811	0.858	0.927	0.960	0.977	0.988
≥ -3	0.758	0.818	0.853	0.898	0.957	0.981	0.991	0.996
≥ -4	0.796	0.854	0.887	0.928	0.977	0.992	0.997	0.999
≥ -5	0.829	0.884	0.914	0.950	0.988	0.997	0.999	1
≥ -6	0.857	0.907	0.934	0.966	0.994	0.999	1	1
≥ -7	0.881	0.926	0.949	0.977	0.997	1	1	1
≥ -8	0.901	0.940	0.960	0.984	0.999	1	1	1
≥ -9	0.917	0.951	0.969	0.990	0.999	1	1	1
≥ -10	0.930	0.960	0.976	0.994	1	1	1	1
≥ -11	0.941	0.967	0.982	0.996	1	1	1	1
≥ -12	0.950	0.972	0.987	0.998	1	1	1	1
≥ -13	0.957	0.977	0.990	0.999	1	1	1	1
≥ -14	0.962	0.980	0.993	0.999	1	1	1	1
≥ -15	0.967	0.984	0.995	0.999	1	1	1	1
≥ -16	0.971	0.987	0.997	1	1	1	1	1
≥ -17	0.975	0.990	0.998	1	1	1	1	1
≥ -18	0.977	0.992	0.998	1	1	1	1	1
≥ -19	0.979	0.994	0.999	1	1	1	1	1
≥ -20	0.981	0.996	0.999	1	1	1	1	1
≥ -25	0.988	0.999	1	1	1	1	1	1
≥ -30	0.993	1	1	1	1	1	1	1
≥ -35	0.998	1	1	1	1	1	1	1
≥ -40	1	1	1	1	1	1	1	1
≥ -45	1	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 5: Probability of achieving at least specified five-year bond returns.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0	0	0	0	0	0	0	0
≥ 95	0	0	0	0	0	0	0	0
≥ 90	0	0	0	0	0	0	0	0
≥ 85	0	0	0	0	0	0	0	0
≥ 80	0	0	0	0	0	0	0	0
≥ 75	0	0	0	0	0	0	0	0
≥ 70	0.001	0	0	0	0	0	0	0
≥ 65	0.001	0	0	0	0	0	0	0
≥ 60	0.002	0	0	0	0	0	0	0
≥ 55	0.003	0	0	0	0	0	0	0
≥ 50	0.006	0	0	0	0	0	0	0
≥ 45	0.011	0.001	0	0	0	0	0	0
≥ 40	0.018	0.002	0	0	0	0	0	0
≥ 35	0.030	0.005	0.001	0	0	0	0	0
≥ 30	0.051	0.013	0.004	0	0	0	0	0
≥ 25	0.085	0.034	0.013	0.002	0	0	0	0
≥ 20	0.146	0.079	0.045	0.015	0.001	0	0	0
≥ 19	0.162	0.093	0.056	0.021	0.002	0	0	0
≥ 18	0.179	0.110	0.070	0.030	0.004	0.001	0	0
≥ 17	0.198	0.129	0.087	0.041	0.007	0.001	0	0
≥ 16	0.219	0.150	0.106	0.056	0.013	0.003	0.001	0
≥ 15	0.242	0.174	0.130	0.076	0.022	0.007	0.002	0.001
≥ 14	0.266	0.201	0.157	0.100	0.036	0.014	0.005	0.002
≥ 13	0.292	0.231	0.187	0.130	0.057	0.027	0.012	0.006
≥ 12	0.319	0.263	0.222	0.165	0.086	0.047	0.027	0.015
≥ 11	0.348	0.298	0.262	0.208	0.126	0.081	0.053	0.035
≥ 10	0.378	0.336	0.304	0.257	0.179	0.131	0.098	0.074
≥ 9	0.409	0.376	0.351	0.312	0.245	0.198	0.166	0.137
≥ 8	0.442	0.419	0.401	0.372	0.323	0.284	0.255	0.232
≥ 7	0.475	0.462	0.452	0.436	0.409	0.387	0.369	0.356
≥ 6	0.509	0.506	0.504	0.503	0.500	0.499	0.498	0.497
≥ 5	0.542	0.551	0.557	0.569	0.592	0.613	0.627	0.641
≥ 4	0.576	0.595	0.609	0.634	0.680	0.715	0.743	0.765
≥ 3	0.609	0.637	0.659	0.696	0.759	0.803	0.835	0.862
≥ 2	0.641	0.678	0.707	0.752	0.826	0.873	0.905	0.927
≥ 1	0.673	0.717	0.751	0.802	0.879	0.924	0.950	0.966
≥ 0	**0.702**	**0.753**	**0.791**	**0.845**	**0.920**	**0.956**	**0.975**	**0.986**
≥ -1	0.731	0.787	0.826	0.882	0.950	0.977	0.989	0.996
≥ -2	0.758	0.817	0.858	0.911	0.970	0.989	0.996	0.998
≥ -3	0.782	0.844	0.886	0.935	0.982	0.995	0.999	1
≥ -4	0.806	0.868	0.908	0.954	0.990	0.998	1	1
≥ -5	0.828	0.890	0.928	0.967	0.995	0.999	1	1
≥ -6	0.847	0.908	0.944	0.977	0.998	1	1	1
≥ -7	0.864	0.924	0.957	0.985	0.999	1	1	1
≥ -8	0.880	0.938	0.967	0.990	0.999	1	1	1
≥ -9	0.894	0.949	0.975	0.994	1	1	1	1
≥ -10	0.907	0.959	0.982	0.996	1	1	1	1
≥ -11	0.919	0.967	0.987	0.998	1	1	1	1
≥ -12	0.929	0.974	0.990	0.999	1	1	1	1
≥ -13	0.938	0.979	0.993	0.999	1	1	1	1
≥ -14	0.945	0.984	0.995	0.999	1	1	1	1
≥ -15	0.952	0.987	0.997	1	1	1	1	1
≥ -16	0.959	0.990	0.998	1	1	1	1	1
≥ -17	0.964	0.993	0.998	1	1	1	1	1
≥ -18	0.969	0.994	0.999	1	1	1	1	1
≥ -19	0.974	0.996	0.999	1	1	1	1	1
≥ -20	0.977	0.997	1	1	1	1	1	1
≥ -25	0.990	0.999	1	1	1	1	1	1
≥ -30	0.996	1	1	1	1	1	1	1
≥ -35	0.999	1	1	1	1	1	1	1
≥ -40	1	1	1	1	1	1	1	1
≥ -45	1	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 6: Probability of achieving at least specified portfolio return, 60-40 per cent.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0	0	0	0	0	0	0	0
≥ 95	0	0	0	0	0	0	0	0
≥ 90	0	0	0	0	0	0	0	0
≥ 85	0	0	0	0	0	0	0	0
≥ 80	0	0	0	0	0	0	0	0
≥ 75	0	0	0	0	0	0	0	0
≥ 70	0	0	0	0	0	0	0	0
≥ 65	0	0	0	0	0	0	0	0
≥ 60	0.001	0	0	0	0	0	0	0
≥ 55	0.002	0	0	0	0	0	0	0
≥ 50	0.003	0	0	0	0	0	0	0
≥ 45	0.006	0	0	0	0	0	0	0
≥ 40	0.012	0.001	0	0	0	0	0	0
≥ 35	0.022	0.003	0	0	0	0	0	0
≥ 30	0.039	0.008	0.002	0	0	0	0	0
≥ 25	0.069	0.023	0.008	0.001	0	0	0	0
≥ 20	0.123	0.060	0.031	0.008	0	0	0	0
≥ 19	0.138	0.072	0.040	0.012	0.001	0	0	0
≥ 18	0.154	0.086	0.050	0.018	0.002	0	0	0
≥ 17	0.172	0.103	0.064	0.026	0.003	0	0	0
≥ 16	0.192	0.123	0.081	0.037	0.006	0.001	0	0
≥ 15	0.213	0.145	0.101	0.053	0.012	0.003	0.001	0
≥ 14	0.237	0.170	0.126	0.073	0.021	0.006	0.002	0.001
≥ 13	0.262	0.198	0.155	0.098	0.035	0.014	0.005	0.002
≥ 12	0.289	0.230	0.188	0.130	0.057	0.027	0.013	0.006
≥ 11	0.319	0.265	0.226	0.169	0.090	0.051	0.030	0.017
≥ 10	0.350	0.304	0.270	0.217	0.136	0.090	0.061	0.042
≥ 9	0.382	0.346	0.317	0.271	0.197	0.149	0.114	0.089
≥ 8	0.416	0.390	0.369	0.334	0.272	0.229	0.195	0.168
≥ 7	0.452	0.436	0.424	0.401	0.360	0.331	0.306	0.288
≥ 6	0.488	0.484	0.480	0.471	0.459	0.448	0.441	0.435
≥ 5	0.525	0.532	0.538	0.545	0.560	0.573	0.581	0.591
≥ 4	0.562	0.581	0.595	0.616	0.658	0.688	0.712	0.735
≥ 3	0.598	0.628	0.650	0.685	0.745	0.789	0.820	0.847
≥ 2	0.634	0.674	0.702	0.747	0.820	0.866	0.898	0.923
≥ 1	0.669	0.717	0.750	0.802	0.879	0.922	0.948	0.966
≥ 0	**0.703**	**0.757**	**0.794**	**0.848**	**0.922**	**0.957**	**0.977**	**0.987**
≥ -1	0.735	0.793	0.832	0.887	0.952	0.979	0.990	0.995
≥ -2	0.764	0.826	0.865	0.917	0.972	0.990	0.997	0.999
≥ -3	0.792	0.854	0.893	0.941	0.985	0.996	0.999	1
≥ -4	0.817	0.879	0.917	0.959	0.992	0.998	1	1
≥ -5	0.840	0.900	0.936	0.972	0.996	0.999	1	1
≥ -6	0.860	0.919	0.951	0.981	0.998	1	1	1
≥ -7	0.878	0.934	0.963	0.988	0.999	1	1	1
≥ -8	0.894	0.947	0.972	0.992	1	1	1	1
≥ -9	0.908	0.957	0.979	0.995	1	1	1	1
≥ -10	0.920	0.966	0.985	0.997	1	1	1	1
≥ -11	0.931	0.973	0.989	0.998	1	1	1	1
≥ -12	0.940	0.979	0.992	0.999	1	1	1	1
≥ -13	0.948	0.983	0.995	0.999	1	1	1	1
≥ -14	0.955	0.987	0.996	1	1	1	1	1
≥ -15	0.961	0.990	0.997	1	1	1	1	1
≥ -16	0.967	0.992	0.998	1	1	1	1	1
≥ -17	0.972	0.994	0.999	1	1	1	1	1
≥ -18	0.976	0.996	0.999	1	1	1	1	1
≥ -19	0.979	0.997	1	1	1	1	1	1
≥ -20	0.982	0.998	1	1	1	1	1	1
≥ -25	0.992	1	1	1	1	1	1	1
≥ -30	0.996	1	1	1	1	1	1	1
≥ -35	0.999	1	1	1	1	1	1	1
≥ -40	1	1	1	1	1	1	1	1
≥ -45	1	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 7: Probability of achieving at least specified portfolio return, 50-50 per cent.

Return in %	1-Year	2-Year	3-Year	5-Year	10-Year	15-Year	20-Year	25-Year
≥ 100	0	0	0	0	0	0	0	0
≥ 95	0	0	0	0	0	0	0	0
≥ 90	0	0	0	0	0	0	0	0
≥ 85	0	0	0	0	0	0	0	0
≥ 80	0	0	0	0	0	0	0	0
≥ 75	0	0	0	0	0	0	0	0
≥ 70	0	0	0	0	0	0	0	0
≥ 65	0	0	0	0	0	0	0	0
≥ 60	0	0	0	0	0	0	0	0
≥ 55	0.001	0	0	0	0	0	0	0
≥ 50	0.002	0	0	0	0	0	0	0
≥ 45	0.004	0	0	0	0	0	0	0
≥ 40	0.008	0	0	0	0	0	0	0
≥ 35	0.016	0.001	0	0	0	0	0	0
≥ 30	0.031	0.005	0.001	0	0	0	0	0
≥ 25	0.058	0.015	0.004	0	0	0	0	0
≥ 20	0.105	0.045	0.020	0.004	0	0	0	0
≥ 19	0.118	0.055	0.027	0.007	0	0	0	0
≥ 18	0.132	0.068	0.035	0.011	0.001	0	0	0
≥ 17	0.148	0.082	0.046	0.016	0.001	0	0	0
≥ 16	0.166	0.099	0.060	0.024	0.003	0	0	0
≥ 15	0.186	0.119	0.078	0.035	0.006	0.001	0	0
≥ 14	0.208	0.142	0.099	0.051	0.011	0.002	0.001	0
≥ 13	0.232	0.168	0.125	0.071	0.020	0.006	0.002	0.001
≥ 12	0.258	0.198	0.155	0.098	0.036	0.014	0.005	0.002
≥ 11	0.287	0.233	0.191	0.134	0.061	0.029	0.015	0.007
≥ 10	0.318	0.271	0.233	0.177	0.098	0.056	0.034	0.020
≥ 9	0.351	0.312	0.280	0.230	0.151	0.102	0.072	0.050
≥ 8	0.386	0.357	0.332	0.291	0.221	0.174	0.139	0.111
≥ 7	0.423	0.406	0.389	0.360	0.308	0.270	0.239	0.211
≥ 6	0.462	0.456	0.448	0.434	0.408	0.387	0.369	0.355
≥ 5	0.502	0.509	0.511	0.513	0.516	0.519	0.518	0.521
≥ 4	0.542	0.561	0.573	0.590	0.622	0.647	0.666	0.683
≥ 3	0.583	0.613	0.635	0.666	0.720	0.761	0.791	0.817
≥ 2	0.623	0.664	0.692	0.734	0.803	0.850	0.883	0.907
≥ 1	0.663	0.711	0.746	0.795	0.869	0.913	0.942	0.959
≥ 0	**0.700**	**0.755**	**0.793**	**0.846**	**0.917**	**0.954**	**0.973**	**0.984**
≥ -1	0.735	0.795	0.834	0.886	0.951	0.978	0.989	0.995
≥ -2	0.768	0.830	0.870	0.919	0.972	0.990	0.996	0.998
≥ -3	0.799	0.861	0.898	0.943	0.985	0.996	0.999	1
≥ -4	0.826	0.886	0.922	0.961	0.992	0.998	1	1
≥ -5	0.850	0.908	0.940	0.974	0.996	0.999	1	1
≥ -6	0.872	0.926	0.955	0.983	0.998	1	1	1
≥ -7	0.890	0.940	0.966	0.989	0.999	1	1	1
≥ -8	0.906	0.952	0.975	0.993	1	1	1	1
≥ -9	0.919	0.962	0.982	0.996	1	1	1	1
≥ -10	0.931	0.970	0.987	0.997	1	1	1	1
≥ -11	0.941	0.976	0.990	0.999	1	1	1	1
≥ -12	0.950	0.981	0.993	0.999	1	1	1	1
≥ -13	0.957	0.985	0.995	1	1	1	1	1
≥ -14	0.963	0.988	0.997	1	1	1	1	1
≥ -15	0.968	0.991	0.998	1	1	1	1	1
≥ -16	0.972	0.993	0.999	1	1	1	1	1
≥ -17	0.976	0.995	0.999	1	1	1	1	1
≥ -18	0.979	0.996	0.999	1	1	1	1	1
≥ -19	0.982	0.997	1	1	1	1	1	1
≥ -20	0.985	0.998	1	1	1	1	1	1
≥ -25	0.992	1	1	1	1	1	1	1
≥ -30	0.996	1	1	1	1	1	1	1
≥ -35	0.999	1	1	1	1	1	1	1
≥ -40	1	1	1	1	1	1	1	1
≥ -45	1	1	1	1	1	1	1	1
≥ -50	1	1	1	1	1	1	1	1

Table 8: Probability of achieving at least specified portfolio return, 40-60 per cent.

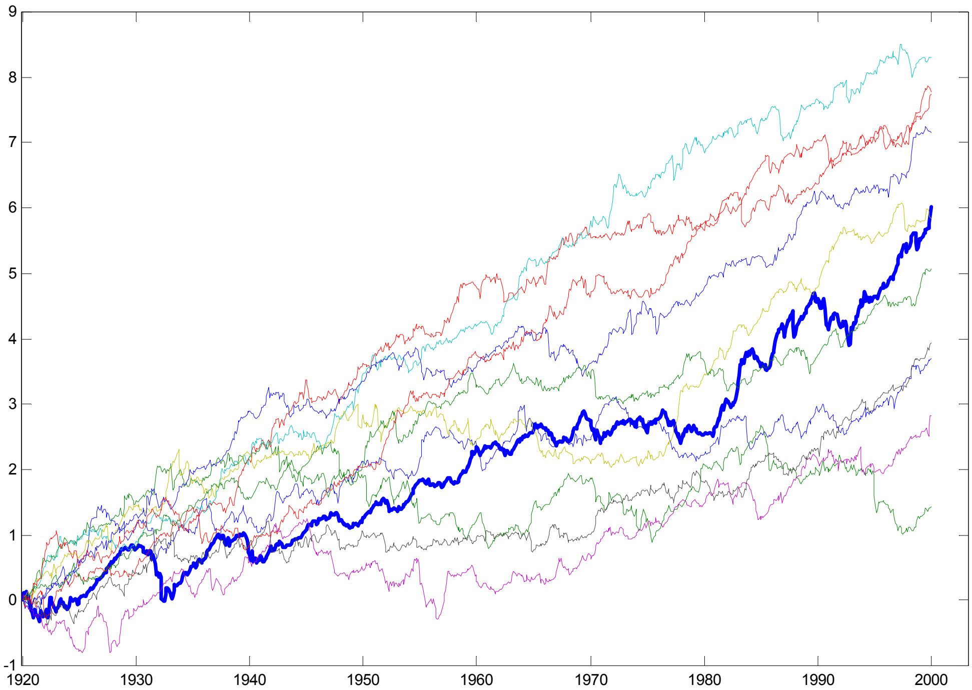

Figure1: Logarithm of resampled real stock price paths vs. Swedish stock market index.

Note: The time scale refers to the Swedish stock market index in real prices, thick line.

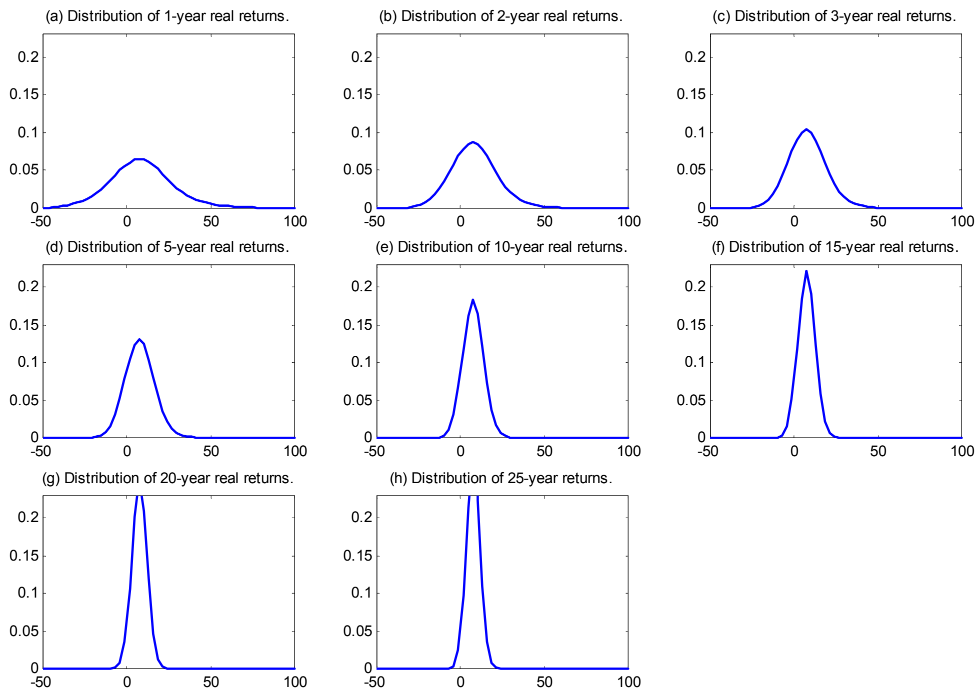

Figure 2: Empirical distributions of real stock returns.

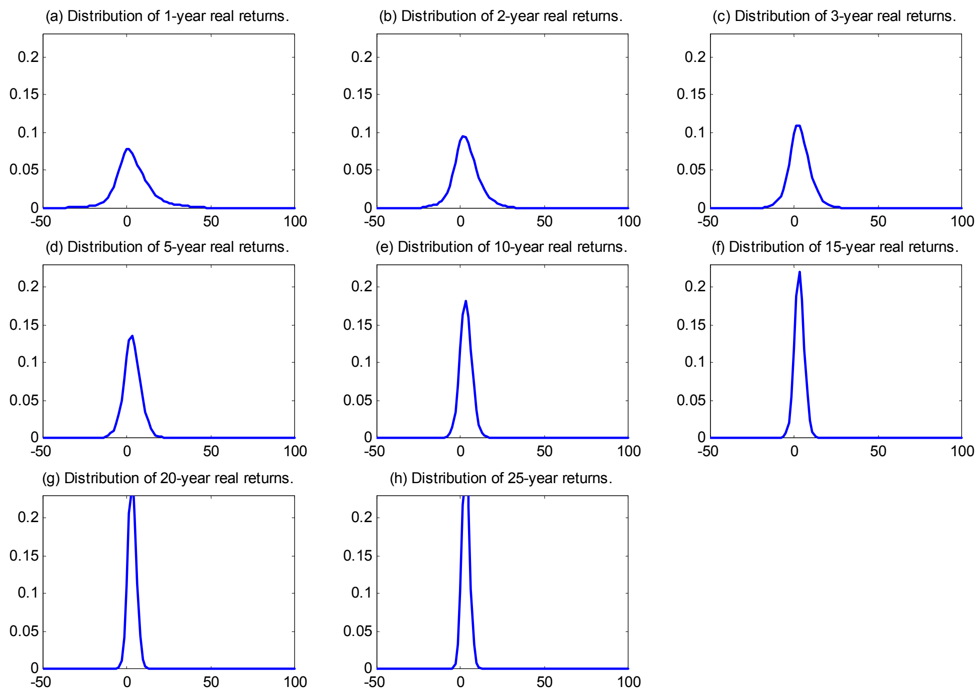

Figure 3: Empirical distributions of real bond returns.

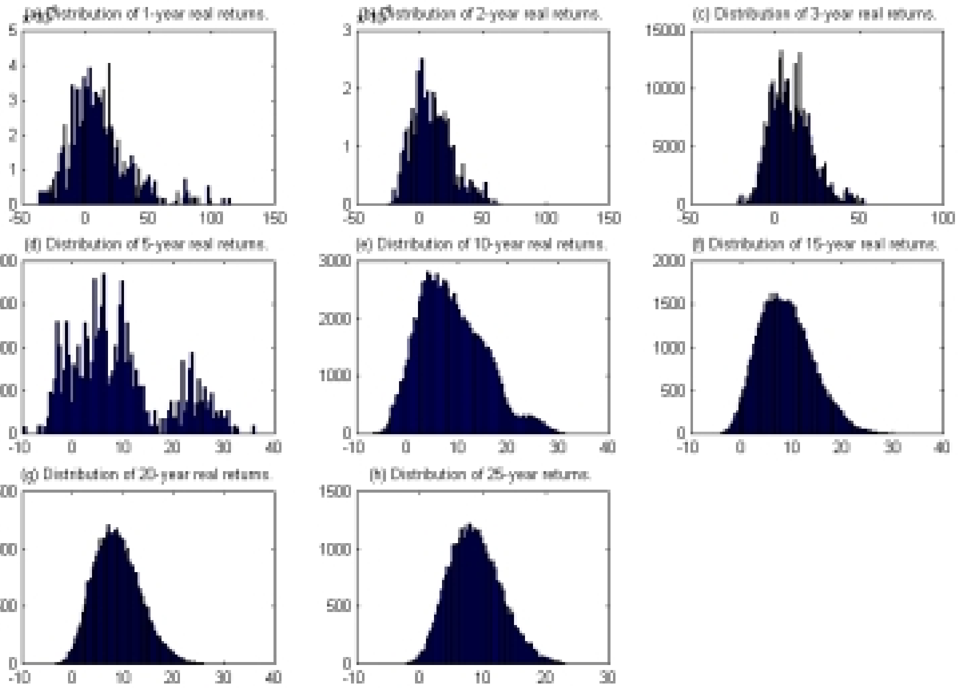

Figure 4: A visualization of the discretization of the empirical distributions of real stock returns for investment horizons shorter than the block length, 60 months.