Skill or Luck?
Search Frictions and Wage Differentials

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Abstract

The paper seeks to explain a collection of empirical regularities concerning inter– and intra–industrial wage differentials. For example, the model is consistent with the following well established set of observations: (i) Controlling for other variables, the wage of displaced workers is strongly related both to the characteristics of the pre–displacement employer and the characteristics of the post–displacement employer, (ii) wage differentials are correlated across occupations, and (iii) the pattern of wage differentials are similar across countries with different market institutions.

Our framework is a search–matching model with heterogeneous worker quality, heterogeneous technology, and endogenous wage policies (wage posting or bargaining). The model has equilibria in which wage bargaining coexists with a variety of posted wages. Higher wages attract more able workers on average. For a given posted wage, there may be an interval of worker productivities such that both the worker and the firm are satisfied with the match. Hence, a worker's wage reflects luck as well as skill.

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1 Introduction

Currently, there are two main schools of thought about wage determination. Following Dickens and Lang (1992) among others, we will refer to them as the *human capital theory* and the *segmented labor market theory.* The first holds that wage differentials by and large reflect productivity differentials: Workers are paid according to their skills. The second holds that wages do not clear all segments of the labor market. There exist worker rents, because some firms systematically pay more than others for the same level of skill. Put differently, luck plays a sifgnificant role for the earnings of individual workers.

Recently, there has been a large number of empirical studies claiming to identify worker rents. (See e.g. Dickens and Katz (1987b), Krueger and Summers (1987,1988), Katz and Summers (1989), and Groshen (1991).) The study of displaced workers by Gibbons and Katz (1992) is arguably the most persuasive. They find that the wage of displaced workers (who change job through no fault or wish of their own) is strongly correlated with the industry affiliation of the post–displacement employer, even after correction for occupation and a variety of skill measures. The composition of their sample rules out self selection. Moreover, the results hold even for workers who switch industries, so the correlations are hardly caused by differences in industry–specific skills.[1]

While there are many theories purporting to explain worker rents, most of them are challenged on the ground that they are inconsistent with some key piece of evidence. The main empirical regularities are: (i) Wage differentials are systematically related to firm characteristics such as capital–labor intensity, size and profit (Brown and Medoff (1989), Groshen (1991), Krueger and Summers (1988), Gibbons and Katz (1992)). (ii) Wage differentials are similar across occupations (Dickens and Katz (1987b), Groshen (1991)). (iii) The patterns repeat themselves in a large number of countries, with different pay systems (Dickens and Katz (1987b), Krueger and Summers (1987)). (iv) Wages of displaced workers correlate with the characteristics of the original firm, even for industry switchers (Gibbons and Katz (1992). (v) Measures of industry wage premia correlate

[1]The interview studies by Wial (1991) presents more direct support for the segmented labor market hypothesis. According to Wial, workers perceive the labor market as a lottery, in which it is important to be at the right place at the right time in order to get the premium jobs. Another frequently cited observation is that quit rates are lower in industries which pay well. However, this need not indicate that the job is attractive *ex ante*.

positively with measured skill levels (Brown and Medoff (1989), Dickens and Katz (1987a), Murphy and Topel (1990)). Our purpose is to present a coherent theory which is consistent with *all* the above observations.[2]

A key question is; why do some firms apparently pay more than they need to in order to fill the position? Here we agree with Carmichael (1990): The most convincing explanation is that a higher wage attracts applicants of higher quality—the selection theory. But we also share the widely held skepticism towards existing selection models, which rely on persistent asymmetric information. For example, in Weiss' (1980) seminal model, it is crucial that the employer *never* learns the ability of his employees. Instead, we argue that, due to search frictions, wage policies shape the pool of suitable applicants. The firm can assess the applicant's quality upon arrival, but have no information about individuals before matching. By promising to pay a high wage, the firm attracts better applicants on average, but also has a higher wage bill.[3] Because of the matching friction, there is a scope for luck. Workers are lucky if they are quickly matched with a firm which pays well, but not too well (in which case the worker is rejected for the job). Similarly, firms are lucky if they can quickly find a worker of high, but not too high productivity. –The trick is to show that there exist equilibria in which firms offer different wages, and that for some wage offers, the firms are willing to employ more than one type of worker.

Unlike earlier search models, we do not rely on exogenous differences in workers' reservation wage. The reservation wage simply reflects the workers' option value from continued search. Our innovation to make sure that this option value differs across workers of different productivity is to allow firms to choose whether to post wages or bargain with workers. In a bargaining firm, the workers' wage is always positively correlated with

[2]The papers which we have already cited contain ample discussion of the strengths and weaknesses of current theories (see in particular Carmichael (1990), Dickens and Katz (1987b), Gibbons and Katz (1992) and Groshen (1991)), and the reader is referred to them for a details. In short, the currently popular view that workers' bargaining power explain why they earn rents (in some profitable but vulnerable industries) is consistent with (i), (ii) and (v), but is harder to reconcile with (iii) and (iv). The effort based efficiency wage theories, on the other hand, has a problem with (ii), and (iv). Besides, these theories tend to predict that workers purchase jobs, and are therefore vulnerable to the "bonding critique." (See Carmichael (1985,1990) for an elaboration of why the absence of entrance fees is evidence against current versions of the efficiency wage theories.) Finally, gift–exchange and fairness theories need some bending in order to explain (iii) and (iv).

[3]The argument would break down if all firms could announce a fully productivity dependent wage schedule. However, we doubt whether such announcements can be credibly made. We return to this issue below.

their productivity, and hence the existence of these firms ensures that the reservation wage differ across workers. In fact, our model provides an explanation for the observed coexistence of wage posting and bargaining: Wage posting has the advantage that the firm captures a large fraction of the gain from trade, whereas bargaining allows more flexibility in adapting the wage to individual applicants. (We explore this trade–off in a companion paper.)

Besides explaining the existence of wage premia, the model is consistent with observations (i)–(v) above: (i) In section 4.2, we show that firms with slightly different technologies can easily end up with very different wages. For example, weak capital–skill complementarities can explain why capital intensive firms try to hire highly skilled workers, and in equilibrium end up paying a premium to a considerable fraction of their employees. (ii) Similarly, weak complementarities between occupations suffice to explain cross–occupational wage differentials. (iii) The reason why the results hold across countries with different market systems is that production technologies are likely to be similar. (iv) Since workers are heterogeneous, skill differentials explain why the wage in the current job correlates with the wage in previous jobs. (v) Finally, as high wages serve precisely to attract high ability applicants, they are offered by the firms which can put high skill to the best use. Hence, it is no coincidence that wage premia and measured skill levels correlate.

Recently, Katz and Summers (1989a,b) have argued that wage premia constitute a rationale for supporting capital intensive industries. It is worth emphasizing, however, that despite the wage premia the role for government intervention is less clear–cut in our model. Subsidies targeted at firm characteristics associated with "good jobs" will promote the characteristic, but may have more modest the effects on wages. For example, a capital subsidy will make firms more capital intensive on average, but could sever the link between capital intensity and high wages (a version the Lucas critique applies).

Before turning to the formal model, let us briefly relate our work to other work pointing to search frictions as the source of luck in the labor market. The paper by Albrecht and Axell (1984) is seminal in offering a dynamic search model in which workers with identical productivity may receive different wages in equilibrium. The idea is that firms are heterogeneous in their cost of waiting, so that impatient firms pay a higher wage.

A similar idea is pursued by Sattinger (1991), who characterizes the wage dispersion in a free entry search equilibrium. It is noteworthy that, unlike Albrecht and Axell, Sattinger allows firms to choose their production technologies. Burdett and Mortensen (1989) also establish analogous results in a model with on–the–job search. Lang (1991) and Montgomery (1991), on the other hand, study static recruiting models and capture the same generic trade-off between the wage level and the probability of hiring.[4] They are also most explicit in relating their results to empirical work on wage differentials.

In all these papers, workers are homogeneous, and high wages stem from high costs of keeping unoccupied vacancies.[5] While they correctly predict that wages should correlate with such variables as capital intensity and profits, it is difficult to believe that a firm's loss from waiting should be so large as to regularly pay a worker in the range of 10–20% more than he would obtain in an average job.[6] Moreover, since workers are homogeneous by assumption, neither of the above papers can account for fact (v); that the firms which pay well also recruit the better workers. Hence, we believe that our assumption of heterogeneous workers is essential to capture the nature as well as the magnitude of real world wage premia.

In many ways the model most similar to ours is Manning (1993). There too, workers are heterogeneous and may be more or less lucky in their search. Besides a number of technical dissimilarities (firms are technologically homogeneous and are confined to post wages; workers differ in their utility of leisure and can search on the job), Manning's paper has a much broader focus than ours. Whereas we go further in explaining the pattern of wage differentials, he also studies issues such as unemployment and skill acquisition.

[4]Weitzman (1989) too argues that wage premia may be caused by the desire by some firms to fill vacancies quickly, but without an explicit model of the search frictions.

[5]Search models with heterogeneous worker productivities include Lockwood (1986) and Sattinger (1992). These papers are however mainly concerned with the efficiency of search equilibria.

[6]Another critique of Albrecht and Axell is that in their model all firms have vacancies at all times, whereas the workers do not learn the wage offer until after they visit the firm. (The search cost is hence the cost of learning the firm's pay policy.) As noted by Lang, Leonard and Lilien (1987), this story is unconvincing if the objective is to explain lasting industry wage differentials, as the workers should learn to predict a firm's wage from its characteristics.

2 The Model

A standard sequential search and matching model is extended in two different directions. First, both technology and labor is heterogeneous, and second, we allow firms to choose whether to post a wage or bargain with the applicants.[7]

2.1 Preliminaries

We consider a subset of the job market (e.g. the market for electrical engineers). There are n candidate workers and m jobs, where m will be endogenously determined. Each job utilizes some technology, $t \in T$. Workers are characterized by their type $i \in I$, where $I = \{1, 2, ..., \bar{i}\}$. Specifically, a worker of type i working in a job with technology t has a value to the employer (productivity) of x_i^t. As a convention, productivity is increasing in i, or formally; $x_i^t < x_{i+1}^t$ for all $i, i+1 \in I$ and $t \in T$.

When a firm opens a vacancy, it also announces a wage policy, j. Either it commits itself to some fixed (productivity independent) wage, or it makes the pay subject to negotiation.[8] In the latter case, the wage is determined by the Nash bargaining solution. We refer to a job type as a pair $(j, t) \in J$.

The worker's productivity is assumed to be observable to the firm once he has applied. Let w_{ij}^t be the wage of a worker of productivity i in a firm with wage policy j and technology t. If a firm commits to a fixed wage, then $w_{ij}^t = w_j$ for any i and t. On the other hand, the wage in bargaining firms, w_{iB}^t, typically depend on worker productivity as well as the firm's technology. Essentially, our formulation of wage policies embodies two assumptions. The first is that any commitment giving the firm a bargaining advantage must be known to the applicant upon his arrival. For example it is not possible to announce a vacancy with the wage subject to negotiation and then make the applicant a take–it–or–leave–it offer when he arrives. The second assumption is that the firm cannot make a commitment to a fixed wage conditional on productivity. This rules out posted

[7]For a detailed exposition of the standard model, with wage bargaining only, see Pissarides (1990).

[8]We are not aware of any other papers in labor economics endogenizing the wage determination mechanism in this way, although it is discussed in Pissarides (1976). Bester (1993), and Spier (1990) are among the few papers which study the choice between price posting and bargaining in a product market context.

wage offers which depends sensitively on the worker's *type*, i.e. his true value to the firm. Our reason is that the type is excessively hard to describe perfectly in a job announcement. We believe that the model could be extended to allow for certain observable characteristics, such as education, which are correlated with the true productivity. In that case, the firms could announce wage policies conditional on these characteristics. (We have chosen not to do this in order to keep down the model's complexity.) What is important is that the firm cannot, at a reasonable cost, commit credibly to a menu containing a perfectly tailored offer to each worker in the economy.[9]

Below, we shall focus exclusively on steady state equilibria, so variables are not indexed by time. Let v_j^t denote the number of vacancies in firms with wage policy j and technology t, and let v be the total number of vacancies. Similarly, let u_i denote the number of unemployed workers of type i, and u the total number of unemployed. The model is set in continuous time. Only the unemployed engage in search. These workers observe vacancies on random. Any given vacancy is observed by some worker, according to a Poisson process, at a rate $q(\theta)$, where $\theta := v/u$. (In other words, the matching function q has constant returns to scale.) It follows that an unemployed worker observes vacancies at a rate $\phi := \theta q(\theta)$. We make the following standard assumption about $q(\theta)$.

Assumption 1 *The matching function satisfies*

$$\begin{aligned} \partial q/\partial\theta &< 0, \\ \lim_{\theta\to 0} q(\theta) &= \infty, \\ \lim_{\theta\to\infty} q(\theta) &= 0. \end{aligned}$$

Consider a worker of type i who sees a vacancy offering the wage w_{ij}^t. In equilibrium, the worker has some reservation wage $\underline{w}_i$ and a job of type (j,t) has a productivity requirement (or reservation productivity) $\underline{x}_j^t$. A match between a worker and a job will be called fruitful if it is acceptable to both parties.

Definition 1 (Fruitful matches) *A match* $(i,(j,t))$ *is fruitful if and only if* $x_i^t \geq \underline{x}_j^t$

[9] Clearly, it is much easier to commit to something simple like; all workers in that job will receive this wage policy than to a more complex policy relating wages to personal characteristics. The latter tend to open for subjective evaluation and hence bargaining.

and $w_{ij}^t \geq \underline{w}_i$.

A suitable job for a worker is one with which he can be fruitfully matched.

Definition 2 (Suitable jobs) *The set of suitable jobs for a worker of type* i *is* $J_S(i) := \{(j,t) \in J | w_{ij}^t \geq \underline{w}_i, x_i^t \geq \underline{x}_j^t\}$.

We define the set of suitable workers for a given job analogously:

Definition 3 (Suitable workers) *The set of suitable workers for a job of type* (j,t) *is* $I_S(j,t) := \{i \in I | x_i^t \geq \underline{x}_j^t, w_{ij}^t \geq \underline{w}_i\}$.

Let us also give a precise definition of a wage premium.

Definition 4 (Wage Premia) *There is a wage premium in the labor market if and only if there exist workers of type* x_i^t *and wage offers* w_{ij}^t *such that* $(w_{ij}^t - \underline{w}_i) > 0$ *and* $(x_i^t - \underline{x}_j^t) \geq 0$.

The movement into and out of employment can then be described as follows. Let ϕ_j^t be the rate at which an unemployed worker observes a vacancy of type (j,t). Formally,

$$\phi_j^t := \frac{v_j^t}{v}\phi.$$

The rate at which a type i worker is offered a *suitable* job is then

$$p_i := \sum_{(j,t) \in J_S(i)} \phi_j^t.$$

We assume that idiosynchratic shocks separate employed workers from their jobs at a rate s.

In a steady state, for each type of worker the movements into and out of unemployment are the same. Letting n_i be the total number of workers of type i, we have the steady state unemployment condition

$$u_i p_i = s(n_i - u_i),$$

or solving for u_i,

$$u_i = \frac{sn_i}{p_i + s}. \tag{1}$$

Similarly, we can derive a steady state vacancy condition for each kind of job. Let q_i be the rate at which vacant positions are observed by workers of type i,

$$q_i := \frac{u_i}{u} q(\theta).$$

The rate at which a vacant position of type (j, t) is observed by a suitable applicant is then

$$\pi_j^t := \sum_{i \in I_S(j,t)} q_i.$$

In the steady state,

$$v_j^t \pi_j^t = s(m_j^t - v_j^t),$$

where m_j^t is the number of jobs of type (j, t). We solve for v_j^t to get

$$v_j^t = \frac{s m_j^t}{\pi_j^t + s}. \tag{2}$$

In order to obtain an explicit solution to the steady state conditions, we must characterize the set of fruitful matches. In other words, we must identify the reservation wage of each type of worker, and the reservation productivity of each type of firm. We will do so in turn.

2.2 Behavior of workers

Workers are either employed or unemployed. For an unemployed worker of type i the present discounted utility is denoted U_i, and for an employed worker in a job of type (j, t) it is denoted E_{ij}^t. Utility is assumed to be money measurable. To convert utilities into flow terms, we multiply by the rate of interest, r. Let z be the flow utility obtained when no job is found. We consider only workers whose productivity, at least in some conceivable job, exceeds their utility of unemployment.

Assumption 2 (Employability) *There is some $t \in T$ such that $z < \min\{x_i^t\}_{i \in I}$.*

In flow terms, the steady state present discounted utility of an unemployed worker, which equals his reservation wage $\underline{w}_i$, can then be written as

$$\underline{w}_i = rU_i = z + \sum_{(j,t)\in J_S(i)} \phi_j^t (E_{ij}^t - U_i), \tag{3}$$

where the second term on the right hand side is simply the expected increase in utility from forming a fruitful match. Analogously, the flow utility of an employed worker is

$$rE_{ij}^t = w_{ij}^t + s(U_i - E_{ij}^t). \tag{4}$$

The solution to these two equations is

$$rU_i = \frac{z(r+s) + \sum_{(j,t)\in J_S(i)} \phi_j^t w_{ij}^t}{r+s+p_i} \tag{5}$$

and

$$E_{ij}^t = \frac{w_{ij}^t + sU_i}{r+s}. \tag{6}$$

An unemployed worker only accepts jobs which pay a wage of at least rU_i. Thus, equation (5) defines the reservation wage—and hence the steady state behavior—of type i workers given the distribution and behavior of firms.

2.3 Behavior of firms

Recall that firms choose whether or not to open a vacancy, and if so what wage policy to adopt. Let V_j^t denote the present discounted value of a vacancy of type (j,t), and let F_{ij}^t denote the present discounted value when it is occupied by a worker of type i. For tractability we assume that all vacancies incur the same cost, c, per unit of time that they are unoccupied. (Whenever jobs differ in their capital/labor ratio, we shall therefore assume that capital is hired after the worker.)

In a steady state, the flow value of a vacancy is

$$rV_j^t = -c + \sum_{i\in I_S(j,t)} q_i (F_{ij}^t - V_j^t), \tag{7}$$

and the flow value of an occupied job is

$$rF_{ij}^t = x_i^t - w_{ij}^t - s(F_{ij}^t - V_j^t). \tag{8}$$

With free entry, the value of a vacancy should be zero for all types of jobs which are offered in equilibrium and negative for all others. Formally, $J^\star$ is an equilibrium set of job types if and only if

$$\forall (j,t) \in J^\star: \quad V_j^t = 0,$$
$$\forall (j,t) \notin J^\star: \quad V_j^t \leq 0.$$

Using (7), (8) and the condition that $V_j^t = 0$, we have for all $j \in J^\star$;

$$\sum_{i \in I_S(j,t)} q_i(x_i^t - w_{ij}^t) = (r+s)c. \tag{9}$$

Conversely, if the left hand side is smaller than the right hand side, the wage w_{ij}^t is not offered in equilibrium. Equation (9) has a natural interpretation. The cost c is an investment, $q_i(x_i^t - w_{ij}^t)$ is the uncertain profit rate, and $(r+s)$ is the required rate of return. The equilibrium value of an occupied job is

$$F_{ij}^t = \frac{x_i^t - w_{ij}^t}{r+s}. \tag{10}$$

For the fixed wage firms, this is a complete description of the steady state, since $w_{ij}^t = w_j$ for all i. For the bargaining firms, we need to derive a wage equation.

The bargaining outcome is assumed to depend on the applicant's productivity. Recall that w_{iB}^t is the outcome when a firm with technology t bargains with a worker of type i. As a solution to the bargaining problem we follow tradition and choose the Nash bargaining solution. Formally, the wage maximizes

$$(E_{iB}^t - U_i)^\beta (F_{iB}^t - V_B^t)^{1-\beta}.$$

From the first order condition, using (6) and (10), we have

$$E_{iB}^t - U_i = \beta(F_{iB}^t + E_{iB}^t - U_i - V_B^t). \tag{11}$$

A wage equation can then be obtained using (6), (10) and the zero profit condition;

$$w_{iB}^t = rU_i + \beta(x_i^t - rU_i). \tag{12}$$

In other words, a worker gets his reservation wage plus a share of the surplus from the match. The share is linear in the worker's bargaining strength, β. We now have one equation for each endogenous variable.

2.4 Equilibrium

Given the matching function, $q(\theta)$, an equilibrium is a solution to the following equations; the zero profit (free entry) conditions, one for each $(j,t) \in J^\star$, given by (9), the steady state unemployment conditions, one for each type of worker i, given by (1), the steady state vacancy conditions, one for each $(j,t) \in J^\star$, described by (2), and the reservation wage equations, one for each type of worker, given by (5). In addition, we need to check that it is not profitable for any firm to switch to another wage policy.

3 Preliminary results

If a firm posts a wage below all the workers' reservation wage, it doesn't attract any applicants. On the other hand, posting a wage above all workers' reservation wage is a waste. More generally, it makes no sense to post a wage which leaves a rent with everyone who is willing to accept it. The employer could then slightly lower the wage without affecting any applicant's decision whether or not to accept the job. Hence, any posted wage must correspond to the reservation wage of some type of worker.

Lemma 1 *All fixed wages w_j such that $(j,t) \in J^\star$ for some $t \in T$, satisfy $w_j = rU_i$ for some $i \in I$.*

PROOF: Suppose $w_j \neq rU_i$ for all i. Clearly, $w_j > \min\{rU_i\}_{i \in I}$. Otherwise, w_j would attract no applicants, and the firm would lose c per unit of time. Hence, there is a largest reservation wage $\underline{w_{\hat{i}}} = rU_{\hat{i}}$ such that $w_j > \underline{w_{\hat{i}}}$. But then there exists an $\epsilon > 0$ such that the lower wage $w = w_j - \epsilon$ attracts the same types of workers as did w_j. ■

Another key observation is that the reservation wage must be weakly increasing in productivity, since a worker of type $i + 1$ is accepted for all jobs in which type i is accepted (look at equation (5)). If there are firms which bargain over the wage, we know from (12) that w^t_{iB} is increasing in i. For later reference, we summarize this observation as follows:

Lemma 2 *The reservation wage, $\underline{w}_i$ is increasing in i and strictly increasing if there is a positive number of firms which bargain over wages.*

Consider now the possibility of equilibria in which all workers get the same wage. The workers' utility of not working, z, is a natural candidate for such a single wage equilibrium. As it turns out, this is also the *only* equilibrium if we restrict attention to posted wages only.

Proposition 1 *If there is a steady state equilibrium in which all firms post wages, then all firms post the wage $w = z$.*

PROOF: Suppose to the contrary that the highest fixed wage which is offered is $w_m > z$. The reservation wage of type i workers, $\underline{w}_i$, is given by (5), and the following two inequalities are easily seen to hold;

$$\underline{w}_i = \frac{z(r+s) + \sum_{(j,t)\in J_S(i)} \phi^t_j w^t_{ij}}{r+s+p_i} \leq \frac{z(r+s) + p_i w_m}{r+s+p_i} < w_m.$$

But if w_m is above the reservation wage of all workers, it is strictly better to post $w_m - \epsilon$, for some $\epsilon > 0$. ■

Appendix 1 demonstrates that a single wage equilibrium exists.

To search theorists, Proposition 1 will have a familiar flavor. The result that firms can use the search friction to extract all gains from trade was first noticed by Diamond (1971), and is sometimes called the Diamond–paradox, because the lack of competitiveness does not depend on the magnitude of the search friction. Monopoly prices (wages) occur whenever the friction is strictly positive.

4 Multiple posted wages

Proposition 1 demonstrates that as long as all firms post wages there is only one posted wage. In a model with homogeneous worker productivity, Albrecht and Axell (1984) generated wage dispersion by assuming that the utility of leisure differ across workers.[10] When workers' productivity is heterogeneous that assumption is unattractive. The wage will only correlate positively with the productivity if productivity correlates positively with the utility of leisure. We think that workers' reservation wages should be shaped by their labor market opportunities, and not mainly by their utility of being unemployed.

If there are some jobs with negotiable wages offers, however, productivity matters: As we saw in Lemma 2, a positive number of bargaining firms ensures that a worker's reservation wage is increasing in his productivity. Firms contemplating to post a wage, must then balance their concern for the wage bill against the desire to hire highly skilled workers.

4.1 Existence: an example

A common problem in search theory is to establish general conditions for the existence of equilibrium. Technically, it is particularly hard when the equilibrium involves price or wage dispersion. Like Sattinger (1991) among others, we have so far failed to find a general existence theorem. On the other hand, in every numerical example that we have examined there is at least one equilibrium with wage dispersion; hence we suspect that sufficient conditions are rather permissive.

We report here the simplest kind of example,the one in which all firms use the same production technology. (The fact that it is possible to generate wage dispersion with homogeneous firms is a strength of the model. Albrecht and Axell (1984) used the heterogeneity of firms to create wage dispersion.) The matching rate q is concave, and there

[10]Besides restricting market mechanisms to give some of the surplus to workers (imposing Nash bargaining on all firms, for example), there are two other standard routes around the Diamond–paradox. First, one may introduce direct competition by departing from the strict sequential search assumption. The key paper in this tradition is Burdett and Judd (1983) who allow the workers to observe more than one offer at a time. A variation on the theme is to allow on–the–job search, so that new offers compete with the worker's original job (see Burdett and Mortensen (1989)). Ranking models, in which several workers may apply for the same job, can also break the single wage equilibrium by modifying the timing.

are 3 types of worker, with productivities $x_1 < x_2 < x_3$. The equilibrium we seek has three different wage policies $\{B, H, L\}$; two different fixed wages, $w_H > w_L$, in addition to the flexible wage, w_B, offered by bargaining firms.

Recall from Lemma 1 that the posted wages must correspond to the reservation wages of some workers. We are going to locate an equilibrium in which the two fixed wages, w_L and w_H, equal the (endogenous) reservation utilities of type 1 and type 2 workers repectively. Moreover, the equilibrium has the property that $w_H < x_1$, so that low productivity workers are hired in all three kinds of firms. (Indeed, for low productivity workers, luck turns out to be an important determinant of their wage.) These characteristics of the equilibrium can be summarized in terms of our suitable jobs and suitable workers definitions. The sets of suitable jobs are

$$J_S(1) = \{B, H, L\}, J_S(2) = \{B, H\}, J_S(3) = \{B\}.$$

(E.g., the suitable jobs for a worker of type 2 are the high posted wage jobs and the negotiable wage jobs.) The sets of suitable workers are

$$I_S(B) = \{1, 2, 3\}, I_S(H) = \{1, 2\}, I_S(L) = \{1\},$$

(which means, inter alia, that the only suitable applicants for a low wage job are the type 1 workers).

From section 2.4 it follows that we need to solve 13 simultaneous equations. (For reference, we repeat these equations in Appendix 2.) Our chosen parametrization of the matching function is

$$q(\frac{v}{u}) = \sqrt{\frac{u}{8v}}.$$

All the other parameters of the model are collected in the following table.

c	β	r	s	z	x_1	x_2	x_3	n_1	n_2	n_3
3	0.5	0.01	0.02	0	1	2	3	10000	1000	100

Table 1: Parameters

Note that the nominal values of r and s are only relevant once we have specified the unit of time. If we think of the time unit as one month, a 1% rate of interest and a 2% chance of separation seem (roughly) internally consistent.

Solutions are found using approximation techniques.[11] One solution is:[12]

w_L	w_H	rU_3	m_B	m_H	m_L	v_B	v_H	v_L	u_1	u_2	u_3	$q(\theta)$	ϕ
0.78	0.94	1.41	2384	4813	3493	94	195	162	724	109	27	0.49	0.26

Table 2: An equilibrium

Mainly the first 3 columns are of interest here. Since $w_H < x_1$, we have confirmation that the firms offering the high fixed wages hire low productivity workers. So, the low productivity workers lucky enough to get such a job (the probability is 43% in this example) earn a sizable rent: The wage w_H is more than 20% above their reservation wage, w_L. Likewise, of course, all types of worker earn a wage premium in the bargaining firms, which in the example supply roughly a fifth of the jobs.

The example demonstrates that different wage policies may coexist in equilibrium. Let us repeat the two trade–offs. On the one hand there is a trade–off between setting a fixed wage and bargain with applicants on arrival. The cost of bargaining is that all workers are paid in excess of their reservation wage. (However, it is noteworthy that in the example type 1 workers are paid less in the bargaining firm, where their wage is 0.89 (use equation (12)), than in the high fixed wage firm, where it is 0.94.) This cost is offset by the recruiting advantage. Since the wage can be adjusted to the applicant's productivity, any type of worker is employable. The other trade–off is that between a high and a low fixed wage. The low wage has the advantage of the firm reaping all gains from trade, but the advantage is exactly offset by the inability to hire high productivity workers (and hire more quickly).

Having given this example that sizable wage differentials can exist, let us now get on to the main characterization result.

[11] More specifically, we have used *Mathematica's* command FindRoot, which employs several standard methods.

[12] We show in Appendix 2 that other wage policies are unprofitable.

4.2 The role of firm characteristics

In the example, we assumed that firms are inherently identical. Hence it is arbitrary which firm choose which wage policy. On the other hand, a striking empirical regularity is that firm and industry characteristics are correlated with the firms' level of pay.

Let us therefore study how firm heterogeneities work in our model. First, suppose that some firms are more capital intensive than others, and that capital is complementary to skill, in the sense that the marginal product of skill is increasing in the amount of capital.

For simplicity we consider Leontief technologies. A firm with technology t needs exactly K^t units of capital per worker. We consider two technologies only, so $T = \{k, l\}$ with k denoting a high capital intensity (i.e. $K^k > K^l$). We also assume that capital can be rented after a worker has been employed (ex ante capital investments would mess up our expressions). Let y_i^t be the gross productivity of a type i worker in a firm with technology t, and let

$$x_i^t := y_i^t - rK^t$$

denote the *net productivity* (the value after capital costs, but before wage payments). A natural definition of capital–skill complementarity is then:

Assumption 3 (Capital–skill complementarity) *For all types of workers $i, i+1 \in I$, the following inequality holds; $x_{i+1}^k - x_{i+1}^l > x_i^k - x_i^l$.*

In other words, the productivity gain from extra capital is greater for the more skillful worker. In order for the technology choice to be non–trivial, the following assumption should also hold.

Assumption 4 (No dominant technology) *There is some type of worker whose net productivity is higher in a capital intensive firm, and some type of worker whose net productivity is higher in a labor intensive firm.*

If this assumption were violated, one technology would strictly dominate the other. Together, the two assumptions have the following implication.

Lemma 3 *(i) There exists a largest $i \in I$ such that $x_i^k \leq x_i^l$ and a smallest i such that $x_i^k \geq x_i^l$. Denote the former i^* and the latter i^{**}. (ii) Then, $i^* \in \{i^{**}, i^{**} - 1\}$.*

Proof: Trivial.■

The result says that, with the possible exception for one type (a type which is equally productive in both types of jobs), we can divide the types of workers into two disjoint and non–empty sets; the lowly skilled workers who have a strictly better net productivity in the labor intensive jobs and the highly skilled workers who are more productive in the capital intensive jobs.

The lemma establishes that capital intensive firms have a comparative advantage in hiring highly skilled labor. Since we know that reservation wages will be positively related to skill, it also seems natural that capital intensive firms post higher wages than the labor intensive firms do. To show it formally, recall that $J^\star$ is an equilibrium set of job types. Let W_j^t be the corresponding set of wages which are offered by firms using technology t. We are then in a position to characterize the relationship between technology and wage policy.

Proposition 2 *Suppose that no type of worker has exactly the same net productivity in both kinds of jobs (i.e., $i^* = i^{**} - 1$). In any multiple wage equilibrium (i) any posted wage $w_j \leq \underline{w}_{i^*}$ must be offered by a labor intensive firm, (ii) any posted wage $w_j \geq x^l_{i^{**}}$ must be offered by a capital intensive firm, and (iii) $\max W_j^l \leq \min W_j^k$; the highest wage posted by a labor intensive firm cannot exceed the lowest wage posted by a capital intensive firm.*

Proof: See Appendix 3. ■

In other words, given that there are some posted wages low enough to deter highly skilled applicants and some high enough to attract them, the former are offered by labor intensive firms and the latter by capital intensive firms. Proposition 2 presents a possible explanation for the observed joint correlations between capital intensity and wage levels and capital intensity and skill levels. Notice in particular that the magnitude of the capital–skill complementarity does not matter for the result.

Firm size is another characteristic which has been shown to correlate with wage. The positive relationship was observed already by Moore (1911), and has received much attention since the work of Lester (1967). More recent contributions confirming the regularity are Mellow (1982), Brown and Medoff (1989) and Oi (1991). Given that there is a complementarity between size and skill (directly or because large firms tend to be more capital intensive), it can be studied in our model in exactly the same way as the capital–skill complementarity. The model provides an explanation to a puzzle uncovered recently by Main and Reilly (1993). In their data large companies tend to hire highly skilled workers, but *for given firm size* the skill premium is very small, in particular in large companies. Their interpretation is that there is no significant size–skill complementarity. However, within our model the paradox is readily explained: Since the wage policy is chosen prior to the arrival of the applicant, the wage reflects the skill level the firms aim to attract: Variations in skill do not affect the level of pay, only the hiring decision.

5 Further implications

The results of the previous section demonstrate that firms may adapt their wage policies in order to recruit from particular segments of the labor pool, and that this choice is affected by the firm's technology. Whereas employers on average get the skills they pay for, there could be considerable scope for luck at the level of individual firms and workers.

Let us now confront the model with the facts and questions which have tended to undermine rival theories of wage differentials.

5.1 Destinies of displaced workers

As mentioned in the introduction, Gibbons and Katz (1992) study the destinies of workers who lost their job through little fault of their own (displaced workers).[13] A remarkable result is that the workers who switch industry due to plant closure "experience wage changes that are of the same sign and of similar magnitude to the difference in the relevant industry differentials estimated in a cross–section." More precisely, they get 75 to 80% of the differential.

To see how this fits with our model, suppose there is a capital intensive industry and a labor intensive industry. (Of course, in reality capital intensity is not perfectly correlated with industry classification.) Consider a potential industry switcher. Clearly, his type, i, satisfies $\underline{w}_i \leq \max W_j^l \leq \min W_j^k \leq x_i^k$ (he is suitable in some labor intensive firms as well as in some capital intensive firm). If he goes from a job in a labor intensive firm to a job

[13]The reason why we focus on the work by Gibbons and Katz is that the earlier studies of job changes by Krueger and Summers (1988) and Murphy and Topel (1990) are prone to self selection biases.

in a capital intensive firm, his wage will increase. Typically however, the worker is only suitable in the labor intensive firms which pay relatively well and in the capital intensive firms which pay relatively badly. This could account for the fact that the worker does not receive the full industry differential. Indeed, a further test of our theory would be to see whether a worker's probability of switching industries is related how his wage departs from the pre–displacement industry's mean wage: A high wage relative to the industry average should imply a higher probability of switching to a better industry.

Gibbons and Katz also find that the pre–displacement wage has a significant impact on the post–displacement wage. That regularity is predicted by our model, as both wages are affected by ability.

5.2 Correlation across jobs

So far, we have studied the market for one occupation only. But as shown by Dickens and Katz (1987b) and Groshen (1991), the kind of firms which pay well for one occupation tend to pay well for others too. How can we accommodate this regularity?

Again weak complementarities do the trick. Let each firm employ workers for several inter–related jobs, and assume that there is some skill complementarity across occupations. For example, suppose that the productivity of the managing director is weakly positively related to capital, and that the productivity of her secretary is weakly complementary to that of his boss, but unrelated to capital. Then, by the logic of Proposition 2, firms with a well qualified manager will also attempt to employ a well qualified secretary, and so choose a policy of high wages. This result contradicts the claim that complementarities cannot account for cross–occupational correlations in wage differentials. That view, taken by Dickens and Lang (1992) and other proponents of the segmented labor market paradigm, rests on the presumption that only very strong complementarities can account for the high degree of correlation. As we have seen, the absolute magnitude of complementarity plays no role in the argument.

5.3 Firm characteristics and measured skill

As emphasized by Brown and Medoff (1989) and Murphy and Topel (1987, 1990) among others, skill levels are significantly higher in high wage industries. This is exactly what is happening in Proposition 2, since high wages attract better applicants. Admittedly, we have derived the result in a model where firms which post wages cannot condition their wage on skill measures. However, there is every reason to expect measured skill to correlate with firm characteristics even if firms were allowed to condition their posted wages on various worker characteristics. After all, in our model it is the desire to recruit highly skilled workers which makes firms pay well in the first place.

5.4 Cross–country correlations

Finally, a remarkable empirical regularity is that the industry wage structure is similar, if not in magnitude at least in sign, across countries with different market systems and for long periods of time. This fact has been seen as troublesome for the rent–sharing theories, since the bargaining power of employees and the way in which it relates to firm characteristics is likely to differ between, say, capitalist and communist systems. But any system other than pure slavery must offer wages that attract workers of the appropriate quality. And since inter–industry differences in technologies (say, capital intensities) are broadly similar across countries our theory is consistent with the cross country evidence as well.

6 Policy implications?

Several economists have argued that the government should facilitate the creation of good jobs, meaning jobs in which workers earn rents. Recent influential papers in this tradition are Katz and Summers (1989a,b).[14] They write:

> Policies that encourage employment in high–wage sectors are likely to transfer labor from low– to high–productivity uses and thereby increase total output.
>
> Katz and Summers (1989a, p212)

[14]The reader is referred to their articles for references to earlier sentiments along the same lines.

In the current model the link between wage premia and public intervention is tenuous. Indeed, the "Lucas critique" has considerable force. To see this most easily, consider again the equilibrium in which technologically identical firms offer different wages. Assume that the firms which offer the high wage share some (otherwise irrelevant) index. Now, suppose that a subsidy is paid to firms with this index. If there are no exit barriers, indexed firms will take over completely. However, the only effect on the equilibrium should be a net increase in the number of jobs (to reflect the subsidy), with no obvious effect on the composition of wage policies.

If firms differ in technology, say in capital intensity as above, the result could be even worse. Support of capital intensive firms increases the net productivity of any worker in such firms, which becomes

$$x_i^k = y_i^k - r(\sigma)K^k,$$

where $r(\sigma) < r$ is the capital cost after a policy σ. As a consequence, the "cut–off" type i^{**} is lowered. This means that after the subsidy some of the workers who are more productive in labor intensive firms, and were previously targeted by such, are targeted by capital intensive firms instead. As a result, there is a mismatch of skills and technology. Although average wages are likely to increase, there is no guarantee that the *wage premia* will increase sufficiently to offset the skill–technology distortion, far less the other distortions arising from having to increase taxes on other items than capital.

7 Conclusion

To sum up, we have provided a new theory of labor market segmentation, accomodating a variety of facts which have earlier required separate explanations. The basic idea is that the heterogeneity of workers induce firms to choose different wage policies. Higher wages attract better workers on average. The higher wage compensates for the *expected* higher skill of applicants, but individual workers may be more or less lucky: Variations in human capital make the labor market into a lottery.

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Appendix 1: Existence of a single wage equilibrium

We restrict attention to the case with only one technology (so we don't need to worry about the distribution of jobs.) To make the analysis as simple as possible, we also make the assumption that all workers are employable, i.e. $x_i > z$ for all i.

Step (i): First, we check that firms and workers use best replies. Given that all firms offer $w = z$, equation (5) implies that each worker's reservation wage is $rU = z$. Since all workers accept this wage, it is irrational for any firm to offer a higher fixed wage, or to bargain with the worker. Step (ii): Rewrite the steady state unemployment condition, (1), as

$$u_i = \frac{sn_i}{\phi + s}.$$

Remember that $x_i > z$ for all i. Inserting into the zero profit condition, (9), using the definition $\phi = \theta q(\theta)$ and rearranging, we get the following expression for the matching rate:

$$q(\theta) = \frac{(r+s)c}{\sum_i (n_i/n)(x_i - z)}.$$

Clearly, the right hand side is a strictly positive constant. It follows from Assumption 1 and the intermediate value theorem that the equation has exactly one solution. Since $q(\theta)$ is monotonic, it can be inverted to obtain an explicit expression for $\theta := v/u$. We can then simply solve for u, v and the equilibrium number of firms m using the equations

$$u := \sum_i u_i = \frac{sn}{\phi + s}$$

and

$$v = \frac{sm}{q(\theta) + s},$$

where the latter equation is implied by (2).

Appendix 2: The 3–type example

Here we restate the 13 equations used in our example. Recall that $u := u_1 + u_2 + u_3$, $v := v_1 + v_2 + v_3$, and that $z = 0$. The arrival rate is

$$q = \sqrt{\frac{u}{8v}}.$$

The zero profit conditions are, from (9),

$$\frac{u_1}{u} q(x_1 - rU_1) - c(r+s) = 0,$$

$$\frac{u_1}{u} q(x_1 - rU_2) + \frac{u_2}{u} q(x_2 - rU_2) - c(r+s) = 0,$$

$$(1-\beta)(\frac{u_1}{u} q(x_1 - rU_1) + \frac{u_2}{u} q(x_2 - rU_2) + \frac{u_3}{u} q(x_3 - rU_3)) - c(r+s) = 0.$$

Steady state unemployment conditions are given by equation (1);

$$u_1 = \frac{sn_1}{\phi + s},$$

$$u_2 = \frac{sn_2}{\phi(v_H/v + v_B/v) + s},$$

$$u_3 = \frac{sn_3}{\phi(v_B/v) + s},$$

The steady state vacancy conditions, given by (2), are

$$v_L = \frac{sm_L}{q(u_1/u) + s},$$

$$v_H = \frac{sm_H}{q(u_1/u + u_2/u) + s},$$

$$v_B = \frac{sm_B}{q + s}.$$

Since the fixed wages correspond to the reservation wages of type 1 and type 2 workers, we have from (5), and using (12), (after some manipulations),

$$w_L = rU_1 = \frac{\phi(rU_2(v_H/v) + \beta x_1(v_B/v))}{r + s + \phi((v_H/v) + \beta(v_B/v))},$$

$$w_H = rU_2 = \frac{\phi\beta x_2(v_B/v)}{r + s + \phi\beta(v_B/v)}.$$

Similarly, the reservation wage of type 3 workers is

$$rU_3 = \frac{\phi\beta x_3(v_B/v)}{r + s + \phi\beta(v_B/v)}.$$

By Lemma 1, the only other candidate for an equilibrium wage policy is $w_3 := rU_3$. Let V_{3i} be the profit from offering w_3 and accepting workers of productivity i and better. It follows that

$$V_{31} = \frac{u_1}{u}q(x_1 - rU_3) + \frac{u_2}{u}q(x_2 - rU_3) + \frac{u_3}{u}q(x_3 - rU_3) - c(r + s),$$

$$V_{32} = \frac{u_2}{u}q(x_2 - rU_3) + \frac{u_3}{u}q(x_3 - rU_3) - c(r + s),$$

$$V_{33} = \frac{u_3}{u}q(x_3 - rU_3) - c(r + s).$$

Inserting the proposed equilibrium values, we have $V_{31} = -0.40$, $V_{32} = -0.06$, and $V_{31} = -0.13$. Hence, it does not pay to deviate to w_3.

Appendix 3: Proof of Proposition 2

Consider the situation of a firm which can choose technology. The value of opening a vacancy with wage policy j is given by equation (7). Fix a wage policy j. Let $V_j^t = 0$, and insert from (10) to get the equilibrium requirement

$$0 = -c + \sum_{i \in I_S(j,t)} q_i \frac{x_i^t - w_j}{r+s}. \tag{13}$$

(i) Suppose $w_j \le \underline{w}_{i^*}$. By Lemma 2, $w_j < \underline{w}_i$ for all $i > i^*$. By Lemma 3, we then have that $x_i^l > x_i^k$ for all workers who accept the wage w_j. It follows that $I_S(j,k) \subset I_S(j,l)$ (at the wage w_j, a worker which is suitable in a capital intensive firm is suitable in a labor intensive firm as well). Consequently,

$$\sum_{i \in I_S(j,l)} q_i(x_i^l - w_j) > \sum_{i \in I_S(j,k)} q_i(x_i^k - w_j),$$

and free entry means that (13) can hold only for $t = l$.

(ii) Suppose $w_j > x_{i^{**}}^l$. The only workers who are hired at this wage are those whose type satisfies $i \ge i^{**}$. These workers are more productive in capital intensive jobs, and a similar argument as above shows that w_j must be offered by a capital intensive firm.

(iii) Let $w_L := \min W_j^k$ and $w_H := \max W_j^l$. We shall show that $w_H > w_L$ leads to a contradiction. Using (i) and (ii), we know that

$$\underline{w}_{i^*} < w_L < w_H < x_{i^*}. \tag{14}$$

In a free entry equilibrium it must also be true that

$$\sum_{i \in I_S(L,k)} q_i(x_i^k - w_L) \ge \sum_{i \in I_S(L,l)} q_i(x_i^l - w_L),$$

and

$$\sum_{i \in I_S(H,l)} q_i(x_i^l - w_H) \ge \sum_{i \in I_S(H,k)} q_i(x_i^k - w_H).$$

These two inequalities imply

$$\sum_{i\in I_S(H,k)} q_i(x_i^k - w_H) - \sum_{i\in I_S(L,k)} q_i(x_i^k - w_L) \leq \sum_{i\in I_S(H,l)} q_i(x_i^l - w_H) - \sum_{i\in I_S(L,l)} q_i(x_i^l - w_L).$$

Let $C_S(\cdot,\cdot)$ denote the complement of $I_S(\cdot,\cdot)$. As will be shown below, any type i which belongs to at least one of the four sets $I_S(H,k), I_S(L,k), I_S(H,l)$ and $I_S(L,l)$ belongs to one of the following non–intersecting sets:

$$\begin{aligned}
I_A &:= I_S(H,k)\cap I_S(H,l)\cap C_S(L,k)\cap C_S(L,l),\\
I_B &:= I_S(H,k)\cap I_S(H,l)\cap I_S(L,k)\cap I_S(L,l),\\
I_C &:= C_S(H,k)\cap C_S(H,l)\cap I_S(L,k)\cap I_S(L,l),\\
I_D &:= C_S(H,k)\cap C_S(H,l)\cap C_S(L,k)\cap I_S(L,l).
\end{aligned}$$

The set I_A contains the types who turn down the wage w_L but accept w_H. The set I_B contains the workers who are suitable in both kinds of firms at either wage. I_C contains the workers who are suitable in both firms at w_L but in neither firm at w_H. Finally, I_D contains the types which are suitable only in a labor intensive firm at the wage w_L and in neither kind of firm at w_H.

We can then rewrite the last inequality as

$$\sum_{i\in I_A} q_i(x_i^k - x_i^l) + \sum_{i\in I_C} q_i(x_i^l - x_i^k) + \sum_{i\in I_D} q_i(x_i^l - w_L) \leq 0.$$

We proceed to show that all three terms are non–negative and that the first is strictly positive, proving (iii) by contradiction: Recall that the endogenous reservation wage is increasing in i. From (14) we then see that for all $i \in I_A, i > i^*$. Hence, by Lemma 3, $x_i^k - x_i^l > 0$ for all $i \in I_A$. Moreover, I_A is nonempty, since the only reason for a labor intensive firm to offer w_H in preference over the lower wage w_L is to attract a worker in I_A. From (14) it is also clear that if $i \in I_C$, then $i < i^*$. Thus, by Lemma 3, $x_i^l - x_i^k > 0$ for all $i \in I_C$. Finally, the third term is non–negative by the definition of I_D.

It remains to prove that we can neglect other sets than the 4 we listed above. There are 12 other non–intersecting sets ($2^4 = 16$). Each of these are straightforwardly ruled out as either being empty or irrelevant using (14) and Lemma 3. For example, if $i \in \{I_S(H,k) \cup I_S(L,k) \cup I_S(H,l) \cup I_S(L,l)\}$, then i does not belong to the set

$$I_E := C_S(H,k) \cap C_S(H,l) \cap C_S(L,k) \cap I_S(L,k).$$

(Here, type i is suitable at a wage w_L only in the capital intensive firm, and in no firm at the wage w_H.) This is seen as follows: Equation (14) implies that $i < i^*$ for all $i \in I_E$. Using Lemma 3 we have that if $i \in I_S(L,k)$, then $i \in I_S(L,l)$. Hence, I_E is empty. ■