Specification Tests for Asymmetric GARCH*

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Abstract

In this paper I present two new Lagrange multiplier test statistics designed for testing the null of GARCH(1,1), against the alternative of asymmetric GARCH. For one test the alternative is the generalized QARCH(1,1) model of Sentana [1995], and for the other the alternative is the logistic smooth transition GARCH(1,1) model of Hagerud [1996], and González-Rivera [1996]. In the study I present small sample properties for the two statistics. The empirical size is show to be equal to the theoretical for reasonable sample sizes. Furthermore, I show that the power of both tests is superior to that of the asymmetry tests proposed by Engle and Ng [1993]. This is true even if the true data generating process is not the GQARCH or LSTGARCH model, but any of the models, EGARCH, GJR, TGARCH, A-PARCH, and VS-ARCH. Thus, the two tests are in fact tests for general GARCH asymmetry.

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1 Introduction

To estimate the unknown parameters of a model in the ARCH/GARCH class, iterative numerical methods are very often required. These procedures are time consuming. Furthermore, if the model in question explains the data badly, the estimation might not converge. Therefore, it is essential to have reliable specification tests. These tests give the econometrician indications of which models can be the data generating process of a time series. This paper presents two Lagrange multiplier tests designed to detect higher order GARCH effects. For both tests, the null hypothesis is the GARCH model proposed by Bollerslev [1986]. In the first test, the alternative hypothesis is the generalized quadratic ARCH (GQARCH) model of Sentana [1995], and in the second test, the alterative is the logistic smooth transition GARCH (LSTGARCH) model presented by Hagerud [1996], and González-Rivera [1996]. The models are only considered in their most simple structure, when the lag lengths are equal to one, but tests for more complex models can easily be derived, using a similar method.

In both the GQARCH and LSTGARCH model, the conditional variance is asymmetric in the sign of lagged innovations. Thus, there is a correlation between current conditional variance and lagged returns. A number of other such *asymmetric* models have been proposed in the literature. The most common of these are: the EGARCH models of Nelson [1991], and the GJR model of Glosten, Jagannathan, and Runkle [1993].

In this paper, results are presented from Monte Carlo simulations performed to investigate the small sample properties of the two statistics. The empirical size is shown to coincide with the theoretical. When the power of the tests is examined, it is determined whether the tests can be used to test for the existence of asymmetry of forms other than those specified in the GQARCH and LSTGARCH models. If that is the case, this will indicate that the tests cannot distinguish between different forms of asymmetry. This is naturally a weakness of the tests. However, it might also be an advantage. If a test can detect other forms of asymmetry, one test can indicate, if the null cannot be rejected, that a large number of ARCH models can be excluded as the data generating test process of a time series. The other asymmetric models considered in this experiment are: EGARCH, GJR, TGARCH, A-PARCH, and VS-ARCH. It is shown that both tests can be used to detect asymmetries generated by these five models.

For plausible parameter values on the data generating process, the empirical power of the tests is always below 100 percent. Therefore, the relative power of the tests is compared to four other asymmetry tests proposed by Engle and Ng [1993]. The procedures of Engle and Ng are the most commonly used tests in the literature. The simulations show that the power properties of the two tests here are superior to that of Engle and Ng's tests. The major contribution of this paper, therefore, is the presentation of two tests for general asymmetry, with superior power properties.

This article is organized as follows. The next section describes the asymmetric ARCH models that will be considered in the Monte Carlo experiment. Section 3 surveys previous literature on specification tests in the ARCH environment. Section 4 contains a presentation of the specification tests. Results from the Monte Carlo experiment are given in Section 5. Finally, Section 6 concludes the paper.

2 Asymmetric GARCH Models

This section presents the GQARCH and the LSTGARCH models, as well as the five other asymmetric GARCH models that will be considered in the Monte Carlo experiment.[1] In all the models presented, and also for the remainder of this article, it is assumed that the return on an asset, r_t, is generated by

$$r_t = \varepsilon_t, \tag{1}$$

where ε_t denotes a discrete-time stochastic process with the form

$$\varepsilon_t = z_t h_t^{1/2}, \tag{2}$$

where $z_t \sim$nid(0,1), and h_t will be the conditional variance at time t.

2.1 GQARCH

Sentana [1995] introduces the Quadratic ARCH model. The term *quadratic* is used since the QARCH model can be interpreted as a second-order Taylor approximation to the unknown conditional variance function. The Generalized QARCH(1,1) model is

$$h_t = \gamma + \zeta\varepsilon_{t-1} + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{3}$$

where γ, ζ, α, and β are constant parameters. Positivity of the variance is achieved if α, $\beta \geq 0$, and $\zeta < 4\alpha\gamma$. The model is covariance stationary if $\alpha + \beta < 1$. Asymmetry is introduced with the parameter ζ.

2.2 LSTGARCH

The logistic smooth transition GARCH model is proposed by Hagerud [1996], and González-Rivera [1996]. In the LSTGARCH(1,1) model, the conditional variance is assumed to be generated by

$$h_t = \gamma + \left[\alpha_1 + \alpha_2 F\left(\varepsilon_{t-1}\right)\right]\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{4}$$

where $F(.)$ is a transition function with the form

$$F\left(\varepsilon_{t-1}\right) = \left(1 + \exp\left[-\theta\varepsilon_{t-1}\right]\right)^{-1} - \frac{1}{2},\ \theta > 0. \tag{5}$$

For positive conditional variance in the LSTGARCH model, it is required that

$$\begin{aligned} \gamma &> 0, \\ \alpha_1 &\geq 0\ , \\ \beta_i &\geq 0\ , \\ \alpha_1 &\geq \frac{1}{2}\left|\alpha_2\right|. \end{aligned}$$

[1] The EGARCH, GJR and A-PARCH models presented below, are more thoroughly surveyed by Hentschel [1996]. Based on his Asymmetric Absolute Value ARCH model, Hentschel develops a general GARCH model, which nests a large number ARCH models. Both symmetric and asymmetric models are nested, but not QARCH and VS-ARCH.

For stationarity of the return process it is required

$$[\alpha_1 - \frac{1}{2}|\alpha_2| + \max(\alpha_2, 0)] + \beta < 1.$$

For the smooth transition GARCH process to be defined, it is required that at least one $\alpha_i > 0$, $i = 1, 2$. In the LSTGARCH(1,1) model, the level of asymmetry is controlled by the parameters α_2 and θ.

2.3 EGARCH

The seminal work in the area of asymmetric ARCH is the exponential GARCH model of Nelson [1991]. In the EGARCH(1,1) model, the natural logarithm of the conditional variance follows the process

$$\ln h_t = \gamma + \beta \ln h_{t-1} + \lambda z_{t-1} + \varphi \left[|z_{t-1}| - \sqrt{2/\pi} \right], \tag{6}$$

where γ, β, λ, and φ are constant parameters, and z_t is defined as in (2). For the process ε_t to be stationary, it is sufficient that $\beta < 1$. Nelson gives three motivations for his model compared to the standard GARCH model of Bollerslev [1986]: (i) The GARCH model cannot explain the asymmetric behavior of the conditional variance in asset price returns. (ii) For the conditional variance to be strictly positive, the parameters of the GARCH models must be non-negative, which is not required in the EGARCH model.[2] (iii) In the GARCH model, it is difficult to evaluate whether or no a shock to variance persists. Persistence of conditional variance in the EGARCH is controlled by the parameter β.

2.4 GJR

In the GJR model of Glosten, Jagannathan, and Runkle [1993] the standard GARCH model is augmented with a term that captures asymmetry. The GJR model is

$$h_t = \gamma + \alpha \varepsilon_{t-1}^2 + \omega S_{t-1}^- \varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{7}$$

where γ, α, β, and ω are constant parameters, and S_{t-1}^- is a variable that takes the value one when $\varepsilon_{t-1} < 0$ and zero otherwise. For positive conditional variance, it is sufficient that the parameters γ, α, and β, and $(\alpha+\omega)$ are non-negative. For the process ε_t to be stationary, it is sufficient that $\alpha+\beta+\omega < 1$.

Note that the GJR model (7) will obtain as a limiting case of the LSTGARCH(1,1) model (4), when the logistic transition function (5), is replaced by the Heaviside function minus one half. In the GJR model, the conditional variance follows one process when the innovations are positive and another process when the innovations are negative. In the LSTGARCH model, however, the transition between states is smooth.

2.5 TGARCH

The Threshold GARCH model is introduced in Zakoïan [1994]. In the TGARCH model, it is not the conditional variance, but the conditional standard deviation, $\sigma_t = h_t^{1/2}$, that is modeled. The TGARCH(1,1)

[2] Nelson and Cao [1992] show that the non-negativity constraint for the GARCH model given by Bollerslev [1986] is only sufficient for strictly positive conditional variance. They demonstrate that weaker conditions can be found.

model is

$$\sigma_t = \gamma + \alpha^+ \varepsilon_{t-1}^+ - \alpha^- \varepsilon_{t-1}^- + \beta\sigma_{t-1}, \tag{8}$$

where $\varepsilon_t^+ = \max(\varepsilon_t, 0)$, and $\varepsilon_t^- = \min(\varepsilon_t, 0)$. For strictly positive conditional standard deviation, it is sufficient that $\gamma > 0$, $\alpha^+ \geq 0$, $\alpha^- \geq 0$, and $\beta \geq 0$. The return series is stationary if

$$\frac{1}{2}\left[(\alpha^+)^2 + (\alpha^-)^2\right] + \beta^2 + \left[\alpha^+ + \alpha^-\right]\sqrt{\frac{2}{\pi}} < 1.$$

Note that (8) can be reparameterized as

$$\sigma_t = \gamma + \alpha\left|\varepsilon_{t-1}\right| + \omega S_{t-1}^- \varepsilon_{t-1} + \beta\sigma_{t-1}.$$

Thus, in the TGARCH(1,1) model, the conditional standard deviation has the same functional form as the conditional variance has in the GJR model (7).

2.6 A-PARCH

Ding, Granger, and Engle [1993] introduce the Asymmetric Power ARCH model. In the A-PARCH(1,1) model, the conditional variance is given by

$$h_t^{\delta/2} = \gamma + \alpha\left(\left|\varepsilon_{t-1}\right| - \eta\varepsilon_{t-1}\right)^{\delta} + \beta h_{t-1}^{\delta/2}, \tag{9}$$

where γ, α, β, η and $\delta > 0$ are constant parameters. Asymmetry is introduced via the parameter $\eta \in (-1, 1)$. For positive conditional variance, it is required that the parameters γ, α, and β, are non-negative. Conditions for stationarity are relatively complex, and can be found in Ding, Granger, and Engle [1993].

The A-PARCH model is a generalization of previous GARCH models. The model includes seven other models as special cases. For example, $\delta = 2$ and $\eta = 0$ will give the GARCH(1,1) model. Letting $\delta = 2$ gives the GJR model. When $\delta = 1$ the dynamics of the model will be similar to that in the TGARCH model. Since these models are nested in the A-PARCH model, likelihood ratio tests can be performed to test the significance of the parameters. Thus, the null of a specific model, against the alternative of A-PARCH, can be tested with relative ease.

2.7 VS-ARCH

The Volatility Switching model is proposed by Fornari and Mele [1996a]. In the VS model the conditional variance follows

$$h_t = \gamma + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} + \xi S_{t-1} v_{t-1}^2, \tag{10}$$

where

$$\begin{aligned} S_t &= 1 \;\text{ if } \varepsilon_t > 0 \\ S_t &= 0 \;\text{ if } \varepsilon_t = 0 \\ S_t &= -1 \;\text{ if } \varepsilon_t < 0, \end{aligned}$$

and v_t^2 is defined as ε_t^2/h_t. The parameters of the model are γ, α, β, and ξ. With the series $\left\{S_t v_t^2\right\}_{t=1}^{T}$, Fornari and Mele introduce what they call *mean reversion* in the conditional variance. v_t^2 measures how much a given squared residual deviates from its expected value, h_t, and S_t indicates the sign of the residual. The model, for example, is able to generate data where unexpectedly large negative returns increase h_t, large positive returns decrease h_t, small negative returns decrease h_t, and small positive returns increase h_t. The level of asymmetry in the model will therefore depend on parameter ξ, and on the relative size of residuals.[3]

3 Previous ARCH Specification Tests

This section is included to introduce the reader to the area of specification tests in the ARCH/GARCH literature. Readers already familiar with this literature can proceed to Section 4.

In the ARCH(q) model of Engle [1982], the conditional variance is given by the process

$$h_t = \gamma + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2, \tag{11}$$

where γ and α_j $(j = 1..q)$ are non-negative constants, with at least one $\alpha_j > 0$. To test the presence of ARCH, i.e. testing $H_0 : \alpha_j = 0\,(j = 1..q)$, against H_1 : at least one $\alpha_j \neq 0$, Engle proposes a Lagrange multiplier test. Since the conditional variance is constant under the null, a Lagrange multiplier test is particularly suitable. Engle shows that the LM statistic can be calculated as $T \cdot R_u^2$, where T is the number of observations, and R_u^2 is the coefficient of multiple correlation from the regression of ε_t^2 on a constant and ε_{t-1}^2,...,ε_{t-q}^2. Under the null, the statistic has an asymptotic χ^2 distribution with q degrees of freedom. As noted by Granger and Teräsvirta [1993], McLeod and Lee's [1983] test of linearity in the conditional mean against unspecified non-linearity is asymptotically equivalent to Engle's test of no ARCH.

In the GARCH(q,p) model of Bollerslev [1986] the conditional variance is given by the process

$$h_t = \gamma + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \tag{12}$$

where γ, α_j $(j = 1..q)$ and β_j $(j = 1..p)$ are non-negative constants, with at least one $\alpha_j > 0$. To test the presence of GARCH, an LM statistic cannot be derived in the way described by Engle. Bollerslev [1986] notes that under the null of no heteroskedasticity, there is no general test for GARCH(q,p). This is because the information matrix is singular if both p>0 and q>0. Based on Bollerslev's finding Lee [1991] derives a *modified LM* statistic for $H_0 : \alpha_j = \beta_i = 0$ $(j = 1..q,\ i = 1..p)$, against H_1 : at least one $\alpha_j \neq 0$ or one $\beta_i \neq 0$. Lee shows that this test is equivalent to the test of no ARCH(q). Thus, under the null of homoskedasticity, the GARCH(q,p) effect and the ARCH(q) effects are locally equivalent alternatives. Lee notes that with his methods for deriving a modified LM statistic for no GARCH, it

[3] In Fornari and Mele[1996b], the authors proposed a slightly different VS-ARCH model, in which two extra parameters are needed.

is possible to derive a test of the null of ARCH(q) against the alternative of GARCH($k_1, q + k_2$), where $k_1 > 0$ and $k_2 > 0$.

Tests of the null of linear ARCH as in (11), or linear GARCH as in (12), against different forms of non-linear ARCH/GARCH, has been proposed by, among others, Bera and Higgins [1992], Higgins and Bera [1992], Engle, and Ng [1993], Rabemananjara, and Zakoïan [1993], and Sentana [1995]. Bera and Higgins [1992] and Higgins and Bera [1992] discuss testing for ARCH against NARCH (Non-linear ARCH). In the NARCH model, the conditional variance is, as in the ARCH and GARCH models, symmetric in the sign of ε_t. Since the GQARCH and the LSTGARCH models are asymmetric, their test is of less importance in this context. It is still worth noting that the test of no LSTGARCH and the test of no NARCH, have the common problem of a non-identified parameter under the null. The methods for solving this problem, however, are quite different.

Engle, and Ng [1993] present four different *LM* type tests for linear ARCH/GARCH against asymmetry. These four statistics will be discussed more thoroughly in Section 5, and a detailed description of how the tests may be calculated appears in the appendix. In this section, a short introduction to the four tests is given. The *Sign bias test* examines the impact of positive and negative shocks on the conditional variance not predicted by the linear model. This is done by investigating whether, in a linear regression model, the variable S_{t-1}^- has any predictive power on squared normalized residuals ε_t^2/h_{0t}, where h_{0t} is the conditional variance under the null. S_{t-1}^- is defined as in the GJR model (7). The test statistic is calculated as a t-ratio in the linear regression model. The other three tests are carried out using similar methods. The *Negative size bias test* investigates whether the linear model can explain the different effects that large and small negative shocks have on the conditional variance. The variable used for this test is $S_{t-1}^-\varepsilon_{t-1}$. In the *Positive size bias test,* different effects of large and small positive shocks are investigated. The variable used for this test is $S_{t-1}^+\varepsilon_{t-1}$, where S_{t-1}^+ is defined analogously to S_{t-1}^-. In the fourth test, the three previous hypotheses are considered simultaneously. In Monte Carlo experiments, Engle and Ng show that the empirical size and power of the tests are reasonable when the sample size is 1000.

The TGARCH model (8) is further developed in Rabemananjara, and Zakoïan [1993]. They allow σ_t to become negative. Thus, σ_t cannot be considered a conditional standard deviation. The TGARCH(q, p) model of Rabemananjara, and Zakoïan [1993] is

$$\sigma_t = \gamma + \sum_{j=1}^{q} \alpha_j \left|\varepsilon_{t-j}\right| + \sum_{j=1}^{p} \beta_j \left|\sigma_{t-j}\right| + \sum_{j=1}^{q} \alpha_j^- \varepsilon_{t-j}^- + \sum_{j=1}^{p} \beta_j^- \sigma_{t-j}^-, \tag{13}$$

where $\varepsilon_t^- = \min(\varepsilon_t, 0)$ and $\sigma_t^- = \min(\sigma_t, 0)$. The null hypothesis for the test of asymmetry in (13) is, $H_0 : \alpha_j^- = \beta_i^- = 0$ $(j = 1..q,\ i = 1..p)$. To test the null against H_1 :at least one $\alpha_j^- \neq 0$ or one $\beta_i^- \neq 0$, Rabemananjara, and Zakoïan derive an *LM* test statistic. Under the null, the statistic is asymptotically distributed χ^2 with $q + p$ degrees of freedom. In a Monte Carlo experiment, the authors show that both the empirical size and power of the test are reasonable for large sample sizes (> 500).

Sentana [1995] presents a number of test procedures that can be used in conjunction with his QARCH model. To test the null of homoskedasticity against the alternative of QARCH(q), Sentana proposes an

LM test based on the regression model

$$\varepsilon_t^2 = a_0 + \sum_{j=1}^{q} a_{1j}\varepsilon_{t-j} + \sum_{j=1}^{q}\sum_{i\leq j} a_{2ij}\varepsilon_{t-j}\varepsilon_{t-i} + \xi_t. \tag{14}$$

The hypothesis is: $H_0 : a_{1j} = a_{2j} = 0$ $(j = 1, ..., q, i = 1, ..., q)$, against H_1 : at least one $a_{1j} \neq 0$ or one $a_{2ij} \neq 0$. The statistic is calculated as $T \cdot R_u^2$ from the regression model (14). Under H_0, the LM statistic has an asymptotic χ^2 distribution with $q(q+3)/2$ degrees of freedom.

The tests described in this section are all derived under the assumption that the residuals are distributed conditionally normal. In practice, this assumption is often not fulfilled. This is particularly the case when the investigated series contains returns of a traded asset. The problem is carefully investigated in Wooldridge [1990] and [1991]. Wooldridge points out that when normality does not hold the asymptotic size of the statistics will be wrong. In the 1990 article, Wooldridge propose a robust version of Engle's test of no ARCH(q). In the 1991 article, he presents a general procedure for robustifying Lagrange multiplier tests of the specification of conditional variance. This method is used by Sentana [1995], when he derives a robust test procedure for the null of ARCH(1), against the alternative of QARCH(1,1). In Wooldridge [1991], the author also presents a method for performing non-nested hypothesis tests for the conditional variance.

The two tests presented in this paper are both derived under conditional normality. Furthermore, the small sample properties are only investigated for normally distributed innovations. However, using the results of Wooldridge [1991], it would be straightforward to derive robust versions of the tests.

4 Specification Tests for Asymmetry

In this section, the two new test statistics are presented. For both tests, the null is the standard GARCH(1,1) model, proposed by Bollerslev [1986]

$$h_t = \gamma + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{15}$$

The test of GARCH(1,1), against the alternative of GQARCH(1,1) is formulated

$$\begin{aligned} H_0 &: \zeta = 0 \\ H_1 &: \zeta \neq 0. \end{aligned}$$

Given that the residual, ε_t, is distributed conditionally normal, a Lagrange multiplier test statistic for the hypothesis is

$$\begin{aligned} &\frac{1}{2}\left\{\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\varepsilon_t^2}{h_{0t}}-1\right]\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right\}'\left\{\sum_{t=1}^{T}\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]'\right\}^{-1} \\ \times\quad &\left\{\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\varepsilon_t^2}{h_{0t}}-1\right]\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right\}, \end{aligned} \tag{16}$$

where

$$\frac{\partial h_t}{\partial\boldsymbol{\beta}'} = \left[\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\varepsilon_{t-i}^2, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{0t-i}, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\varepsilon_{t-i}\right],$$

h_{0t} is the conditional variance under the null of GARCH(1,1), $\boldsymbol{\beta}'$ is the vector of parameters $(\gamma, \alpha, \beta, \zeta)$, and $\widehat{\beta}$ is the estimated parameter β in the GARCH(1,1) model. The derivation of (16) is given in the appendix. In the appendix, it can also be seen that based on test (16), it is possible to derive the asymptotically equivalent test $T \cdot R_u^2$ from the regression

$$\left\{ \frac{\varepsilon_t^2}{h_{0t}} - 1 \right\}$$

on

$$\left\{ \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{0t-i}}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}}{h_{0t}} \right\}. \tag{17}$$

Before a complete test procedure is presented, first a problem concerning the estimation of the model (15) must be considered. Given that the series of conditional variance under the null is estimated with maximum likelihood, the normalized residuals, $\upsilon_{0t} \equiv \varepsilon_t / h_{0t}^{1/2}$, should be orthogonal to

$$\left\{ \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{0t-i}}{h_{0t}} \right\}. \tag{18}$$

This should be true independent of whether or not the null is true. However, in practice, exact orthogonality cannot always be guaranteed. If orthogonality does not hold, the empirical size of the statistic might be distorted. To overcome this complication, the customary procedure is to replace υ_{0t} with a quantity that is guaranteed to be orthogonal to (18), (see e.g. Eitrheim and Teräsvirta [1996]). The following procedure will accomplish that:

1. Regress
$$\left\{ \frac{\varepsilon_t^2}{h_{0t}} - 1 \right\}$$
on (18). Let $\{\widetilde{e}_t\}_{t=1}^T$ be the series of residuals from the regression. These residuals will by construction be orthogonal to (18).

2. Regress $\widetilde{e}_t$ on (17). The statistic is set equal $T \cdot R_u^2$ from this regression.

In the Monte Carlo experiments, it was seen that the empirical size of the statistic (16), and its asymptotically alternative, were slightly above the theoretical significance level. By using the method described above, it was possible to correct the size. However, it was also found that a slightly simplified method gave almost the same result. In this method, the vector (17) was replaced by

$$\left\{ 1, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}}{h_{0t}} \right\}$$

Base on this conclusion, it is proposed that the null hypothesis of GARCH(1,1), against the alternative of GQARCH(1,1), should be tested using the following procedure:

1. Estimate a GARCH(1, 1) model. Form the vectors

$$\mathbf{c}_t = \left\{1, \frac{\sum_{i=1}^{t-1} \widehat{\beta}_1^{i-1} \varepsilon_{t-i}}{h_{0t}}\right\}$$

2. Regress

$$\left\{\frac{\varepsilon_t^2}{h_{0t}} - 1\right\}$$

on $\mathbf{c}_t$, and calculate

$$LM_1 = T \cdot R_u^2 \tag{19}$$

from the regression.

The test statistic LM_1 is under the null asymptotically distributed χ^2 with one degree of freedom.

The test of GARCH(1,1), against the alternative of LSTGARCH(1,1) is formulated

$$\begin{aligned} H_0 &: \alpha_2 = 0 \\ H_1 &: \alpha_2 \neq 0. \end{aligned}$$

Given that the residual, ε_t, is distributed conditionally normal, a Lagrange multiplier test statistic for the hypothesis is

$$\begin{aligned} &\frac{1}{2}\left\{\sum_{t=1}^{T} \frac{1}{2h_{0t}} \left[\frac{\varepsilon_t^2}{h_{0t}} - 1\right] \frac{\partial h_t}{\partial \boldsymbol{\alpha}}\right\}' \left\{\sum_{t=1}^{T} \left[\frac{1}{h_{0t}} \frac{\partial h_t}{\partial \boldsymbol{\alpha}}\right] \left[\frac{1}{h_{0t}} \frac{\partial h_t}{\partial \boldsymbol{\alpha}}\right]'\right\}^{-1} \\ \times \quad &\left\{\sum_{t=1}^{T} \frac{1}{2h_{0t}} \left[\frac{\varepsilon_t^2}{h_{0t}} - 1\right] \frac{\partial h_t}{\partial \boldsymbol{\alpha}}\right\}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} \frac{\partial h_t}{\partial \boldsymbol{\alpha}'} = &\left\{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}^2, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} F(\varepsilon_t|\theta)\varepsilon_{t-i}^2, \right. \\ &\left. \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i}, \alpha_2 \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \frac{e^{-\theta\varepsilon_{t-i}}}{\left(1 + e^{-\theta\varepsilon_{t-i}}\right)^2} \varepsilon_{t-i}^3 \right\} \end{aligned} \tag{21}$$

h_{0t} is the conditional variance under the null of GARCH(1,1), $\boldsymbol{\alpha}'$ is the vector of parameters $(\gamma, \alpha_1, \alpha_2, \beta, \theta)$, $F(\varepsilon_t|\theta)$ is the value of the transition function at time t, and $\widehat{\beta}$ is the estimated parameter β in the GARCH(1,1) model. The derivation of (20) is done with the same methods used for the derivation of (16).

The statistic (20), however, is not operational, since the vector (21) is dependent on the transition function (5), which under the null has a non-identified parameter θ. Following Luukkonen, Saikkonen, and Teräsvirta [1988], this problem is solved by making a second-order Taylor expansion of the transition function, around zero.[4] The obtained approximation of $F(.)$ is then inserted into formula (4), and this

[4] A test of GARCH(1,1) against LSTGARCH(1,1) has previously been developed by González-Rivera [1996]. However, she solves the problem with the non-identified parameter θ somewhat differently, using a method proposed by Davies [1977].

results in an approximate version of the conditional variance equation. Since $F(0) = 0$, the transition function (5) can be approximated by

$$T_l = F'(0)x = \frac{\theta}{4}x \tag{22}$$

The LSTGARCH(1,1) model can therefore be approximated by

$$\widetilde{h}_t = \gamma + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \frac{\theta}{4} \varepsilon_{t-1}^3 + \beta h_{t-1}.$$

The hypothesis of GARCH(1,1), against LSTGARCH(1,1) can therefore be written

$$\begin{aligned} H_0 &: \alpha_2 \theta / 4 = 0 \\ H_1 &: \alpha_2 \theta / 4 \neq 0. \end{aligned}$$

A Lagrange multiplier for this test is equal to

$$\begin{aligned} &\frac{1}{2}\left\{\sum_{t=1}^{T} \frac{1}{2h_{0t}} \left[\frac{\varepsilon_t^2}{h_{0t}} - 1\right] \frac{\partial \widetilde{h}_t}{\partial \widetilde{\boldsymbol{\alpha}}}\right\}' \left\{\sum_{t=1}^{T} \left[\frac{1}{h_{0t}} \frac{\partial \widetilde{h}_t}{\partial \widetilde{\boldsymbol{\alpha}}}\right] \left[\frac{1}{h_{0t}} \frac{\partial \widetilde{h}_t}{\partial \widetilde{\boldsymbol{\alpha}}}\right]'\right\}^{-1} \\ &\times \left\{\sum_{t=1}^{T} \frac{1}{2h_{0t}} \left[\frac{\varepsilon_t^2}{h_{0t}} - 1\right] \frac{\partial \widetilde{h}_t}{\partial \widetilde{\boldsymbol{\alpha}}}\right\}, \end{aligned} \tag{23}$$

where

$$\frac{\partial \widetilde{h}_t}{\partial \widetilde{\boldsymbol{\alpha}}'} = \left[\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}^2, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}^3, \sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{0t-i}\right],$$

h_{0t} is the conditional variance under the null of GARCH(1,1), $\widetilde{\boldsymbol{\alpha}}'$ is the vector of parameters $(\gamma, \alpha_1, \alpha_2\theta/4, \beta)$, and $\widehat{\beta}$ is the estimated parameter β in the GARCH(1,1) model. As in the GQARCH test, to test the null of GARCH(1,1), against the alternative of LSTGARCH(1,1), the procedure proposed is:

1. Estimate a GARCH$(1,1)$ model. Form the vectors

$$\mathbf{d}_t = \left(1, \frac{\sum_{i=1}^{t-1} \widehat{\beta}_1^{i-1} \varepsilon_{t-i}^3}{h_{0t}}\right)$$

2. Regress

$$\left\{\frac{\varepsilon_t^2}{h_{0t}} - 1\right\}$$

on $\mathbf{d}_t$, and calculate

$$LM_2 = T \cdot R_u^2 \tag{24}$$

from the regression.

LM_2 is under the null asymptotically distributed χ^2 with one degree of freedom.

5 Monte Carlo Experiment

The Monte Carlo experiment for testing the empirical size of the test statistics (19) and (24) is based on a GARCH(1,1) data generating process

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= z_t h_t^{1/2} \\ h_t &= \gamma + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \tag{25}$$

where z_t ~nid$(0,1)$. Four combinations of the constant parameters γ, α, and β are studied. These values are shown in Tables 1 and 2. For each set of parameter values 2,500 samples with 250 and 1000 observations are generated. The test statistics are calculated, and compared to the critical values for one, five and ten percent confidence levels.

In Table 1, the actual rejection frequencies of the test procedure (19) are reported. The empirical size of the test is relatively close to the theoretical size, both when the number of observations is 1000 and 250. However, the simulated size seems to be somewhat less accurate for the smaller sample size. Table 2 reports the simulation results for test procedure (24). For the larger sample the empirical size is quite close to the theoretical size. When the sample size is 250 the empirical size seems to be lower than the theoretical. It is therefore concluded that for both statistics, the simulated size is fairly accurate for the larger sample size, and the simulated size is reasonable for a sample of 250 observations.

As noted in the introduction, the two tests' ability to detect asymmetry will be compared, to that of Engle and Ng's [1993] four tests, the Sign bias test (SB), the Negative size bias test (NSB), the Positive size bias test (PSB), and the test of the joint hypothesis of SB, NSB, and PSB. Such a comparison can only be made if it is known that the empirical size properties of Engle and Ng's tests are similar to those reported for tests (19) and (24). A number of different alternative formulations of Engle and Ng's tests were tried before appropriate size properties were received . The procedures that where found most promising, and which are used in the remainder of this article, are described in the appendix. Tables 3 to 6 report the actual rejection frequencies for the four alternative tests. Based on the results reported in Tables 1 to 6, it is concluded that for the larger sample size, the empirical size properties for the six tests are quite similar. When the number of observations is 250, the empirical size properties fluctuate more across the different test procedures. The remainder of this study will therefore focus on the results based on the larger sample size.

To investigate the power of test (19) the data generating process considered is

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= z_t h_t^{1/2} \\ h_t &= 1.25 \cdot 10^{-6} - 1.68 \cdot 10^{-4}\varepsilon_{t-1} + 0.0355\varepsilon_{t-1}^2 + 0.952 h_{t-1}, \end{aligned} \tag{26}$$

where z_t ~nid$(0,1)$. The parameters of the model (26) have been obtained from an estimation performed on daily observations for the Financial Times All-share Index. The sample period is January 1991 to July 1996. The estimated power of the test is reported on the rows labeled LM_1 in Table 7. From the results when the sample size is 250, it can concluded that for such a small sample, the test has very low power. When the sample size is 1000 the estimated power is increased, but is still low. However, compared to the estimated power for the four tests of Engle and Ng, also reported in Table 7, the test constitutes a marked improvement. The result is not surprising, since the test (19) is designed to detect the kind of asymmetry caused by (26), whereas the tests of Engle and Ng are designed to detect general GARCH asymmetry. But surprisingly, the test of no LSTGARCH (24) also outperforms the tests of Engle and Ng.

To investigate the power of test (24) the data generating process considered is

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= z_t h_t^{1/2} \\ h_t &= 5.2 \cdot 10^{-7} + [0.295 - 0.258 \cdot F(\varepsilon_{t-1})]\varepsilon_{t-1}^2 + 0.70 \cdot h_{t-1} \\ F(\varepsilon_{t-1}) &= (1 + \exp[-200 \cdot \varepsilon_{t-1}])^{-1} - \frac{1}{2}, \end{aligned} \tag{27}$$

where z_t ∼nid$(0,1)$. The parameters of the model (27) are in part taken from the paper by Engle and Ng [1993], when they consider the GJR model (7). If the parameter θ, which has been set to 200, is allowed to increase towards infinity, the data generating process (27) will coincide with the GJR model considered in Engle and Ng's article. By setting θ to 200, the transition function $F(\varepsilon_{t-1})$ will not just take on its extreme values $-1/2$ and $1/2$. Table 8 reports the power properties of the six statistics when the true data generating process is (27). The power of the test (24) is reasonable when the sample size is 1000. The power of the test is reduced considerably for the smaller sample size. The test (19) also proves to have power to detect asymmetry generated by (27), but, as expected, the power is significantly lower than for the test (24). The tests of Engle and Ng are a clear disappointment. It is important to keep in mind that the reported power is a function of the parameter values in the data generating process. One should therefore not compare the figures in Table 7 and 8. The parameters of the data generating process (27) clearly give rise to more asymmetry than the process (26).

The two statistics will now be further evaluated by investigating the empirical power of the tests for detecting asymmetry caused by the other five GARCH models presented in Section 2. To investigate the power of the tests, when the true model is EGARCH, the data generating process considered is

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= z_t h_t^{1/2} \\ \ln h_t &= -0.7395 + 0.90 \cdot \ln h_{t-1} - 0.075 \cdot z_{t-1} + 0.25 \cdot \left[|z_{t-1}| - \sqrt{2/\pi}\right], \end{aligned} \tag{28}$$

where z_t ∼nid$(0,1)$. Except for the value of the parameter γ, the parameters of the model (28) are those used by Engle and Ng [1993], when they consider the EGARCH model (6). The simulated power of the six tests are presented in Table 9. The highest power is reported for test (19), followed by test (24). Among the tests of Engle and Ng, the Negative size bias test perform best, but the power is still significantly lower than the power for the two tests presented in this paper. The results for the smaller sample size is again disappointing.

The power of the tests, when the true model is GJR, is investigated considering the data generating process

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= z_t h_t^{1/2} \\ h_t &= 5.20 \cdot 10^{-7} + 0.166 \cdot \varepsilon_{t-1}^2 + 0.2576 \cdot S_{t-1}^- \varepsilon_{t-1}^2 + 0.70 \cdot h_{t-1}, \end{aligned} \tag{29}$$

where $z_t \sim$nid$(0,1)$. Except for the value of the parameter γ, the parameters of the model (29) are those used by Engle and Ng [1993], when they consider the GJR model (7). Results of the simulations are in Table 10. These parameter values appear to generate a very marked asymmetry. For the larger sample size, the simulated power for tests (19) and (24) are strikingly high , whereas the tests of Engle and Ng perform less efficiently.

To investigate the power of the tests, when the true model is TGARCH, the data generating process considered is

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= z_t \sigma_t \\ \sigma_t &= 6.54 \cdot 10^{-4} + 0.111 \cdot \varepsilon_{t-1}^+ - 0.192 \cdot \varepsilon_{t-1}^- + 0.833 \cdot \sigma_{t-1}, \end{aligned} \tag{30}$$

where $z_t \sim$nid$(0,1)$. Except for the value of the parameter γ, the parameters of the model (30) have been obtained from an estimation performed on daily observations for the French CAC 240 Index, reported by Zakoïan [1994]. The sample period is 1976 to 1990. Results from the simulations are shown in Table 11. Tests (19) and (24) perform well once again. The power properties of test (19) must , in these circumstances, be considered very good. The results for the tests SB, NSB, PSB, and the joint test are, compared to the results for the other two tests, unsatisfactory.

When the true model is A-PARCH, the situation is investigated considering the data generating process

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= z_t h_t^{1/2} \\ h_t^{1.43/2} &= 9.22 \cdot 10^{-6} + 0.083 \cdot (|\varepsilon_{t-1}| - 0.373 \cdot \varepsilon_{t-1})^{1.43} + 0.92 \cdot h_{t-1}^{1.43/2}, \end{aligned} \tag{31}$$

where $z_t \sim$nid$(0,1)$. Except for the value of the parameter γ, the parameters of the model (31) have been obtained from an estimation performed on daily observations for the S&P 500 Index, reported by Ding, Granger, and Engle [1993]. The sample period is 1928 to 1991. The results of the simulations are shown in Table 12. It is interesting to note that the parameters of the models, which have be estimated on this very large sample, apparently give rise to a marked asymmetry. This can be seen from the high power reported for tests (19) and (24). In this case, the results for the tests of Engle and Ng are even more disappointing than when the TGARCH model was the true data generating process.

Finally, the power of tests when the true model is VS-ARCH is investigated. In this case, the data generating process studied is

$$\begin{aligned} r_t &= \varepsilon_t \\ \varepsilon_t &= z_t h_t^{1/2} \\ h_t &= 3.9 \cdot 10^{-6} + 0.043 \cdot \varepsilon_{t-1}^2 + 0.918 h_{t-1} + 2.22 \cdot 10^{-6} \cdot S_{t-1} v_{t-1}^2, \end{aligned} \tag{32}$$

where $z_t \sim$nid$(0,1)$. Except for the value of the parameter γ, the parameters of the model (32) have been obtained from an estimation performed on daily observations for the S&P 500 Index, reported by Fornari and Mele [1996a]. The sample period is January 1990 to September 1994. Table 13 reports the simulation results. The power of test (19) and of test (24) almost coincide, and are at reasonable levels for the larger sample size. The tests of Engle and Ng prove to have almost no power at all.

6 Summary and Conclusion

In the paper two new Lagrange multiplier test procedures have been presented. The procedures are developed for testing the null hypothesis that the conditional variance follows a GARCH(1,1) process, against the alternative that the conditional variance follows an asymmetric GARCH process. In the alternative hypotheses, well specified parametric models are considered. In test number one, the conditional variance follows a GQARCH(1,1) process under the alternative. In the second test, the alternative model is the LSTGARCH(1,1).

Small sample properties for the two tests have also been presented. These have been obtained from a number of Monte Carlo experiments. In those experiments two sample sizes are considered, 1000 and 250 observations. It is shown that the empirical size of the two tests is quite accurate for the larger sample size, and reasonable for the smaller. Since asymmetric GARCH specifications are primarily used for modeling high frequency financial data, a sample size of 1000 observations is not at all unusual.

The power of the tests is naturally a function of the parameters of the data generating process under the alternative. If the level of asymmetry in the data is low, the power falls considerably. To evaluate the power properties of the two tests, the power of the tests is compared to those of four other GARCH asymmetry tests, previously proposed in the literature. The four tests are: the Sign bias test (SB), the Negative size bias test (NSB), the Positive size bias test (PSB), and the test for the joint hypothesis of SB, NSB, and PSB. These test are all developed by Engle and Ng [1993]. The Monte Carlo simulations show that the power of the two tests presented in this paper is much higher than the power of the four alternative tests. Furthermore, it is shown that the power properties of the two tests are also superior when the true data generating process is not the GQARCH(1,1) model or the LSTGARCH(1,1) model. The other data generating processes considered are: EGARCH(1,1), GJR, TGARCH(1,1), A-PARCH(1,1), and VS-ARCH. The test for which the alternative is the GQARCH(1,1) model generally proves to have slightly better power properties than the test for which the alternative is the LSTGARCH(1,1) model. It is therefore concluded that the two tests are in fact tests for general GARCH asymmetry, with reasonable power properties. This finding should be of importance for any econometrician working with GARCH models. The disappointing results of the tests of Engle and Ng [1993] are most likely a function of the fact that the tests have been developed without a well specified parametric alternative.

That the two tests can detect asymmetry caused by many parametric GARCH model is, however, not only good news. The results show that, using these tests, it is very hard, to actually decide which asymmetric model might have been the data generation process of a time series. Nevertheless, the tests will indicate whether any model in the family of asymmetric GARCH models could or could not have been the data generation process. Since the tests presented above have relatively low power, I am pessimistic about the possibility of designing powerful LM tests for testing the different models against each other. This subject still calls for further research.

Both test statistics presented in the paper are derived under conditional normality. The small sample properties are also investigated when the innovations of the data process are drawn from a Gaussian distribution. Many empirical investigations have shown that the assumption that financial data is distributed conditionally normal is most likely incorrect. Under such circumstances, the simulated size results are of less importance. Research in the area of specifications tests under non-normality is therefore strongly called for.

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Appendix

1. Derivation of the LM statistic (16)

Assume that we have the observed time series $\varepsilon_0, \varepsilon_1, ..., \varepsilon_T$. The conditional variance is under the alternative assumed to be generated by

$$h_t = \mathbf{w}_t'\boldsymbol{\beta},$$

where $\mathbf{w}_t' = (1, \varepsilon_{t-1}^2,\ h_{t-1}, \varepsilon_{t-1})$, and $\boldsymbol{\beta}' = (\gamma, \alpha, \beta, \zeta)$. The test is $H_0 : \zeta = 0$, against $H_1 : \zeta \neq 0$. The Lagrange multiplier statistic has the general form

$$LM = T\overline{q}_T(\boldsymbol{\beta}_0)' I(\boldsymbol{\beta}_0)^{-1}\overline{q}_T(\boldsymbol{\beta}_0),$$

where $\boldsymbol{\beta}_0$ is the vector of parameters under the null. $\overline{q}_T(\boldsymbol{\beta})$ is the average score and $I(\boldsymbol{\beta})$ is the information matrix. If we assume that the innovations are Gaussian, the log likelihood of one observation is equal to

$$l_t = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln h_t - \frac{1}{2}\frac{\varepsilon_t^2}{h_t}.$$

Assuming that h_1 is fixed such that $\partial h_1/\partial\boldsymbol{\beta} = \mathbf{0}$, it can be shown that the average score is equal to

$$\overline{q}_T(\boldsymbol{\beta}_0) = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\varepsilon_t^2}{h_{0t}} - 1\right]\frac{\partial h_t}{\partial\boldsymbol{\beta}}, \tag{33}$$

where

$$\frac{\partial h_t}{\partial\boldsymbol{\beta}'} = \left[\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\varepsilon_{t-i}^2, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{t-i}, \sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\varepsilon_{t-i}\right]. \tag{34}$$

In (33) h_{0t} is the estimated conditional variance under the null of GARCH, and $\widehat{\beta}$ in (34) is estimated under the null. The information matrix is the negative expectation of the Hessian averaged over all observations

$$I(\boldsymbol{\beta}) = -E\left[\frac{1}{T}\sum_{t=1}^{T}\frac{\partial^2 l_t}{\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}'}\right].$$

The Hessian for one observation can be shown to be equal to

$$\frac{\partial^2 l_t}{\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}'} = \left[\frac{1}{2} - \frac{\varepsilon_t^2}{h_t}\right]\left[\frac{1}{h_t}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]\left[\frac{1}{h_t}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right],$$

which implies that the information matrix becomes

$$I(\boldsymbol{\beta}) = \frac{1}{2T}\sum_{t=1}^{T}E\left[\frac{1}{h_t}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]\left[\frac{1}{h_t}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]'.$$

The information matrix under the null is consistently estimated by

$$\widehat{I}(\boldsymbol{\beta}_0) = \frac{1}{2T}\sum_{t=1}^{T}\left[\frac{1}{h_{0t}}\frac{\partial h_{0t}}{\partial\boldsymbol{\beta}}\right]\left[\frac{1}{h_{0t}}\frac{\partial h_{0t}}{\partial\boldsymbol{\beta}}\right]'.$$

The Lagrange multiplier test of GARCH against GQARCH can therefore be written

$$\frac{1}{2}\left\{\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\varepsilon_t^2}{h_{0t}} - 1\right]\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right\}'\left\{\sum_{t=1}^{T}\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right]'\right\}^{-1}$$

$$\times \quad \left\{\sum_{t=1}^{T}\frac{1}{2h_{0t}}\left[\frac{\varepsilon_t^2}{h_{0t}} - 1\right]\frac{\partial h_t}{\partial\boldsymbol{\beta}}\right\},$$

which corresponds to formula (16). ■

2. Derivation of asymptotically equivalent statistic $\mathbf{T} \cdot \mathbf{R}_u^2$

Consider the statistic (16). The equation can be rewritten as

$$\frac{1}{2}\left\{\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{h_{0t}}-1\right]\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial \boldsymbol{\beta}}\right]\right\}'\left\{\sum_{t=1}^{T}\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial \boldsymbol{\beta}}\right]\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial \boldsymbol{\beta}}\right]'\right\}^{-1} \times \left\{\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{h_{0t}}-1\right]\left[\frac{1}{h_{0t}}\frac{\partial h_t}{\partial \boldsymbol{\beta}}\right]\right\}, \qquad (35)$$

Define $\mathbf{y}' = (y_1, ..., y_T)$ and $\mathbf{X}' = (\mathbf{x}_1, ..., \mathbf{x}_T)$, where

$$y_t = \left\{\frac{\varepsilon_t^2}{h_{0t}} - 1\right\}$$

and

$$\begin{aligned}\mathbf{x}_t &= \frac{1}{h_{0t}}\frac{\partial h_t}{\partial \boldsymbol{\beta}'} \\ &= \left\{\frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\varepsilon_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}h_{t-i}}{h_{0t}}, \frac{\sum_{i=1}^{t-1}\widehat{\beta}^{i-1}\varepsilon_{t-i}}{h_{0t}}\right\}.\end{aligned}$$

Then, it is straightforward to rewrite (35) as

$$\frac{1}{2}\mathbf{y}'\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y}$$

Note, that given that $\varepsilon_t \sim N(0, h_{0t})$ then

$$\operatorname*{plim}_{T\to\infty} \mathbf{y}'\mathbf{y} = \operatorname*{plim}_{T\to\infty} \sum_{t=1}^{T}\left(\frac{\varepsilon_t^2}{h_{0t}} - 1\right)^2 = 2 \cdot T.$$

This suggests that an asymptotically equivalent statistic is

$$LM_2 = T\frac{\mathbf{y}'\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y}}{\mathbf{y}'\mathbf{y}} = T \cdot R_u^2$$

where R_u^2 is the squared multiple correlation between $\mathbf{y}$ and $\mathbf{X}$. Thus, the statistic is equal to $T \cdot R_u^2$ from the regression y_t on $\mathbf{x}_t$. ■

3. Test procedure used for the Sign bias test

The sign bias test statistic is defined as the t-ratio for the coefficient b in the regression equation

$$v_t^2 = a + b \cdot S_{t-1}^- + \boldsymbol{\tau}'\frac{1}{h_{0t}}\frac{\partial h_t}{\partial \boldsymbol{\beta}_0} + e_t \qquad (36)$$

where $v_t^2 = \varepsilon_t^2/h_{0t}$, h_{0t} is the conditional variance under the null, $\boldsymbol{\tau}$ is a constant parameter vector, $\boldsymbol{\beta}_0$ are the parameters under the null, and e_t is the residual. When the null is the GARCH(1,1) model, the test procedure used is:

1. Estimate a GARCH$(1,1)$ model. Form the vectors

$$\upsilon_t^2 = \left\{ \frac{\varepsilon_t^2}{h_{0t}} \right\},$$

and

$$\frac{1}{h_{0t}} \frac{\partial h_t}{\partial \boldsymbol{\beta}_0'} = \left[\frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i}}{h_{0t}} \right]$$

2. Run the regression (36), and calculate the statistic SB, which is equal to t-ratio for the estimate of the parameter b.

The test statistic SB is under the null asymptotically distributed standard normal.

4. Test procedure used for the Negative size bias test

The negative size bias test statistic is defined as the t-ratio for the coefficient b in the regression equation

$$\upsilon_t^2 = a + b \cdot S_{t-1}^{-} \varepsilon_{t-1} + \boldsymbol{\tau}' \frac{1}{h_{0t}} \frac{\partial h_t}{\partial \boldsymbol{\beta}_0} + e_t \tag{37}$$

where $\upsilon_t^2 = \varepsilon_t^2 / h_{0t}$, h_{0t} is the conditional variance under the null, $\boldsymbol{\tau}$ is a constant parameter vector, $\boldsymbol{\beta}_0$ are the parameters under the null, and e_t is the residual. When the null is the GARCH(1,1) model, the test procedure used is:

1. Estimate a GARCH$(1,1)$ model. Form the vectors

$$\upsilon_t^2 = \left\{ \frac{\varepsilon_t^2}{h_{0t}} \right\},$$

and

$$\frac{1}{h_{0t}} \frac{\partial h_t}{\partial \boldsymbol{\beta}_0'} = \left[\frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i}}{h_{0t}} \right]$$

2. Run the regression (37), and calculate the statistic NSB, which is equal to t-ratio for the estimate of the parameter b.

The test statistic NSB is under the null asymptotically distributed standard normal.

5. Test procedure used for the Positive size bias test

The positive size bias test statistic is defined as the t-ratio for the coefficient b in the regression equation

$$\upsilon_t^2 = a + b \cdot S_{t-1}^{+} \varepsilon_{t-1} + \boldsymbol{\tau}' \frac{1}{h_{0t}} \frac{\partial h_t}{\partial \boldsymbol{\beta}_0} + e_t \tag{38}$$

where $\upsilon_t^2 = \varepsilon_t^2 / h_{0t}$, h_{0t} is the conditional variance under the null, $\boldsymbol{\tau}$ is a constant parameter vector, $\boldsymbol{\beta}_0$ are the parameters under the null, and e_t is the residual. When the null is the GARCH(1,1) model, we used the test procedure:

1. Estimate a GARCH$(1,1)$ model. Form the vectors

$$v_t^2 = \left\{ \frac{\varepsilon_t^2}{h_{0t}} \right\},$$

and

$$\frac{1}{h_{0t}} \frac{\partial h_t}{\partial \boldsymbol{\beta}_0'} = \left[\frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1}}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} \varepsilon_{t-i}^2}{h_{0t}}, \frac{\sum_{i=1}^{t-1} \widehat{\beta}^{i-1} h_{t-i}}{h_{0t}} \right]$$

2. Run the regression (38), and calculate the statistic PSB, which is equal to t-ratio for the estimate of the parameter b.

The test statistic PSB is under the null asymptotically distributed standard normal.

6. Test procedure used for the Joint test

The test for the joint hypothesis of SB, NSB, and PSB is formulated as

$$\begin{aligned} H_0 &: \quad b_1 = b_2 = b_3 = 0, \\ H_1 &: \quad b_i \neq 0, \quad i = 1, 2, 3, \end{aligned}$$

in the regression

$$v_t^2 = a + b_1 \cdot S_{t-1}^- + b_2 \cdot S_{t-1}^- \varepsilon_{t-1} + b_3 \cdot S_{t-1}^+ \varepsilon_{t-1} + \boldsymbol{\tau}' \frac{1}{h_{0t}} \frac{\partial h_t}{\partial \boldsymbol{\beta}_0} + e_t$$

where $v_t^2 = \varepsilon_t^2 / h_{0t}$, h_{0t} is the conditional variance under the null, $\boldsymbol{\tau}$ is a constant parameter vector, $\boldsymbol{\beta}_0$ are the parameters under the null, and e_t is the residual. Since $\partial h_t / \partial \boldsymbol{\beta}_0$ should be orthogonal to v_t^2, the test statistic could be calculated as $T \cdot R_u^2$ from the regression. However, the simulations showed that the empirical size of such a statistic is severely distorted. To achieve an appropriate size, v_t^2 was adjusted, and $\partial h_t / \partial \boldsymbol{\beta}_0$ was replaced with a slightly simplified vector. When the null is the GARCH(1,1) model, the test procedure used was:

1. Estimate a GARCH$(1,1)$ model. Form the vector

$$v_t^2 = \left\{ \frac{\varepsilon_t^2}{h_{0t}} \right\}.$$

   Run the regression v_t^2 on $\left\{1, \varepsilon_{t-1}^2, h_{0t-1}\right\}$, and calculate the series of residuals $\widetilde{v}_t^2$.

2. Calculate the statistic as $T \cdot R_u^2$ from the regression $\widetilde{v}_t^2$ on

$$\left\{ 1, S_{t-1}^-, S_{t-1}^- \varepsilon_{t-1}, S_{t-1}^+ \varepsilon_{t-1}, \frac{1}{h_{0t}}, \frac{\varepsilon_{t-i}^2}{h_{0t}}, \frac{h_{t-i}}{h_{0t}} \right\}.$$

The test statistic should be compared to a χ^2 distribution with three degrees of freedom.

Table 1. Simulated Size for the Test of no GQARCH

The table shows results from a Monte Carlo experiment where the size of test statistic (19) is investigated. In the experiment, the data generating process is model (25), with the four different parameter combinations shown in column one. The column labeled Actual Rejection Frequencies report the simulated empirical size at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, thus a 95 % confidence interval is given by $\widehat{\alpha} + 2 \cdot \sqrt{\widehat{\alpha}(1-\widehat{\alpha})/2500}$, where $\widehat{\alpha}$ is the empirical size.

		Nominal Significance Level		
	Sample	1 %	5 %	10%
Parameter Values	Size	Actual Rejection Frequencies		
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.25, \beta = 0.70$	1000	1.36	4.88	9.92
	250	1.16	6.04	12.04
$\gamma = 1.0 \cdot 10^{-5}, \alpha = 0.05, \beta = 0.85$	1000	0.88	4.76	9.64
	250	0.76	5.16	10.12
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.05, \beta = 0.90$	1000	1.20	5.44	10.20
	250	0.88	5.00	11.32
$\gamma = 1.0 \cdot 10^{-6}, \alpha = 0.09, \beta = 0.90$	1000	1.12	5.64	11.96
	250	0.84	5.96	12.20

Table 2. Simulated Size for the Test of no LSTGARCH

The table shows results from a Monte Carlo experiment where the size of test statistic (24), is investigated. In the experiment, the data generating process is model (25), with the four different parameter combinations shown in column one. The columns labeled Actual Rejection Frequencies report the simulated empirical size at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, thus a 95 % confidence interval is given by $\widehat{\alpha} + 2 \cdot \sqrt{\widehat{\alpha}(1-\widehat{\alpha})/2500}$, where $\widehat{\alpha}$ is the empirical size.

		Nominal Significance Level		
	Sample	1 %	5 %	10%
Parameter Values	Size	Actual Rejection Frequencies		
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.25, \beta = 0.70$	1000	0.88	5.16	10.84
	250	0.72	3.56	8.12
$\gamma = 1.0 \cdot 10^{-5}, \alpha = 0.05, \beta = 0.85$	1000	0.88	4.40	9.16
	250	0.40	3.04	6.64
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.05, \beta = 0.90$	1000	0.92	5.16	10.20
	250	0.68	4.24	8.80
$\gamma = 1.0 \cdot 10^{-6}, \alpha = 0.09, \beta = 0.90$	1000	1.20	5.28	10.16
	250	0.76	5.20	11.24

Table 3. Simulated Size for the Sign Bias Test

The table shows results from a Monte Carlo experiment where the size of the Sign bias test, is investigated. In the experiment, the data generating process is model (25), with the four different parameter combinations shown in column one. The columns labeled Actual Rejection Frequencies report the simulated empirical size at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, thus a 95 % confidence interval is given by $\widehat{\alpha} + 2 \cdot \sqrt{\widehat{\alpha}(1-\widehat{\alpha})/2500}$, where $\widehat{\alpha}$ is the empirical size.

		Nominal Significance Level		
	Sample	1 %	5 %	10%
Parameter Values	Size	Actual Rejection Frequencies		
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.25, \beta = 0.70$	1000	0.96	4.80	9.44
	250	0.84	5.08	10.68
$\gamma = 1.0 \cdot 10^{-5}, \alpha = 0.05, \beta = 0.85$	1000	0.84	5.28	10.60
	250	0.72	5.24	10.84
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.05, \beta = 0.90$	1000	1.44	6.04	10.96
	250	1.00	5.64	11.12
$\gamma = 1.0 \cdot 10^{-6}, \alpha = 0.09, \beta = 0.90$	1000	1.04	5.40	9.72
	250	0.76	5.20	11.24

Table 4. Simulated Size for the Negative Size Bias Test

The table shows results from a Monte Carlo experiment where the size of the Negative size bias test, is investigated. In the experiment, the data generating process is model (25), with the four different parameter combinations shown in column one. The columns labeled Actual Rejection Frequencies report the simulated empirical size at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, thus a 95 % confidence interval is given by $\widehat{\alpha} + 2 \cdot \sqrt{\widehat{\alpha}(1-\widehat{\alpha})/2500}$, where $\widehat{\alpha}$ is the empirical size.

		Nominal Significance Level		
	Sample	1 %	5 %	10%
Parameter Values	Size	Actual Rejection Frequencies		
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.25, \beta = 0.70$	1000	1.08	4.24	8.96
	250	0.60	4.28	8.00
$\gamma = 1.0 \cdot 10^{-5}, \alpha = 0.05, \beta = 0.85$	1000	0.92	5.12	9.76
	250	0.60	4.28	9.20
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.05, \beta = 0.90$	1000	0.88	5.88	11.44
	250	0.72	3.68	8.80
$\gamma = 1.0 \cdot 10^{-6}, \alpha = 0.09, \beta = 0.90$	1000	0.72	4.68	9.60
	250	1.04	4.88	9.08

Table 5. Simulated Size for the Positive Size Bias Test

The table shows results from a Monte Carlo experiment where the size of the Positive size bias test, is investigated. In the experiment, the data generating process is model (25), with the four different parameter combinations shown in column one. The columns labeled Actual Rejection Frequencies report the simulated empirical size at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, thus a 95 % confidence interval is given by $\widehat{\alpha} + 2 \cdot \sqrt{\widehat{\alpha}(1-\widehat{\alpha})/2500}$, where $\widehat{\alpha}$ is the empirical size.

		Nominal Significance Level		
	Sample	1 %	5 %	10%
Parameter Values	Size	Actual Rejection Frequencies		
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.25, \beta = 0.70$	1000	0.80	4.20	9.28
	250	0.72	4.32	8.60
$\gamma = 1.0 \cdot 10^{-5}, \alpha = 0.05, \beta = 0.85$	1000	0.96	4.36	9.56
	250	0.84	4.64	9.32
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.05, \beta = 0.90$	1000	1.20	5.24	9.84
	250	0.56	4.20	8.64
$\gamma = 1.0 \cdot 10^{-6}, \alpha = 0.09, \beta = 0.90$	1000	1.12	5.68	10.92
	250	0.80	4.28	8.72

Table 6. Simulated Size for Engle and Ng's Joint Test

The table shows results from a Monte Carlo experiments, where the size of the Joint test of Engle and Ng[1993], is investigated. In the experiment, the data generating process is model (25), with the four different parameter combinations shown in column one. The columns labeled Actual Rejection Frequencies report the simulated empirical size at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, thus a 95 % confidence interval is given by $\widehat{\alpha} + 2 \cdot \sqrt{\widehat{\alpha}(1-\widehat{\alpha})/2500}$, where $\widehat{\alpha}$ is the empirical size.

		Nominal Significance Level		
	Sample	1 %	5 %	10%
Parameter Values	Size	Actual Rejection Frequencies		
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.25, \beta = 0.70$	1000	1.44	4.80	8.76
	250	0.80	3.64	7.64
$\gamma = 1.0 \cdot 10^{-5}, \alpha = 0.05, \beta = 0.85$	1000	1.12	5.08	9.72
	250	0.88	4.16	8.48
$\gamma = 5.0 \cdot 10^{-6}, \alpha = 0.05, \beta = 0.90$	1000	1.36	5.44	11.20
	250	0.76	4.00	8.92
$\gamma = 1.0 \cdot 10^{-6}, \alpha = 0.09, \beta = 0.90$	1000	1.44	5.04	9.88
	250	1.00	4.24	9.12

Table 7. Actual Rejection Frequencies when the True Model is GQARCH(1,1)

The table shows results from a Monte Carlo experiment where the empirical power of six specification tests are investigated. In the experiment, the data generating process is model (26). The columns labeled Actual Rejection Frequencies report the simulated empirical power at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, which give an estimated standard error of the estimated power, $\widehat{p}$, equal to $\sqrt{\widehat{p}(1-\widehat{p})/2500}$. The abbreviations are: LM_1 refers to the test of no GQARCH, LM_2 to the test of no LSTGARCH, SB to the Sign bias test, NSB to the Negative size bias test, PSB to the Positive size bias test, and Joint refers to the test of the joint hypothesis of SB, NSB, and PSB.

		Nominal Significance Level		
		1 %	5 %	10%
Test	Sample Size	Actual Rejection Frequencies (%)		
LM_1	1000	6.28	20.72	32.56
	250	1.44	6.48	13.80
LM_2	1000	3.92	13.20	22.56
	250	0.84	4.60	11.36
SB	1000	1.28	5.52	10.92
	250	0.88	5.28	9.92
NSB	1000	1.60	6.00	10.92
	250	0.96	4.28	8.88
PSB	1000	1.00	4.88	10.76
	250	0.48	4.44	9.68
$Joint$	1000	1.24	4.92	10.48
	250	1.00	4.12	8.00

Table 8. Actual Rejection Frequencies when the True Model is LSTGARCH(1,1)

The table shows results from a Monte Carlo experiment where the empirical power of six specification tests is investigated. In the experiment, the data generating process is model (27). The columns labeled Actual Rejection Frequencies report the simulated empirical power at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, which give an estimated standard error of the estimated power, $\widehat{p}$, equal to $\sqrt{\widehat{p}(1-\widehat{p})/2500}$. The abbreviations are: LM_1 refers to the test of no GQARCH, LM_2 to the test of no LSTGARCH, SB to the Sign bias test, NSB to the Negative size bias test, PSB to the Positive size bias test, and Joint refers to the test of the joint hypothesis of SB, NSB, and PSB.

		Nominal Significance Level		
		1 %	5 %	10%
Test	Sample Size	Actual Rejection Frequencies (%)		
LM_1	1000	15.36	34.72	47.36
	250	5.12	15.36	25.12
LM_2	1000	36.12	64.40	75.60
	250	7.12	21.64	32.92
SB	1000	5.40	15.00	24.04
	250	1.64	7.76	14.48
NSB	1000	6.36	16.68	26.48
	250	1.72	7.12	12.20
PSB	1000	6.44	21.20	34.40
	250	1.00	7.40	18.08
$Joint$	1000	6.08	17.08	27.44
	250	2.40	6.96	12.72

Table 9. Actual Rejection Frequencies when the True Model is EGARCH(1,1)

The table shows results from a Monte Carlo experiment where the empirical power of six specification tests is investigated. In the experiment, the data generating process is model (28). The columns labeled Actual Rejection Frequencies report the simulated empirical power at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, which give an estimated standard error of the estimated power, $\widehat{p}$, equal to $\sqrt{\widehat{p}(1-\widehat{p})/2500}$. The abbreviations are: LM_1 refers to the test of no GQARCH, LM_2 to the test of no LSTGARCH, SB to the Sign bias test, NSB to the Negative size bias test, PSB to the Positive size bias test, and Joint refers to the test of the joint hypothesis of SB, NSB, and PSB.

		Nominal Significance Level		
		1 %	5 %	10%
Test	Sample Size	Actual Rejection Frequencies (%)		
LM_1	1000	51.08	74.00	82.80
	250	7.72	22.08	33.44
LM_2	1000	31.32	57.00	70.56
	250	4.08	15.52	24.56
SB	1000	9.16	24.44	36.80
	250	2.08	9.20	16.40
NSB	1000	12.44	29.56	41.08
	250	1.88	8.32	14.96
PSB	1000	9.52	27.84	40.68
	250	2.20	9.92	18.36
$Joint$	1000	7.32	20.84	32.12
	250	1.16	6.12	11.88

Table 10. Actual Rejection Frequencies when the True Model is GJR

The table shows results from a Monte Carlo experiment where the empirical power of six specification tests is investigated. In the experiment, the data generating process is model (29). The columns labeled Actual Rejection Frequencies report the simulated empirical power at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, which give an estimated standard error of the estimated power, $\widehat{p}$, equal to $\sqrt{\widehat{p}(1-\widehat{p})/2500}$. The abbreviations are: LM_1 refers to the test of no GQARCH, LM_2 to the test of no LSTGARCH, SB to the Sign bias test, NSB to the Negative size bias test, PSB to the Positive size bias test, and Joint refers to the test of the joint hypothesis of SB, NSB, and PSB.

		Nominal Significance Level		
		1 %	5 %	10%
Test	Sample Size	Actual Rejection Frequencies (%)		
LM_1	1000	84.16	95.00	97.44
	250	19.08	43.00	56.92
LM_2	1000	73.56	90.52	95.12
	250	14.68	37.04	51.12
SB	1000	26.68	49.76	62.20
	250	5.12	15.16	24.08
NSB	1000	18.72	39.40	51.28
	250	4.00	11.72	19.20
PSB	1000	22.16	49.08	62.96
	250	3.28	14.68	25.40
$Joint$	1000	20.24	44.36	58.20
	250	3.16	10.96	18.48

Table 11. Actual Rejection Frequencies when the True Model is TGARCH

The table shows results from a Monte Carlo experiment where the empirical power of six specification tests is investigated. In the experiment, the data generating process is model (30). The columns labeled Actual Rejection Frequencies report the simulated empirical power at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, which give an estimated standard error of the estimated power, $\widehat{p}$, equal to $\sqrt{\widehat{p}(1-\widehat{p})/2500}$. The abbreviations are: LM_1 refers to the test of no GQARCH, LM_2 to the test of no LSTGARCH, SB to the Sign bias test, NSB to the Negative size bias test, PSB to the Positive size bias test, and Joint refers to the test of the joint hypothesis of SB, NSB, and PSB.

		Nominal Significance Level		
		1 %	5 %	10%
Test	Sample Size	Actual Rejection Frequencies (%)		
LM_1	1000	56.76	79.28	86.00
	250	10.24	26.60	38.16
LM_2	1000	32.72	58.80	71.64
	250	5.44	17.48	27.24
SB	1000	11.00	27.12	37.60
	250	2.28	8.48	15.36
NSB	1000	10.48	25.76	36.36
	250	2.76	7.80	14.52
PSB	1000	9.00	26.48	39.04
	250	2.16	8.36	16.16
$Joint$	1000	7.24	21.44	32.92
	250	1.32	5.68	11.44

Table 12. Actual Rejection Frequencies when the True Model is A-PARCH

The table shows results from a Monte Carlo experiment where the empirical power of six specification tests is investigated. In the experiment, the data generating process is model (31). The columns labeled Actual Rejection Frequencies report the simulated empirical power at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, which give an estimated standard error of the estimated power, $\widehat{p}$, equal to $\sqrt{\widehat{p}(1-\widehat{p})/2500}$. The abbreviations are: LM_1 refers to the test of no GQARCH, LM_2 to the test of no LSTGARCH, SB to the Sign bias test, NSB to the Negative size bias test, PSB to the Positive size bias test, and Joint refers to the test of the joint hypothesis of SB, NSB, and PSB.

		Nominal Significance Level		
		1 %	5 %	10%
Test	Sample Size	Actual Rejection Frequencies (%)		
LM_1	1000	76.28	91.56	95.40
	250	12.76	32.40	45.84
LM_2	1000	64.16	85.36	91.32
	250	8.92	27.12	41.08
SB	1000	6.72	17.40	26.88
	250	1.48	7.36	13.16
NSB	1000	8.32	18.92	27.60
	250	2.12	8.16	14.08
PSB	1000	5.72	18.72	30.60
	250	1.20	6.88	14.00
$Joint$	1000	5.80	16.36	25.84
	250	1.12	5.76	11.56

Table 13. Actual Rejection Frequencies when the True Model is VS-ARCH

The table shows results from a Monte Carlo experiment where the empirical power of six specification tests is investigated. In the experiment, the data generating process is model (32). The columns labeled Actual Rejection Frequencies report the simulated empirical power at the three different theoretical significance levels of one, five, and ten percent. The figures are based on 2,500 samples, which give an estimated standard error of the estimated power, $\widehat{p}$, equal to $\sqrt{\widehat{p}(1-\widehat{p})/2500}$. The abbreviations are: LM_1 refers to the test of no GQARCH, LM_2 to the test of no LSTGARCH, SB to the Sign bias test, NSB to the Negative size bias test, PSB to the Positive size bias test, and Joint refers to the test of the joint hypothesis of SB, NSB, and PSB.

		Nominal Significance Level		
		1 %	5 %	10%
Test	Sample Size	Actual Rejection Frequencies (%)		
LM_1	1000	20.96	43.32	55.92
	250	3.00	12.04	20.52
LM_2	1000	19.56	42.08	56.04
	250	1.96	10.04	17.20
SB	1000	1.60	7.52	13.68
	250	1.04	5.56	11.60
NSB	1000	2.12	9.48	17.48
	250	0.88	6.08	12.84
PSB	1000	2.52	9.64	16.96
	250	1.12	5.44	10.80
$Joint$	1000	1.40	7.80	15.24
	250	0.84	4.88	10.76