Delegation of Bargaining and Power

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Abstract

Two principals simultaneously appoint one agent each and decide how much power to give to their agents. The agents' task is to bargain over the provision of a public good. Power here means the right to decide the own side's provision if negotiations break down. In equilibrium the principals delegate to agents that are relatively disinterested in the public good and give them all power. The fact that both principals have the possibility to delegate is, in equilibrium, harmful to at least one of them. The equilibrium may even be Pareto dominated by the outcome under autarchy. *Journal of Economic Literature* Classification Numbers: C71, and C72.

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1. Introduction

The nature of many important decisions is such that they cannot be made by those who are most concerned. In the case of negotiations between countries it is usually impractical, if not impossible, to gather the citizens for a referendum every

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time a decision has to be made. Instead we delegate that kind of decision making to political institutions. The main concern of the individual citizen is then not the decision making itself but to choose a policymaker with the appropriate ideology (preferences) and to give this policymaker a well-balanced amount of power. The natural question is: what characterizes a good combination of ideology and power? Is it in the interest of the citizen to have a policymaker who is ideologically very different from herself and to give this policymaker limited power, or does she want the policymaker to be more powerful and ideologically close to herself? This problem of ideology and power is the problem addressed in this study. As it turns out, the citizen wants the policymaker to be ideologically different from herself and she wants him to have an extensive authority.

The framework studied is international negotiations but the model and its logic apply to a much broader spectrum of situations. A few examples are given later in the introduction. The basic model is similar to the one used in Segendorff (1998); there are two countries with one unit of resources each that can be allocated between the production of two goods. One of the goods is private for the producing country and the other good is public between the countries. The private good can be thought of as health care and the public good as reduction of the emission of carbon dioxide into the atmosphere. Every citizen in each country prefers a particular national resource allocation (output combination of the two goods). This ideal allocation is determined by her taste parameter which is continuously distributed across the population.

In each country there is a particular citizen called *the principal* (henceforth *she*) who delegates the national allocation decision to an especially selected citizen, *the agent* (henceforth *he*). She also decides on how much power, in a sense explained below, to give to her agent. There is no voting in the model but it may be helpful to think of the principal as some decisive voter and the agent as some elected policymaker such as a president or a prime minister. Alternatively, we may think of the principal as a prime minister appointing a member of the cabinet. The payment scheme of such an agent is usually low-incentive powered (see also Perry and Samuelson (1994)) and here the agent is given a fixed wage normalized to zero. There is no monetary aspect of the national allocation decision and the agent's decision is consequently based on his preferences directly over the resource allocation of his country. We like to think of a politician as driven by ideological motives and this is how we can think of the agent.[1] This basic model is

[1] High-powered incentive schemes are not ruled out but they require a different interpretation of the model. Suppose that agents only care about money and that the chosen agent is rewarded in proportion to the principal's utility. The agents are uninformed of the principal's preferences and instead of differing in interest for the public good they differ in their belief over the principal's preferences. Agents who are relatively disinterested in the private good believe the principal to

the foundation of two delegation games that are used to study strategic delegation of bargaining and power. The resulting sets of Nash equilibria are compared to a benchmark called *autarchy.*

By autarchy is meant a situation where the two principals simultaneously decide on their national output combinations. The unique Nash equilibrium is for the two principals to implement their ideal allocations. This equilibrium is not Pareto efficient with regard to the principals because not one of them internalizes the effect of her decision on the other principal. Therefore, the principals have an incentive to coordinate on some mutually preferred resource allocation and this is what motivates the delegation games.

The first delegation game is a two-stage game. In the first stage the principals simultaneously choose agents and the amount of power to give to the agents. In the second stage the agents meet and bargain over the global resource allocation, i.e., the output combinations of the two countries. The bargaining is modelled by way of the Nash bargaining solution and the disagreement point is constituted by the agents' utilities of some alternative global allocation that is implemented in the case of a break down in negotiations (see Binmore *et al.* (1986) for a more extensive discussion on the subject). By power we mean the agent's influence on the resource allocation of his country in the case of a break down. If the agent is given no (all) power then he has no (total) influence on the break-down allocation which then becomes the principal's (agent's) ideal allocation. In the case of intermediate power the break-down allocation is somewhere between the two ideal allocations. Power can thus be thought of as a politician's ability to implement his preferred policy after a break down in negotiations. However, delegation of power is only important if it is credible, i.e., if the principal can commit to such delegation. In the model we assume that delegation is credible.

Delegation of power is just as important as the preferences of the agent because it is the combination of power and preferences that allows the principal to threat the agent of the other country. If she appoints an agent with less taste for the public good than herself then this agent would like to allocate relatively less resources to production of the public good in the case of a break down. Giving the agent power to influence the break-down allocation lowers (increases) the disagreement utility of the agent of the other (own) country and thus works as a threat. An increase in the disagreement utility of an agent induces an increase in that agent's payoff. This is called *disagreement point monotonicity* (Thomson (1987)) and in the model it implies a decrease in the provision of public good of that agent's country and an increase in the other country's provision of public good. This is beneficial for both the principal and her agent since the amount of resources allocated to production of their private good increases without the

care more about the public good than about the private good, and vice versa.

corresponding amount of resources being withdrawn from production of the public good. In equilibrium the principals give total power to agents who are less interested in the public good than the principals themselves. At least one off the principals is worse of in equilibrium compared to a situation where the two principals bargain themselves. Moreover, the negative effects of delegation may more than offset the gains from coordination and both principals may be worse off than in autarchy.

The second delegation game is a three-stage game where the principals simultaneously decide on the agents' power in the first stage and thereafter, in the second stage, simultaneously choose agents after having observed the choices made in the first stage. In the third stage the agents meet and bargain. We show that in any subgame-perfect Nash equilibrium to this game the principals delegate to agents who are relatively disinterested in the public good. The main contribution of the three-stage delegation game is thus to show the robustness of the principals' incentive to delegate strategically to agents with little interest in the public good.

The model does not only apply to international negotiations but to a much broader class of situations where delegation of bargaining and power occurs. One example is two firms that can gain from cooperation in R&D. The firm owners choose managers who meet and negotiate. Managers differ in what they believe maximizes profits; investment in the sales organization (private good) or investment in R&D (public good). This example presumes the reasonable assumption of spill-over of knowledge between the firms in the case of no cooperation. Bargaining between local governments over the provision of a (locally) public good, say libraries, is another example. One more example is bargaining between interest groups who have partly coinciding objectives and who try to convince a third party about something, e.g., a union and an employers' association lobbying for subsidies to an industry or protection from international competition by trade barriers.

This study is closely related to Segendorff (1998) who lets the power of the agents be exogenously given and who studies two delegation games differing in the amount of power given to the agents. In the *weak delegation game* the agents have no power and in the *strong delegation game* they have total power. The main findings are that the equilibrium of the weak delegation game Pareto dominates autarchy while the equilibrium of the strong delegation game may be Pareto dominated by autarchy. The study presented here is different in one important respect; the amount of power given to the agents is determined endogenously. A principal will thus only give her agent power if it is in her interest to do so.

Jones (1989) studies a situation where two principals choose agents to bargain over the division of two private goods. The bargaining is modelled by way of the

Nash bargaining solution and the disagreement point is normalized to zero. The main finding is that there can never be a utility gain for both principals compared to a situation where the principals bargain themselves. Fershtman *et al.* (1991) let two principals delegate a bargaining to two agents. Each principal signs a contract (payment scheme) with her agent where the payment is determined by the bargaining outcome. The principal is free to design the contract and the agent has preferences over the payment only. Their main result is that when allowing for a broad class of contracts, any cooperative outcome of the bargaining game without delegation can be made the unique subgame-perfect equilibrium of the delegation game. The delegation game studied below is concerned with a problem very different from the problem studied by Jones since the bargaining in this study is over a public good and the Nash bargaining solution is interpreted differently. It is different from the study by Fershtman *et al.* because the agents' incentives are non-monetary.

Finally, Crawford and Varian (1979), Sobel (1981), and Burtraw (1992) recognize that the Nash and related solution concepts to the bargaining problem presume information that is unobservable in practice and that a bargainer may gain from misrepresenting her true preferences. In the context of the Nash bargaining solution, the unique dominant-strategy Nash equilibrium is for both parties to report risk-neutral utility functions. These studies, even though distortion of preferences and delegation are related to each other, cannot capture some important aspects of delegation such as delegation of power.

The basic model and the autarchy benchmark are given in Section 2. Power and the Nash bargaining solution are defined in Section 3 and the two delegation games are given in Section 4. A numeric example is given in Section 5 and Section 6 contains the summary and comments. All proofs are given in the Appendix.

2. The Basic Model

The basic model is similar to the two-country model in Segendorff (1998). Each country has one unit of resources to allocate between the production of two goods of which one is private for the producing country and the other is public between the two countries. Every citizen has preferences over her country's production of the private good and the total production of the public good. These preferences are determined by a taste parameter, θ, and in both countries the taste parameters are continuously distributed over the interval $[a, 1]$ where $a \in (0, 1)$. An arbitrary citizen of country $k = 1, 2$ is denoted $\theta \in [a, 1]$ and the amount of country k's resources that is allocated to production of the public good is denoted $x_k \in [0, 1]$. The preferences of θ for $x_k < 1$ are represented by the von Neuman-Morgenstern utility function

$$v_k(\mathbf{x}, \theta) = \theta \ln(1 - x_k) + x_1 + x_2$$

where $\mathbf{x} = (x_1, x_2)$ and if $x_k = 1$, then $v_k(\mathbf{x}, \theta) = -\infty$. (Bold-face will in the following be used to denote vectors.) Every θ has an ideal resource allocation $x^*(\theta) = 1 - \theta$ that θ prefers to any other x_k. The assumption $0 < a$ can thus be interpreted as every citizen receives some utility from consumption of the private good. The two countries' allocations are strategically neutral ($v_{k12} = v_{k21} = 0$) and θ's ideal allocation is the same for all resource allocations of the other country.

In each country there is a citizen called the *principal* (she) with taste parameter $a < \theta_k^P < 1$. Agents with stronger taste for the private good than the principal, $\theta_k > \theta_k^P$, will in the following be said to be *to the right* and agents with less taste for the private good than the principal, $\theta_k < \theta_k^P$, will consequently be said to be *to the left*. In *autarchy* the two principals simultaneously decide on the national output combination and they implement the allocation $\mathbf{x}^P = (x_1^P, x_2^P) = (x^*(\theta_1^P), x^*(\theta_2^P))$ which is not Pareto efficient from their point of view. Hence, any such pair of principals would benefit from coordinating on some other mutually preferred allocation. In the delegation game presented below, the countries coordinate their allocation decisions through bargaining. In reality negotiations are often carried out by delegates who represent the bargaining parties and below delegation is introduced in order to capture that important aspect of negotiations.

3. Delegation of Bargaining and Power

Consider the following two-stage game. In the first stage the principals simultaneously delegate the task of deciding on the national allocation to an especially selected citizen, the *agent* (he). At the same time each principal chooses how much power to give to her agent. Let θ_1 be the agent from country 1 and θ_2 the agent from country 2. The agent may be any citizen including the principal herself. The latter case is called *self-representation*. In the second stage the appointed agents meet and bargain over the resource allocations of the two countries, i.e., the provision of the public good. The resulting agreement is assumed to be binding. If no agreement is reached, some alternative break-down allocation is implemented.

The break-down allocation of a country is determined by the principal and her agent. Let the break-down allocation of country k be

$$b_k(\alpha_k, \theta_k) = x_k^P + \alpha_k(x^*(\theta_k) - x_k^P) = 1 - \theta_k^P + \alpha_k(\theta_k^P - \theta_k)$$

where $\alpha_k \in [0, 1]$ represents the power of the agent. If no (all) power is given to the agent, $\alpha_k = 0$ ($\alpha_k = 1$), then the ideal allocation of the principal (agent) is implemented in the case of a break-down in negotiations. In the intermediate case the break-down allocation is a linear combination of the two ideal allocations. In the following $b_k(\alpha_k, \theta_k)$ is viewed as a compromise (some bargaining outcome) between the principal and the agent where their relative bargaining strength is determined by the power of the agent. This simplification is easily justified since any Pareto efficient bargaining outcome between the principal and her agent can be described as a linear combination of the ideal allocations x_k^P and $x^*(\theta_k)$. The variable α_k can then be thought of as reflecting properties of the underlying bargaining game.[2]

Let $\mathbf{b}(\boldsymbol{\theta}, \boldsymbol{\alpha}) = (b_1(\theta_1, \alpha_1), b_2(\theta_2, \alpha_2))$ where $\boldsymbol{\theta} = (\theta_1, \theta_2)$ and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$. The bargaining between the two agents is modelled by way of the Nash bargaining solution.

Definition 1. *Let*

$$N(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\theta}) = (v_1(\mathbf{x}, \theta_1) - v_1(\mathbf{b}(\boldsymbol{\alpha}, \boldsymbol{\theta}), \theta_1))(v_2(\mathbf{x}, \theta_2) - v_2(\mathbf{b}(\boldsymbol{\alpha}, \boldsymbol{\theta}), \theta_2)).$$

Then the Nash bargaining solution is

$$\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}) = \arg \max_{\mathbf{x} \in Z(\boldsymbol{\alpha}, \boldsymbol{\theta})} N(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\theta}) \tag{3.1}$$

where

$$Z(\boldsymbol{\alpha}, \boldsymbol{\theta}) = \left\{\mathbf{x} \in [0,1]^2 \mid \mathbf{x} \succsim_{\theta_1} \mathbf{b}(\boldsymbol{\alpha}, \boldsymbol{\theta}) \text{ and } \mathbf{x} \succsim_{\theta_2} \mathbf{b}(\boldsymbol{\alpha}, \boldsymbol{\theta})\right\}$$

is the contract zone of the two agents.

The bargaining outcome $\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta})$ is unique and the described two-stage delegation game can be reduced to a one-stage simultaneous-move game played by the principals.

[2] An alternative but not adopted view is to think of $b_k(\alpha_k, \theta_k)$ as the expected break-down allocation and α_i as the probability of the agent winning a political struggle after a break down. The winner of the struggle implements his/her ideal allocation. Thus, the power of the agent determines the probabilities of the two allocations x_k^P and $x^*(\theta_k)$. Taking the preferences of the principal and her agent as given, the break-down allocation is interpreted as a lottery induced by α_k. The principal and her agent are risk averse with respect to the own country's allocation. Their expected utilities from participating in the lottery are thus lower than their utilities from implementing the corresponding compromise. Moreover, the utility of the other country's agent is the same under both interpretations of b_k since his utility is linear in country k's provision of public good. In the model, an increase in the disagreement utility of country k's agent induces an increase in the amount of public good provided by the other country and a decrease in the amount of public good provided by the own country. This is beneficial for country k's agent.

4. The Delegation Game

In this section we formalize and analyze the two-stage delegation game. Let $D = (N, S, \boldsymbol{\pi})$ denote the delegation game where $N = \{1, 2\}$ is the set of principals that play the game. The principals simultaneously choose agents and decide on how much power to give to their agents. The set from which principal k chooses her agent's power is $A_k = [0, 1]$ and the set of agents available to her is $\Theta_k = [a, 1]$. A strategy for principal k thus a pair $s_k = (\alpha_k, \theta_k) \in S_k$ where $S_k = A_k \times \Theta_k$ is her set of strategies. Let $S = S_1 \times S_2$. The pair of payoff functions is $\boldsymbol{\pi} = (\pi_1, \pi_2)$ where π_k denotes the payoff to principal k as a function of the strategy profile $\mathbf{s} = (s_1, s_2) \in S$, i.e.,

$$\pi_k(\mathbf{s}) = v_k\left(\mathbf{x}^{NB}(\mathbf{s}), \theta_k^P\right).$$

For the moment, treat the strategy of principal $l \neq k$ and the power of agent k as given. Let $\xi_k(\alpha_k, s_l)$ be the set of agents that maximize principal k's utility given $\alpha_k \in A_k$ and $s_l \in S_l$

$$\xi_k(\alpha_k, s_l) = \arg\max_{\theta_k \in \Theta_k} \pi_k(\mathbf{s}). \tag{4.1}$$

Studying principal k's maximization problem it can be shown that any agent that maximizes her utility against $s_l \in S_l$ must be to the right of her and this is true for all levels of power given to the agent.[3]

Lemma 1. $\xi_k(\alpha_k, s_l) \subseteq \left(\theta_k^P, 1\right]$ $\forall \alpha_k \in A_k$, $\forall s_l \in S_l$.

Proof. See the Appendix.

The logic behind Lemma 1 is the following. Because the bargaining outcome is Pareto efficient it will stipulate each country to provide more public good than prescribed by the agents' ideal allocations, $\mathbf{x}^{NB}(\mathbf{s}) > (x^*(\theta_1), x^*(\theta_2))$ $\forall \mathbf{s} \in S$. In the case of self-representation, $\theta_k = \theta_k^P$, country k has to provide more of the public good than the principal wishes, $x_k^{NB}((\alpha_k, \theta_k^P), s_l) > x_k^P$, and she consequently has an incentive to lower her country's provision of public good. If she appoints an agent to her left then she can not achieve this reduction since this agent is even more interested in the public good. Delegation aiming at reducing the own production of public good to x_k^P must therefore be to an agent who is relatively disinterested in the public good, i.e., who is to the right of the principal. The welfare of principal k does not only depend on her country's resource allocation but also on the resource allocation of the other country which in turn depends

[3] The weak delegation game in Segendorff (1998) is the mixed extension of D where $A_k = \{0\}$ and $\Theta_k = [0, 1]$ for $k = 1, 2$. Lemma 1 can be extended to provide a strengthening of Proposition 2(ii) in Segendorff (1998) by making it possible to say that delegation is made to the right.

on principal k's choice of agent and the amount of power given to that agent. In Lemma 1 we learn that principal k gains from strategic delegation to some agent to her right. The eventual utility loss from a decrease in the other country's provision of public good is outweighed by the utility gain from the reduction in the own country's provision of public good.

The Nash bargaining solution has a property called *disagreement point monotonicity* (Thomson (1987)); an increase in the disagreement utility of an agent induces an increase in that agent's payoff. Suppose principal k delegates to an agent to her right, $\theta_k > \theta_k^P$. By increasing the power of her agent she increases the disagreement utility of the agent and lowers the disagreement utility of the other agent. Because of the disagreement point monotonicity this change induces a change in the agreement making country k provide less of the public good and country l provide more.

Lemma 2. *Let $\theta_k > \theta_k^P$. Then $\frac{dx_k^{NB}}{d\alpha_k} < 0$ and $\frac{dx_l^{NB}}{d\alpha_k} > 0$ $\forall \alpha_k \in A_k$, $\forall s_l \in S_l$.*

Proof. See the Appendix.

Changes in the power of the agent does not affect the slope of the agents' (here linear) contract curve but moves $\mathbf{x}^{NB}(\mathbf{s})$ along the contract curve. Let $\beta_k(s_l)$ denote the set of strategies that maximize π_k against $s_l \in S_l$, $l \neq k$

$$\beta_k(s_l) = \arg\max_{s_k \in S_k} \pi_k(\mathbf{s}).$$

The set $\beta_k(s_l)$ is nonempty because S_k is compact and convex and π_k is continuous in s_k. From Lemma 1 we have that $\theta_k > \theta_k^P$ for all $s_k \in \beta_k(s_l)$ and using Lemma 2 together with the properties of the Nash bargaining solution gives $\alpha_k = 1$. In order to see this, suppose first that $s_k \in \beta_k(s_l)$ is such that $\alpha_k = 0$. Then $x_k^{NB}(\mathbf{s}) > x_k^P$ and increasing α_k unambiguously increases π_k by Lemma 2. Hence, any best reply for principal k includes giving her agent some power. Now, suppose s_k is such that $\alpha_k \in (0,1)$. Principal k's utility function is quasi-concave in $\mathbf{x}$ and her highest feasible indifference curves is thus tangent to the agents' contract curve at $\mathbf{x}^{NB}(\mathbf{s})$. By playing $\theta_k' = \theta_k - \varepsilon$ for some small $\varepsilon > 0$ instead of θ_k she can marginally shift the contract curve upward so that it cuts through her old indifference curve and becomes tangent to a new indifference curve representing a higher level of utility. The new tangency point $\mathbf{x}'$ is feasible since ε is arbitrary small, i.e., there exists an $\alpha_k' < 1$ such that $\mathbf{x}^{NB}(\alpha_k', \theta_k', s_l) = \mathbf{x}'$. This argument is illustrated in Figure 4.1 and it applies to every $\alpha_k \in (0,1)$. Hence, $\alpha_k = 1$. Finally, if $\alpha_k = 1$ then π_k is strictly concave in θ_k which implies that the utility maximizing strategy s_k is unique.

Lemma 3. *Let $s_l \in S_l$, $l \neq k$. Then $\beta_k(s_l)$ is a singleton set and if $\beta_k(s_l) = \{(\alpha_k, \theta_k)\}$ then $\alpha_k = 1$ and $\theta_k > \theta_k^P$.*

Proof. See the Appendix.

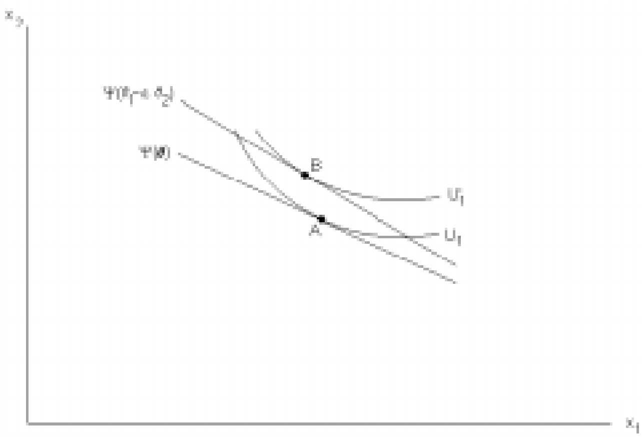

Figure 4.1: Principal 1's indifference curve U_1 tangent to the agents' contract curve $\Psi(\boldsymbol{\theta})$ at point A and agent 1 is given intermediate power, $0 < \alpha_1 < 1$. By delegating to an agent who is slightly more interested in the public good principal 1 shifts the agents' contract curve upwards. For a small enough change there exists a new intermediate level of power such that the point B is reached. At B, the new higher contract curve is tangent to an indifference curve representing a higher level of utility than U_1.

By using Lemma 3 we can show the existence of a Nash equilibrium to the game D. Principal k's best-reply correspondence, β_k, is a continuous function since the best reply always is unique by Lemma 3 and the payoff function π_k is continuous in $\mathbf{s}$. Hence, the combined best-reply function $\beta = \beta_1 \times \beta_2$ has a fixed point $\mathbf{s}^{NE} = (s_1^{NE}, s_2^{NE}) \in S$ which is a Nash equilibrium to the game D. Let $NE(D)$ denote the set of Nash equilibria to the game D.

Proposition 1. $NE(D) \neq \emptyset$.

Proof. See the Appendix.

We now turn our attention to the qualitative properties of $NE(D)$. Any Nash equilibrium to the game D must, by Lemma 3, be constituted by a strategy profile such that both principals delegate to agents to their right and give them all power.[4]

[4] The strong delegation game in Segendorff (1998) is the mixed extension of D where $A_k = \{1\}$ and $\Theta_k = [0, 1]$. Propositions 1 and 2 above can be extended to the strong delegation game, i.e., there exists a pure-strategy Nash equilibrium with the properties stated in Proposition 2 to the strong delegation game.

Proposition 2. *Let* $\mathbf{s}^{NE} \in NE(D)$ *where* $s_k^{NE} = (\alpha_k^{NE}, \theta_k^{NE})$*. Then* $\alpha_k^{NE} = 1$ *and* $\theta_k^{NE} > \theta_k^P$ *for* $k = 1, 2$.

Proof. See the Appendix.

In Lemma 4 we have reformulated Lemma 1 from Segendorff (1998). It says that if principal k gives all power to her agent then country k's (country l's) provision of the public good decreases (increases) with the agent's taste for the private good. This is due to the disagreement point monotonicity and the Pareto efficiency of the Nash bargaining solution.

Lemma 4. *Let* $\alpha_k = 1$*, then* $\frac{dx_k^{NB}}{d\theta_k} < 0$ *and* $\frac{dx_l^{NB}}{d\theta_k} > 0$*,* $l \neq k$*.*

Proof. See the proof of Lemma 1 in Segendorff (1998).

The equilibrium agreement $\mathbf{x}^{NE}(\mathbf{s}^{NE})$ and the principals' welfare in equilibrium can be studied by applying Lemma 4 to the first-order conditions of principal k's maximization problem. Suppose principal k does not delegate to her right most agent in equilibrium, i.e., $\theta_k^{NE} < 1$. Then Lemma 4 tells us that principal k's marginal utility of her own provision of public good is positive, $dv_k/dx_k > 0$, which implies that the share of her country's resources that is allocated to production of the public good in equilibrium is smaller the principal's ideal allocation, $x_k^{NB}(\mathbf{s}^{NE}) < x_k^P$. This necessarily means that principal $l \neq k$ is worse of in equilibrium than in autarchy because even if she implements her ideal allocation, this can not compensate for the reduction of country k's provision of public good. Consequently, if both principals delegate such agents then they both provide less public good than in autarchy and they both worse off in equilibrium than in autarchy. Let $\boldsymbol{\theta}^P = (\theta_1^P, \theta_2^P)$ and let $A = A_1 \times A_2$.

Corollary 1. *Let* $\mathbf{s}^{NE} \in NE(D)$*. Then:*
(i) If $\theta_k^{NE} < 1$ *then* $x_k^{NB}(\mathbf{s}^{NE}) < x_k^P$ *and* $\pi_l(\mathbf{s}^{NE}) < v_l(\mathbf{x}^P, \theta_l^P)$ *for* $l \neq k$*.*
(ii) If $\theta_k^{NE} < 1$ *for* $k = 1, 2$*, then* $\pi_k(\mathbf{s}^{NE}) < v_k(\mathbf{x}^P, \theta_k^P)$*.*
(iii) $\pi_k(\mathbf{s}^{NE}) < v_k(\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P), \theta_k^P)$ *for some* $k = 1, 2$*,* $\forall \boldsymbol{\alpha} \in A$*.*

Proof. See the Appendix.

The third statement in Corollary 1 comes from Proposition 2 and the Pareto efficiency of the Nash bargaining solution. In the case of mutual self-representation the bargaining outcome lies on the two principals' contract curve. Lemma 2 states that $\theta_k^{NE} > \theta_k^P$ for $k = 1, 2$ and this implies that the agents' contract curve lies below the contract curve of the two principals. The equilibrium agreement, $x^{NB}(\mathbf{s}^{NE})$, can therefore not be on principals' contract curve. It follows that at least one of the principals is worse off in equilibrium compared to the case of mutual self-representation.

4.1. Separate Choices of Agents and Power

In international negotiations the power of a delegate is sometimes defined by the constitution of his country. Because of the often complex political process required to change a constitution it is natural to view the choice of constitution as a long-term choice and the choice of delegate as a short-term choice. It is also natural to think of the constitutions of the concerned countries as common knowledge when the delegates are chosen. This situation can be modelled by a three-stage delegation game where the principals simultaneously choose how much power to give to their agents in the first stage - this is their choice of constitutions. In the second stage the principals observe the chosen amounts of power and simultaneously choose agents. In the third stage, the agents meet and bargain over the two countries' resource allocations.

Let $D' = (N, S', \boldsymbol{\pi}')$ be the reduced three-stage delegation game. The set of principals is the same as in the definition of the game D but the strategy sets and the payoff functions are different. The strategy of principal k is a pair (α_k, φ_k) where φ_k is a function that to every $\boldsymbol{\alpha} \in A$ assigns a probability distribution $F_k(\cdot \mid \boldsymbol{\alpha})$ over principal k's set of agents, Θ_k. Hence, principal k's strategy set is $S'_k = A_k \times \Phi_k$ where Φ_k is the set of all functions φ_k from A to the set of all probability distributions over Θ_k. Let $S' = S'_1 \times S'_2$ and let $F(\cdot \mid \boldsymbol{\alpha})$ be the joint (product) probability distribution. The expected payoff for principal k from the strategy profile $\mathbf{s} \in S'$ is

$$\pi'_k(\mathbf{s}) = \int_{\boldsymbol{\theta} \in \Theta} v_k(\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}), \theta_k^P) dF(\boldsymbol{\theta} \mid \boldsymbol{\alpha})$$

and $\boldsymbol{\pi}' = (\pi'_1, \pi'_2)$.

A subgame-perfect equilibrium is a strategy profile $\mathbf{s}^{SPE} \in S'$ such that (i) $F_k^{SPE}(\cdot \mid \boldsymbol{\alpha})$ is a best reply against $F_l^{SPE}(\cdot \mid \boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in A$, and (ii) α_k^{SPE} is a best reply against α_l^{SPE} given $\boldsymbol{\varphi}^{SPE} = (\varphi_1^{SPE}, \varphi_2^{SPE})$, $l \neq k$. Let $SPE(D')$ denote the set of subgame-perfect Nash equilibria to the game D'. From Lemma 1 it follows that for all choices of power $\boldsymbol{\alpha}$, $F_k^{SPE}(\cdot \mid \boldsymbol{\alpha})$ assigns positive probability only to agents who are less interested in the public good than their principals.

Proposition 3. *If* $\mathbf{s}^{SPE} \in SPE(D')$ *then* $F^{SPE}(\boldsymbol{\theta}^P \mid \boldsymbol{\alpha}^{SPE}) = 0$.

Proof. See the Appendix.

In a subgame-perfect equilibrium we only observe bargaining between agents who are less interested in the public good than their principals. The difference in taste induces a bargaining outcome in which at least one of the principals provides less of the public good than in the case of mutual self-representation. This is true for all pair of agents in the support of $F^{SPE}(\cdot \mid \boldsymbol{\alpha})$, for all $\boldsymbol{\alpha} \in A$, and thus also true for the expected bargaining outcome. Hence, at least one of the principals is worse off in equilibrium than under mutual self-representation.

Corollary 2. *Let* $\mathbf{s}^{SPE} \in SPE(D')$. *Then:*
(i) $x_k^{NB}(\mathbf{s}^{SPE}) < x_k^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P)$ *for some* $k = 1, 2,\ \forall \boldsymbol{\alpha} \in A$.
(ii) $\pi_k'(\mathbf{s}^{SPE}) < v_k(\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P), \theta_k^P)$ *for some* $k = 1, 2,\ \forall \boldsymbol{\alpha} \in A$.

Proof. See the Appendix.

It is clear from Proposition 3 and Corollary 2 that separating the choices of power and agents in time can not eliminate the incentives for the principals to delegate strategically to agents to their right.

5. An Example

In the numerical example below we compute the Nash equilibrium $\mathbf{s}^{NE}$ to the two-stage delegation game for three pairs of principals and compare the welfare properties of each equilibrium with the cases of autarchy and mutual self-representation. For each of the three pairs of principals there exists an unique Nash equilibrium. Figure 5.1 is based on deriving the Nash equilibria for a large number of pairs of principals and in every case the Nash equilibrium is unique. The set of Nash equilibria to D is therefore likely to be a singleton set for every pair of principals.

Throughout the example, let $a = 0.05$. First, let $\theta_1^P = 0.3$ and let $\theta_2^P = 0.2$. Then is $\mathbf{s}^{NE} = ((1, 0.69), (1, 0.58))$ and both principals are worse off than in the cases of autarchy and mutual self-representation. If we increase principal 2's taste for the private good, say $\theta_2^P = 0.7$, then $\mathbf{s}^{NE} = ((1, 0.7), (1, 1))$ and principal 1 prefers the equilibrium to autarchy but prefers mutual self-representation to the equilibrium. The reason to why principal 1 prefers the equilibrium to autarchy is that the restriction $\theta_2 \leq 1$ is binding. This means that principal 2, being relatively disinterested in the public good, can not lower her provision of the public good much compared to her provision in autarchy. The difference in country 2's provision of public good in equilibrium and under mutual self-representation is still to large to make principal 1 prefer the equilibrium to mutual self-representation. Again, Principal 2 is worse of than in autarchy.

If we increase principal 2's taste parameter further to $\theta_2^P = 0.9$, then the Nash equilibrium is the same as above, $\mathbf{s}^{NE} = ((1, 0.7), (1, 1))$. This is because the best reply of principal 1 does not depend on θ_2^P but only on θ_2. However, the change in θ_2^P changes the benchmarks and now principal 1 is better of in equilibrium than in the case of mutual self-representation. The difference in country 2's provision of public good in equilibrium and under mutual self-representation is smaller than before since θ_2^P has increased. Principal 2 is still worse off than in autarchy.

In Figure 5.1 the equilibrium utilities of all possible combinations of principals are compared with the cases of autarchy and mutual self-representation. In the example above, we move from region A to region B, and then to region E.

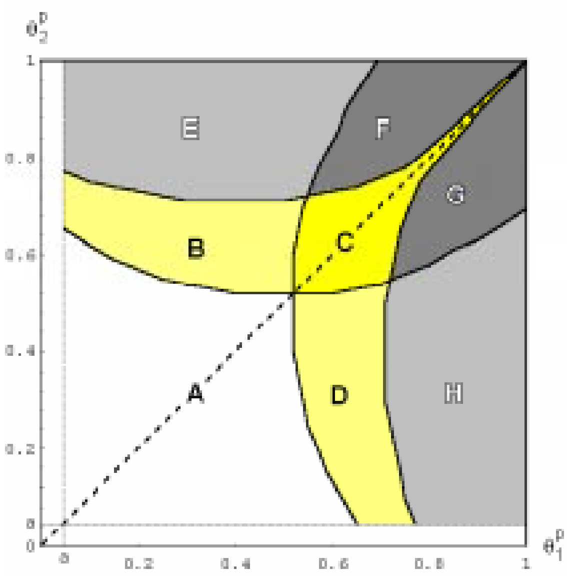

Figure 5.1: The welfare properties of the equilibrium provision of public good depends on the principals' taste. Principal 1 is prefers $\mathbf{s}^{NE}$ to mutual self-representation in regions E and F, and $\mathbf{s}^{NE}$ to autarchy in regions B, C, E, F, and G. Principal 2 analogously prefers $\mathbf{s}^{NE}$ to mutual self-representation in regions G and H, and $\mathbf{s}^{NE}$ to autarchy in regions C, D, F, G, and H. Both principals worse off in equilibrium than in autarchy in region A.

6. Summary and Comments

Any Nash equilibrium to the studied two-stage delegation game is such that each principal delegates the bargaining to an agent to her right and gives this agent all power. Each principal does so in order to reach a more favorable agreement than if she had bargained herself. However, since both principals reason in the same way and delegate as described above, they may end up in an equilibrium in which they both are worse off than under autarchy. In equilibrium, at least one of them is worse of in equilibrium compared to mutual self-representation.

In equilibrium, each principal correctly anticipates the strategy played by her opponent and plays a best reply against it. The principals therefore realize the bad nature of the equilibrium and they would benefit from coordinating on some institutional set-up in which the agents are given no or little power or/and in which mutual self-representations is played in equilibrium. One natural and important question is if such institutional set-ups can be achieved as the equilibrium outcome of some (political) delegation game and if so, what set of rules characterizes that game? Finally, we show that strategic delegation to the right is a part of every subgame-perfect Nash equilibrium of the three-stage delegation game. This suggests that the incentive to delegate strategically to the right is a robust result.

The results arrived to in this study partly depend on the chosen utility function. In order to determine the qualitative properties of the set of Nash equilibria to the delegation game one must have clear-cut results from the comparative statics carried out on the Nash bargaining solution. With a general utility function, this is not possible without imposing several restrictions on the form of the utility function. The explicit utility function used in this study was chosen for the reasons of simplicity and clearness. It keeps the model fairly simple and still provides some important insights. Even though a more general utility function is desirable we argue that this should not be considered a major drawback.

Appendix: Proofs

Lemma 1. $\xi_k(\theta_l, \boldsymbol{\alpha}) = \arg\max_{\theta_k \in \Theta_k} v_k \left(x^{NB}(\boldsymbol{\theta}, \boldsymbol{\alpha}), \theta_k^P\right)$ and the derivative of θ_k^P's maximization problem w.r.t. θ_k is

$$\frac{\partial \pi_k}{\partial \theta_k} = \frac{\partial v_k^P}{\partial x_k} \frac{dx_k^{NB}}{d\theta_k} + \frac{dx_l^{NB}}{d\theta_k} \tag{6.1}$$

where v_k^P indicates the principal's utility function. Lemma 1 is proved by showing that Equation 6.1 is strictly positive for all $\theta_k \leq \theta_k^P$ and the proof is carried out in two steps. First we derive the expressions for $\frac{dx_k^{NB}}{d\theta_k}$ and $\frac{dx_l^{NB}}{d\theta_k}$ where $l \neq k$ and then we use the derived expressions to determine the sign of Equation 6.1.

Step 1: Let

$$B_k(\boldsymbol{\theta}, \boldsymbol{\alpha}) = \left(v_k(\mathbf{x}, \theta_k) - v_k(\mathbf{b}(\boldsymbol{\theta}, \boldsymbol{\alpha}), \theta_k)\right) \tag{6.2}$$

and the first-order conditions to the Nash bargaining solution are

$$\frac{\partial NB}{\partial x_k} = \frac{\partial v_k}{\partial x_k} B_l(\boldsymbol{\theta}, \boldsymbol{\alpha}) + B_k(\boldsymbol{\theta}, \boldsymbol{\alpha}) = 0. \tag{6.3}$$

The system 6.3 defines the unique bargaining outcome $\mathbf{x}^{NB}(\boldsymbol{\theta}, \boldsymbol{\alpha})$. The outcome $\mathbf{x}^{NB}(\boldsymbol{\theta}, \boldsymbol{\alpha})$ is, by the Implicit function theorem, locally continuous in $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$. In the following we consider principal 1's problem, i.e., $k = 1$. Differentiation of the system 6.3 w.r.t. x_1, x_2, and θ_1 and rearranging gives

$$\begin{bmatrix} \frac{\partial^2 NB}{\partial x_1^2} & \frac{\partial^2 NB}{\partial x_2 \partial x_1} \\ \frac{\partial^2 NB}{\partial x_1 \partial x_2} & \frac{\partial^2 NB}{\partial x_2^2} \end{bmatrix} \begin{bmatrix} \frac{dx_1^{NB}}{d\theta_1} \\ \frac{dx_2^{NB}}{d\theta_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 NB}{\partial \theta_1 \partial x_1} \\ -\frac{\partial^2 NB}{\partial \theta_1 \partial x_2} \end{bmatrix}.$$

The determinant of the Hessian to NB, denoted $\mathbb{H}$, is positive since NB is a concave function evaluated at its maximum. Cramer's rule gives

$$\frac{dx_1^{NB}}{d\theta_1} = -\frac{\begin{pmatrix} \frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial^2 v_2}{\partial x_2^2} B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) + 2\frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial v_2}{\partial x_2} B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) \\ -\frac{\partial v_1}{\partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial b_1}{\partial \theta_1} + \frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial B_1}{\partial \theta_1} B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) \end{pmatrix}}{\det(\mathbb{H})} \tag{6.4}$$

and

$$\frac{dx_2^{NB}}{d\theta_1} = \frac{2\frac{\partial^2 v_1}{\partial\theta_1 \partial x_1} B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) + \frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial b_1}{\partial\theta_1} - \frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial v_2}{\partial x_2}\frac{\partial B_1}{\partial \theta_1} B_2(\boldsymbol{\theta},\boldsymbol{\alpha})}{\det(\mathbb{H})} \tag{6.5}$$

Step 2: Let $\boldsymbol{\alpha} \in A = A_1 \times A_2$, $\theta_2 \in \Theta_2$ and $\theta_1 \leq \theta_1^P$. Then is $\partial B_1/\partial\theta_1 < 0$. Substituting Equations 6.4 and 6.5 into Equation 6.1 and gives

$$\begin{aligned} \frac{\partial \pi_1}{\partial \theta_1} &= -\frac{\frac{\partial v_1^P}{\partial x_1}}{\det(\mathbb{H})} \begin{pmatrix} \frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial^2 v_2}{\partial x_2^2} B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) + 2\frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial v_2}{\partial x_2} B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) \\ -\frac{\partial v_1}{\partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial b_1}{\partial \theta_1} + \frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial B_1}{\partial \theta_1} B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) \end{pmatrix} \\ &+ \frac{1}{\det(\mathbb{H})}\left(\frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial b_1}{\partial \theta_1} - \frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial v_2}{\partial x_2}\frac{\partial B_1}{\partial \theta_1} B_2(\boldsymbol{\theta},\boldsymbol{\alpha})\right) + \frac{2B_2(\boldsymbol{\theta},\boldsymbol{\alpha})}{\det(\mathbb{H})}\frac{\partial^2 v_1}{\partial\theta_1\partial x_1}. \end{aligned} \tag{6.6}$$

The sign of Equation 6.6 is ambiguous because the last term is negative while the first two terms are positive. Rewriting Equation 6.6 slightly gives

$$\begin{aligned} \frac{\partial \pi_1}{\partial \theta_1} &= -\frac{\frac{\partial v_1^P}{\partial x_1}}{\det(\mathbb{H})} \begin{pmatrix} \frac{\partial^2 v_1}{\partial\theta_1 \partial x_1}\frac{\partial^2 v_2}{\partial x_2^2} B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) B_2(\boldsymbol{\theta},\boldsymbol{\alpha}) \\ -\frac{\partial v_1}{\partial x_1}\frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial b_1}{\partial \theta_1} + \frac{\partial^2 v_2}{\partial x_2^2}\frac{\partial B_1}{\partial \theta_1} B_1(\boldsymbol{\theta},\boldsymbol{\alpha}) \end{pmatrix} \\ &+ \frac{1}{\det(\mathbb{H})}\left(\frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial b_1}{\partial \theta_1} - \frac{\partial^2 v_1}{\partial x_1^2}\frac{\partial v_2}{\partial x_2}\frac{\partial B_1}{\partial \theta_1} B_2(\boldsymbol{\theta},\boldsymbol{\alpha})\right) \\ &+ \frac{2B_2(\boldsymbol{\theta},\boldsymbol{\alpha})}{\det(\mathbb{H})}\frac{\partial^2 v_1}{\partial\theta_1\partial x_1}\left(-\frac{\partial v_1^P}{\partial x_1}\frac{\partial v_2}{\partial x_2} + 1\right). \end{aligned} \tag{6.7}$$

Equation 6.7 is positive if the last term is positive, i.e., if

$$-\frac{\partial v_1^P}{\partial x_1}\frac{\partial v_2}{\partial x_2} + 1 < 0. \tag{6.8}$$

From the system of first-order conditions (system 6.3) we have $\frac{\partial v_1}{\partial x_1}\frac{\partial v_2}{\partial x_2} = 1$. Because $\theta_1 < \theta_1^P$ by assumption and $x_1^{NB}(\boldsymbol{\theta},\boldsymbol{\alpha}) > x^*(\theta_1) > b_1(\theta_1,\alpha_1)$ we have $\frac{\partial v_1^P}{\partial x_1}\frac{\partial v_2}{\partial x_2} > 1$. Hence, $\frac{\partial \pi_1}{\partial \theta_1} > 0 \ \forall \theta_1 \leq \theta_1^P, \forall \boldsymbol{\alpha} \in A$.

It follows that if $\theta_1 \leq \theta_1^P$ then $\theta_1 \notin \xi_1(\theta_2,\boldsymbol{\alpha})$ and it follows from the continuity of v_1 that this is true also for $\theta_1 = a$. Hence $\xi_1(\theta_2,\boldsymbol{\alpha}) \subseteq \left(\theta_1^P, 1\right]$. By analogy is $\xi_2(\theta_1,\boldsymbol{\alpha}) \subseteq \left(\theta_2^P, 1\right]$. ■

Lemma 2. Lemma 2 is proved by showing by first deriving the expressions $\frac{dx_k^{NB}}{d\alpha_k}$ and $\frac{dx_l^{NB}}{d\alpha_k}$, $l \neq k$. Thereafter we determine their signs using that $\theta_k > \theta_k^P$.

Step 1: In the following we consider country 1, i.e., $k = 1$. Differentiating the system of first-order conditions to the Nash bargaining solution (system 6.3) w.r.t. x_1, x_2, and α_1 gives

$$\begin{bmatrix} \frac{\partial^2 NB}{\partial x_1^2} & \frac{\partial^2 NB}{\partial x_2 \partial x_1} \\ \frac{\partial^2 NB}{\partial x_1 \partial x_2} & \frac{\partial^2 NB}{\partial x_2^2} \end{bmatrix} \begin{bmatrix} \frac{dx_1^{NB}}{d\alpha_1} \\ \frac{dx_2^{NB}}{d\alpha_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 NB}{\partial \alpha_1 \partial x_1} \\ -\frac{\partial^2 NB}{\partial \alpha_1 \partial x_2} \end{bmatrix}.$$

Using Cramer's rule and simplifying gives

$$\frac{dx_1^{NB}}{d\alpha_1} = \frac{-(\theta_1 - \theta_1^P)\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_1}{\partial x_1}\right)\frac{\partial^2 v_2}{\partial x_2^2} B_1(\boldsymbol{\theta}, \boldsymbol{\alpha})}{\det(\mathbb{H})}. \tag{6.9}$$

and

$$\frac{dx_2^{NB}}{d\alpha_1} = \frac{-(\theta_1 - \theta_1^P)\frac{\partial^2 v_1}{\partial x_1^2}\left(\frac{\partial v_1}{\partial x_1} + \frac{\partial v_1}{\partial x_1}\right)\frac{\partial v_2}{\partial x_2} B_2(\boldsymbol{\theta}, \boldsymbol{\alpha})}{\det(\mathbb{H})}. \tag{6.10}$$

Step 2: If $\theta_1 > \theta_1^P$ then (i) $(\theta_1 - \theta_1^P) > 0$ and (ii) $\frac{\partial v_1}{\partial x_1} \leq 0$ because $b_1(\theta_1, \alpha_1) \geq x^*(\theta_1)$. $\frac{\partial^2 v_2}{\partial x_2^2} < 0$ since $\theta_2 > 0$. Hence, $\frac{dx_1^{NB}}{d\alpha_1} < 0$ and $\frac{dx_2^{NB}}{d\alpha_1} > 0$. ■

Lemma 3. This proof is carried out in three steps. First we show that $\alpha_k = 1$ in all best replies. Thereafter this is used to show β_k is a singleton set.

Step 1: First it is shown that $\alpha_k > 0$ and thereafter it is shown that $\alpha_k = 1$. The first-order condition of $\pi_k(\mathbf{s})$ w.r.t. α_k is:

$$\frac{\partial \pi_k}{\partial \alpha_k} = \frac{\partial v_k^P}{\partial x_k}\frac{dx_k^{NB}}{d\alpha_k} + \frac{dx_l^{NB}}{d\alpha_k} = 0 \tag{6.11}$$

for interior solutions where v_k^P denotes principal k's utility function. Now, let $\alpha_k = 0$. Since $\theta_k \in \xi_k(0, s_l)$ we have $\mathbf{x}^{NB}((0, \theta_k), s_l) \succ_{\theta_k^P} \mathbf{x}^{NB}((0, \theta_k^P), s_l)$. Assume that $\mathbf{x}^{NB}((0, \theta_k), s_l) < \mathbf{b}((0, \theta_k), s_l)$, then $\mathbf{b}((0, \theta_k), s_l) \succ_{\theta_k^P} \mathbf{x}^{NB}((0, \theta_k), s_l)$ which contradicts $\theta_k \in \xi_k(0, s_l)$. Hence, $\mathbf{x}^{NB}((0, \theta_k), s_l) > \mathbf{b}((0, \theta_k), s_l)$. Then, by Lemma 2, can principal k increase her utility by increasing α_k which decreases x_k^{NB} and increases x_l^{NB}, $l \neq k$. Hence, $(0, \theta_k) \notin \beta_k(s_l)$ for any $\theta_k > \theta_k^p$.

The set of Pareto efficient allocations for θ_1 and θ_2 is

$$\Psi(\boldsymbol{\theta}) = \left\{\mathbf{x} \in [0,1]^2 \mid x_2 = (1 - \theta_2) + \frac{\theta_2}{\theta_1}(1 - x_1)\right\}$$

and an indifference curve for principal k is

$$U_k(\mathbf{s}) = \left\{\mathbf{x} \in [0,1]^2 \mid v_k\left(\mathbf{x}, \theta_k^P\right) = \pi_k(\mathbf{s})\right\}.$$

Let $s_l \in S_l$ and let $s_k \in \beta_k(s_l)$. From Lemma 1 we know $\theta_k > \theta_k^P$ and from Step 1 above we know $\alpha_k > 0$. Suppose that $\alpha_k < 1$, then $U_k(\mathbf{s})$ is tangent to $\Psi(\boldsymbol{\theta})$ at $\mathbf{x}^{NB}(\mathbf{s})$. Let $\theta_k' = \theta_k - \varepsilon$ for some small $\varepsilon > 0$. Then $\Psi(\theta', \theta_l)$ intercepts $U_k(\mathbf{s})$ twice since v_k is quasi-concave in $\mathbf{x}$. Due to the continuity of principal k's first-order condition with respect to α_k (Equation 6.11) we can find some $\varepsilon > 0$ and some $\alpha_k' \in (0,1)$ such that $\Psi(\theta', \theta_l)$ is tangent to $U_k(\alpha_k', \theta_k', s_l)$ where $\pi_k(\alpha_k', \theta_k', s_l) > \pi_k(\mathbf{s})$. Hence, by contradiction we showed that if $(\alpha_k, \theta_k) \in \beta_k(s_l)$ then $\alpha_k = 1$.

Step 2: Let

$$graph(s_l) = \left\{\mathbf{x}^{NB}(\mathbf{s}) \in [0,1]^2 \mid s_k = (1, \theta_k),\ 0 \leq \theta_k \leq 1\right\}.$$

In this part of the proof we show that for any two points on $graph(s_l)$ the part of the graph between these two points lies above the line segment joining the two points. From this we show that $\beta_k(s_l)$ is singleton. Throughout the proof is $l \neq k$.

Let the strategy profile $s = (\boldsymbol{\alpha}, \boldsymbol{\theta})$ describe a pair of agents and a pair of constitutions where $\alpha_k = 1$. Let $\varepsilon > 0$ be arbitrary small and let $\mathbf{s}' = (\boldsymbol{\alpha}, (\theta_k + \varepsilon, \theta_l))$ and $\mathbf{s}'' = (\boldsymbol{\alpha}, (\theta_k + 2\varepsilon, \theta_l))$. We then have three corresponding agreements; $\mathbf{x} = \mathbf{x}^{NB}(\mathbf{s})$, $\mathbf{x}' = \mathbf{x}^{NB}(\mathbf{s}')$, and $\mathbf{x}'' = x^{NB}(\mathbf{s}'')$. From the Pareto efficiency of the Nash bargaining solution it follows that $\mathbf{x}' \in \Psi(\theta_k + \varepsilon, \theta_l)$ and that $\mathbf{x}'' \in \Psi(\theta_k + 2\varepsilon, \theta_l)$. Let $dx_k' = x_k' - x_k = \frac{\partial x_k^{NB}}{\partial \theta_k}\varepsilon < 0$ and let $dx_k'' = x_k'' - x_k = 2\frac{\partial x_k^{NB}}{\partial \theta_k}\varepsilon < 0$. Divide dx_k' into two parts $a', b' < 0$, i.e. $dx_k' = a' + b'$, where b' is such that $dx_l' = -\frac{\theta_l}{\theta_k + \varepsilon}b > 0$ and a' is such that $(1 - \theta_l) + \frac{\theta_l}{\theta_k + \varepsilon}(1 - x_k - a') = x_l$, i.e., a' is such that $(x_k + a', x_l) \in \Psi(\theta_k + \varepsilon, \theta_l)$ and b' is such that $(x_k + a' + b', x_l') \in \Psi(\theta_k + \varepsilon, \theta_l)$. Solving the latter expression for a' gives $a' = \frac{\varepsilon}{\theta_k}(1 - x_k)$ when using that $x_l = (1 - \theta_l) - \frac{\theta_l}{\theta_k}(1 - x_k)$. Analogously, letting $dx_k'' = a'' + b''$ where $a'', b'' < 0$ and where $dx_l'' = -\frac{\theta_l}{\theta_k + 2\varepsilon}b'' > 0$ and $(1 - \theta_l) + \frac{\theta_l}{\theta_k + 2\varepsilon}(1 - x_k - a'') = x_l$ gives $a'' = \frac{2\varepsilon}{\theta_k}(1 - x_k)$. Thus, $a'' = 2a'$ and since $dx_k'' = 2dx_k'$ we have $b'' = 2b'$. This implies that $dx_l'' < 2dx_l'$ and hence $graph(s_l)$ describes x_l locally as a concave function of x_k. The same reasoning applies for all $\mathbf{x} \in graph(s_l)$ and π_k is locally a concave function of θ_k around $\mathbf{x}^{NB}(\mathbf{s})$ when holding s_2 fixed and $\alpha_k = 1$.

It follows that $\beta_k(s_l)$ is a singleton set. Suppose that this is not true and that $s_k = (1, \theta_k)$ and $s_k' = (1, \theta_k')$ both belong to $\beta_k(s_l)$. Let $s_k'' = (1, (\theta_k + \theta_k')/2)$. Then $x_k^{NB}(s_k'', s_l) = \lambda x_k^{NB}(\mathbf{s}) + (1 - \lambda)x_k^{NB}(s_k', s_l)$ for some $0 < \lambda < 1$ and $x_l^{NB}(s_k'', s_l) > \lambda x_l^{NB}(\mathbf{s}) + (1 - \lambda)x_l^{NB}(s_k', s_l)$. Since $\lambda\mathbf{x}^{NB}(\mathbf{s}) + (1 - \lambda)\mathbf{x}^{NB}(s_k', s_l) \succeq_{\theta_k^P} \mathbf{x}^{NB}(\mathbf{s})$, $\mathbf{x}^{NB}(s_k', s_l)$ we have that $\mathbf{x}^{NB}(s_k'', s_l) \succeq_{\theta_k^P} \mathbf{x}^{NB}(\mathbf{s})$, $\mathbf{x}^{NB}(s_k', s_l)$ which is a contradiction. Hence, $\beta_k(s_l)$ is a singleton set. ■

Proposition 1. Let the correspondence $\gamma_k : S_l \twoheadrightarrow S_k$ be defined by $\gamma_k(s_l) = S_k$ for all $s_l \in S_l$ and notice that let $\beta_k(s_l) = \arg\max_{s_k \in \gamma(s_l)} \pi_k(\mathbf{s})$. Lemma 3 states that $\beta_k(s_l)$ is singleton and hence closed at s_l. γ_k is a continuous correspondence and $\gamma_k(s_l)$ is compact. From the properties of γ_k and β_k it follows that β_k is continuous at s_l (Border (1985), pp. 59, Proposition 11.21(b)) and hence that β_k is continuous function. Define the function $\beta : S \to S$ by $\beta(\mathbf{s}) = \beta_1(s_2) \times \beta_2(s_1)$. From above we have that S is convex and compact and that β is a continuous function. By Brouwer's fixed point theorem β has a fixed point (Border (1985), pp. 29, Corollary 6.6). Hence, the game D has a Nash equilibrium $\mathbf{s}^{NE} = (s_1^{NE}, s_2^{NE}) \in NE(D) \neq \emptyset$. ■

Proposition 2. Follows directly from Lemma 3. ■

Corollary 1. (i) Let v_k^P denote the principals utility function. If $\theta_k^{NE} < 1$ then $\frac{\partial v_k^P}{\partial x_k}\frac{dx_k^{NB}}{d\theta_k} + \frac{dx_l^{NB}}{d\theta_k} = 0$ which by Lemma 4 implies $\frac{\partial v_k^P}{\partial x_k} < 0$. Hence, $x_k^{NB}(\mathbf{s}^{NE}) < x_k^P$. If $x_k^{NB}(\mathbf{s}^{NE}) < x_k^P$ then $\pi_l(\mathbf{s}^{NE}) \leq v_l(x_k^{NB}(\mathbf{s}^{NE}), x_l^P, \theta_l^P) < v_l(\mathbf{x}^P, \theta_l^P)$ by definition of x_l^P.

(ii) If $\theta_k^{NE} < 1$ for $k = 1, 2$, then $\mathbf{x}^{NB}(\mathbf{s}^{NE}) < \mathbf{x}^P$ by (i) above. From (i) it also follows that $\mathbf{x}^P \succ_{\theta_k^P} \mathbf{x}^{NB}(\mathbf{s}^{NE})$.

(iii) Let $s_k^P = (\alpha_k, \theta_k^P)$ and let $\mathbf{s}^P = (s_1^P, s_2^P)$. By the Pareto efficiency of the Nash bargaining solution we have that $\mathbf{x}^{NB}(\mathbf{s}^P) \in \Psi(\boldsymbol{\theta}^P)$ where $\boldsymbol{\theta}^P = (\theta_1^P, \theta_2^P)$ and $\Psi(\boldsymbol{\theta})$ is defined as in the proof of Lemma 3. We also have that $\mathbf{x}^{NB}(\mathbf{s}^{NE}) \in \Psi(\boldsymbol{\theta}^{NE})$. Because $\theta_k^{NE} > \theta_k^P$ for $k = 1, 2$ we have $\Psi(\boldsymbol{\theta}^P) \cap \Psi(\boldsymbol{\theta}) = \emptyset$ by the definition of Ψ in the proof of Lemma 3. Hence, $\mathbf{x}^{NB}(\mathbf{s}^P) \neq \mathbf{x}^{NB}(\mathbf{s}^{NE})$ and one of the principals are worse off. ■

Proposition 3. Suppose $\mathbf{s}^{SPE} = (\boldsymbol{\alpha}^{SPE}, \boldsymbol{\varphi}^{SPE}) \in SPE(D')$. Then $(l \neq k)$

$$\int_{\theta_l \in \Theta_l} \frac{\partial \pi_k'}{\partial \theta_k} dF_l(\theta_l \mid \boldsymbol{\alpha}^{SPE}) = 0 \tag{6.12}$$

for all $\theta_k \in (0, 1)$. By Lemma 1, the right-hand side of Equation 6.12 is strictly positive for all $\theta_k \leq \theta_k^P$. Moreover, $F_k^{SPE}(a \mid \boldsymbol{\alpha}^{SPE}) = 0$ by the continuity of π_k'. Hence, $F_k^{SPE}(\theta^P \mid \boldsymbol{\alpha}^{SPE}) = F^{SPE}(\boldsymbol{\theta}^P \mid \boldsymbol{\alpha}^{SPE}) = 0$. ■

Corollary 2. (i) Let $C^{SPE} \subseteq \left(\theta_1^P, 1\right] \times \left(\theta_2^P, 1\right]$ be the set of pairs of agents that are assigned positive probability by $\mathbf{s}^{SPE}$. The set of possible bargaining outcomes under $\mathbf{s}^{SPE}$ is

$$X^{SPE} = \left\{\mathbf{x}^{NB}(\boldsymbol{\alpha}^{SPE}, \boldsymbol{\theta}) \in [0,1]^2 \mid \boldsymbol{\theta} \in C^{SPE}\right\}.$$

Let $con(X^{SPE})$ be the convex hull of X^{SPE}. We have that $\mathbf{x}^{NB}(\mathbf{s}^{SPE}) \in con(X^{SPE})$. By the definition of Ψ in the proof of Lemma 3 and by Proposition 3 is $\Psi\left(\boldsymbol{\theta}^P\right) \cap con(X^{SPE}) = \emptyset$. Hence, $\mathbf{x}^{NB}(\mathbf{s}^{SPE}) \neq \mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P)$. Moreover, $x_k^{NB}(\mathbf{s}^{SPE}) < x_k^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P)$ for some $k = 1, 2$ since $con(\mathbf{x}^{SPE})$ lies below $\Psi\left(\boldsymbol{\theta}^P\right)$.

(*ii*) The vN-M utility function v_k is concave in $\mathbf{x}$. Then, from $\mathbf{x}^{NB}(\mathbf{s}^{SPE}) \neq \mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P)$ it follows that $v_k(\mathbf{x}^{NB}(\boldsymbol{\alpha}, \boldsymbol{\theta}^P), \theta_k^P) > v_k(\mathbf{x}^{NB}(\mathbf{s}^{SPE}), \theta_k^P) > \pi'_k(\mathbf{s}^{SPE})$. ■

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