A Note on Contingent Claims Pricing with Non-Traded Assets*

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Abstract

One of the main objections to applying contingent claims analysis outside the area of derivatives pricing, such as to the pricing of corporate (or sovereign) debt, has been that it is not possible to trade in the relevant state variable, e.g. the assets of a firm. Consequently, replicating portfolios can not be formed and preference free pricing does not result.

The aim of this paper is to show that assuming traded assets, as is routinely done, is inconsistent with the presence of stocks and bonds. It is also unnecessary. We argue that a superior alternative to obtain a complete markets setting, is to assume that at least one of the firm's securities, e.g. equity, is traded.

Keywords: corporate bonds, sovereign debt, contingent claims, traded assets, non-traded state variable

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An important application of contingent claims theory, as pioneered by Black, Scholes and Merton in the early seventies, is the valuation of corporate liabilities. Their insight was that stocks and bonds could be viewed and valued as derivatives on the assets of the firm. However, practical use of the theory in this area has been questioned on the grounds that this state variable neither is traded nor easily observable. For example Leland (1994) states that

> "We leave unanswered the delicate question of whether V, which could be associated with the value of an unlevered firm, is a traded asset."

and Jarrow et al. (1997) argue that

> "This approach is difficult to implement in practice because all of the firm's assets are not tradeable nor observable."

As a result, the simple arbitrage argument breaks down and pricing is no longer preference-free. The perceived necessity of a traded state variable dates back to the seminal papers in the field; Merton (1974) assumes that "Trading in /the firm's/ assets takes place continuously in time." In applied work, one appears to be left with one of two choices: either estimate market preferences or retain the assumption of a physically traded underlying asset – an "asset security". While the first approach is inherently difficult, the second, albeit more common, is even less satisfying as it suggests the existence of two sets of securities with conflicting claims to the same physical assets. On the one hand, the total value of the outstanding corporate securities is by construction equal to the value of the firm's assets.[1] But on the other, it is assumed that it is possible to buy the asset security, which by definition also constitutes a claim

[1] For simplicity of exposition, we abstract from costs of financial distress and corporate taxation. Nonetheless, the proposed argument is robust to such an extension.

on all assets. In essence, each dollar of profit generated by the assets is assumed to be distributed twice: first divided into dividends and coupons to shareholders and creditors, and second in its entirety to the asset security holder. Note that this inconsistency does not appear in standard derivative pricing since a stock option does not constitute a claim on the firm's shareholders, but on the writer of the option. Thus, the total market value of stock options is not limited by a firm's market capitalization. In sharp contrast, the total market value of corporate securities is inextricably tied to the underlying asset value.

Clearly, the benefit of assuming the existence of a traded claim is that any analysis can be carried out in a complete markets setting. However, as we have argued, assuming that the firm's assets are traded is inconsistent with the presence of traded securities on the firm's balance sheet. The aim of this paper is to point to a third, more satisfying, alternative: to assume that at least one of the firm's securities, e.g. the stock, is traded, while the assets are not. Now instead of the assets, that security completes the market and, as a result, the pricing of any other claim on the assets, such as a corporate bond, does not require information about investors' risk preferences.

Perhaps less obviously, we argue that an appropriately defined fictive security can be used as a basis for pricing all claims, and that this security has the natural interpretation of capturing the value of the firm's assets. By setting up a replicating portfolio consisting of stocks and risk free bonds, one would be able to track the value of this security without actually buying it. Thus, although it is inappropriate to assume that a firm's assets are traded, the dynamics of the replicated asset security are the same as those of traded assets. As a result, most models in this field can be applied without modification.[2]

[2] Examples are Merton (1974), Nielsen et al. (1993), Longstaff & Schwartz (1995), Anderson & Sundaresan (1996) and Leland & Toft (1996). Corporate bond models of this type are often referred to as structural, or firm value based, models. An alternative class of models are the reduced form, or intensity based, models. The latter class does not model the value of the

To understand the intuition of the replication argument, consider an analogy with an ordinary stock option model. Fundamentally, the option can be priced precisely because we can replicate its payoff using the stock and risk free bonds. However, we can just as well value the stock by replicating *its* payoff using the (traded) option. In the same fashion, we can value the firm's assets using stocks and risk free bonds.

To be more specific, let x_t denote cash flow, or net earnings, generated by the firm's assets at time t, of which security i receives a fraction determined by the pricing function S^i. Assuming that the value of securities is determined by cash flows only, we can write $S_t^i = S^i\left(\{x_s\}_{s\geq t}\right)$. The crucial assumption is that one of these securities (S^0) is traded and thus spans the space of payoffs – stocks would be a natural security to think of. Presuming that the interest rate market is also complete, the valuation of the firm's securities is preference-free.

We can, in particular, price a (fictive) security with a claim on *all* cash flow. This security is appropriately termed *the value of the assets,* denoted ω_t. To see how to use this variable as a basis for pricing all securities, first define β_t as the fraction of asset value generated as cash flow at each point in time: $\beta_t \equiv \frac{x_t}{\omega_t}$. Second, simply express the pricing functions of securities in terms of asset value: $S_t^i = S^i\left(\{\beta_s\omega_s\}_{s\geq t}\right)$. Trivial as this may seem, it is more than a mere substitution of variables since the asset value is a *price* with corresponding preference-free drift under the risk-adjusted probability measure $(r_t - \beta_t)\,\omega_t\Delta t$ (where r_t denotes the appropriate risk free rate at time t).[3]

In summary, if we are willing to assume that one of the firm's securities, such as stocks, is traded, the following must hold:

firm, the assets or the common stock and hence will never encounter the problem addressed in this paper.

[3] A price process is one for which today's level is an (appropriately discounted) present value. Temperatures, company earnings (x) and interest rates do not follow price processes, whereas stock prices without dividends and gold prices do.

- A variable exists that reflects the value of the assets.
- The evolution of this variable under the (unique) risk-adjusted probability measure – as defined by the traded security S^0 – is governed by "preference free" parameters.
- This *asset value* can be used as a pricing tool for all of the firm's securities.

For an illustration of the above in a familiar setting, consider the following example:

Example 1 *Let the cash flow be determined by a geometric Brownian motion*

$$dx_t = \alpha x_t dt + \gamma x_t dW_t \tag{1}$$

where α *is the expected increase in profits and* γ *their volatility. Furthermore, assume that the common stocks of the firm are traded at a price* S_t^0.

$$dS_t^0 = \mu(x_t, t) S_t^0 dt + \sigma(x_t, t) S_t^0 dW_t$$

The risk-adjusted probability measure Q *is defined as the one where traded securities have an expected return equal to the constant risk-free interest rate* r. *Thus, the relevant Girsanov kernel* (λ) *is*

$$\lambda \equiv \frac{\mu(x_t, t) - r}{\sigma(x_t, t)}$$

The Girsanov kernel, interpreted as the market price for risk, will be a constant in this case. The process for the state variable under the risk-adjusted measure is

$$dx_t = (\alpha - \lambda\gamma) x_t dt + \gamma x_t dW_t^Q \tag{2}$$

where W_t^Q *is a* Q*-Wiener process. The* ***value of the assets*** (ω_t) *that generate* *x must equal the (risk-adjusted) expected value of all future cash flows:*[4]

$$\omega_t \equiv \int_t^\infty E^Q\left[e^{-r(s-t)} \cdot x_s\right] ds$$

$$= \frac{1}{r+\lambda\gamma-\alpha} x_t \quad (3)$$

Next define $\beta_t \equiv \frac{x_t}{\omega_t}$ *as the "cash flow as a fraction of asset value". Using (3) we find that*

$$\beta_t \equiv \frac{x_t}{\omega_t} = r + \lambda\gamma - \alpha \quad (4)$$

Thus β *is a constant in this case. Finally, it follows from (3) and (4) that* ω_t *has the following dynamics:*

$$d\omega_t = (r - \beta)\,\omega_t dt + \gamma\omega_t dW_t^Q \quad (5)$$

This process is the starting point for the structural models mentioned in the introduction.[5] *However, it is usually accompanied by an assumption of tradeability. Some models do indeed start from (1) but assume risk neutral investors in order to arrive at (5).*[6] *In contrast, we get there by the assumption of traded stocks.*

Finally, note that it is much easier to estimate $(r-\beta)$ *than* $(\alpha - \lambda\gamma)$*. In the first*

[4] To prevent infinite asset values, we assign the condition $\alpha < r + \lambda\gamma$.

[5] To use the same arguments for models with no net cash payouts (e.g. Leland (1994)) consider the corresponding gain process $\widehat{\omega}$

$$\widehat{\omega}_t \equiv \omega_t + \int_0^t \beta\omega_s ds$$

with dynamics

$$d\widehat{\omega}_t = d\omega_t + \beta\omega_t dt$$

$$= r\widehat{\omega}_t dt + \sigma\widehat{\omega}_t dW_t^Q$$

[6] E.g. Mella-Barral & Perraudin (1997).

case you can use empirical observations on interest rates r, while using earnings and dividends to pin down the estimate of β; the second case involves market preference parameters, the estimation of which is next to impossible (Merton (1980)).

In this paper, we have shown that contingent claims models applied to the pricing of corporate securities, cannot rely on the assumption that the assets are traded. However, we argue that one can assume that at least one of the firm's securities is traded and remain in a complete market setting. In such a framework, although a firm's assets are not traded, their value can be replicated. In practice, models can, as in the past, be based on the risk neutral dynamics for the asset value. However, they should not be accompanied by an assumption that is both unnecessary and economically questionable. Finally, note that the argument presented in this paper could be applied to the pricing of sovereign debt as well; a country's "assets" need not be traded for contingent claims pricing to be useful.

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