Variability and average profits - does Oi's result generalize?*

Richard Friberg[†], Kaj Martensen[‡]

Stockholm School of Economics

SSE Working paper series in Economics and Finance No 402

September 14, 2000

Abstract

Average profits of a price taker are increasing in the variability of the output price (Oi, 1961). We show that, for the same reason, average profits of the price taker are increasing in the variability of the price of inputs. We proceed to establish that the same holds for a firm with a downward sloping demand curve. Unless the inverse demand curve of the firm with market power is very convex, the profit function of the price taker forms an upper limit for the convexity of profit (assuming constant curvature of costs).

Keywords: cost uncertainty, convexity of profit function

JEL: D0. D80

*We thank Marcus Asplund and Tore Ellingsen for valuable comments. Financial support from The Wallander and Hedelius Foundation and the Swedish Competition Authority is gratefully acknowedged.

[†]E-mail: Richard.Friberg@hhs.se

[‡]E-mail: Kaj.Martensen@hhs.se

1 Introduction

Dealing with variability in underlying conditions such as inflation, interest rates, real exchange rates and commodity prices is crucial to many lines of business. Variability presents problems for risk averse firms but also offers opportunities for increasing average profits. The seminal article by Oi (1961) demonstrates that the average profits of a price taker are increasing in the variability of the output price. A natural question is: Does Oi's result generalize to firms that face downward sloping demand curves?

In one sense, the answer is trivially no, since it is only to the price taker that price is an exogenous stochastic variable.[1] In this paper we first establish that the profits of the price taker are strictly convex in the variability of input costs. Is this true also for firms facing a downward sloping demand curve? One could conjecture that the ability of the price taker to expand and contract quantities without affecting price is central for being flexible enough to benefit from variability. Despite the prominent place given to Oi's (1961) result in the literature and the importance of risk management for firms there has to the best of our understanding been no attempts to formally prove if results generalize.

In the next section we examine how average profits depend on a shock to costs. Average profits of both a price-taking firm and a firm faced with downward sloping demand are shown to be increasing in the variability of the cost shock. We proceed to establish the conditions under which the profits of the price taker have a greater degree of convexity in the cost shock than those of a firm with market power (under some additional conditions).

[1] There is one more sense in which the answer is trivially no - all else equal, profits will be concave in changes in the price for a price setting firm, otherwise the second order condition for profit maximization would not hold.

2 What is the curvature of profits?

The timing, which we will use throughout the paper, is as follows: First, the firm first observes the cost or price shift and then it decides on what quantity to produce. We confine attention to internal solutions and assume cost and demand functions (for the firm with market power) to be twice continuously differentiable.

2.1 The price taker

Let us first establish Oi's result as a point of reference. Let a variable $\theta' > 0$ affect the market price faced by a price-taking firm. Using Oi's notation denote the market price in a competitive industry with P and the quantity of the price taker with x. Costs of production are given by $c(x)$ with $c_x > 0$, $c_{xx} > 0$ - throughout subindexes denote partial derivatives. Strictly convex costs are necessary to ensure that there exists an optimal quantity. Thus the profit maximization problem of the firm is Eq. (1), which yields an optimal quantity, denoted x^*

$$\Pi\left(\theta'\right) = \max_{x>0} \left[\theta' P x - c(x)\right]. \tag{1}$$

Proposition 1 *(Oi, 1961) Let a variable $\theta' > 0$ affect the market price facing a price taking firm. Average profits are increasing in the variability of θ'.*

Proof. Average profits are increasing in the variability of θ' if and only if $d^2\Pi\left(x^*\left(\theta'\right)\right)/d\theta'^2 > 0$. Twice totally differentiating profits yields

$$\frac{d^2\Pi\left(x^*\left(\theta'\right)\right)}{d\theta'^2} = \Pi_{xx}\left(\frac{dx^*}{d\theta'}\right)^2 + 2\Pi_{x\theta'}\frac{dx^*}{d\theta'} + \Pi_x\frac{dx^{*2}}{d\theta'^2} + \Pi_{\theta'\theta'}.$$

Totally differentiating the first order condition establishes that $dx^*/d\theta' = P/c_{xx}$. Further use $\Pi_{x\theta'} = P$, $\Pi_{\theta'\theta'} = 0$, $\Pi_x = 0$ around optimum and that $c_{xx} > 0$ to establish that

$$\frac{d^2\Pi\left(x^*\left(\theta'\right)\right)}{d\theta^2} = \frac{P^2}{c_{xx}} > 0.$$

■

Oi (1961) established the result graphically.[2]

Now, denote the profits of the competitive firm under cost variability by Π^c and let the variable $\theta > 0$ affect the vector of input prices w such that the maximization problem facing the firm is

$$\Pi^c(\theta) = \max_{x>0} \left[Px - c(x, \theta w)\right].$$

Since the cost function is homogeneous of degree 1 in input costs given competitively supplied inputs, we can write the maximization problem as

$$\Pi^c(\theta) = \max_{x>0} \left[Px - \theta c(x, w)\right].$$

Proposition 2 *Let a variable $\theta > 0$ affect the price of inputs for the price-taking firm. Average profits are increasing in the variability of θ.*

Proof. The proof is analogous to the proof of Proposition 1.

$$\frac{d\Pi^c\left(x^*(\theta)\right)}{d\theta^2} = \Pi^c_{xx}\left(\frac{dx^*}{d\theta}\right)^2 + 2\Pi^c_{x\theta}\frac{dx^*}{d\theta} + \Pi^c_x\frac{dx^{*2}}{d\theta^2} + \Pi^c_{\theta\theta}$$

[2] A formal proof for that the profit function of a price taker is convex in P is found in for instance Shepard (1970) and many graduate textbooks. Despite claims to the contrary by for instance Varian (1992), this only proves that average profits are non-decreasing in the variability of the price shock, not that they are increasing.

Totally differentiating the first order condition establishes that $dx^*/d\theta = c_x/\Pi^c_{xx}$. Use that $\Pi^c_{x\theta} = -c_x$, $\Pi^c_{\theta\theta} = 0$, that $\Pi^c_x = 0$ around optimum and that $\Pi^c_{xx} < 0$ by the second order conditions for profit maximization to establish that

$$\frac{d^2\Pi^c\left(x^*\left(\theta\right)\right)}{d\theta^2} = -\frac{\left(c_x\right)^2}{\Pi^c_{xx}} > 0.$$

■

Shephard (1970), among others, provides a proof that with competitively supplied inputs the cost function is concave in input costs. Since strict concavity is not shown, this is a necessary but not sufficient condition for the Proposition to hold.

Profits are strictly convex in the cost of inputs for the same reason as they are strictly convex in the price of output. The first order conditions for profit maximization are, respectively, $\theta' P = c_x$ and $P = \theta c_x$. In both cases quantity is set optimally - by making the best of favorable conditions and by cutting back in less favorable, profits are increasing in variability.

2.2 The firm with market power

Now turn to the issue of how profits of firms facing downward sloping demand are affected by variability. Here the case where there is no strategic interaction, no price discrimination and where the firm faces a non stochastic demand curve is considered. Denote profits of this firm with π, quantity with q, costs with $c(q, \theta w)$ and inverse demand with $p(q)$ where $p_q < 0$. The firms maximization problem is

$$\pi\left(\theta\right) = \max_{q>0}\left[p(q)q - c\left(q, \theta w\right)\right]$$

with first order condition (using homogeneity of degree 1 in costs)

$$p_q q + p - \theta c_q = 0.$$

Proposition 3 *Let a variable $\theta > 0$ affect the price of inputs for a firm with downward sloping demand. Average profits of the firm are then increasing in the variability of θ.*

Proof. The proof is analogous to the proof of Proposition 1.

$$\frac{d^2\pi(q^*(\theta))}{d\theta^2} = \pi_{qq}\left(\frac{dq^*}{d\theta}\right)^2 + 2\pi_{q\theta}\frac{dq^*}{d\theta} + \pi_q\frac{d^2q^*}{d\theta^2} + \pi_{\theta\theta}.$$

Totally differentiating the first order condition yields $dq^*/d\theta = c_q/\pi_{qq}$. Further use that $\pi_{q\theta} = -c_q$, $\pi_{\theta\theta} = 0$, $\pi_q = 0$ around optimum and that $\pi_{qq} < 0$ by second order conditions for profit maximization to establish that

$$\frac{d^2\pi(q^*(\theta))}{d\theta^2} = -\frac{(c_q)^2}{\pi_{qq}} > 0$$

■

Note the very general nature of the result - no specific functional forms are assumed. For a (monopoly) firm faced with cost shocks and downward sloping demand, average profits will be increasing in the variability of the cost of the input. In this sense the result of Oi (1961) generalizes - not only price takers benefit from variability. Our initial interest was motivated by the feeling that although it is possible that all firms may improve average profits as a result of variability, the ability of a price taker to change quantity without affecting price should put it in a superior position to benefit from variability. We therefore proceed with a comparison of the curvature of profits.

2.3 Which profit function has the greater curvature?

Is the price taker more "flexible" than the monopolist and can she therefore achieve higher average profits than the monopolist? By choosing for example the degree of differentiation a firm influences the way that price will be affected by quantity changes - and thus chooses how profits will respond to shocks in the underlying environment.[3]

To make a comparison assume that both the price taker and the firm with downward sloping demand have the same cost function and that the third derivative of the cost function is zero.[4] The profit of the competitive firm is more convex than the firm with market power if

$$\frac{d^2\Pi^c}{d\theta^2} > \frac{d^2\pi}{d\theta^2} \tag{2}$$

where the expressions are evaluated at their optimal levels of x and q respectively. Then, for Eq. (2) to hold it has to be true that

$$\frac{-(c_x)^2}{\Pi^c_{xx}} > \frac{-(c_q)^2}{\pi_{qq}}$$

Inserting the expressions for the second order conditions and marginal costs yields

$$\frac{p_{qq}q + 2p_q - \theta c_{qq}}{-\theta c_{xx}} > \left(\frac{c_q}{c_x}\right)^2. \tag{3}$$

[3] Flexibility thus matters also on the demand side. Most attention on flexibility has focused on costs, see Carlsson, 1989 for a survey or Athey and Schmutzler, 1995, for a recent contribution.

[4] The last assumption amounts to making a second order Taylor approximation of the cost function.

By standard theory $q < x$ so that $c_q < c_x$ by convexity and consequently the right hand side of Eq. (3) is less than 1. Use that $c_{qq} = c_{xx}$ and rewrite to establish that

$$\frac{d^2\Pi^c}{d\theta^2} > \frac{d^2\pi}{d\theta^2}$$

$\Leftrightarrow$

$$\frac{p_{qq}q + 2p_q}{-\theta c_{qq}} > \left(\frac{c_q}{c_x}\right)^2 - 1 \tag{4}$$

On the left hand side of Eq. (4) is the ratio between the curvature of revenue and the curvature of costs. The right hand side is negative. Clearly this condition is satisfied for a concave demand function ($p_{qq} \leq 0$). It will hold also if p_{qq} is positive but not too large. For sufficiently large degree of convexity it will not hold however, since by the second order condition $p_{qq}q + 2p_q$ is allowed to be as great as θc_{qq} which reverses the inequality above.

Proposition 4 *A competitive firm gains more from variability in the price of inputs than a monopolist as long as demand is concave or not too convex. For sufficient convexity of demand the monopolist gains more.*

Proof. The second order condition requires $p_{qq}q < \theta c_{qq} - 2p_q$ but (4) requires $p_{qq}q < \theta c_{qq}\left(1 - \left(\frac{c_q}{c_x}\right)^2\right) - 2p_q$ which is a tighter condition. Thus when

$$p_{qq}q \in \left(-\infty\,,\, \theta c_{qq}\left(1 - \left(\frac{c_q}{c_x}\right)^2\right) - 2p_q\right)$$

the competitive firm earns more on average and when

$$p_{qq}q \in \left[\theta c_{qq}\left(1 - \left(\frac{c_q}{c_x}\right)^2\right) - 2p_q\,,\, \theta c_{qq} - 2p_q\right)$$

the monopoly earns more on average. ■

Again, the result is general in that it does not consider any specific functional forms. Consider a firm faced with the choice of selling its product on a competitive world market or differentiate it and sell as a local monopoly (with linear demand). For small enough $\pi(\overline{\theta}) - \Pi^c(\overline{\theta}) > 0$ ($\overline{\theta}$ denoting the average θ) and high enough variability, average profits of the price taker will be larger than for the monopolist.[5] A related result is found in Chang and Harrington (1996) who show that under a linear demand duopoly with constant marginal costs and asymmetric cost shocks, the degree of product differentiation that maximizes industry profits is lowered by cost variability. The reason is precisely the above - the better substitutes that the products are, the more can quantity be expanded following a beneficial shock. Proposition 4 shows that the results of Chang and Harrington depend on the assumption that demand is linear. To see the intuition for why the curvature of demand matters consider the case when the inverse demand curve is very convex. When quantity is expanded there is little price effect, in a similar manner as the price taker the firm is thus able to expand quantities to benefit from decreased costs without affecting price much. Conversely when costs rise and quantity is being cut down this is associated with a large increase in the price that the monopolist receives if demand is very convex.

[5] It deserves to be pointed out though that if there is free entry into the "price taking" industry expected profits will be 0. This is explored in Sheshinski and Drèze (1976).

3 Conclusions

It is worth emphasizing that in deriving the above results we did not specify any specific demand function or cost function. To a considerable extent therefore Oi's result does generalize. We also would like to stress that we have not attempted to provide an analysis of whether society as a whole benefits from variability or not.[6]

A number of avenues for future research present themselves - investigating generalizability to oligopoly is clearly one. We reached our conclusions by assuming that (residual) demand was unaffected by changes in the price of inputs. Results under oligopoly are likely to be influenced by conjectures about responses (Stackelberg, Bertrand), and the potential for implicit collusion (sequencing of moves, observability) and are left for future research.[7] Profits should be more convex the greater the scope for adjustment (this follows from the "LeChatelier principle"). It is therefore clear that changing the timing of the response will affect results. In future work we intend to address such issues but then with a focus on risk management and short- versus longterm hedging.

4 References

Athey, Susan and Armin Schmutzler, 1995, Product and process flexibility in an innovative environment, RAND Journal of Economics 26, 557-574.

Carlsson, Bo, 1989, Flexibility and the theory of the firm, International Journal

[6] Oi's paper did indeed spur several investigations of welfare implications of commodity price variability and price stabilizing schemes. See Samuelsson (1972) or Turnovski et al. (1984).

[7] Some preliminary results show that profits are increasing in the variability of input costs for firms that engage in Cournot competition (for a simple specification of the cost function) when shocks have a positive correlation. For some values of negative correlation the relation between variability and average profits is reversed and Oi's result does not generalize.

of Industrial Organization 7, 179-203.

Chang, Myong-Hun and Joseph E. Harrington, Jr, 1996, The interactive effect of product differentiation and cost variability on profit, Journal of Economics and Management Strategy 5, 175-193.

Oi, Walter Y, 1961, The desirability of price instability under perfect competition, Econometrica 29, 58-64.

Samuelson, Paul A., 1972, Feasible price stability, The Quarterly Journal of Economics LXXXVI, 476-94.

Sheshinski, Eytan and Jacques H Drèze, 1976, Demand fluctuations, capacity utilization, and costs, The American Economic Review 66, 731-743

Shephard, Ronald W, 1970, Theory of cost and production functions, Princeton University Press, Princeton, New Jersey.

Turnovski, Stephen J, Haim Shalit and Andrew Schmitz, 1984, Consumer's surplus, price instability, and consumer welfare, Econometrica 48, 135-152.

Varian, Hal, 1992, Microeconomic Analysis, Third Edition, Norton, New York.