Vintage Capital and Expectations Driven Business Cycles*

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Abstract

This paper demonstrates that increased optimism about future productivity can generate an immediate economic expansion in a neoclassical model with vintage capital and variable capacity utilization. Previous research has documented that standard neoclassical models cannot generate a simultaneous increase in consumption, investment, and hours in response to news shocks, and that optimism in these models tends to reduce investment and hours. When technology is vintage specific, however, expectations of higher future productivity raise the demand for new vintages of capital relative to old capital. Capital depreciates faster when utilization is high, but this depreciation only affects installed capital. The cost of high depreciation therefore falls when the value of installed capital falls. It is demonstrated here that with standard parameter values, more optimism raises utilization, consumption, investment, hours, and output.

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1 Introduction

Optimism and pessimism in the economy are often, particularly in the non-academic world, mentioned as important sources of business cycle fluctuations. While traditional Keynesian business cycle theories typically were vague about the sources of demand fluctuations and did not explicitly model expectations, more recent theories (starting with Diamond, 1980, and Cooper and John, 1988) allowed for expectations that were rational but still not related to fundamental developments in the economy. Recent studies, however, have highlighted the importance of information or expectations about fundamental developments and argued that changes in such expectations are related to business cycle fluctuations. In particular, Rotemberg (2003) argues that technological innovations diffuse slowly into production and Beaudry and Portier (2004) find that technological developments are reflected in stock market prices several years before the developments can be measured in production data.

An important empirical business cycle regularity is that consumption, investment, and employment are procyclical, i.e. that they are positively correlated with output. If changing expectations are an important source of business cycle fluctuations, a theory of the business cycle should be able to generate such positive comovements in response to changing expectations. In a recent paper, however, Beaudry and Portier (2005) demonstrate that in typical neoclassical models, shocks that affect expectations but not the current technology cannot generate positive comovements between consumption, employment, and investment.[1] Using their terminology, these models cannot explain Expectations Driven Business Cycles.

Beaudry and Portier then show that Expectations Driven Business Cycles can be generated in neoclassical settings if there are more than two production sectors and if there are cost complimentarities for firms that supply goods to several sectors. In a related paper, Jaimovich and Rebelo (2006) show that changing expectations can generate business cycle fluctuations in a neoclassical model with variable capital utilization, a particular form of adjustment costs for capital, and habit persistence in the utility function. In another recent paper Christiano, Motto and Rostagno (2006) add sticky nominal prices and an inflation targeting central bank to a framework with habit persistence and adjustment costs, and demonstrate that expectations then generate larger and longer fluctuations.

This paper demonstrates that Expectations Driven Business Cycles can be generated in a neoclassical growth model that is more standard than those previously proposed. This model has standard preferences and only one production sector, but adds two realistic and commonly used features to the most basic model. The two additions are variable capital utilization and capital-embodied technological change (or more loosely "vintage capital").[2] These model ingredients were proposed already by Greenwood, Hercowitz and Huffman (1988) and have since been used widely in the business cycle literature. In their

[1] This problematic reaction to news was noted earlier by Cochrane (1994), Danthine, Donaldson and Johnsen (1998) and Manuelli (2000).

[2] Although not crucial in their setting, Jaimovich and Rebelo (2006) also allow for vintage capital, but with a slightly different interpretation that results in a difference in the timing convention. The model used here is therefore almost identical to Jaimovich and Rebelo's, but with standard preferences (following King, Plosser and Rebelo, 1988), and without adjustment costs for capital.

survey of the real business cycle literature, King and Rebelo (1999) argue that variable capital utilization is both a realistic and important ingredient in business cycle models. Greenwood, Hercowitz and Krusell (1997, 2000) further analyze the implications of capital-embodied technological change in neoclassical settings, and Fisher (2005) finds that U.S. business cycle fluctuations are generated by invetment-specific technological innovations to a larger extent than by neutral innovations.

To understand why vintage capital and variable utilization are important, consider the most basic neoclassical model. The essence of the argument is best understood if we abstract from labor supply. Suppose that there are positive news about future productivity but that today's technology is unaffected. Production is then initially fixed so if consumption increases, investment must fall, and if investment increases consumption must fall. By allowing for variable capital utilization, it is possible to raise both consumption and investment even if the technology and the capital stock are fixed. But Beaudry and Portier (2005) show that the planner would never choose to simultaneously raise consumption and investment in typical neoclassical models. That would require higher utilization which would result in higher depreciation of capital. There is therefore still a trade-off between higher consumption today and a higher capital stock tomorrow.

This trade-off is relaxed when the technology is vintage specific. Consider a planner who receives positive news about the future productivity of capital built today. As before, the positive news raises demand both for investment and consumption. But higher capital utilization, implying faster depreciation of installed capital, is now less costly since installed capital will not benefit from the higher future productivity. The planner may therefore choose to simultaneously raise investment (to benefit from high future productivity) and consumption (because of the income effect) by utilizing old capital more intensively.

The next section presents the full dynamic model and discusses how it differs from the frameworks analyzed by Beaudry and Portier (2005) and Jaimovich and Rebelo (2006). Section 3 then analyzes a two-period version of the model and demonstrates that Expectations Driven Business Cycles (EDBC) are generated if capacity utilization and labor supply are sufficiently elastic, and if the depreciation rate of capital is sufficiently high. EDBC can be generated even if labor supply is perfectly inelastic, but only if the intertemporal elasticity of substitution is smaller than unity.[3] However, EDBC cannot be generated if capacity utilization is perfectly inelastic. Section 4 provides numerical examples based on the full dynamic model. These examples demonstrate that EDBC are generated when the model is calibrated with standard parameter values. The examples also support the theoretical results from the two-period model; EDBC are stronger and more likely if capacity utilization and labor supply are more elastic, if the intertemporal elasticity of substitution is low, and if the depreciation rate of capital is high. Section 5 concludes.

[3]When labor supply is inelastic, I define EDBC as a simultaneous increase in consumption and investment in response to an expectational shock.

2 The Model

2.1 Production and Capital

Consider an economy where the productivity of capital is vintage specific so that capital first used in production in period t has productivity q_t in all periods.[4] Production is of the Cobb-Douglas form,

$$y_t = \left(\sum_{s=0}^{\infty} \nu_t q_{t-s} k_{t,s} \right)^{\theta} h_t^{1-\theta},$$

where $k_{t,s}$ denotes the capital introduced in period $t - s$ that is still available in period t, h denotes labor supply, i investment, ν the capital utilization rate, and θ the capital share in production.

The vintages of capital develop according to

$$k_{t+1,0} = i_t, \tag{1}$$

and, for $s \geq 1$,

$$k_{t+1,s} = [1 - d(\nu_t)] \, k_{t,s-1}. \tag{2}$$

The depreciation rate d depends on capital utilization, and we assume that $d(\nu)$ is strictly increasing and convex.

The production side of this economy can be formulated more compactly if we let κ denote the capital stock in efficiency units,

$$\kappa_t = \sum_{s=0}^{\infty} q_{t-s} k_{t,s}, \tag{3}$$

so that

$$y_t = (\nu_t \kappa_t)^{\theta} h_t^{1-\theta}.$$

Note that (1) and (2) together with (3) imply that[5]

$$\kappa_{t+1} = [1 - d(\nu_t)] \, \kappa_t + q_{t+1} i_t. \tag{4}$$

2.2 Households

The economy is populated by a large number of identical households with expected lifetime utility

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

[4] The technology is thus embodied in the different vintages of capital, and these models are often refered to as models with capital-embodied technological change (see Greenwood, Hercowitz and Krusell, 1997).

[5] Greenwood, Hercowitz and Krusell (1997) start from this specification and interpret q as the productivity of investments. They show in an appendix that the vintage capital interpretation is analytically identical.

where c is consumption. The instantaneous utility function belongs to the class of utility functions that King, Plosser and Rebelo (1987) demonstrate is consistent with balanced growth, i.e.

$$u(c,h) = \frac{[cx(h)]^{1-\mu} - 1}{1-\mu}$$

where $\mu > 0$, $x > 0$, $x_h < 0$, and $x_{hh} < 0$. When $\mu \to 1$ this utility function becomes $u(c,h) = \ln c + \ln x(h)$.

The planner maximizes the households' expected utility subject to the resource constraint

$$c_t + i_t = y_t,$$

the production function

$$y_t = (\nu_t \kappa_t)^{\theta} h_t^{1-\theta},$$

and the evolution of efficient capital, equation (4).

2.3 Interpretation

In the present setting different vintages of capital have different productivity in production. Greenwood, Hercowitz and Krusell (1997) however note that this setting is identical to one where all vintages of capital are equally productive, but where the cost of producing the different vintages of capital varies. The term q in equation (4) can therefore either be interpreted as the productivity of the new vintage of capital or as the efficiency in production of investment goods. The timing of information about q may however differ for these two interpretations. It is natural to assume that much information is available about the present production function. If focus is on the latter interpretation, as in Jaimovich and Rebelo (2006), equation (4) will then be replaced by

$$\kappa_{t+1} = [1 - d(\nu_t)]\,\kappa_t + q_t i_t$$

where κ is raw capital and q_t is known in the beginning of period t. If, as in the present setting, technologies are vintage specific, a natural interpretations is that the productivity of new capital is not perfectly observed until the capital is implemented in production.

Note that the model falls outside the class of models analyzed by Beaudry and Portier (2005). They require that the resource constraint can be written as

$$c_t = G(\kappa_t, h_t, \kappa_{t+1}; q_t)$$

where G is some function (that can include the optimal utilization ν_t), and where the variables κ_t, h_t, κ_{t+1}, and q_t are known or determined in period t. In the present model, however, tomorrow's effective capital stock κ_{t+1} depends on tomorrow's productivity q_{t+1}, and the optimal utilization of capital depends on expectations about future productivity.

3 A Two-Period Model

Let us now examine under what conditions Expectations Driven Business Cycles can arise in a two-period version of the model above. Capital utilization is variable in the first period but fixed at unity in the second period, and uncertainty is ignored so that future productivity q_2 is known already in the first period. The planner solves

$$\max \frac{[c_1 x(h_1)]^{1-\mu} - 1}{1-\mu} + \beta \frac{[c_2 x(h_2)]^{1-\mu} - 1}{1-\mu}$$

subject to

$$c_1 + i_1 = (v_1 \kappa_1)^{\theta} h_1^{1-\theta} \tag{5}$$

$$c_2 = \kappa_2^{\theta} h_2^{1-\theta} \tag{6}$$

$$\kappa_2 = [1 - d(v_1)] \kappa_1 + q_2 i_1 \tag{7}$$

$$d(v_1) = \alpha_1 + \alpha_2 \nu_1^{\eta}$$

with κ_1 given, and assuming $x > 0$, $x_h < 0$, $x_{hh} < 0$, $-\mu x x'' > (1-2\mu)(x')^2$, $\mu > 0$, $\eta > 1$, and $\alpha_2 > 0$.[6]

The first-order conditions to this problem are

$$c_1 x_{h1} = -x_1 (1-\theta)(v_1 \kappa_1)^{\theta} h_1^{-\theta} \tag{8}$$

$$c_2 x_{h2} = -x_2 (1-\theta) \kappa_2^{\theta} h_2^{-\theta} \tag{9}$$

$$q_2 \theta v_1^{\theta-1} \kappa_1^{\theta} h_1^{1-\theta} = \alpha_2 \eta \nu_1^{\eta-1} \kappa_1 \tag{10}$$

and

$$\beta \theta q_2 c_2^{-\mu} x_2^{1-\mu} \kappa_2^{\theta-1} h_2^{1-\theta} = c_1^{-\mu} x_1^{1-\mu}. \tag{11}$$

Without loss of generality, we can set $\kappa_1 = 1$ and choose parameter values (α_1, α_2, and in the utility function x) so that when $q_2 = 1$ we get $\nu_1 = h_1 = 1$ and $d(\nu_1) = \delta$ for some chosen depreciation rate δ. The solution to this problem can then be characterized by equation (7) and the following five equations

$$h_1^{\mu\theta\omega_2} x_1^{1-\mu} = \sigma^{\mu} \Omega \kappa_2^{-\omega_4} q_2^{1+\theta\mu\omega_1} \tag{12}$$

$$c_1 = (1-\theta) \sigma q_2^{\theta\omega_1} h_1^{-\theta\omega_2} \tag{13}$$

$$i_1 = [h_1 - (1-\theta)\sigma] q_2^{\theta\omega_1} h_1^{-\theta\omega_2} \tag{14}$$

$$\nu_1 = q_2^{\omega_1} h_1^{\omega_3} \tag{15}$$

and

$$h_2 = \frac{-(1-\theta) x_2}{x_{h2}}, \tag{16}$$

where we let $\omega_1 = (\eta - \theta)^{-1}$, $\omega_2 = (\eta - 1)\omega_1$, $\omega_3 = (1-\theta)\omega_1$, $\omega_4 = 1 + \theta(\mu - 1)$, $\sigma = -x_1/x_{h1}$, and $\Omega = \beta\theta(1-\theta)^{\mu} \left(h_2^{1-\theta} x_2\right)^{1-\mu}$.

The first equation, the Euler equation, determines first-period hours as a function of future productivity, q_2, and parameters. The following equations then determine first-period consumption, investment, and capacity utilization as functions of h_1, q_2, and parameters. Finally, equation (16) shows that second-period hours worked only depend on the utility function x and the capital share in production.

[6] We let x_t denote $x(h_t)$ and x_{ht} and x_{hht} denote the first and second derivatives of $x(h_t)$ with respect to h_t, etc.

3.1 Expectations Driven Business Cycles

Suppose that the economy is in an equilibrium where productivity is expected to be constant at unity ($q_2 = 1$) when these expectations become more optimistic (expectations of q_2 rise in the beginning of period 1). We now analyze conditions under which such an increase in optimism can generate an economic expansion and raise consumption, investment, and hours.

Totally differentiate (12) at $h_1 = q_2 = 1$ to get

$$h_q = \frac{\mathrm{d}h_1}{\mathrm{d}q_2} = \frac{N}{D}$$

where

$$N = (1 + \mu\theta\omega_1)\,\kappa_2 + \omega_1\omega_4\,(\theta - \eta i_1)\,, \tag{17}$$

$$D = \left(\frac{\mu - 1 - \mu\sigma_h}{\sigma} + \mu\theta\omega_2\right)\kappa_2 + \omega_4\left[1 - \theta\omega_3 - (1-\theta)\,\sigma_h - \theta\omega_2 i_1\right], \tag{18}$$

and $\sigma_h = -\,(1 + \sigma x_{hh}/x_h) < 0$. The denominator D is always positive. To see this, note that concavity of the utility function implies that $\mu - 1 - \mu\sigma_h > 0$, and $c_1 > 0$ implies that $i_1 < 1$.

From (15) it is clear that capacity utilization will rise in response to higher future productivity if this productivity increase raises hours worked (i.e. if $h_q > 0$). To see how consumption and investment are affected, totally differentiate (13) and (14) to get

$$c_q = \frac{\mathrm{d}c_1}{\mathrm{d}q_2} = \theta\omega_1 c_1 + \left[(1-\theta)\,\sigma_h - \theta\omega_2 c_1\right] h_q \tag{19}$$

and

$$i_q = \frac{\mathrm{d}i_1}{\mathrm{d}q_2} = \theta\omega_1 i_1 + \left[1 - (1-\theta)\,\sigma_h - \theta\omega_2 i_1\right] h_q. \tag{20}$$

The model is consistent with expectations driven business cycles (EDBC) if h_q, c_q and i_q are positive. Propositions 1–3 below demonstrate conditions under which EDBC are generated, and conditions under which EDBC cannot be generated.[7] Proposition 1 first demonstrates that any parameterization of the model will generate EDBC if the utility function is separable in consumption and leisure ($\mu = 1$) and the depreciation rate of capital is sufficiently high. The proposition further demonstrates that EDBC can be generated for a broader set of depreciation rates if labor supply and capacity utilization are more elastic. For Proposition 1, it will be useful to define $\gamma = -\sigma_h/\sigma$ and note that $\gamma > 0$, and that γ is the inverse of the Frisch labor supply elasticity when $\mu = 1$.[8]

Proposition 1 *If $\mu = 1$, then*

(a) for any parameter values $(\beta, \gamma, \eta, \theta)$ that result in $i_1 > 0$, there is a $\delta^* < 1$ such that EDBC are generated for all $\delta > \delta^*$.

[7] The proofs of these propositions are in the appendix.

[8] The Frisch labor supply elasticity is defined as $w\mathrm{d}h/\left(h\mathrm{d}w\right)|_{u_c}$, i.e. the elasticity of labor with respect to the wage holding marginal utility fixed.

(b) less elastic labor supply ($\gamma\uparrow$) and less elastic capacity utilization ($\eta\uparrow$) make δ^* more restrictive,[9]

$$\frac{\partial\delta^*}{\partial\gamma}|_{\delta^*>0} > 0$$

and

$$\frac{\partial\delta^*}{\partial\eta}|_{\delta^*>0} > 0$$

(c) EDBC do not exist if either labor supply or capacity utilization is infinitely inelastic

$$\lim_{\gamma\to\infty}\delta^* = \lim_{\eta\to\infty}\delta^* = 1.$$

Part (c) of Proposition 1 indicates that elastic capacity utilization and elastic labor supply are important for the existence of EDBC. Proposition 2 demonstrates that elastic capacity utilization is indeed a necessary condition for EDBC in this framework, while Proposition 3 demonstrates that elastic labor supply is not necessary; if $\mu > 1$, EDBC are generated if capacity utilization is sufficiently elastic (low η) and depreciation sufficiently high.

Proposition 2 *If capacity utilization is exogenously fixed, EDBC cannot exist.*

Proposition 3 *If labor supply is exogenously fixed, EDBC cannot exist if $\mu \leq 1$. For any $\mu > 1$, there is a pair (δ^*, η^*) such that all $\delta > \delta^*$ and $\eta < \eta^*$ generate EDBC.*

To better understand the importance of vintage capital, let us consider a specification that nests normal and embodied technological change by replacing (7) with

$$\kappa_2 = \left[(1 - d(v_1))\,\kappa_1 + q_2^{\phi} i_1\right] q_2^{1-\phi} \tag{7'}$$

where $\phi \in [0, 1]$ is the degree of embodiedness. The equations characterizing the solution then change to

$$h_1^{\mu\theta\omega_2} x_1^{1-\mu} = \sigma^{\mu}\Omega\kappa_2^{-\omega_4} q_2^{\phi(1+\theta\mu\omega_1)} \tag{12'}$$

$$c_1 = (1-\theta)\,\sigma q_2^{\phi\theta\omega_1} h_1^{-\theta\omega_2} \tag{13'}$$

$$i_1 = [h_1 - (1-\theta)\,\sigma]\, q_2^{\phi\theta\omega_1} h_1^{-\theta\omega_2} \tag{14'}$$

and

$$\nu_1 = q_2^{\phi\omega_1} h_1^{\omega_3}. \tag{15'}$$

We are interested in comparing the effects of changes in q_2 for different degrees of embodiedness. Note however that the direct effect of a change in q will depend on ϕ. In particular, a given increase in q_2 has a larger impact on the second-period efficient capital stock if technology is neutral, since then both old and new capital benefits from the technological improvement. To analyze technological changes with similar immediate effects,

[9] The first part of this statement and the first part of the statement in part (c) of the proposition require that γ can be treated as a parameter. This will be the case for some standard utility functions, for example the one used in Section 4.

let $q_2 = q + m(\phi)\varepsilon$ where $m(\phi) = i_1 / [i_1 + (1-\phi)(1-\delta)]$. Then $\partial\kappa_2/\partial\varepsilon$ is independent of ϕ if we fix i_1 and ν_1, and evaluate derivatives at $q = 1$ and $\varepsilon = 0$. Totally differentiating (12') at $q = 1$ and $\varepsilon = 0$ we get

$$h_\varepsilon = \frac{\mathrm{d}h_1}{\mathrm{d}\varepsilon} = \frac{m(\phi)\,n(\phi)}{D} \tag{21}$$

where the denominator is still given by (18) and where

$$n(\phi) = \phi N - (1-\phi)\,\omega_4\kappa_2. \tag{22}$$

From (13') and (14') we further get

$$c_\varepsilon = \frac{\mathrm{d}c_1}{\mathrm{d}\varepsilon} = \phi m(\phi)\,\theta\omega_1 c_1 + [(1-\theta)\,\sigma_h - \theta\omega_2 c_1]\,h_\varepsilon \tag{23}$$

and

$$i_\varepsilon = \frac{\mathrm{d}i_1}{\mathrm{d}\varepsilon} = \phi m(\phi)\,\theta\omega_1 i_1 + [1-(1-\theta)\,\sigma_h - \theta\omega_2 i_1]\,h_\varepsilon. \tag{24}$$

Proposition 4 summarizes some results derived from these equations in the subsequent discussion.

Proposition 4 *(a) Positive news about future productivity reduces hours ($h_\varepsilon < 0$) and investment ($i_\varepsilon < 0$) and raises consumption ($c_\varepsilon > 0$) when technological development is neutral ($\phi = 0$).*

(b) More embodiedness ($\phi \uparrow$) raises the response of hours to positive news shocks ($\partial h_\varepsilon/\partial\phi > 0$).

When technological change is neutral, the model falls into the class of models analyzed by Beaudry and Portier (2005) and consequently h_ε, i_ε, and c_ε can then not simultaneuosly be positive. That result is confirmed here, and we can make a stronger statement: in the basic neoclassical model with neutral technological change ($\phi = 0$) and standard preferences (as in King, Plosser and Rebelo, 1988), positive news about future productivity raises consumption but reduces labor supply and investment on impact. This statement follows since $D > 0$ and since equation (22) implies that $n(0) < 0$ and consequently $h_\varepsilon < 0$ when $\phi = 0$. Furthermore, $1-(1-\theta)\,\sigma_h - \theta\omega_2 i_1 > 0$ implies that $i_\varepsilon < 0$ when $h_\varepsilon < 0$ while $\sigma_h < 0$ implies that $c_\varepsilon > 0$ if $h_\varepsilon < 0$, all under the assumption that $\phi = 0$.

Fisher (2006) finds that hours respond more strongly to investment specific shocks than to neutral shocks that immediately raise productivity. A similar result holds in the present framework. Differentiating equation (21) we get $\partial h_\varepsilon/\partial\phi > 0$, i.e. hours respond more strongly to embodied technological shocks than to neutral news shock. Intuitively, present leisure is more expensive relative to future leisure when technology is embodied so that only new investments benefit from the technological developments.

One may suspect that a similar argument implies that consumption is less responsive to embodied technological shocks than to neutral shocks, but that need not be the case. The second term on the right hand side in equation (23) captures the effect that embodiedness reduces the responsiveness of consumption if hours become more responsive. The first term is however more positive if technology is embodied. The key to understanding this effect is equation (15'). If $\phi > 0$, higher future productivity implies higher utilization even if hours are unaffected. The intuition is that the cost of high utilization in terms of high depreciation of installed capital is smaller when installed capital does not benefit from technological improvements. The higher utilization may imply that consumption becomes more responsive to embodied shocks than to neutral shocks, and also reinforces the increase in responsiveness of investment.

4 Numerical Examples

Let us now return to the infinite-horizon model specified in Section 2, and analyze numerically how the economy reacts to changing expectations. Let $x(h) = \exp\left(-\zeta h^{1+\gamma}/(1+\gamma)\right)$ so that the utility function is[10]

$$u(c,h) = \frac{\left[c\exp\left(\frac{-\zeta h^{1+\gamma}}{1+\gamma}\right)\right]^{1-\mu} - 1}{1-\mu}$$

when $\mu \neq 1$ and

$$u(c,h) = \log c - \frac{\zeta h^{1+\gamma}}{1+\gamma}$$

when $\mu = 1$.

Except for the utility function, the parameterization of the model mostly follows Jaimovich and Rebelo (2006). In the benchmark specification, we then have unit risk aversion, $\mu = 1$, and to get a labor supply elasticity of 2.5 we set $\gamma = 1/2.5 = 0.4$. The capital share in production is set to $\theta = 0.36$, and the parameter determining the elasticity of depreciation to utilization is set to $\eta = 1.20$. This choice is rather arbitrary, and alternative values will be considered. Furthermore, one model period is one quarter of a year, and the time-discount factor is set to $\beta = 0.985$. The parameters ζ, α_1, and α_2 are chosen so that the economy converges to a steady state with $h = \nu = 1$ and $d(1) = 0.02$ when technology is constant at $q = 1$. Table 1 summarizes the parameter values used in the benchmark economy, and also reports the implied steady state values for the variables.[11]

4.1 The Economy's Response to News

To examine how the economy reacts to news about future productivity, the following experiment is considered. In period zero the economy is in a steady state without technological change. In the beginning of period one agents get unanticipated news that technology will permanently improve by one percent from period two and on, i.e. $q_t = 1.01q_1$ for all $t \geq 2$.

[10] This function is not always concave when $\mu < 1$. Only specifications where $\mu \geq 1$ are therefore considered.

[11] See the appendix for a description of the solution to this model.

Table 1: Benchmark Values

Parameter values		Initial steady state	
α_1	-0.0094	ν	1.0000
α_2	0.0294	κ	37.7748
β	0.9850	h	1.0000
γ	0.4000	y	3.6965
δ	0.0200	c	2.9410
ζ	0.8044	i	0.7555
η	1.2000		
θ	0.3600		
μ	1.0000		

Figure 1 shows how the economy reacts to this one percent increase in productivity, and Table 2 reports the impact reaction when the news about future productivity improvements arrive. For this benchmark parameterization, the conditions for EDBC are fulfilled; there is an economic expansion already in the first period although the technology is not affected until in the second period. From Table 2 we also see that the response to changing expectations can be quantitatively important. Investment increases by almost four percent and production by almost one percent in the first period when productivity is expected to increase by one percent.

Columns (ii) to (vi) in Table 2 show the impact responses to the news shock under alternative parameterizations. In column (ii), $\gamma = 2.0$ so that the labor-supply elasticity is 0.5. As expected, the impact responses are smaller when labor supply is less elastic. When risk aversion is higher (column (iii)), the willingness to intertemporally substitute is lower and consumption smoothing is more important. Consumption therefore increases faster towards the new equilibrium level and as a consequence the impact response of investment is smaller. The analysis in Section 3 demonstrated that elastic capital utilization is crucial for obtaining a simultaneous first-period increase in consumption and investment. As expected, therefore, the impact responses are smaller when capital utilization is less elastic as in column (iv). The impact response of consumption is then negligible but still positive. Column (iv) indicates that a higher capital share in production raises the impact responses while column (v) indicates that a higher steady-state depreciation rate raises the impact response of consumption but reduces the response of the other variables.

To further examine the validity of the two-period analysis for the fully dynamic setting, Figure 2 displays combinations of parameter values that generate EDBC. In the first panel, all parameters are held at the benchmark values except the elasticities of capacity utilization and labor supply. When capacity utilization is less elastic (higher η) labor supply must be more elastic (lower γ) for expectations driven business cycles to be generated. This finding is in line with Proposition 1b and Proposition 2. The second panel shows that EDBC can be generated with less elastic labor supply if risk aversion is high, which was also indicated by Proposition 3. The final panel shows that also a high depreciation rate of capital allows for EDBC under less restrictive assumptions of the labor-supply elasticity, as was indicated by Proposition 1a and Proposition 3.

Table 2: Impact response to news

	Benchmark	$\gamma = 2.0$	$\mu = 2.0$	$\eta = 1.50$	$\theta = 0.40$	$\delta = 0.03$
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Δc_1	0.16	0.05	0.27	0.00	0.19	0.18
Δh_1	0.55	0.18	0.31	0.56	0.62	0.52
Δi_1	3.94	2.71	2.44	3.87	4.04	3.22
$\Delta \nu_1$	1.61	1.33	1.43	1.20	1.72	1.59
Δy_1	0.93	0.59	0.72	0.79	1.06	0.90

Note: The table shows the percentage change in the variables in response to a one percent permanent increase in q_t, $t \geq 2$, when news about this change arrive in the beginning of period $t = 1$. Column (i) shows the outcome under the benchmark parameterization. The following columns show results under alternative parameterizations.

5 Concluding Discussion

This paper has demonstrated that optimism and pessimism of future productivity can generate business cycle fluctuations in a neoclassical growth model with vintage capital and variable capacity utilization. To isolate the mechanisms, only exogenous changes in expectations have been considered and uncertainty has not explicitly been modeled. Future work needs to model the processes for technological innovations, the implementation of these innovations in production, and the information and uncertainty about how these innovations affect productivity.

In the present paper, expectations are formed one quarter ahead and investments are transformed into capital in one quarter. The evidence reported both by Rotemberg (2003) and by Beaudry and Portier (2004) however indicates that technological developments diffuse slowly into production and that news of innovations may affect expectations several years before total factor productivity is affected. A more realistic model specification should therefore allow for a longer lag between information shocks and implementation, maybe by allowing for "time-to-build" as in Kydland and Prescott's (1982) original real business cycle model.

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Appendix A: Proofs of Propositions

A.1 Proof of Proposition 1

Proof. When $\mu = 1$, (17) reduces to

$$N/\omega_1 = \eta (1 - \delta) + \theta$$

and (18) reduces to

$$D/\omega_1 = \theta (\eta - 1) (1 - \delta) + (-\sigma_h/\sigma) (\eta - \theta) (2 - \delta) + \eta - \theta - \theta (1 - \theta) .$$

Note that

$$\eta - \theta - \theta (1 - \theta) = \eta - 1 + (1 - \theta)^2 > 0$$

where the inequality follows from our assumption that $\eta > 1$ and $1 > \theta > 0$. Note also that $\sigma > 0$ and $\sigma_h < 0$ since by assumption $x > 0$, $x_h < 0$, and $x_{hh} < 0$. We then see that both N and D are positive, and consequently $h_q > 0$.

From (20) we get

$$\frac{i_q}{\omega_1} = \theta i_1 + [\eta + (\eta - \theta) (-\sigma_h) + (\eta - 1) \theta\sigma] (1 - \theta) h_q$$

which is positive if $h_q > 0$ and $i_1 > 0$.

Equation (19) implies that

$$\frac{c_q}{\omega_1 c_1} = \theta - [\gamma (\eta - \theta) + \theta (\eta - 1)] h_q$$

which in turn implies that

$$\begin{aligned} c_q &> 0 \iff \theta - [\gamma (\eta - \theta) + \theta (\eta - 1)] h_q > 0 \\ &\iff \phi < \theta \end{aligned}$$

where we define

$$\phi = \frac{[\gamma (\eta - \theta) + \theta (\eta - 1)] [\eta (1 - \delta) + \theta]}{\theta (\eta - 1) (1 - \delta) + \gamma (\eta - \theta) (2 - \delta) + \eta - \theta - \theta (1 - \theta)}.$$

We can then derive

$$\phi < \theta \iff \delta > \delta^* = 1 - \frac{\theta (1 - \theta)}{\gamma (\eta - \theta) + \theta (\eta - 1)}. \tag{A.1}$$

This demonstrates part (a) of the proposition, i.e. for any parameter values $(\beta, \gamma, \eta, \theta)$ there is a $\delta^* < 1$ such that all $\delta > \delta^*$ result in EDBC.

Taking derivatives of δ^* as defined in (A.1) we get

$$\frac{\partial \delta^*}{\partial \gamma} = \frac{(\eta - \theta) (1 - \delta^*)}{\gamma (\eta - \theta) + \theta (\eta - 1)}$$

and

$$\frac{\partial \delta^*}{\partial \eta} = \frac{(\gamma + \theta)(1 - \delta^*)}{\gamma(\eta - \theta) + \theta(\eta - 1)}.$$

If $\delta^* > 0$, both these derivatives are positive, which demonstrates part (b) of the proposition.

From (A.1) it is also clear that

$$\lim_{\gamma \to \infty} \delta^* = \lim_{\eta \to \infty} \delta^* = 1$$

which demonstrates part (c) of the proposition. ■

A.2 Proof of Proposition (2)

When $\eta \to \infty$, equation (13) reduces to

$$c_1 = (1 - \theta)\,\sigma h_1^{-\theta}.$$

Totally differentiating we get

$$c_q = [(1 - \theta)\,\sigma_h - \theta c_1]\,h_q.$$

Since $\sigma_h < 0$, we see that c_q and h_q have different signs, and consequently cannot simultaneously be positive.

A.3 Proof of Proposition 3

Proof. If labor supply is fixed at $h_1 = h_2 = 1$, the equilibrium is characterized by the Euler equation

$$c_1^{-\mu} = \beta\theta q_2 \left[1 - \alpha_1 - \alpha_2 v_1^{\eta} + q_2 v_1^{\theta} - q_2 c_1\right]^{-\omega_4} \tag{A.2}$$

and the budget constraint $i_1 = \nu_1^{\theta} - c_1$ where capacity utilization is $v_1 = q_2^{\omega_1}$.Totally differentiating these equations at an initial equilibrium where $q_2 = \nu_1 = 1$, we get

$$c_q = \frac{(1 - c_1)\,\omega_4 - \kappa_2}{\omega_4 + \mu\kappa_2/c_1} \tag{A.3}$$

and

$$i_q = \frac{\theta}{\eta - \theta} - \frac{\mathrm{d}c_1}{\mathrm{d}q_2}. \tag{A.4}$$

Note that the denominator in (A.3) is positive. We then get

$$\begin{aligned} c_q \quad &> \quad 0 \iff (1 - c_1)\,[1 + \theta(\mu - 1)] > \kappa_2 \\ &\iff \quad i_1\,[1 + \theta(\mu - 1)] > 1 - \delta + i_1 \\ &\iff \quad \theta(\mu - 1)\,i_1 > 1 - \delta \end{aligned} \tag{A.5}$$

From (A.5) we immediately see that c_q cannot be positive if $\mu \leq 1$. It remains to show that for any $\mu > 1$, there are parameter values δ and η such that both c_q and i_q are positive.

The proof proceeds in two steps. It is first demonstrated that $c_q > 0$ for sufficiently high δ. It is then demonstrated that $i_q > 0$ for sufficiently low η.

In the initial equilibrium, the Euler equation (A.2) reduces to

$$c_1^{-\mu} = \beta\theta \left[2 - \delta - c_1\right]^{-\omega_4}.$$

Totally differentiating with respect to c_1 and δ, we get $\mathrm{d}c_1/\mathrm{d}\delta < 0$ and thus $\mathrm{d}i_1/\mathrm{d}\delta > 0$ for any recalibration that holds capacity utilization fixed at $\nu_1 = 1$. Consequently, for any $\mu > 1$, as δ is raised towards unity, the left hand side of (A.5) becomes larger (starting from a positive value) while the right hand side approaches zero. There is therefore a δ^* such that the inequality is satisfied for all $\delta > \delta^*$.

From (A.4) we get

$$\begin{aligned} i_q &> 0 \iff \frac{\theta}{\eta - \theta} > c_q \\ &\iff \frac{\theta}{\eta - \theta}\left[\frac{\omega_4}{\kappa_2} + \frac{\mu}{c_1}\right] > \frac{(1 - c_1)\,\omega_4}{\kappa_2} - 1 \\ &\iff \left(\frac{\theta\mu}{(\eta - \theta)\,c_1} + 1\right)\kappa_2 > \left[1 + \theta\,(\mu - 1)\right]\left(\frac{\eta - 2\theta}{\eta - \theta} - c_1\right) \end{aligned}$$

Note that

$$\kappa_2 = 1 - \delta + i_1 > i_1 = 1 - c_1 > \frac{\eta - 2\theta}{\eta - \theta} - c_1.$$

Consequently, if

$$\frac{\theta\mu}{(\eta - \theta)\,c_1} + 1 > 1 + \theta\,(\mu - 1)$$

then $i_q > 0$ for all $i_1 > 0$. Note that

$$\begin{aligned} \frac{\theta\mu}{(\eta - \theta)\,c_1} + 1 &> 1 + \theta\,(\mu - 1) \\ &\iff \frac{\mu}{(\eta - \theta)\,c_1} > \mu - 1. \end{aligned}$$

If $i_1 > 0$, we have $c_1 < 1$ and thus $\mu/\left[(1 - \theta)\,c_1\right] > \mu > \mu - 1$. There is therefore always a value η^* such that the inequality is fulfilled for all $\eta < \eta^*$. ■

Appendix B: Model Solution

This appendix describes the solution to the model analyzed in Section 4. The relevant first order conditions are

$$u_{h_t} = -\,(1 - \theta)\,(\nu_t\kappa_t)^{\theta}\,h_t^{-\theta}u_{ct} \tag{B.6}$$

$$\nu_t^{\eta - \theta} = \frac{\theta}{\eta\alpha_2}\left(\frac{h_t}{\kappa_t}\right)^{1-\theta} q_{t+1} \tag{B.7}$$

and

$$\beta\left[\theta\nu_{t+1}^{\theta}\kappa_{t+1}^{\theta-1}h_{t+1}^{1-\theta}q_{t+2} + (1 - d\,(\nu_{t+1}))\right]\frac{u_{ct+1}}{q_{t+2}} = \frac{u_{ct}}{q_{t+1}}. \tag{B.8}$$

B.1 Steady State

Consider first a steady state where q is constant. This steady state is described by the budget constraint $d(\nu)\kappa = qi$ and the first order conditions (B.6) to (B.8) which reduce to

$$u_h = -(1-\theta)(\nu\kappa)^{\theta} h^{-\theta} u_c \tag{B.9}$$

$$\nu^{\eta-\theta} = \frac{\theta}{\eta\alpha_2}\left(\frac{h}{\kappa}\right)^{1-\theta} q \tag{B.10}$$

and

$$\beta\left[\theta\nu^{\theta}\kappa^{\theta-1}h^{1-\theta}q + (1-d(v))\right] = 1. \tag{B.11}$$

We want to calibrate the model so that $h = \nu = 1$ when $q = 1$. By using $\kappa = i/\delta$ in (B.11) we see that the marginal product of efficient capital is $\theta\kappa^{\theta-1} = 1/\beta - 1 + \delta$ so that the efficient capital stock can be calculated as a function of known parameters. Using this expression in (B.10) we get

$$\alpha_2 = \frac{\theta\kappa^{\theta-1}}{\eta} = \frac{1}{\eta}\left(\frac{1}{\beta} - 1 + \delta\right).$$

Calculating u_c and u_h and using those expressions in (B.9) we get $\zeta h^{\gamma+\theta}c = (1-\theta)(\nu\kappa)^{\theta}$. To get $h = 1$ we must therefore have $\zeta c = (1-\theta)\kappa^{\theta}$. Note that $c + i = \kappa^{\theta}$ and $i = \delta\kappa$, which implies that

$$\zeta = \frac{1-\theta}{1-\delta\kappa^{1-\theta}}.$$

In a steady state where $q \neq 1$ (B.10) and (B.11) imply that

$$\beta\left[\eta\alpha_2\nu^{\eta} + (1-d(v))\right] = 1$$

which demonstrates that ν is unaffected by q in steady state. We thus still have $\nu = 1$, and using this in (B.10) we get

$$\kappa = \left(\frac{\theta q}{\eta\alpha_2}\right)^{\frac{1}{1-\theta}} h.$$

One can also show that h is also unaffected but this also follows from the properties of the utility function. We thus have $h = 1$ and then $c = (1-\theta)\kappa^{\theta}/\zeta$, etc.

B.2 Transition

Suppose that the economy is in this steady state in the beginning of period 1, and suppose that agents then learn that from period 2 and on, productivity will be $q_t = \hat{q}$. To solve for the transition to the new steady state, guess some path $\{h_t\}_{t=1}^{T}$ for some large T. Then follow this procedure: (i) Set $s = 1$. (ii) Use (B.7) to solve for ν_s. (iii) Use (B.6) to solve for c_s. (iv) Use the production function to calculate y_s, and use the resource constraint to calculate i_s. (v) Use (4) to calculate κ_{s+1}. (vi) Raise s by one, and iterate from (ii) if $s \leq T$. (vii) Use the calculated paths to evaluate the Euler equation (B.8) in all periods. If these equations are not satisfied, use an equation solver to update the guess for $\{h_t\}_{t=1}^{T}$ and iterate from (i).

Appendix C: Derivations

(This section will be removed in future versions of the paper)

Deriving equations (12) to (16):

Use $\kappa_1 = 1$ in (10) to get

$$v_1^{\eta-\theta} = \frac{q_2\theta}{\alpha_2\eta} h_1^{1-\theta}.$$

The model is calibrated to get $v_1 = 1$ when $q_2 = h_1 = 1$. We therefore get $\alpha_2 = \theta/\eta$, and (15) follows.

Use $\kappa_1 = 1$ and (15) in (8) to get

$$c_1 x_{h1} = -x_1 (1-\theta) q_2^{\theta\omega_1} h_1^{\theta\omega_3} h_1^{-\theta}.$$

Note that $\theta\omega_3 - \theta = -\theta\omega_2$, and (13) follows. Use $\kappa_1 = 1$, (15), and (13) in (5) to get (14), and use (6) in (9) to get (16). Finally use these results in (11) to get (12).

Deriving equation (17) and (18):

Differentiate (12) to get

$$\frac{\mu\theta\omega_2 dh_1}{h_1} + \frac{(1-\mu) dx_1}{x_1} = \frac{\mu d\sigma}{\sigma} - \frac{\omega_4 d\kappa_2}{\kappa_2} + \frac{(1+\theta\mu\omega_1) dq_2}{q_2}. \tag{C.12}$$

Differentiating $x(h)$, we get $dx_1 = x_{h1} dh_1$, and differentiating $\sigma = -x_1/x_{h1}$ we get $d\sigma = \sigma_h dh_1$ where $\sigma_h = -(1 + \sigma x_{hh}/x_h)$. Differentiating (6) at $q_2 = h_1 = \nu = 1$ we get

$$\begin{aligned} d\kappa_2 &= -\theta(\omega_1 dq_2 + \omega_3 dh_1) + [dh_1 - (1-\theta)\sigma_h dh_1] \\ &\quad + [1 - (1-\theta)\sigma](\eta\omega_1 dq_2 - \theta\omega_2 dh_1) \\ &= [\eta - (1-\theta)\sigma\eta - \theta]\omega_1 dq_2 + \\ &\quad [1 - \theta\omega_3 - (1-\theta)\sigma_h - \theta\omega_2 + (1-\theta)\sigma\theta\omega_2] dh_1 \\ &= (\eta i_1 - \theta)\omega_1 dq_2 + [1 - \theta\omega_3 - (1-\theta)\sigma_h - \theta\omega_2 i_1] dh_1 \end{aligned}$$

where the final step used $i_1 = 1 - (1-\theta)\sigma$ in the initial equilibrium. Use this and $q_2 = h_1 = \nu = 1$ in (C.12) to get

$$\begin{aligned} \mu\theta\omega_2 dh_1 + \frac{(1-\mu) x_{h1} dh_1}{x_1} &= \frac{\mu\sigma_h dh_1}{\sigma} - \frac{\omega_4}{\kappa_2}(\eta i_1 - \theta)\omega_1 dq_2 - \\ &\quad \frac{\omega_4}{\kappa_2}[1 - \theta\omega_3 - (1-\theta)\sigma_h - \theta\omega_2 i_1] dh_1 + (1+\theta\mu\omega_1) dq_2 \end{aligned}$$

or

$$\frac{dh_1}{dq_2} = \frac{\hat{N}}{\hat{D}}$$

where

$$\hat{N} = 1 + \theta\mu\omega_1 - \frac{\omega_4}{\kappa_2}(\eta i_1 - \theta)\omega_1$$

$$\hat{D} = \mu\theta\omega_2 + \frac{(1-\mu) x_{h1}}{x_1} - \frac{\mu\sigma_h}{\sigma} + \frac{\omega_4}{\kappa_2}[1 - \theta\omega_3 - (1-\theta)\sigma_h - \theta\omega_2 i_1].$$

Note that
$$\kappa_2 \hat{N} = (1 + \mu\theta\omega_1)\,\kappa_2 - \omega_1\omega_4\,(\eta i_1 - \theta)$$
and use $\sigma = -x_1/x_{h1}$ to get
$$\kappa_2 \hat{D} = \left[\frac{\mu - 1 - \mu\sigma_h}{\sigma} + \mu\theta\omega_2\right]\kappa_2 - \omega_4\left[\theta\omega_3 + (1-\theta)\,\sigma_h + \theta\omega_2 i_1 - 1\right].$$
Let $N = \kappa_2 \hat{N}$ and $D = \kappa_2 \hat{D}$, and (17) and (18) follow.

Deriving equations (12') to (15'):

The Lagrangean is now (assuming $\kappa_1 = 1$)
$$\begin{aligned} L &= \frac{(c_1 x_1)^{1-\mu}}{1-\mu} + \beta\frac{(c_2 x_2)^{1-\mu}}{1-\mu} + \\ &\quad \lambda_1\left[v_1^{\theta} h_1^{1-\theta} - c_1 - i_1\right] \\ &\quad + \beta\lambda_2\left[\kappa_2^{\theta} h_2^{1-\theta} - c_2\right] \\ &\quad + \lambda_3\left[1 - d(\nu_1) + q_2^{\phi} i_1 - \kappa_2\right] q_2^{1-\phi} \end{aligned}$$
and the first order conditions are
$$c_t^{-\mu} x_t^{1-\mu} = \lambda_t$$
$$x_{ht} c_t^{1-\mu} x_t^{-\mu} = -(1-\theta)\,(\nu_t\kappa_t)^{\theta}\, h_t^{-\theta}\lambda_t \text{ (with } \kappa_1 = \nu_2 = 1)$$
$$\theta\nu^{\theta-1} h_1^{1-\theta}\lambda_1 = \alpha_2\eta\nu_1^{\eta-1}\lambda_3 q_2^{1-\phi}$$
$$\lambda_1 = q_2\lambda_3$$
and
$$\theta\beta k_2^{\theta-1} h_2^{1-\theta}\lambda_2 = q_2^{1-\phi}\lambda_3.$$
Note that for $\nu_1 = 1$ when $q_2 = h_1 = 1$ we still get $\alpha_2 = \theta/\eta$. Substituting out λ_i and rearranging, equations (12') to (15') follow.

C.1 Model solution

This section describes the solution to the model analyzed in Section 4. The Lagrangean is
$$\begin{aligned} L &= \sum \beta^t u(c_t, h_t) + \sum \beta^t \lambda_t\left[(v_t\kappa_t)^{\theta} h_t^{1-\theta} - c_t - i_t\right] + \\ &\quad \sum \beta^t \phi_t\left[(1 - d(v_t)\,\kappa_t) + q_{t+1} i_t - \kappa_{t+1}\right]. \end{aligned}$$
The first order conditions w.r.t. c_t, h_t, ν_t, i_t, and κ_{t+1} are then
$$u_{ct} = \lambda_t$$
$$u_{h_t} = -(1-\theta)\,(\nu_t\kappa_t)^{\theta}\, h_t^{-\theta}\lambda_t$$
$$\theta\nu_t^{\theta-1}\kappa_t^{\theta} h_t^{1-\theta}\lambda_t = d'(\nu_t)\,\kappa_t\phi_t$$

$$\lambda_t = q_{t+1}\phi_t$$

and

$$\beta\left[\theta\nu_t^{\theta}\kappa_t^{\theta-1}h_t^{1-\theta}\lambda_{t+1} + \left(1 - d\left(\nu_{t+1}\right)\right)\phi_{t+1}\right] = \phi_t.$$

Appendix B describes how these first order conditions are used to calibrate and solve the model.

Figure 1: Response to permanent increase in q_t ($t \geq 2$) announced at $t = 1$

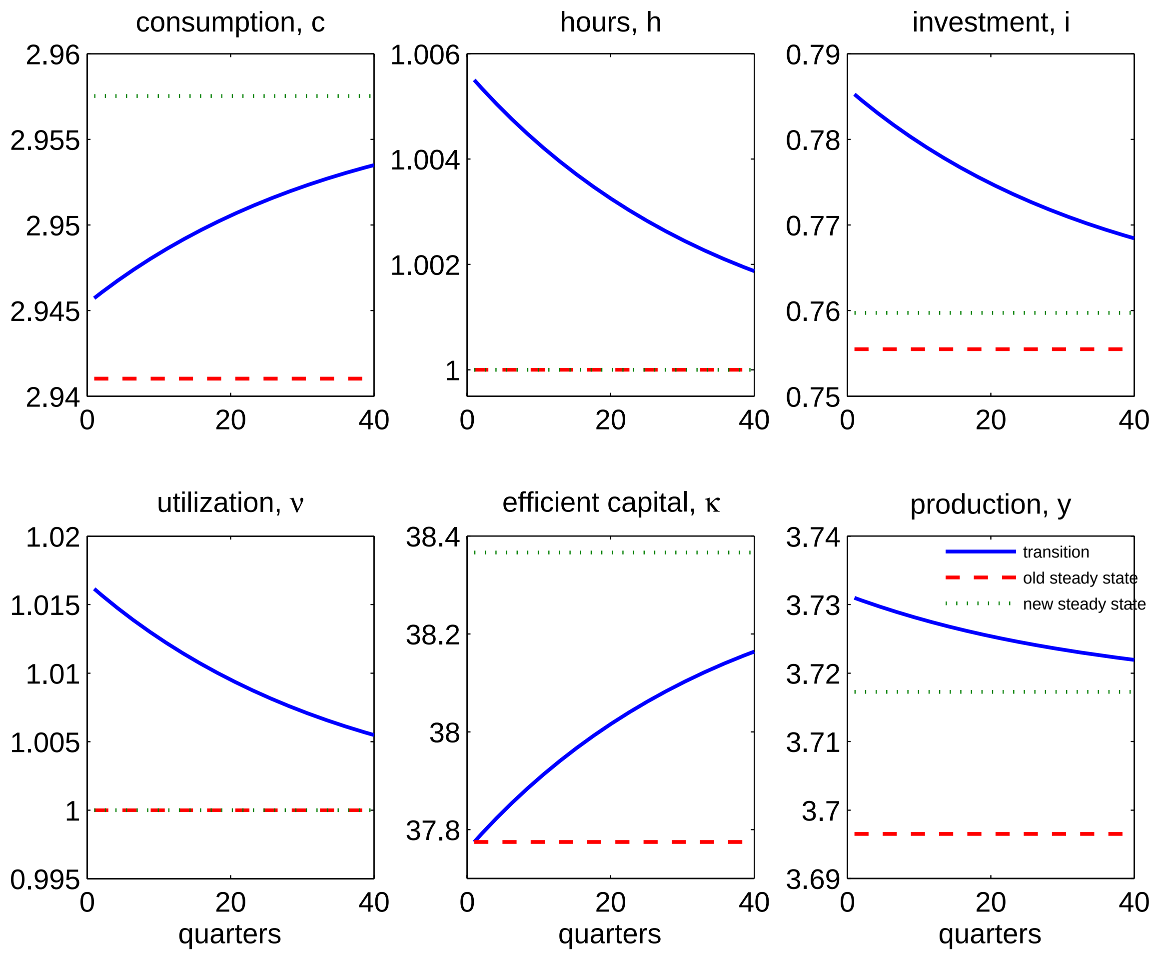
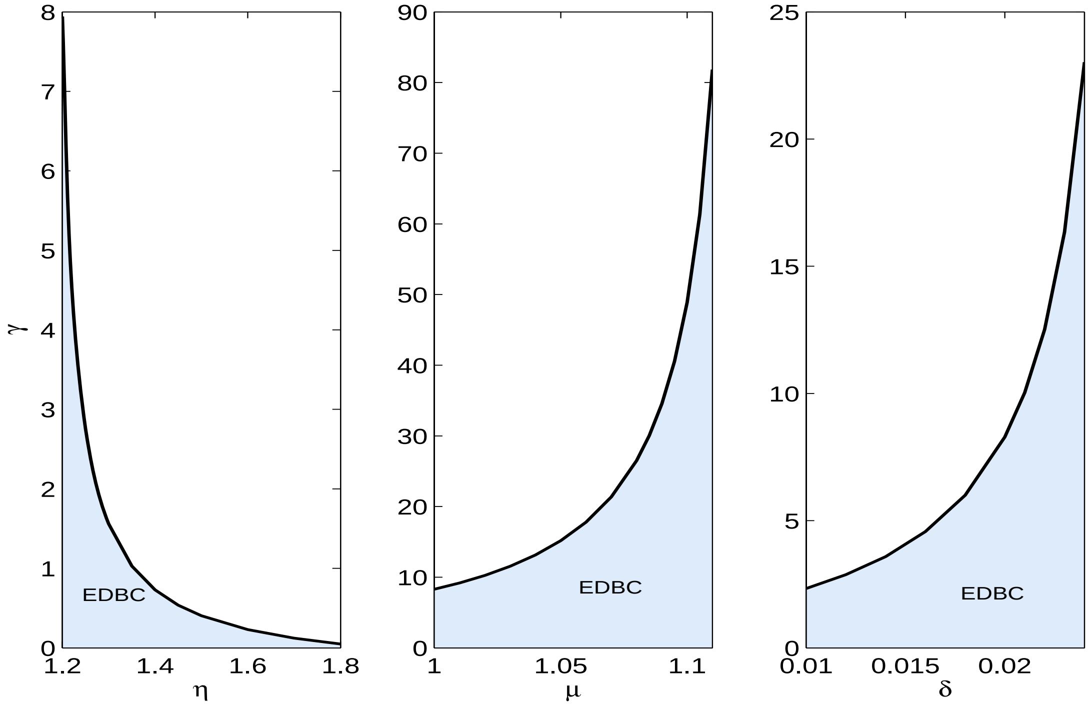


Figure 2: Combinations of parameter values that generate EDBC



Note: The shaded areas show combinations (γ, η), (γ, μ), and (γ, δ) that generate EDBC when the other parameters are set to the benchmark values.