Destructive Creation

Emilio Calvano[1]

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[1]ecalvano@fas.harvard.edu. Toulouse School of Economics and Department of Economics, Harvard University. Comments are always welcome. I apologize in advance for eventual errors, inaccuracies and typos.

I. Introduction

This study is part of a large body of research on the incentives to innovate in durable goods industries. In most of the existing formulations, the decision to invest in research and development is driven by the prospect of monopoly profits on the incremental value that the new vintages provide. Thus, in much of the existing literature, innovation goes hand-in-hand with value creation.

In this paper, I reexamine the manufacturers' incentives assuming that the *mere* introduction of new vintages affects the usage value of all vintages previously sold. In particular, I study a standard durable good pricing model in which a monopolist has the option, at the beginning of each period, to destroy[1] the usage value of all units previously sold and simultaneously introduce a new, perhaps improved, vintage at some cost $c \geq 0$, a practice which I refer to as "destructive creation". Such cost is interpreted as any expenditure incurred in the process of destruction as well as in the process of creating, developing and marketing the new versions. In equilibrium *innovation cycles* of finite length, consisting in the periodic introduction of successive, non-overlapping vintages arise. In this framework I address three basic questions. First, how do the incentives to innovate affect the equilibrium prices and sales? Second, is this practice desirable from a profit maximizing perspective? Since rational consumers anticipate opportunistic behavior and adjust their willingness to pay accordingly, manufacturers may actually want to build a reputation for not doing this kind of things. And third, what are the welfare consequences?

By allowing innovation to affect the value of the existing stock of durable goods, we highlight the role of *destruction* rather than *creation* in driving innovative activity. The formal analysis shows that destructive creation unambiguously leads to higher profits whatever the innovation cost. On second thought this shouldn't come as a surprise. If the "problem", from a profit maximizing perspective, is the durability of the output then it follows that any (cheap enough) mechanism that reduces or eliminates it would put the monopolist in a stronger position (i.e. "closer" to the rental outcome). The power to "wreck" the value of old versions of a product ends up serving much the same purpose and hence the profit restoral.

This result comes with important strings attached, due to the fact that new introductions are always determined *ex-post*. This distinctive feature of this mechanism generates a link between market prices and consumers expectations: the price *itself* affects the willingness to pay for it. The reason is that the incentives to innovate depend on the existing stock of durable. Thus current sales affect the expected duration of the good. In equilibrium a unique continuation profile is associated to

[1] The assumption of full destruction is made for expositional convenience. One can allow for partial destruction as long as destruction is anyway *significant*.

every price such that the higher the price for the current vintage, the farther away innovation and thus the higher the willingness to pay. Manufacturers are shown to sometimes exploit this linkage to extract higher rents.

Finally welfare effects are in general ambiguous both for consumers and for aggregate efficiency. The analysis shows that remedies aiming at increasing the cost of destructive creation, and thus at discouraging its practice, can backfire. They can lead to an increase in the discounted amount of resources invested in the practice and/or to distortion of the equilibrium prices (thus affecting consumers' surplus). This result is particularly intriguing since it holds even if the new vintages are *not* of increased value.

Crucial in the analysis is the role of destruction. One of the primary ways in which it can be accomplished is through product design or restrictive aftermarket practices. For instance software writers usually limit backward compatibility while manufacturers usually cease after a while to supply essential after-sales services or spare parts for their old products. Examples and applications include aftermarket practices that hinder prolonged usage;[2] excessive add-on pricing;[3] markets characterized by network externalities and/or compatibility issues;[4] standard setting; social consumption.[5] Kodak, Prime Computer, Data General, Unisys and Xerox, for example, have been repeadetly alleged of monopolizing the maintenance market refusing to deal with independent service organizations (ISOs). In fact Borenstein *et al* (1995; pp. 470) argue that in some of these classic court cases was presented

> "...evidence that manufacturers introduce price increases for parts and service on old equipment -or refuse to service old models altogether- specifically to induce customers to migrate to a newer model."

More recently Microsoft, contextually to the launch of its new O.S. Vista, has discontinued the provision of security support for older versions as part of its "Life Cycle Support Policy", forcing customers still running these old editions (actually millions) to upgrade.[6] Similarly Turbine Ent., the publisher of Asheron's Call 2, a

[2] e.g., prohibitive maintenance, repair, consumables, spare parts prices; discontinued provision of essential complementary services such as security updates (OSs, antiviruses) or on-line platforms (video games, e-services).

[3] High add-on prices may be interpreted as a mean to encourage customers to migrate to newer, perhaps richer models.

[4] e.g. software upgrades, textbooks revisions, consumer electronics.

[5] e.g., fashion clothes (Pesendorfer 1996), conspicuous consumption (Bagwell and Bernheim 1996), prosocial behavior (Benabou and Tirole 2006).

[6] Windows Vista was originally promised for the second half of 2006. Effective July 11, 2006, Windows 98, Windows 98 Second Edition, and Windows Me (and their related components) have transitioned to a non-supported status (http://www.microsoft.com/windows/support/endofsupport.mspx). In December 2005, 22% of Pc Users were still running Windows 98/ME according to Elizabeth Montalbano's "Older Windows OS Users: Kiss Tech Support Good-Bye"; Thursday, April 13, 2006; www.pcworld.com.

popular multiplayer on-line game has recently shutdown its servers (thus actually "killing" thousands of virtual characters) as a reaction to tepid sales of their latest expansion packs. Despite many remonstrances (that sometimes degenerated in "virtual" but otherwise real in-game riots), the publisher did not make any effort to commit to prolonged service or to guarantee some sort of backward compatibility.[7]

As a byproduct, the model is thus able to explain in a unified manner a number of business practices related to secondary markets which have a long history of scrutiny under the antitrust laws but which have been seldomly related both in theory and in practice. The plaintiffs' arguments in most cases relied either on some sort of leverage theory (for firms with substantial market power in the primary market) or on the lack of commitment power (or imperfect contractibilities) that prevented competitive pressure in the primary market from restoring cost based pricing in the aftermarkets. This research instead interprets these practice as a mean of encouraging customers migrating to newer models. High spare parts prices are not meant to be paid but to be avoided through substitution. It thus gives an additional reason for monopolizing aftermarkets even when the seller has substantial market power in the primary market which does not rely on any leverage hypothesis.

Interestingly, experimental evidence suggests that destruction can also be obtained through marketing techniques. For instance, Okada (2001) shows that trade-in pricing, gift opportunities, and low external reference prices significantly increase the likelihood of replacement of old vintages with new ones by negatively affecting the *perceived* residual value of the old products.

Lastly theoretical applications of destructive creation also include Schumpeterian growth models i.e., endogenous growth models in which whoever succeeds in the R&D race reaps the full monopoly profits "as if" the value of the previous (lower quality) goods were completely destroyed (e. g. Aghion and Howitt 1992).[8]

The paper is organized as follows. Section 2 introduces the main ingredients and solves the so called "Shutdown Game", establishing the conditions under which a monopolist will continue to sell an "older" version when he can instead opt out of the market, (i.e. shutdown) and receive a reward $s \geq 0$. The fact that the buyers' valuation of the good is endogenous (it depends on how long the seller stays on the market before shutting down) raises issues related to the existence and uniqueness of an equilibrium which differ from the ones discussed in classic intertemporal price

[7] "The end is virtually nigh", The Economist, 12/10/2005, Vol. 377, Issue 8456, Special Section pp14.

[8] Consumers' expectations of future "destruction" are typically neglected in these models. This is because the monopolist's good is either assumed to be a consumption good or to be a durable (capital) good that is rented rather than sold. However since US vs United Shoe machinery corp., the courts have declared the latter policy illegal when employed by a monopolist such as a patent holder. The fact that rational consumers anticipate "destruction" then ends up affecting the incentives to innovate which are crucial in these models.

discrimination models.[9] Section 3 (the Innovation Game) endogenizes the outside option s which is defined as the continuation value of the game following an "empty" innovation net of the future R&D fixed cost. Section 4 extends the positive analysis by considering a broader equilibrium concept. The latter gives rise to interesting additional equilibria labeled "innovation traps" and "cycling cycles" whose main characteristic is that the value of a particular vintage depends directly on calendar time. Section 5 discusses welfare. It also discusses to what extent the mere introduction of superior products can be interpreted as a means of destroying the value of previous versions. Section 6 concludes. All proofs are presented in appendix.

Related literature Since consumers are heterogeneous this model combines standard intertemporal price discrimination and obsolescence. This article is thus related to the seminal papers of Waldman (1993), Choi (1993), Waldman (1996) and Fishman and Rob (2000)[10] on new product introductions and to the subsequent debate prompted by these works. The first two articles, like this one, are concerned with the effects of new *destructive* product introductions. In both papers the combination of incompatibility between successive product generations and network externalities generates a destructive effect, since, in equilibrium, as more and more consumers upgrade, the value of the old product decreases. Waldman (1996) and Fishman and Rob (2000) instead study the monopolist's incentives to introduce a superior product in a later period. However, all these works abstract away from Coasian issues assuming either non-overlapping cohorts of homogeneous consumers or high heterogeneity[11] and hence they don't look at the interplay between introduction policies and Coasian dynamics. Fudenberg and Tirole (1998), Lee and Lee (1998) and Nahm (2004) relax the assumption of homogeneous consumers and present a two period model of technological innovation. Neither article captures the effects described here for reasons discussed at the end of section 4. Hendel and Lizzeri (1999) and Morita and Waldman (2004) propose a model in which a monopolist can affect the value of used units restricting consumers' abilities to maintain the good. The former article assumes away Coasian dynamics[12] and identifies an additional reason for why a seller may still want to affect the value of used units. Since consumers have heterogeneous valuations for quality and used goods are imperfect substitutes of new goods, by controlling the rate at which goods deteriorate though both product design and restrictive aftermarket practices the seller can segment the market in used unit users and new unit users. Morita and Waldman (2004) instead relate aftermarket practices to Coasian dynamics. They present a

[9] E.g. Fudenberg, Levine and Tirole (1985) and Gul, Sonnenschein and Wilson (1986).

[10] These papers in turn build on Coase (1972) and Bulow (1982, 1986)

[11] In Waldman (1996) there is only one "type" whose valuation exceeds the marginal cost of production.

[12] The seller can commit to an output plan at the beginning of the game.

two-period model of a market in which durable goods naturally deteriorate[13] (i.e. destruction is assumed), and the seller can costlessly[14] *restore* the value of used units say, by allowing for competition in aftermarkets. When the scrap value of an used unit is higher than the consumption value their model can be interpreted as a model of destructive creation with $c = 0$ since destroying on purpose is equivalent to not restoring on purpose. The two models are complementary since Morita and Waldman assume destruction (or that destruction is costless) and vary the rate at which the durable goods deteriorate while I assume that the old versions are worthless if destroyed and let the cost of doing so vary. This distinction is crucial. Varying the cost of destruction generates the intertemporal conflict that constitutes the core of this paper. In contrast Morita and Waldman's results would not change if the monopolist could commit at the beginning of the game to a behavior in the aftermarket. Furthermore all these analyses do not investigate the properties of the resulting *cycles* as they typically study a finite, two-period model.

Pesendorfer (1996) illustrates an interesting mechanism by which the introduction of new "styles" of no additional value destroys the value of the previous ones.[15] In his model a matching market that sorts people by the fashion they use creates a signaling/screening role of consumption. As "high" types upgrade to new, more exclusive products the value of the old product decreases.[16] This paper takes the destructive mechanism as given (i.e. endows the seller with a destructive wand) and investigates issues related to its optimal employment and to its performance relative to other means to fight the Coasian inclination to lower prices over time.

Finally this paper is also related to models of cyclic pricing such as Conlisk *et al.* (1984) and Sobel (1991). In this model sales (or recurrent periods of low prices) occur over time as consumers anticipate opportunistic behavior and are thus willing to pay less for goods expected to be destroyed sometime soon.

II. The Shutdown Game

The elements of the formal model are as follows. A Seller has an infinite number of units for sale. Storage is costless and the Seller derives no utility from having such objects in his inventory. There is a unit measure of non-atomic buyers indexed by $b \in [\underline{b}, \overline{b}]$ with $\underline{b} > 0$ who derive a positive utility, in a way discussed below, from consumption. The Seller cannot discriminate among different buyers but it

[13] Used, non maintained, units are less productive than new or maintained units.

[14] They assume that units costs of maintenance are sufficiently low so that used units are always restored at the competitive price.

[15] The astute reader should have noticed that I've already celebrated Pesendorfer's intuition in the opening including "fashions" and signal provision as potential applications and examples of destructive creation.

[16] By the same logic any model of (dynamic) signal provision is a model of destructive creation as long as the introduction of new signals affects the (equilibrium) value of the signals already sold.

is common knowledge that b is a random variable *i.i.d.* across consumers with a smooth[17] cumulative distribution function $F(b)$ and bounded density $f(b) \in (0, \bar{f}]$. Time is indexed by periods $t = 1, 2, ..., \infty$ and $\delta < 1$ is a common discount factor. At the beginning of each period the monopolist can either continue to serve the residual demand and hence propose a price p_t or shut down. When the latter option is preferred the game ends and the monopolist obtains $s \geq 0$. Buyers, right after purchasing, derive a per period utility b in every period before shut down occurs. I will refer to the number of sale's periods before shut down in equilibrium as the (residual) "length of the game".

Assume that the monopolist shuts down in period $T+1$ and hence let T denote the last period of sales. The utility from purchasing the good evaluated at $t \leq T$ is then given by:

$$\sum_{i=t}^{T} \delta^{i-t} b - p_t. \tag{1}$$

Each Buyer maximizes his expected utility while the Seller maximizes the expected present value of its revenue stream. A behavior strategy for the Seller is a mapping from histories to probability distributions over shutdown decisions and prices whereas a pure strategy for a buyer is a mapping from those histories in which he didn't purchase to purchase decisions.[18] A Perfect Bayesian Equilibrium of this game is a pair of strategies and a set of beliefs satisfying the usual optimality conditions and Bayes rule. As we shall see the case $s = 0$ raises additional issues. To avoid them in what follows I assume that there is an epsilon sunk cost that should be paid to provide an extra period of durability. This guarantees that the monopolist always shutdowns in every subgame in which he has already sold one unit to everybody. The case $s = 0$ is postponed to section 4 as an extension.

Let us initially consider the situation where the Seller can commit, at the beginning of the game, to any time path of prices and shutdown policy. The associated payoff constitutes an upperbound on what the seller can extract and therefore a natural benchmark.

Proposition 1 *The optimal precommitment strategy is to charge a fixed price equal to the (static) monopoly price and to never shutdown as long as the shutdown reward is lower or equal than the associated profits and to shutdown immediately otherwise.*

[17] Formally, I assume that the cumulative distribution function is differentiable and has a differentiable inverse.

[18] Lastly, I assume that the equilibrium actions of each agent are constant on histories in which prices are the same and the sets of agents accepting at each point of time differs at most by sets of measure 0, which is a natural requirement in all intertemporal price discrimination models (a good discussion of this issue can be found in Gul *et al.* 1986, note 6.2).

The precommitment payoff in case of permanent service is given by

$$\pi_{fc}(\delta) = [1 - F(p_{fc}(1-\delta))]\, p_{fc},$$

where p_{fc} is any optimal precommitment price. Such payoff equals the "rental solution" of a standard durable good model where a buyer b gets utility $b/(1-\delta)$ upon purchasing. It can hence be rewritten as $\max_r [1 - F(r)]\, r/(1-\delta)$ where r is the rental price. Committing to permanent service maximizes all buyers' willingness to pay as it removes concerns over durability. At the same time a fixed price strategy permits to restore market power as it removes cheaper substitutes in the future. Yet, if the outside option s is greater than the resulting profits, the Seller would trivially stay out of the market and cash s. That shutdown in finite time cannot be optimal follows from the fact that the full commitment payoff coincides with the rental solution. As the problem of whether to rent or shutdown is clearly stationary then the monopoly either rents every period or never rents.

Let us now turn to the equilibrium analysis, absent any commitment power. The fact that the value of the good depends positively on how long the seller stays raises issues related to the existence and uniqueness of an equilibrium. An important one is that multiplicity could arise due to self-fulfilling expectations of the form: the higher the expected durability, the higher the willingness to pay, the lower the incentive to actually shutdown. Another concern is whether shutdown always occurs in finite time in any equilibrium. The following lemma establishes that any equilibrium should be characterized by negotiations of finite nature.

Lemma 1 *If $s \geq 0$ then shutdown always occurs in finite time.*

Starting from the "last period of sales" is then possible to "work backwards" to construct an equilibrium.

Proposition 2 *An equilibrium exists and is unique. Furthermore*

(i) There exists a finite, decreasing sequence of outside options $\{s_n(\delta)\}_n$ such that, in equilibrium, if $s \in (s_{n+1}(\delta), s_n(\delta))$ then there will be n periods of sales before shutdown whereas for $s = s_{n+1}(\delta)$ there will be either n or $n+1$ periods of sales depending on the seller's initial choice.

(ii) The monopolist enters (i.e. makes at least one offer before shutting down operations) if and only if the shutdown reward is less than the full commitment profits.

As in standard intertemporal price discrimination models, the consumers' set in every t will be partitioned in two, possibly empty, convex and disjoint subsets:

owners and not owners. Define as b_t the owner with the lowest valuation at time t.[19] To see that expectations over durability cannot self-fulfill, let $\widetilde{b}_1(s)$ be the level of b_t (or residual demand) that makes the seller indifferent between staying one more period or not in the *last* period of sales (i.e. given that shutdown occurs tomorrow with probability one). The key to the proof is that for any history such that $b_t < \widetilde{b}_1(s)$ in any equilibrium the seller must shutdown with probability one. Suppose that this is not the case. By lemma 1 a last period always exists in this continuation game and, by definition of $\widetilde{b}_1(s)$, the seller will always shutdown before any such last period, a contradiction. Given such termination condition the proof proceeds by inductive hypothesis on b_t employing a dominance argument. It analogously defines a unique sequence of thresholds $\{\widetilde{b}_n(s)\}$ such that the (continuation) equilibrium is characterized by shutdown after at most $n-1$ periods of sales whenever $b_t < \widetilde{b}_n$. Because the monopolist always sells to everybody in a finite number of periods the induction should eventually stop and therefore an equilibrium exists. Uniqueness is then established up to the seller initial choice. To any first period price is associated a unique sequence of offers by the seller and acceptance decisions by the buyers. Since the program need not be convex, it can be the case that the seller is indifferent between two (or more) first period prices. The monopolist therefore "selects" the equilibrium path through his initial choice.

Corollary (i) characterizes the relationship between the equilibrium durability and the outside option. Intuitively an higher s, ceteris paribus, increases the temptation to shutdown and therefore weakly reduces the equilibrium durability. The latter part of the statement accounts for the possibility that $\bar{b}$ is exactly equal to $\widetilde{b}_n(s)$ for some n. If this is the case then also the equilibrium length depends on the seller's initial choice as, by definition, $\widetilde{b}_n(s)$ leaves the seller indifferent between n or $n-1$ periods of sales.

Let $\pi(\delta, s)$ denote the (discounted) equilibrium profits, which is a well defined, continuous function by proposition 2. (ii) implies that the equilibrium profits are always greater than s whenever $s < \pi_{fc}$ and are always equal to s otherwise, a property which will be useful later on.

Since in section 3 s is interpreted as the net value of an innovation, it remains to be established whether the equilibrium profits increase with s. Also, whether there is any conflict between what the seller would like to commit to and what he ends up doing in equilibrium. Interestingly the answer to both questions is no, not necessarily. The fact that the profits can indeed *decrease* with s is somewhat surprising. One would have conjectured that a higher outside option would have

[19] Intuitively as the game advances b_t weakly declines towards $\underline{b}$ since more and more consumers join the owners group. In the remaining part of the paper I will improperly refer to b_t as the "residual demand".

put the seller in a "stronger" position. If the profits are not increasing in s, then the value of the outside option, defined as $\pi(\delta, s) - \pi(\delta, 0)$, could be negative as well: it can be advantageous, ex-ante, to "drop" an outside opportunity whenever offered one. In order to build the intuition behind these results, I turn to the analysis of a simple, well-behaved two period game. In light of proposition 2 this two-period illustration can be interpreted as the actual equilibrium of a game in which the support is "narrow enough".

A. A two-period illustration

Consider a simple (and rather sad) world inhabited by a seller whose output, lasts no more than two periods. At the beginning of each period $t = 1, 2$, the seller can propose a price that all buyers evaluate or cash $s \geq 0$ and leave the market. When the latter option is chosen the game ends and the value of all units previously sold, if any, is destroyed. If shutdown has not previously occurred, it will occur at $t = 3$ with probability one. To simplify the exposition assume also, in this illustration, that there is enough concavity in the problem that equilibrium prices are unique for almost every s[20] and consider those cases in which the $\max_r [1 - F(r)]r > \underline{b}$.[21] If $\overline{b}$ is (relatively) low enough then, by virtue of proposition 2, an equilibrium exists and is unique. It is worth considering here two different cases: in the first case the monopolist can commit to shutdown at any time $T + 1 \in \{1, 2, 3\}$ (or can guarantee any "durability" T he wants) although he cannot pledge himself to predetermined prices, whereas in the second case he is unable to do so.

Commitment on duration.—Under commitment, the monopolist's profits can be trivially decomposed in two parts: the discounted sum of the per period revenues and the discounted shutdown reward, as the amount of the latter does not affect the equilibrium prices. Consider firstly the impact of increased durability, say from one to two periods, on the buyers' willingness to pay and therefore on the equilibrium revenues. First, increased durability raises the utility that all buyers get upon purchase in period 1 (durability effect). On the contrary, the seller's inability to commit to future prices creates a cheaper substitute in the future and hence, all things being equal, this reduces their willingness to pay (*Coasian* effect).[22] The former effect always dominates the latter and therefore postponing shutdown always

[20] A rectangular distribution would guarantee this. Later on it will be clear what "almost" means in this context.

[21] This condition restricts the analysis to non-trivial cases in which the seller can profitably exercise his market power.

[22] The presence of a Coasian effect is the main difference between this case and the full commitment case. Under full commitment, deferring shutdown increases the value of sales by a factor of δ^T whereas here the value of sales increases less due to the seller's dynamic inconsistency.

increases revenues despite the seller's dynamic inconsistency.[23] To see this, consider the first period of sales. The indifferent buyer b between purchasing today at price p or tomorrow at some price p_2 in the two-period game (i.e. the game where the seller commits to two periods of service) solves:

$$b(1+\delta) - p = \delta(b - p_2). \tag{2}$$

Uniqueness comes from the fact that the optimal second period price (i.e. $\arg\max_p [F(\beta) - F(p)]p + \delta s$) is a non decreasing function of residual demand. It follows that at any given first period price p strictly more consumers, if any, purchase in the two-period game (buy iff $b \geq p - \delta p_2$) than in the one period game (buy iff $b \geq p$).[24]

Conversely, postponing shutdown entails a loss as the quantity s is cashed later due to standard discounting (deferral effect). The seller hence trades-off the incremental gains due to extended durability with the incremental losses due to the deferral of the outside option. The higher the outside option, the lower the optimal precommitment durability.

No commitment.—Now consider the more interesting case where the firm is unable to commit. Since in period 3 shutdown occurs with probability one then those buyers still on the market in period 2 behave accordingly and buy iff $b \geq p$. At the beginning of period 2 the Seller has to evaluate what are the potential benefits of serving the residual demand compared with the sure option of s. Trivially, there is a threshold level$\widetilde{b}(s)$, defined by:

$$\widetilde{b}(s) = \max\left\{\beta \leq \overline{b} : \max_p [F(\beta) - F(p)]p + \delta s \leq s\right\}$$

such that the seller always stays whenever $\beta > \widetilde{b}(s)$, always shutdowns whenever $\beta < \widetilde{b}(s)$ and is indifferent whenever $\beta = \widetilde{b}(s)$, which is increasing in s up to $\overline{b}$.

Consider now the first period. Since buyers care about "durability" their willingness to pay depends on whether the next period the seller will stay or leave, that is on whether residual demand will exceed $\widetilde{b}(s)$ or not. The indifferent buyer is now defined by:

$$b(1+\delta\psi) - p = \delta\psi(b - p_2) \tag{2'}$$

where ψ is the (equilibrium) probability that the seller doesn't shutdown and makes another offer p_2 in period two.

Intuitively if the first period price is sufficiently low then, even if buyers are

[23] Notice that, in standard models of intertemporal discrimination, the converse result holds since there is no "durability effect".

[24] The assumption that $\max_r [1 - F(r)]r > \underline{b}$ guarantees that the seller never sells to everybody in the first period.

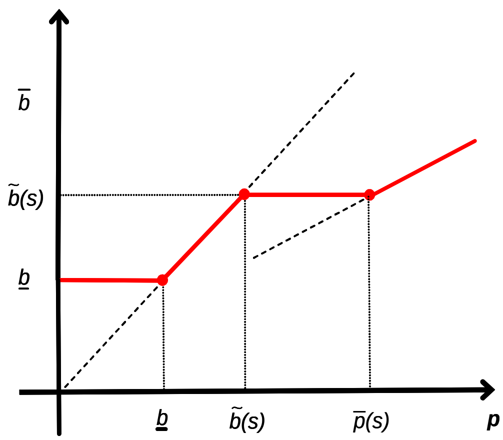

Figure 1: Indifferent Buyer under Uniform Distribution

"pessimistic" (they expect only one period of service), enough of them purchase and hence tomorrow the seller will actually shutdown with probability one. On the contrary if the first period price is high enough, then even if buyers are "optimistic", few of them purchase and tomorrow the seller will stay with probability one. Figure 1 captures this intuition and depicts the indifferent buyer as a function of p, in the simplest case of uniformly distributed buyers. Interestingly, for intermediate values of p, a pure strategies equilibrium does not exist. To see this notice that if buyers are optimistic, too many of them end up buying whereas if they are pessimistic, too few of them purchase. In this intermediate price range for each first period price there exists a unique continuation equilibrium in which: 1) only buyers with valuation greater or equal than $\widetilde{b}(s)$ purchase, 2) the seller randomizes between staying one more period and shutting down and 3) the equilibrium probability of staying one more period increases with the first period price. The upper bound of the interval, denoted $\overline{p}(s)$, solves (2') when $\psi = 1$, $b = \widetilde{b}(s)$ and $p_2 = \arg\max_p[F(\widetilde{b}(s)) - F(p)]p + \delta s$.

Letting $\beta(p)$ be the indifferent buyer as a function of the first period price and $\pi_1(\beta, s) \equiv \max\{\max_p[F(\beta) - F(p)]p + \delta s, s\}$ be the continuation value of the game at the beginning of period 2, the seller program is given by:

$$\max_p [1 - F(\beta(p))]p + \delta\pi_1(\beta(p), s). \quad (3)$$

Let p^* denote any solution of (3). By inspection it is possible to immediately exclude that the optimal price falls in the region $\left[\widetilde{b}(s), \overline{p}(s)\right)$ since demand is insensitive to price in this range. Such observation is a consequence of the seller's inability to charge low prices ($p < \overline{p}(s)$) and all together maintain buyers' confidence over durability due to his own inconsistency problem. Therefore, depending on s, two kinds

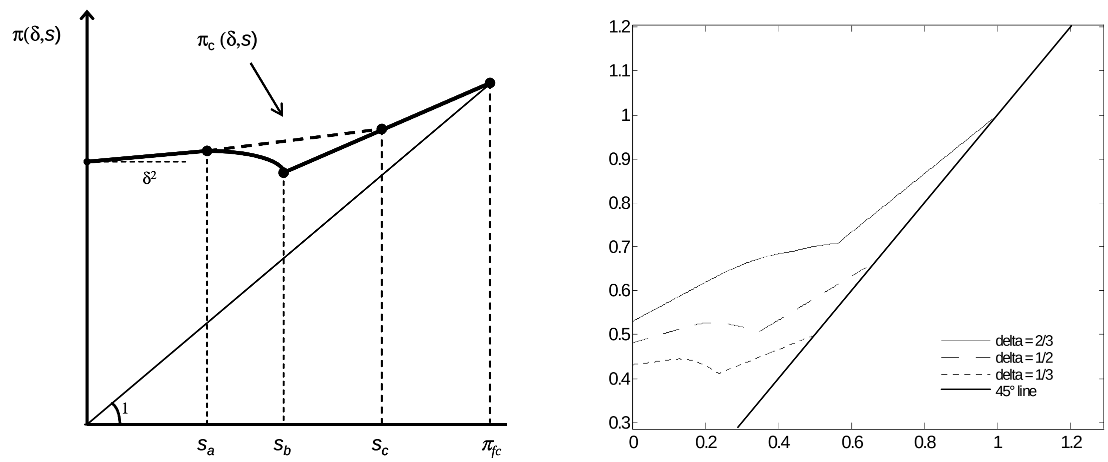

Figure 2: (a) Value Function (b) Simulation

of equilibrium can arise: a "one-period equilibrium" in which the seller proposes a low price $p^* < \widetilde{b}(s)$ and then shuts down; or a "two-period equilibrium" in which the seller proposes a high price $p^* \geq \overline{p}(s)$ and then a lower price, conditional on selling. By convexity, in this simple illustration it is possible to prove that for almost any $s \geq 0$ the equilibrium is unique. The "almost" qualifier accounts for the possibility that the Seller can be indifferent, for some s, between a two-period and a one-period equilibrium.

It is now possible to compare the two solutions. Figure 2a depicts the equilibrium profits as a function of the shutdown reward both in the commitment and in the no commitment case. Consider the latter. An higher s affects the game in two respects. As in the commitment case, it raises the continuation value of the game, since s is cashed at some point (direct effect). Furthermore it increases the temptation to shutdown, since it raises the threshold $\widetilde{b}(s)$ and, as a consequence, the minimum price $\overline{p}(s)$ that guarantees that tomorrow shutdown will *not* occur with probability one (indirect effect). Let p_1^c denote the optimal first period price when the seller commits to two periods of sales. For relatively low values of s the seller charges p_1^c, thus replicating the commitment solution as dynamic inconsistency is not a concern. However if, for some values of s, $\overline{p}(s)$ exceeds p_1^c whereas the optimal precommitment durability is 2, then it is possible to prove that the seller finds worthwhile, in some range (s_a, s_b), to distort his first period price upwards (up to $\overline{p}(s)$) to preserve his own incentives to stay on the market and therefore to maintain buyer's confidence over durability. If the indirect effect (negative) of a marginal increase of s overwhelms the direct effect (δ^2) then the value function could well decrease in this range. Eventually ($s \in (s_b, s_c)$) the seller switches to a low price

($< \widetilde{b}(s)$) and therefore shuts down at the beginning of period 2, even tough he would prefer to commit to two periods of sales.

The following proposition (proved in the appendix) contains this section main insights. It establishes that, for any parametrization of the model, there always exists a δ low enough (possibly greater than one) such that this is actually the case in equilibrium.

Proposition 3 *In the two period game, if players are sufficiently impatient then there exists an open, convex and non empty subset of shutdown values s such that:*
(i) The precommitment profits exceed the equilibrium profits.
(ii) The seller charges higher prices than in the precommitment case in both periods.[25]
(iii) Equilibrium profits decrease with the shutdown reward.

Figure 2b plots the value function for different values of the discount factor when b is uniformly distributed in $[1, 3]$. Values on both axes are expressed in % of the rental solution $\pi_{fc}(2/3)$. Notice that for $\delta = 2/3$ the value function increases with the shutdown reward whereas for $\delta = 1/2$ and $\delta = 1/3$ this is no longer the case. In particular for delta equal to 1/3 the value of the outside option becomes actually negative for some $s > 0$ and therefore the seller would get more if s were equal to zero.

B. Discussion

So far I only addressed the issue of existence and uniqueness. Before moving on, it is worth taking a few lines to comment on and summarize a number of features of the equilibrium. First, the unique restriction invoked so far is the so called "gap case" that is $\underline{b} > 0$. It is possible to show that nothing changes if we let $\underline{b} = 0$ whenever $s > 0$. However, when both parameters are equal to zero, it is not possible to bound the number of sales periods and therefore to have a complete characterization of the equilibrium set. In this case durability is always infinite[26] but the actual allocation of gains from trade varies over the equilibrium set.

The shutdown reward can be interpreted either as the opportunity cost of staying on a given market or as an alternative elastic demand. Therefore one can find examples where this theory directly applies. For instance in the video games and software industries firms typically have to maintain essential complementary services such as on-line platforms or after sales support that typically don't generate

[25] Clearly (ii) $\Rightarrow$(i), however there are values of s such that only (i) holds, this is why the two statements are apart.

[26] The seller will never charge a price equal to zero.

enough revenues to cover their costs. However the buyers' willingness to pay for the main product crucially depends on the availability of such services. Sound economic reasoning would suggest these firms to build a reputation for keeping their servers up. According to the shutdown model, if the price at which their titles sell on the market do not justify further investments *even if* the publishers were able to build such a reputation, for instance because the opportunity cost is particularly high, then their shutdown policy could be interpreted as a means to maximize profits rather than as an expression of a dynamic inconsistency issue.

III. Innovation Cycles

This section completes the theoretical investigation. It presents an equilibrium analysis of a market in which a seller can, at a cost c, destroy the value of all units previously sold and simultaneously introduce a new version. The model focuses on the pure "destructive" aspect of innovation by assuming that the new versions are of no additional value (the effect of relaxing this assumption are discussed in section 5). The cost c can be interpreted as any expenditure incurred in the process of destruction.[27] It can also be interpreted as the cost of creating, developing and marketing the new versions. Clearly if such cost is sufficiently low then *innovation cycles* of finite length endogenously arise.

The extensive form described in section 2 is modified replacing the shutdown option with an *innovate* option in the way described below. Innovating destroys the value of the old products, if any (and in this respect is equivalent to shutdown). However a fixed cost $c \geq 0$ must be paid each time an innovation takes place. The utility from purchasing the *latest* version evaluated at $t \leq T$ is still given by (1) where T is now interpreted as the last period of sales before a new version is introduced. The timing of the game is as follows. At the beginning of period 1 the monopolist faces an entry decision.[28] He can either stay out of the market or pay c, create a product and fix a price p_1 that all buyers evaluate. When the former option is chosen the game ends and the monopolist obtains 0. Conditional on entry, at the beginning of each subsequent period $t = 2, ..., \infty$ the monopolist can either continue to serve residual demand and hence propose a price p_t or innovate. When the latter option is selected the monopolist pays c and starts to sell his new product, which is in all respects identical to the old one, by fixing a new price and so on. Define the period between two successive innovations as a "cycle".

[27] E.g., the costs of restrictive after market practices, the cost of setting a new standard or of shutting down existing platforms.

[28] Alternatively one could assume that the seller is "endowed" with his first product and pays c only to innovate (there is no entry decision). To simplify thinghs I also assume w.l.o.g. that the seller always enters whenever indifferent.

As the outcome of each cycle is uniquely determined by the continuation payoffs after an innovation takes place (proposition 2), the problem has the flavour of a repeated game. That is: after each innovation the seller and the buyers play a game which is equivalent in all respects to the previous one save for the expected continuation payoff following an innovation which varies with the profile under scrutiny. Let $\tau = 1, 2, ..,$ index the innovations and let s^e_τ be the expected value of the game after the τ-th innovation net of the (future) innovation costs. An equilibrium of the innovation game is defined[29] as any sequence $\{s^e_\tau\}_{\tau=1}^\infty$ that satisfies:

(a) $s^e_\tau + c \in \pi(\delta, s^e_{\tau+1})$

(b) $s^e_\tau \in [0, \pi_{fc} - c]$

since to any such sequence it is possible to associate the corresponding "stage game" equilibrium profiles. The first condition simply reflects the requirement of subgame perfection. The latter condition remarks the upper and lower bounds of the seller's profits. If an equilibrium sequence is constant over τ then the associated equilibrium is said to be *stationary*. Stationary equilibria are obviously more attractive as the value of an innovation does not depend directly on time t. For this reason in most of what follows I focus attention on this class of equilibria. However it turns out that (some) non stationary equilibria unveil aspects of the seller's problem that could be equally interpreted in economic terms and hence are worth exploring. Indeed there are cases where non stationary equilibria dominate the stationary ones. Their discussion is postponed to section 4.

A. Stationary Equilibrium

I now proceed to characterize the stationary equilibrium of the game. An important issue is that the link between the seller's eagerness to innovate and his continuation payoff can generate multiplicity. For instance, suppose that the Seller is *optimistic* in that he expects the upcoming cycles to be short. If shorter cycles are more profitable, the Seller will be eager to innovate and hence the current cycle is expected to end soon in equilibrium. On the other hand the same reasoning applies for *pessimistic* sellers: beliefs over the future might self-fulfill. The proof of the following theorem breaks down this argument arguing that this full-filling effect would require the expected continuation value of the game to decrease (or increase) "without bounds".

Theorem 1 *A stationary equilibrium exists and is unique in the class of stationary equilibria. Moreover if the cost of innovation is low enough ($c < \pi(\delta, 0)$) entry*

[29] Details on strategies and beliefs are given in the proof of theorem 1.

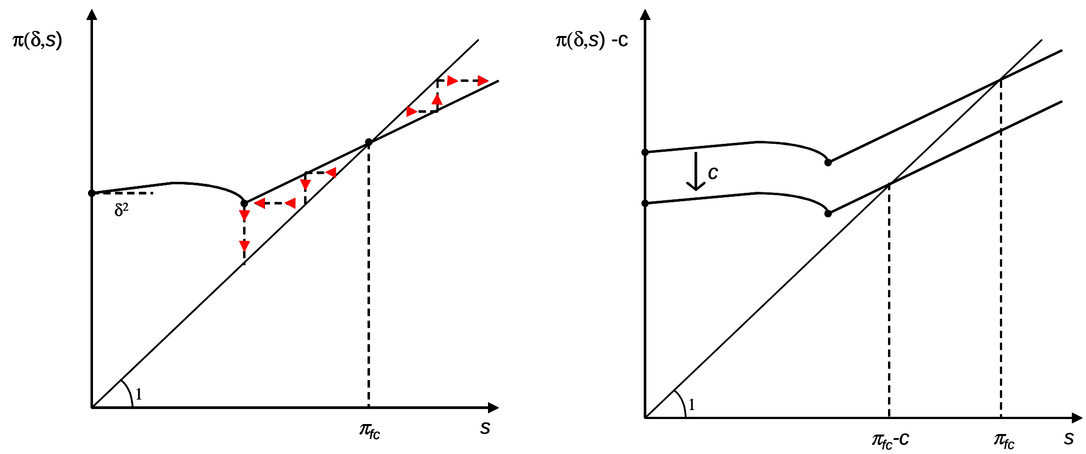

Figure 3: (a) $c = 0$ (b) $c > 0$

occurs and the equilibrium is characterized by an infinite replication of symmetric cycles.

The proof looks for any fixed point s^* of the correspondence $\pi(\delta, s) - c$ that satisfies (b). To see that no more than one fixed point can exist it is sufficient to consider what happens when any s^e_τ is arbitrarily set below (above) a fixed point. For instance if the buyers and the seller are *pessimistic* over the future, i.e. $s^e_\tau < s^*$, then such future would self-fulfill if and only if such pessimism grows over time without bounds. The point can be intuitively explained using figure 3a. Assume (for simplicity) that π has modulus of continuity lower than one in absolute terms. Recall from section 2 that for every $s^e < \pi_{fc}$ the seller stays at least one period on the market as it gets more than just grabbing s. Therefore if $s^e_\tau < \pi_{fc}$ then it should be that $s^e_\tau > s^e_{\tau+1}$ for s^e_τ to be fulfilled in equilibrium. Iterating this reasoning, eventually $s^e_{\tau+i}$ will jump outside the payoff bounds in (b) and hence it is not possible to construct an equilibrium around any such trajectory.

Let

$$\Pi(\delta, c) = \pi(\delta, \Pi(\delta, c)) - c \tag{4}$$

be the value of the game. The theorem says that there exists only one perfect equilibrium that satisfies the stationarity assumption. If the average[30] cost of innovation is low enough, the model reproduces a crucial feature of these durable markets: the cyclical introduction of non-overlapping generations of goods or, alternatively, the cyclical destruction and contextual introduction of a new generation of products.

[30] Since profits are measured per-capita, c can be interpreted as the \$cost of destruction per buyer.

B. Properties of Innovation Cycles

I shall now ask what are the properties of these innovation cycles, what is the (relative) profitability of destructive creation so as compared with the full and partial commitment solutions and how does this profitability vary with the innovation cost c. Finally I shall ask what is the role of pricing and hence what kind of predictions it is possible to make on the *within* cycles equilibrium prices. Much of what follows capitalizes the investment made in section 2.

Proposition 4 *If innovation is costless then equilibrium profits equal the rental solution profits. Furthermore the equilibrium is unique*[31] *and characterized by a new innovation in every period.*

That when $c = 0$ the equilibrium profits equal the rental (or full commitment) solution comes at no surprise. Selling a good that is expected (and is actually) costlessly destroyed after each period is trivially equivalent to renting in a world with no marginal costs of production. For low enough values of c the "always innovating" solution continues both to exist and to be unique with the monopolist now paying a (little) fee every period: $\Pi(\delta, c) = \pi_{fc} - c/(1-\delta)$.

Proposition 5 *Consider the stationary equilibrium and suppose that the cost of innovation is such that entry occurs.*
(i) Equilibrium profits decrease with the cost of innovation.
(ii) The equilibrium length of each cycle (weakly) increases with the cost of innovation.
(iii) The (within) cycle price path exhibits Coasian dynamics.
Moreover consider the two period illustration of section 2. If the cost of innovation is such that an intertemporal conflict arises then:
(iv) Innovation occurs weakly too soon, from an ex-ante, profit maximizing perspective.
(v) A high enough first period price is followed by delayed innovation and this can be used to maintain buyers' confidence over "durability" in those instances where an intertemporal conflict arises.

Rather than cluttering the remaining part of the paper with formal statements about the relationship between the cost c and the equilibrium length of each cycle (or equilibrium durability) I limit myself to observe that there is a one to one relationship between the outside option's thresholds defined in section 2 and the cost thresholds in the innovation game. By (i) higher costs lower the value of an innovation. Hence, in terms of the shutdown game, higher costs lower the outside

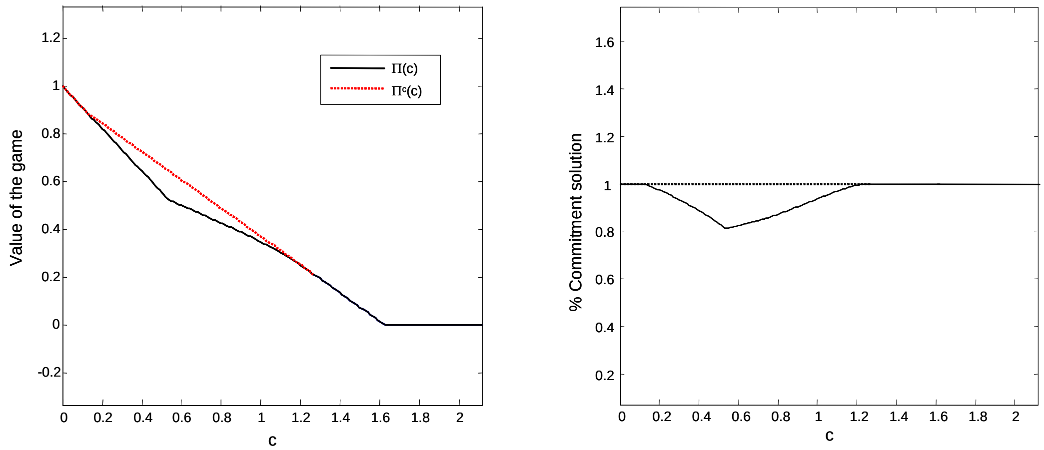

Figure 4: Simulation of the value function: (a) $\Pi(c)/\pi_{fc}$ (b) $\Pi(c)/\Pi^c(c)$

option and therefore increase the equilibrium durability of each and every product.

More interesting is the comparison between the equilibrium profits and the profits the seller would make under commitment to introduction policies. Figure 6a plots the both value functions when b is uniformly distributed in $[1, 3]$ and $\delta = 1/2$. The value is expressed in % of the rental solution. Figure 6b plots the value of the game as a % of the value under a commitment to an optimal introduction policy. The discussion parallels section 2's analysis as these are clearly two facets of exactly the same problem. For low enough values of c the commitment solution coincides with the no commitment one, since the seller's dynamic inconsistency is not a concern. For intermediate values of c an intertemporal conflict arises. Under commitment the seller switches sooner to two-period cycles since it doesn't have to pay the extra-cost of persuading the buyers that he will indeed wait one more period before introducing his new product. Hence in equilibrium, for relatively low values of c, the seller continues to innovate every period even tough he would prefer to commit *not* to. The higher c, the lower the attractiveness of a one-period equilibrium as compared to a two-period one. Eventually the seller will find profitable to implement the "high pricing scheme" to save on innovation costs. In this range he charges higher prices than the precommitment ones in both periods to maintain the buyers confidence over durability. Lastly for c high enough the equilibrium matches once again the commitment solution up to the point where the innovation costs are so high that the seller prefers to stay out of the market altogether.

In the stationary equilibrium, Waldman's intuition about the seller's tendency to innovate "too much" from an ex-ante, profit maximizing perspective still applies

[31] Actually I prove that the stationary equilibria is the unique equilibrium of the game for c equal or "close enough" to zero.

(Although with some caveats as for some values of c there is no conflict[32]), but his conjecture that

> "...the firm might be able to mitigate the effects of this time inconsistency problem by choosing a technology that is excessively costly to improve." (Waldman 1996 pp. 593)

does not hold when one allows for heterogeneous consumers. In this model, were the seller able to choose from more than one production technology at the beginning of the game, he will always choose the most efficient one despite his (eventual) time inconsistency problem. This can help explain why firms such as "Microsoft, do not seem to be taking any such actions" (Waldman 2003, pp.147) that constrain their own ability to introduce upgrades.

Why Waldman's conjecture does not apply? As I have shown that the value of the shutdown game can indeed decrease with the continuation value of the game due to the seller's dynamic inconsistency, it is legitimate to question whether increasing the innovation cost can actually *raise* the seller profitability in this extended framework as it *mitigates* the time inconsistency problem, if any.

The intuition goes as follows. The marginal impact of innovation costs can be decomposed in two effects. First, higher costs decrease the continuation value of the game since they are actually incurred in equilibrium (negative effect). Second, in those instances where an intertemporal conflict arises, they reduce the relative cost of implementing the high pricing regime because they reduce the temptation to innovate (positive effect). In other words, lower expected earnings from innovation relax the "credibility constraint" and permit to safely expand supply in earlier periods. The combined effect is always negative as the higher expenses incurred due to raising costs outweigh the marginal benefits which are bounded above by one. So a firm would *never* gain by constraining his own ability to practice destructive creation through increasing its costs.

Finally, the model reproduces the cyclic price patterns that characterize most durable goods markets. Within each cycle, prices decrease due to standard Coasian dynamics. However when a new model is created, the introduction (or first period) price jumps up to the price at which the previous version was introduced. In this particular framework introductory prices do not reflect the incremental value of the new products (as one would expect if the new products were of higher quality). They are instead correlated with the introductory prices of previous product lines, something that could be amenable to empirical investigation.

[32] The fact that the seller ex-ante and ex-post incentives to innovate are aligned for some c is deliberately not stressed throughout the paper as it is a consequence of the discrete nature of the model.

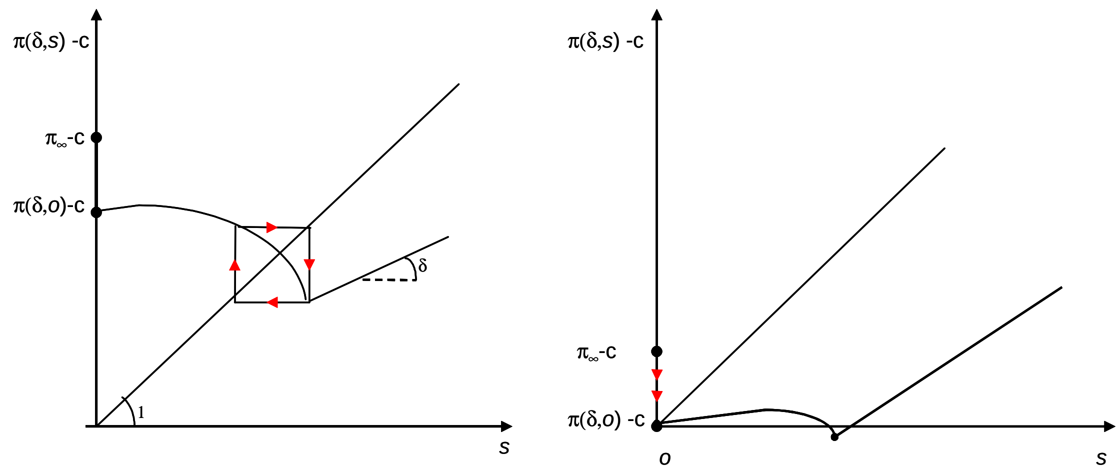

Figure 5: Non-stationary equilibria: (a) Cycling cycles (b) Innovation trap

IV. Robustness and extensions: Innovation Traps and Cycling Cycles.

This section discusses a number of features of the equilibrium that have been previously assumed away. First, it considers the effect of relaxing the assumption that there is an epsilon sunk cost that should be paid to provide an extra period of durability. Then it characterizes the non stationary equilibria of the (extended) model. Finally it provides a sufficient condition for the stationary equilibrium to be unique.

Consider the stationary equilibrium. The assumption that staying idle on the market is costly has bite only when the (expected) value of an innovation is equal to zero (or when $s = 0$ in section 2 notation). Therefore it has bite only when the innovation costs are high enough to drain all the revenues from future sales. Let π_∞ denote the revenues that the seller makes if he commits to never innovate.[33] If we replace the assumption above with a milder one, namely that lower (last period) prices are never "cue" of higher durability[34] (AA), then Lemma 1 and hence proposition 2 continue to hold. For every c a stationary equilibrium exists and is still unique but characterized by entry for cost levels up to $\pi_\infty(> \pi(\delta, 0))$. The higher the cost, the lower the frequency of innovation that keeps the continuation profits equal to zero. When $c \in [\pi(\delta, 0), \pi_\infty)$ the seller is caught in an "innovation trap". He is unable to make any profits even if he could make some by simply abstaining from innovating which is a weakly dominated strategy. In this cost region the mere

[33] Notice that π_∞ is nothing else then the value of the (unique) equilibrium of a standard durable good model in which the utility that a type b gets upon purchase is constant over time and given by $b/(1-\delta)$ and there are no fixed costs.

[34] Formally, it is sufficient to require the "expected" residual durability at the time everybody purchases one unit (i.e. in the last period of sales) to be non decreasing in the last period of sales' price.

fact that the seller is expected to behave opportunistically prevents him from asking higher prices on the market.[35]

Other (non-stationary) equilibria may arise for two mutually exclusive reasons depending on the cost level c. First, since the slope of the value function can exceed 1 in absolute terms then other sequences $\{s^e_\tau\}_\tau$ that satisfy conditions (a) and (b) may exist in addition to the fixed point. For instance there could exist converging patterns, i.e. sequences of expectations that converge asymptotically to the fixed point[36] as well as cycling patterns as the one depicted in figure 5a. Second, if the cost of innovation is in $[\pi(\delta,0),\pi_\infty(\delta,0)]$ then any sequence of the form $(s_1,0,0,..)$ with $s_1\in[0,\pi_\infty-c]$ can be supported as an equilibrium outcome (figure 5b). The expectation of making no profits out of innovating may still justify to make some positive profits in the first cycle. For instance if $c\in[\pi(\delta,0),\pi_\infty)$ there always exists an equilibrium in which the seller never innovates (as the value of an innovation is zero) whose associated profits are given by $\pi_\infty-c>0$ where c is the entry cost.

Notice that these latter outcomes dominate the stationary equilibrium in which the seller is caught in an innovation trap. Although stationary equilibria are typically more attractive, innovation traps generated by "pessimistic" expectations over durability are somewhat less likely to arise since in the repeated game the seller can frustrate these expectations and provide higher durability at no cost. The following corollary of theorem 1 gives a sufficient condition for the stationary equilibrium to be unique.

Corollary 1 *If $c\in[0,\pi(\delta,0))$ and $|\partial\pi(\delta,s)/\partial s|<1$ $\forall s$ then the equilibrium of the innovation game is unique.*

The first condition eliminates equilibria of the latter type as it insures that the continuation value of the game is always positive. The second condition guarantees that π^{-1} is a contraction mapping and therefore that any sequence starting at any value other than the fixed point will diverge.

V. Discussion

An interesting question is what would happen if the government could affect the incentives to practice destructive creation, for instance, mandating the provision of security support for a minimum number of years. For this purpose consider the problem of a regulator who can affect the cost c. Define welfare as the discounted

[35] In a previous version of this paper (available on request) I show that the above analysis remains valid even absent any restriction for the case $s=0$, at the cost of higher complexity.

[36] If $\partial\pi(\delta,s)/\partial s<-1$ for some s then one can find a cost range in which $\pi(\delta,s)-c$ is a contraction mapping for s close enough to the fixed point.

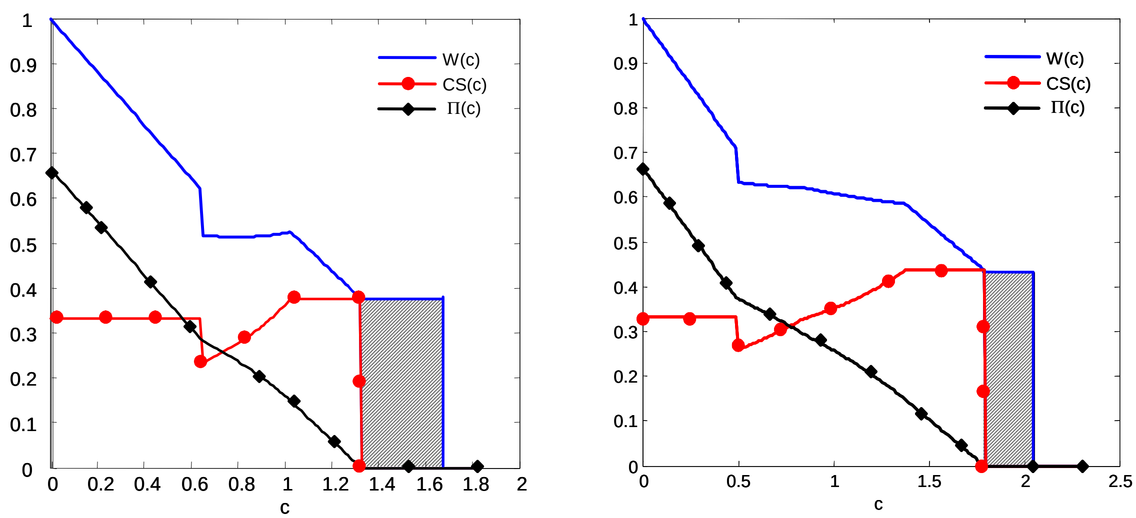

Figure 6: Total Surplus and its decomposition (a) $\delta = 1/5$ (b) $\delta = 2/3$

total surplus, that is, the sum of the consumers' surplus and the seller's profits gross of innovation costs.

Since we already know that equilibrium profits are non increasing in c, consider consumer surplus. A higher c affects consumers' surplus in two, different ways depending on whether it increases or not the equilibrium durability. Suppose that this is not the case, i.e. suppose that a marginal increase of the cost does not trigger any reduction in durability. If the seller is operating in the high pricing region then such increase relax the "credibility" constraint and therefore result in lower prices and in higher consumers' surplus. In all other cases higher costs simply lower profits as the seller keeps on charging the precommitment prices.

Conversely a marginal increase in c may trigger higher durability. In this case the impact on consumers' surplus is typically ambiguous. Increasing the length of each cycle clearly increases the number of equilibrium offers and therefore the number of consumers who have access to the good, but at the same time alters the relative price and period at which different consumers join the owners' group. Therefore consumers' surplus can again increase with the equilibrium length of each cycle depending on the parametrization of the model. If the resulting increase in consumers' surplus outweighs the seller's losses then total welfare may well *increase* with the innovations costs.

Figure 6 depicts total surplus as a function of c for the two-period game introduced earlier.[37] Consider consumers' surplus. The vertical drop, mirrored in total

[37] The welfare function is simulated using the same parametrization for the two-period game employed throughout the article. Values are in % of the total welfare under full commitment

welfare, corresponds to the case in which a marginal increase in c triggers higher durability. In this specific case, for both discount factors, consumers are on average worse off when the equilibrium length of each cycle increases from one period to two periods. The subsequent increasing portion of the curve corresponds to the case in which higher costs relax the dynamic inconsistency constraint. Interestingly, figure 6(a) (as opposed to figure 6(b)) illustrates a case in which the resulting increase in consumers' surplus outweighs the seller's losses with the result that total welfare *increases* with c despite the social waste due to higher costs. Worth noticing is also the shaded region on the right side of both plots. It reflects the seller's indifference between entering or not when $c \in [\pi(\delta, 0), \pi_\infty]$. Conditional on entry, the seller is caught in the "innovation trap" described above in which expected profits are equal to zero. Since consumers always benefit from having access to the monopolist's product, in this region welfare depends trivially on the equilibrium probability of entry.

These considerations suggest that the regulator's task of fine tuning the cost c is particularly arduous in this framework. Irrespective of how much consumers' surplus is weighed relative to the monopolist's profits, increasing the cost of destruction can backfire, leading to lower surplus for all parties involved.

A. Non destructive creation

Clearly allowing for "non empty" (or improved) innovations won't change much as long as the seller can likewise destroy the value of old units. Improved versions increase the temptation to practice destructive creation as consumers value more the new generations of goods. However the qualitative results remain unchanged.[38] Yet with one important conceptual difference. In this paper destroying old products *restores* market power preventing Coasian dynamics which usually ends up hurting consumers' welfare. When innovations are not empty, destructive creation serves a second, more noble, purpose. As Fishman and Rob (2000) point out, when a durable good monopolist introduces a new, improved version[39] he can only charge for the *incremental* value the new product provides until its replacement by a yet-better model, as otherwise consumers would wait the next generation of goods. On the other hand such incremental value is enjoyed forever which implies that the incentives to invest are below the social optimum level. Destructive creation helps the seller to recover the full value of the innovation and thus preserves the incentives to innovate. This argument adds a further cautionary note for the regulator who wants to forbid this practice.

calculated in $c = 0$.

[38] A suitable renormalization of the model would preserve all the qualitative results.

[39] In their model the new version incorporates all the previous improvements.

Consider now the more general case of genuine, non destructive innovation (i.e. "Creative Destruction"). An important question is whether the *mere* introduction of superior products can be itself a means of destroying the value of previous versions. If rational buyers are willing to pay less for the current product whenever they expect future innovations then it is "as if" the sellers are actually destroying (at least part of) the value of their previous versions with their new introductions. In such cases one would expect, among other things, analogous effects to arise.

One reason why this could happen is due to a "replacement effect" on the willingness to pay. More precisely, consider a simple extension of the two-period illustration of section 2 with the twist that the seller can introduce a superior product rather than shutting down and that old products continue to be fully functional. The shutdown reward can be interpreted as the expected discounted value of the innovation. In a two-period world with no overlapping innovations, no second hand markets and anonymous buyers, the mere expectation of *replacing* the old good (i.e. substituting your old laptop with a new one) lowers the willingness to pay of repeated purchasers to the equivalent value of one period of service. The (classic) case of the Osborne Computer Corporation illustrates one instance of this phenomenon. At the beginning of the 80's the company, (that invented the first mass-produced portable computer) went unexpectedly bankrupt as the announcement of a new line of revolutionary products killed the demand for the company's existing products, causing financial distress.

Previous related works in this literature include Fudenberg and Tirole (1998) and Lee and Lee (1998) who present a two-period model of technological innovation with heterogeneous buyers in which a seller introduces an improved product in period 2. However in both articles technological progress is exogenously given[40] and hence there is no role for destructive creation. Nahm (2004) endogenizes the R&D decision to capture the interactions between the outstanding stock of the old product and the incentives to introduce a new one. In his paper (net sales case) the existence of perfect second hand markets compensates repeated purchasers and thus offsets the "replacement effect" described above (there is no destruction). In analogy, one can conjecture than any mechanism that permits to compensate repeated purchasers (i.e. offer discounts) will have the effect of mitigating (or even assume away) the "destructive" downside of the innovative activity.

VI. Concluding remarks

Destructive creation offers a stylized description of many markets and industries. In particular it offers an interpretive key for recent business cases of destruction

[40] In Lee and Lee (1998) innovation is actually a choice variable but the seller commits to R&D before the first period of sales.

as well as for other classic court cases concerning the issue of aftermarket monopolization. In principle banning altogether the use of this mechanism would have the desirable effects of restoring Coasian dynamics and of preventing the social waste that comes with empty innovations. This could be done, for instance, by requiring after-sale services, extending warranties, mandating compatibility or preventing and monitoring the institution of new industrial standards of little (or no) incremental value which trigger waves of mass replacements. However, such plan of action would require value judgments on the nature of innovations and assessments of the firms' cost schedules in the after-markets. Furthermore, it seems more reasonable to think that a regulator can, at best, increase the cost of practicing destructive creation. As we have seen, an intervention in this direction can potentially reduce both the total and the consumers' surplus even if new vintages are indeed of no additional value.

One compelling research avenue is to identify under what conditions the mere introduction of superior products can generate a similar dynamics in the more complex setting with recurrent, *non destructive* innovations. On the empirical side event studies could be performed to assess what is the impact (if any) of the introduction (or of the expected introduction) of a new generation of products on the value of the old ones through measurements, for instance, of prices' variations in the primary and secondary markets. These and other related issues remain the topic of future research.

VII. Appendix

The proof of lemma 1 and Proposition 2 are adapted from Fudenberg, Levine and Tirole (1985). Subscripts attached to mappings denote the number of sales' periods left before shutdown in equilibrium unless otherwise indicated.

A. Proof of Proposition 1

If the seller can commit to shutdown time (or can guarantee any durability he wants) then the utility that type b gets from purchasing in the first period a good that lasts up to T is given by $b\sum_{i=1}^{T}\delta^{i-1}$ In a related context Stokey (1979) and Riley and Zeckhauser (1983) proved that the optimal precommitment strategy is to "hold firm" and charge a fixed price throughout the horizon. Type b buys in the first period iff $b \geq p/\sum_{i=1}^{T}\delta^{i-1}$ or doesn't buy at all. The seller's program is therefore given by $\max_p \left[1 - F\left(p/\sum_{i=1}^{T}\delta^{i-1}\right)\right] p + \delta^T s$ equivalent to:

$$\max_r \left[1 - F\left(r\right)\right] r \sum_{i=1}^{T} \delta^{i-1} \tag{5}$$

since $p = b\sum_{i=1}^{T}\delta^{i-1}$. From (5) it is clear that the optimal price does not depend on T and that it should be equal to $\sum_{i=1}^{T}\delta^{i-1}$ times the rental price. Consider the associated envelope $\pi_{fc}(T) = \left[1 - F\left(r^*\right)\right] r^* \sum_{i=1}^{T}\delta^{i-1} + \delta^T s$ where r^* is any solution of (5). $\pi_{fc}(T+1) - \pi_{fc}(T) = \delta^T(\left[1 - F\left(r^*\right)\right] r^* - (1-\delta)s)$. The latter expression is greater than zero whenever $s < \frac{[1-F(r^*)]r^*}{1-\delta}$ which establishes the result.

B. Proof of lemma 1

The proof is divided in three parts.

1) [sorting condition] Notice that in any equilibrium the residual set of buyers following an offer p_t is the prior set $[\underline{b}, b_t]$ truncated from above at some point $b_{t+1} \leq b_t$ (i.e. a sorting condition holds) where b_t is defined as the owner with the lowest valuation at time t. To see this let $\boldsymbol{\xi}_m : \mathbf{H} \longrightarrow \Delta\left[\mathbb{R}_+ \times \{0,1\}\right]$ denote a (behavior) strategy for the seller and $\boldsymbol{\xi}_b : \mathbf{H} \longrightarrow \{0,1\}$ denote a (pure) strategy for type b where $\mathbf{H}$ represents the set of all possible histories in every period in which shut down has not occurred and $\Delta(\cdot)$ is the set of all probability distributions over prices and shutdown decisions. Given any seller's profile type b buys at time t iff

$$\sum_{i=t}^{T} \delta^{i-t} b - p_t \geq \delta V_b(b, H_t, T) \tag{6}$$

Where $V_b(b, H_t, T)$ is his valuation at time $t+1$ given history H_t when the monopolist is expected to shutdown in period $T+1$ which is equal to:

$$\sum_{j=t+1}^{T} \delta^{j-(t+1)} \xi_j(b, H_t) \left[\sum_{i=j}^{T} \delta^{i-j}(b - p_i) \right]$$

if $T \geq t+1$ or 0 otherwise. The term in brackets is the discounted utility flow conditional on purchase at time j; $\xi_j(b, H_t)$ is the equilibrium probability, conditional on today's information H_t that purchase is made at time j. Let $b' > b$. Since type b can always mimic b''s optimal strategy, that is, accept exactly when b' accepts, then it should be that:

$$V_b(b, H_t, T) \geq \sum_{j=t+1}^{T} \delta^{j-(t+1)} \xi_j(b', H_t) \left[\sum_{i=j}^{T} \delta^{i-j}(b - p_i) \right]$$

which in turn implies an upper bound on the difference:

$$V_b(b', H_t, T) - V_b(b, H_t, T) \leq \sum_{j=t+1}^{T} \delta^{j-(t+1)} \xi_j(b', H_t) \left[\sum_{i=j}^{T} \delta^{i-j}(b' - b) \right]$$

Intuitively, the difference between the two continuation values cannot be greater because otherwise type b would obtain more by mimicking type b'. This observation coupled with the fact that $\delta < 1$ and that $\sum \xi_j = 1$ implies that

$$\delta \left[V_b(b', H_t, T) - V_b(b, H_t, T) \right] < \sum_{i=j}^{T} \delta^{i-j}(b' - b) \tag{7}$$

Subtracting side to side (6) into (7) we obtain:

$$\sum_{i=t}^{T} \delta^{i-t} b' - p_t > \delta V_b(b', H_t, T) \tag{8}$$

that is if type b finds optimal to buy today at price p_t (i.e. (6) holds) then any other buyer $b' > b$ accepts the same price with probability 1.

2) [lowerbound on prices] Notice that the seller never (i.e. in no subgame) charges a price below $\underline{b}$[41] whenever $s > 0$. To see this let $\underline{p}$ denote the infimum of the prices offered by the monopolist in any subgame. At this price everybody buys as a better deal in the future cannot be expected. Observe that 1) $\underline{p}$ should necessarily be lower or equal than $\underline{b}$ as otherwise type $\underline{b}$ would never accept it, as shutdown always occurs in any subgame in which he has already purchased; 2) $\underline{p}$ should necessarily be greater than $-\infty$ since gains from trade are finite. To see that $\underline{p} < \underline{b}$ cannot be an equilibrium offer assume that tomorrow's price is expected to be $\underline{p}$ (i.e. minimize

[41] this trivially extends to the case $s = 0$ under our additional assumption that there is an epsilon sunk cost to stay one more period on the market.

all buyers willingness to pay) and consider type $\underline{b}$'s problem. Type $\underline{b}$ prefers to buy today iff $\underline{b} - p \geq \delta(\underline{b} - \underline{p})$ i.e. iff $p \leq \underline{p} + (1-\delta)(\underline{b} - \underline{p})$. This implies that as long as $\underline{p} < \underline{b}$ there always exists a price $p > \underline{p}$ such that type $\underline{b}$ and thus any other type buys. Therefore the Seller can raise $\underline{p}$ by a discrete amount and still have full demand in every such subgame, a contradiction.

3) [finite nature] Given the sorting condition and the lowerbound on prices I want to show that there always exists a given posterior b^* beyond which charging $\underline{b} > 0$ and selling to everybody is a dominant strategy. Assume that buyers $b > b^e$ have already bought the good and that all the remaining buyers play myopically and accept all prices less than their valuation. Moreover assume that everybody expects an infinite durability. If a monopolist wishes to charge $\underline{b}$ against this optimistic and myopic buyers he will do so against any other buyer, since any other (rational) buyer would be less likely to purchase. Myopic and optimistic buyers buy iff $p \leq \frac{b}{1-\delta} \Leftrightarrow b \geq p(1-\delta)$. Profits are given by

$$\pi(p) = [F(b^e) - F(p(1-\delta))]\, p + \frac{\delta}{1-\delta} \int_{\underline{b}}^{p(1-\delta)} bf(b)db$$

where $\frac{\delta}{1-\delta} \int_{\underline{b}}^{p(1-\delta)} bf(b)db$ is clearly an upperbound of the continuation value (it has been obtained assuming that tomorrow the monopolist would be able to perfectly discriminate against all remaining myopic and optimistic buyers). Differentiating with respect to p yields:

$$\frac{\partial \pi}{\partial p} = [F(b^e) - F(p(1-\delta))] - f(p(1-\delta))\,[1-\delta]^2 \tag{9}$$

since $f > 0$ there always exists a threshold $b^* > \underline{b}$ such that $\forall b^e < b^*$ $\frac{\partial \pi}{\partial p} < 0$ for any p and thus $p^* = \underline{b}$ and game ends since (9) is increasing in b^e. Lastly I show that there always exists a $T < \infty$ such that residual demand drops below b^* within T periods when $s = 0$. Suppose not. Notice that, by charging $p = \underline{b}$ the Seller can guarantee himself at least $[F(b^*) - F(\underline{b})]\, \underline{b} \equiv k > 0$ that implies that the continuation value of the game $V_s(b^e)$ should always be at least k as otherwise deviating to $p = \underline{b}$ would be profitable. Given b^e an upperbound on what the seller can get selling would be given by:

$$B(b^e) = \int_{b^*}^{b^e} \frac{b}{1-\delta} f(b)db$$

once again assuming that buyers are both myopic and optimistic. Since the latter expression strictly decreases with b^e, there exists another threshold b^{**} such that $B(b) < k$ iff $b^e < b^{**}$. Therefore the posterior should never drop below b^{**} as otherwise V_s drops below k. Similarly one can construct a sequence $\{b_n\}_n$ of such thresholds with the property that:

$$\|b_n - b_{n-1}\| \geq \frac{2(1-\delta)k}{\overline{f}} > 0$$

which in turn implies that eventually $b_n > b^e$, i.e. that the posterior cannot drop below b^e as otherwise the seller would find profitable to deviate to $p = \underline{b}$ and game ends.

C. Proof of proposition 2

Wlog prices are always restricted in some compact set $[\underline{b}, \overline{p}]$ as nobody would accept a price $p > \overline{b}/(1-\delta)$. A pure strategy for the seller is a mapping $\xi : \mathbf{H} \to [\underline{b}, \overline{p}] \times \{0, 1\}$; a continuation strategy, given history h_t is a mapping $\xi : \mathbf{H}_t \to [\underline{b}, \overline{p}] \times \{0, 1\}$ where $\mathbf{H}_t \subseteq \mathbf{H}$ is the set of all possible continuation histories given h_t. As the Seller may well randomize, any continuation strategy specifies a probability distribution over prices and shutdown probabilities for every subgame. Since all players care only about expected prices and shutdown probabilities, any two (behavior) strategies that generate the same expectations for the same history are considered as equivalent. To perform some comparative statics it is necessary to specify one ordering of sets which will be useful later on. Consider a generic compact and real valued correspondence $\phi : X \subseteq \mathbb{R} \to Y$. According to Veinott (1989) ordering of sets, I will say that, given any two x, x' such that $x' \geq x$ with $x' \neq x$, for any $a' \in \phi(x')$, $a \in \phi(x)$, ϕ is non decreasing whenever $\max\{a', a\} \in \phi(x')$ and $\min\{a', a\} \in \phi(x)$; is strictly increasing whenever $a' \geq a$ and is strongly increasing whenever $a' > a$.

Define the maximization problem $P(b^e, \beta, \pi, s, \overline{p})$ as it follows:

$$\max_{\psi \in [0,1]} \left[\max_{p \leq \overline{p}} (F(b^e) - F(\beta(p))p + \delta\pi(\beta(p), s) \right] \psi + (1-\psi)s$$

where β and π are well defined continuous functions. Let $\sigma(b^e, s, \overline{p})$ denote the associated solution correspondence with respect to p, $\widehat{\sigma}(b^e, s, \overline{p})$ its convex hull and $\psi(b^e, s, \overline{p})$ the associated solution correspondence with respect to ψ. The argument $\overline{p}$ will be omitted whenever $\overline{p} = \overline{b}/(1-\delta)$ (unrestricted program). Notice that both σ and ψ and therefore $\widehat{\sigma}$ correspondences are non empty, compact valued and upper hemi-continuous by Berge's theorem. Since the objective function is continuous and $(F(b^e) - F(\beta(p))p$ is strictly supermodular in b^e ($f(b^e) > 0$) then $\sigma(b^e, s)$ and therefore $\widehat{\sigma}(b^e, s)$ are strictly increasing in b^e (Topkis 1978). Lastly notice that the associated value function increases with b^e and s whenever $\pi(\beta(p), s)$ increases with s. Notice that this modified[42] specification of the objective function permits to separate the pricing strategy from the shutdown policy. It is then possible to define

[42] I'm taking the $\max_\psi (1-\psi)(\max_p(\cdot))$ rather than $\max_\psi \max_p(\cdot)$.

a strategy as any pair of functions $\xi \equiv (\boldsymbol{\sigma}(H), \boldsymbol{\psi}(H))$ that specify an (expected) price and a probability of shutdown for every subgame.

Let $\pi_0(\beta(p), s) = s$. Define recursively the n-period game and its associated problem $P(b^e, \beta_n, \pi_{n-1}, s, \overline{p})$ as the game where the seller is constrained to shutdown after at most n periods of sales where β_n maps prices into indifferent buyers and $\pi_{n-1}(\beta_n(p), s)$ is the value of the $n-1$-period game. If β_n and π_{n-1} are well defined functions then the problem is well defined. Denote with σ_n, $\widehat{\sigma}_n$ and ψ_n the associated solution correspondences. Lastly let $\xi_n^*(H) \equiv (\boldsymbol{\sigma}_n(H), \boldsymbol{\psi}_n(H))$ denote an equilibrium profile for the n-period game.

Consider the one-period game. Obviously $\beta_1(p) = p$ as long as $p \leq b^e$ and hence its associated problem is well defined. Let $\widetilde{b}_1(s) = \{\max b^e \in \mathbb{R}_+ : \pi_1(b^e, s) = s\}$ be the threshold on residual demand that leaves the seller indifferent between shutdown and one period of sales. Clearly shutdown is a strictly dominant strategy whenever $b^e < \widetilde{b}_1(s)$, a weakly dominant strategy whenever $b^e = \widetilde{b}_1(s)$ and a dominated strategy whenever $b^e > \widetilde{b}_1(s)$. Let $\widetilde{p}_1(s) \equiv \widetilde{b}_1(s)$.

Lemma 2 *For any history such that $b_t < \widetilde{b}_1(s)$, $\xi^*(H_t) = \xi_1^*(H_t) = (\boldsymbol{\sigma}_1(b_t), \mathbf{0})$.*

The claim is that shutdown is actually a dominant strategy for any history such that $b_t < \widetilde{b}_1(s)$. To see this recall from lemma 1 that b^* is the posterior such that it is always optimal to charge $p_t = \underline{b}$ whenever $b_t \in [\underline{b}, b^*]$. If $b^* \geq \widetilde{b}_1(s)$ (case "s low") then, conditional on staying, the seller charges $p_t = \underline{b}$ which results in $F(b_t)\underline{b}$ profits less than s by definition of $\widetilde{b}_1(s)$. Consider now the complementary case: s such that $b^* < \widetilde{b}_1(s)$. If $b_t \in [\underline{b}, b^*]$ then, by the same logic, shutdown is a dominant strategy. Choose an ε such that $[F(b+\varepsilon) - F(b)]\,(b+\varepsilon)/(1-\delta) < (1-\delta)s$ and $b+\varepsilon < \widetilde{b}_1(s)$ for every $b \in \left[b^*, \widetilde{b}_1(s)\right]$. The claim is that if $b_t \in [\underline{b}, b^* + \varepsilon]$ then shutdown is a dominant strategy. To see this notice that if the seller does not shutdown at time t then either $b_{t+1} \leq b^*$ or $b_{t+1} \in [b^*, b^* + \varepsilon]$. In the former case shutdown takes place at time $t+1$ with probability one and therefore $\beta = \beta_1$, $\pi = s$ and the seller gets at most $\pi_1(b^e, s)$ which is less than s by definition of $\widetilde{b}_1(s)$. In the latter case an upperbound on what the seller can get is given by $[F(b^* + \varepsilon) - F(b^*)]\,(b^*+\varepsilon)/(1-\delta) + \delta s$ which is less than s by definition of ε. Therefore, for any history h_t such that $b_t \in [\underline{b}, b^* + \varepsilon]$ shutdown is always a dominant strategy. The same is true by induction for any $b_t \in \left[b^*, \widetilde{b}_1(s)\right)$.

Lemma 3 *If $p_t < \widetilde{p}_1(s)$ then $b_{t+1} < \widetilde{b}_1(s)$.*

The claim is that when a price $p_t < \widetilde{p}_1(s)$ is charged then shutdown occurs tomorrow with probability one. Suppose not. Then it should be that $b_{t+1} \geq \widetilde{b}_1(s)$ and that $\psi_{t+1} > 0$ as otherwise type $\beta_1(p_t) < \widetilde{b}_1(s)$ should have accepted p_t. Type $\widetilde{b}_1(s)$

prefers to buy today rather than tomorrow iff $\widetilde{b}_1(s) \geq p_t - \delta p_{t+1}\psi_{t+1}$. The latter is always lower or equal than $p_t - \delta \underline{b}$ since by lemma 1 the seller never charges a price lower than $\underline{b}$. Therefore type $\widetilde{b}_1(s)$ strictly prefers to buy at time t rather than at $t+1$ whatever $p_{t+1}\psi_{t+1}$. For the same reason he would not buy later on and therefore $b_{t+1} < \widetilde{b}_1(s)$, a contradiction.

Consider the two-period game. $\beta_2(p)$ is nothing else than the (set) of types b that, given price p, satisfies[43]:

$$b\left[1 + \delta\psi_1(b)\right] - p \in \delta\left[b - \widehat{\sigma}_1(b)\right]\psi_1(b)$$

which is equivalent to

$$p \in b + \delta\psi_1(b)\widehat{\sigma}_1(b) \tag{10}$$

The right hand side of (10) represents the (net) utility that type b gets from purchasing today if he were the indifferent buyer as a function of next period price correspondence $\widehat{\sigma}_1(b)$ and the probability that there will be no future $\psi_1(b)$.
Any (continuation) strategy of the Seller specifies a curve $f : [\underline{b}, b^e] \rightarrow P$ such that[44]:

$$f(b) \subseteq b + \delta\widehat{\sigma}_1(b)\psi_1(b) \qquad \forall b \tag{11}$$

As $\widehat{\sigma}_1(b)\psi_1(b)$ is a strictly increasing, compact and convex valued, uhc correspondence then $b + \delta\widehat{\sigma}_1(b)\psi_1(b)$ is strongly increasing and uhc. In other words $b + \delta\widehat{\sigma}_1(b)\psi_1(b)$ depicts an increasing curve with some vertical but no horizontal traits. Hence there exists a unique curve that satisfies property (11) which is invertible; define such inverse as $f^{-1} = \beta_2(p)$. β_2 is therefore a well defined, non decreasing function. Notice that $\forall p < \widetilde{p}_1(s)$, $\beta_2(p) = \beta_1(p)$ as the right hand side of (10) collapses to b.
The function $\beta_2(p)$ has therefore some "flat spots" where increasing the current price does not reduce the number of buyers as higher prices today are associated with higher prices tomorrow. Notice moreover that for any p_t there exists a unique pair $(b, p_{t+1} \in \psi_1(b)\widehat{\sigma}_1(b))$ that satisfies $\beta_2(p_t) = p_t - \delta p_{t+1}$ and therefore that the choice of any price p_t uniquely determines both the indifferent buyer and the (expected) next period price.

Recall that $\widetilde{p}_1(s) \equiv \widetilde{b}_1(s)$ and define recursively $\widetilde{b}_n(s) \equiv \{\max b^e \in \mathbb{R}_+ : \min \sigma_n(b^e, s) < \widetilde{p}_{n-1}(s)\}$ and $\widetilde{p}_n(s) \equiv \widetilde{b}_n(s) + \delta \min \sigma_n(\widetilde{b}_n, s)$. Notice that $\beta_n(p_t) < \widetilde{b}_n(s)$ for every $p_t < \widetilde{p}_n(s)$ by definition of $\widetilde{p}_n(s)$. Suppose that:

[43] To save notation the argument s is dropped in the following paragraph.
[44] For instance one strategy (for $b \geq b_1$) may consist in charging tomorrow the highest price whatever today's price or: $f(b) = b + \delta \max\{\widehat{\boldsymbol{\sigma}}_1(b)\} \qquad \forall b$

a. $\beta_n(p_t) = p_t - \delta p^e_{t+1}(p_t)$ is a well defined, continuous and non decreasing function where $p^e_{t+1}(p_t) \in \psi_{n-1}(\beta_n(p_t))\widehat{\sigma}_{n-1}(\beta_n(p_t))$ is the unique solution to $p_t - \delta p^e_{t+1} = \beta_n(p_t)$.

b. $\pi_{n-1}(b^e, s)$ is continuous and increasing in both b^e and s.

c. $\sigma_{n-1}(b^e)$, $\sigma_{n-1}(b^e, \widetilde{p}_{n-1}(s))$ and $\psi_{n-1}(b^e)$ are non empty, compact valued, uhc and strictly increasing correspondences.

d. For any history h_t such that $b_t < \widetilde{b}_{n-1}(s)$, then $p_t < \widetilde{p}_{n-2}(s)$ and shutdown occurs after at most $n-2$ periods of sales with probability one which implies that $\xi^*(H_t) = \xi^*_{n-2}(H_t)$[45].

e. If $p_t < \widetilde{p}_{n-1}(s)$ then b_{t+1} is necessarily lower than $\widetilde{b}_{n-1}(s)$ and shutdown occurs after at most $n-1$ periods of sales.

I shall show that the same properties hold for the n-period game.

Consider the problem $P(b^e, \beta_n, \pi_{n-1}, s, \overline{p})$. Notice that since σ_n is a strictly increasing correspondence then for every $b_t < \widetilde{b}_n(s)$ any element of $\sigma_n(b_t)$ should be lower than $\widetilde{p}_{n-1}(s)$ by definition of $\widetilde{b}_n(s)$. If a price $p_t \in \sigma_n(b_t)$ such that $p_t < \widetilde{p}_{n-1}(s)$ is charged then by inductive hypothesis $\pi_{n-1} = \pi_{n-2}$, $\beta_n(p) = \beta_{n-1}(p)$ and therefore p_t should also be an element of $\sigma_{n-1}(b_t)$.

Lemma 4 *For any history h_t such that $b_t < \widetilde{b}_n(s)$, then shutdown occurs after at most $n-1$ periods of sales with probability one.*

To see this note that by (e.) it is sufficient to show that whenever $b_t < \widetilde{b}_n(s)$, charging any price greater or equal than $\widetilde{p}_{n-1}(s)$ cannot be optimal. Observe that by inductive hypothesis the seller can always guarantee himself $\pi_{n-1}(b_t, s)$ charging some price $p_t < \widetilde{p}_{n-1}(s)$ with $p_t \in \sigma_n(b_t)$ and moreover that any other price $p_t \notin \sigma_n(b_t)$ such that $b_{t+1} < \widetilde{b}_{n-1}(s)$ would lead to lower profits. It remains to be proved that given any price $p_t \geq \widetilde{p}_{n-1}(s)$ such that $b_{t+1} \geq \widetilde{b}_{n-1}(s)$ the seller cannot get anything better or equal. To see this, consider an ε such that $[F(b+\varepsilon) - F(b)]\,(b+\varepsilon)/(1-\delta) + \delta\pi_{n-1}(b, s) < \pi_{n-1}(b, s)$ for every b in $\left[\widetilde{b}_{n-1}(s), \widetilde{b}_n(s)\right)$ and such that $b + \varepsilon < \widetilde{b}_n(s)$. Consider $b = \widetilde{b}_{n-1}(s) + \varepsilon$. Clearly for every p_t is such that $b_{t+1} \geq \widetilde{b}_{n-1}(s)$ the seller gets at most $\left[F(\widetilde{b}_{n-1}(s) + \varepsilon) - F(b)\right](\widetilde{b}_{n-1}(s) + \varepsilon)/(1-\delta) + \delta\pi_{n-1}(\widetilde{b}_{n-1}(s), s)$ less than $\pi_{n-1}(\widetilde{b}_{n-1}(s), s)$. The same is true by induction for every b in $\left[\widetilde{b}_{n-1}(s), \widetilde{b}_n(s)\right)$.

Lemma 5 *If $p_t < \widetilde{p}_n(s)$ is charged then b_{t+1} should be necessarily lower than $\widetilde{b}_n(s)$.*

[45] For the case $n = 2$ let $p_0(s) = \underline{b} + \varepsilon$ be a dummy variable and assume (innocuously) that the seller is contrained to prices lower than p_0 whenever he shut downs ($\psi = 0$).

To prove this result it is sufficient to show that type $\beta_n(p_t) < \widetilde{b}_n(s)$ always accepts any such price. Suppose not. Then it should be that $b_{t+1} \geq \widetilde{b}_n(s)$. Consider p_{t+1}. By a straightforward revealed preference argument $p_{t+1} \geq \min\{\sigma_{n-1}(b_{t+1}, \widetilde{p}_{n-1}(s))\} \geq \min\left\{\sigma_{n-1}(\widetilde{b}_n(s))\right\} \geq \max\{\sigma_{n-1}(\beta_n(p_t))\} \geq p^e_{t+1}(p_t)$. At this price type $\beta_n(p_t)$ is "at most" indifferent between buying today at price p_t or tomorrow which implies that all types $b > \beta_n(p_t)$ strictly prefer to buy today rather than tomorrow (lemma 1) and hence that $b_{t+2} \geq \widetilde{b}_n(s)$. By the same logic they would not buy later on and hence they should all buy today at price p_t and therefore $b_{t+1} < \widetilde{b}_n(s)$.

Lastly I work backward one period to show that $\beta_{n+1}(p)$ is well defined. The valuation of a buyer who is just indifferent between paying p and waiting in the $n+1$ period game must satisfy $b\left[1+\delta\psi_n(b)\right] - p \in \delta\left[b - \widehat{\sigma}_n(b)\right]\psi_n(b)$ or $p \in b + \delta\psi_{n-1}(b)\widehat{\sigma}_{n-1}(b)$. Once again as $\widehat{\sigma}_n(b)\psi_n(b)$ is a strictly increasing, compact and convex valued, uhc correspondence then $b + \delta\widehat{\sigma}_n(b)\psi_n(b)$ is strongly increasing and uhc. Therefore it has a unique, continuous and non decreasing inverse function $\beta_n(p) = p - \delta p^e(p)$ where $p^e(p) \in \psi_n(\beta_n(p))\widehat{\sigma}_n(\beta_n(p))$ is the unique solution to $p_t - \delta p^e = \beta_n(p_t)$.

By lemma 1, there always exist a T^*+1 high enough such that $b_{T^*}(s) \leq \overline{b} < b_{T^*+1}(s)$ and the inductive process comes to an endpoint. Since to each first period choice $p_1 < p_{T^*}(s)$ is associated, by inductive hypothesis, a unique sequence of prices such that $\{p^e_i < p_{T^*-i}(s)\}_{i=1}^{T^*}$ then the equilibrium is unique up to the seller initial choice. The value of the game is therefore given by $\pi_{T^*}(\overline{b}, s)$ which is continuous and increasing in s by inductive hypothesis.

The threshold $\widetilde{b}_n(s)$ can be alternatively defined as $\{\max b^e \in \mathbb{R}_+ : \pi_{n-1}(b^e, s) \leq \pi_n(b^e, s)\}$ and hence interpreted as the threshold that leaves the seller indifferent between an n-period and an $n-1$-period game. The thresholds in corollary 1 are uniquely defined by $s_n = \left\{s \in \mathbb{R}_+ : \widetilde{b}_n(s) = \overline{b}\right\}$ since $\widetilde{b}_n(s)$ is increasing in s. Moreover notice that s_n decreases with n since $\widetilde{b}_n(s) < \widetilde{b}_{n+1}(s)$ for every s. For $s = s_n$ then the seller is actually indifferent between charging a price $p_t \in \sigma_n(\overline{b})$ with $p_t < p_{n-1}$ that results in $n-1$ periods of sales and $p_t \in \sigma_n(\overline{b})$ with $p_{n-1} \leq p_t < p_n$ that results in n periods of sales. In particular notice that s_1 is such that $\pi_1(\overline{b}, s_1) = \max_r[1 - F(r)]r + \delta s_1 = \pi_0(\overline{b}, s_1) \equiv s_1$which is nothing else than the value of the rental solution π_{fc}. Corollary 2 follows from the fact that $\widetilde{b}_1(s) < \overline{b}$ whenever $s < s_1$ and therefore shutdown is a dominated strategy.

D. Proof of Proposition 3

First notice that $(ii) \Rightarrow (i)$ since the seller can always commit to charge the equilibrium prices. (ii)'s proof is divided in two parts: 1) analogously to the no-commitment case, the optimal precommitment durability weakly decreases with the value of the outside option and 2) there always exists a positive and small enough δ such that for some values of s, $\overline{p}(s)$ exceeds the first period optimal precommitment price whereas both the precommitment and the equilibrium durability is 2.

(a) If the seller can commit to any durability he wants, the optimal pricing strategy does not depend on s. Let $\pi_0^c(\beta(p), s) = s$. Define recursively $\pi_n^c(b, s)$ and $p_n^c(b)$ for $n \in \{1, 2\}$ as respectively the value function and the (unique) solution of the maximization problem $P(b, \beta_n^c, \pi_{n-1}^c, s, \overline{b}/(1-\delta))$ where $\beta_1^c(p) = p$ and $\beta_2^c(p)$ is the unique solution of (2) when $p_2 = p_1^c(\beta_2^c(p))$. Define the thresholds $s_n^c = \left\{s \in \mathbb{R}_+ : \pi_n^c(\overline{b}, s) = \pi_{n-1}^c(\overline{b}, s)\right\}$ for $n \in \{1, 2\}$. The optimal durability weakly decreases with s if

(i) $s_2^c, s_1^c > 0$ and

(ii) $s_2^c < s_1^c$

First notice that $\beta_2^c(p) < p$ for every $p > \underline{b}$ which implies that $p_2^c(\overline{b}) \geq p_1^c(\overline{b}) = \arg\max_r[1 - F(r)]r > \underline{b}$ and therefore that $\pi_2^c(\overline{b}, 0) \geq \max_p[1 - F(\beta_2^c(p))]p > \max_r[1 - F(r)]r = \pi_1^c(\overline{b}, 0) > \pi_0^c(\overline{b}, 0)$. The latter coupled with the fact that $\partial\pi_n^c(\overline{b}, s)/s = \delta^n < 1$ implies (i). Let $J_n^c(\overline{b})$ denote the "value of sales" of the n-period game, that is $\pi_n^c(\overline{b}, s) - \delta^n s$. Showing that s_2^c is strictly less than s_1^c is equivalent to show that $J_2^c(\overline{b}) < J_1^c(\overline{b})(1 + \delta)$ since $s_2^c = (J_2^c - J_1^c)/\delta(1 - \delta)$ and $s_1^c = J_1^c/(1 - \delta)$. That the latter holds follows from the fact that $J_1^c(\overline{b})(1 + \delta)$ is equal to the full commitment payoff which should be necessarily greater than the no commitment one under the assumption that $\max_r[1 - F(r)]r > \underline{b}$.

(b) Since it is more convenient to work with marginal buyers rather than prices let $b_1^c(\beta) \equiv \arg\max_b[F(\beta) - F(b)]b$ and $b_2^c(\overline{b}) \equiv \arg\max_b[1 - F(b)](b + \delta p_1^c(b)) + \delta\pi_1^c(b, s)$ be the optimal precommitment indifferent buyers which are unique by assumption. Recall that $\forall s \in [0, s_2^c)$ the seller strictly prefers a two period game (and hence chooses $b_2^c(\overline{b})$) rather than a one period game. This implies, by a straightforward revealed preference argument, that the seller would do so even when he cannot resort to any commitment device. But since $\widetilde{b}(s)$ increases with s, if $\widetilde{b}(s) = b_2^c(\overline{b})$ for some $s < s_2^c$ then the seller cannot replicate the commitment solution in some open neighborhood of s_2^c. $\widetilde{b}(s) = b_2^c(\overline{b})$ whenever s is equal to:

$$\frac{[F(b_2^c(\overline{b})) - F(b_1^c(b_2^c(\overline{b})))]b_1^c(b_2^c(\overline{b}))}{1 - \delta} \tag{12}$$

($\equiv s_a$ in figure 2's notation). The issue is to find conditions under which the latter expression is lower than s_2^c and therefore conditions under which the seller cannot replicate the commitment solution for some $s < s_2^c$. Recall that s_2^c is such that $\pi_2^c(\bar{b}, s) = \pi_1^c(\bar{b}, s)$ or:

$$s_2^c = \frac{[1-F(b_2^c(\bar{b}))](b_2^c(\bar{b}) + \delta b_1^c(b_2^c(\bar{b}))) + \delta[F(b_2^c(\bar{b})) - F(b_1^c(b_2^c(\bar{b})))]b_1^c(b_2^c(\bar{b})) - [1-F(b_1^c(\bar{b}))]b_1^c(\bar{b})}{\delta(1-\delta)}$$

I want to find conditions on δ such that $s_2^c >$ (12) which is equivalent to:

$$\frac{[1-F(b_2^c(\bar{b}))]b_2^c(\bar{b})}{[1-F(b_1^c(\bar{b}))]b_1^c(\bar{b})} > \frac{1}{1+\delta b_1^c(b_2^c(\bar{b}))/b_2^c(\bar{b})} \tag{13}$$

From the definition of $b_2^c(\bar{b})$ notice that if $\delta = 0$ then $b_2^c(\bar{b}) = b_1^c(\bar{b})$ (the two period game coincides with the one period game since there is no future) and hence at $\delta = 0$ condition (13) is not satisfied. Using the fact that $1 + \delta b_1^{c\prime}(b_2^c(\bar{b})) \equiv \alpha$ is equal to one when $\delta = 0$, it follows that the derivative of the left hand side of (13) calculated in $\delta = 0$ is equal to zero since:

$$\left[\frac{\partial \frac{[1-F(b_2^c(\bar{b}))]b_2^c(\bar{b})}{[1-F(b_1^c(\bar{b}))]b_1^c(\bar{b})}}{\partial\delta}\right]_{\delta=0} = \left[\frac{1}{[1-F(b_1^c(\bar{b}))]b_1^c(\bar{b})}\left[\frac{\left[b_2^c(\bar{b})\right]^2 b_1^{c\prime}(b_2^c(\bar{b}))f(b_2^c(\bar{b}))}{\alpha^2[1-hr'_{-1}(b_2^c(\bar{b}))\alpha + \delta b_1^{c\prime\prime}(b_2^c(\bar{b}))/\alpha]}(1-\alpha)\right]\right]_{\delta=0} = 0$$

where $hr'_{-1}(b_2^c(\bar{b}))$ denotes the derivative of the inverse of the hazard rate of the distribution calculated in $b_2^c(\bar{b})$. On the other hand

$$\left[\frac{\partial - \frac{1}{1+\delta b_1^c(b_2^c(\bar{b}))/b_2^c(\bar{b})}}{\partial\delta}\right]_{\delta=0} = \left[\frac{1}{(1+\delta)^2}\right]_{\delta=0} = 1$$

which implies that, for any distribution F, condition (13) is always satisfied for some δ close enough to zero. Lastly it remains to show that $s_a < s_2$. To see this recall that the equilibrium profits $\pi(\bar{b}, s)$ are continuous in s by proposition 2 which together with the fact $\pi(\bar{b}, s_a) = \pi_2^c(\bar{b}, s_a) > \pi_1^c(\bar{b}, s_a)$ necessarily imply that there exists an open, non empty subset of $\mathbb{R}_+$ such that the equilibrium durability is two and the seller charges higher than precommitment prices in both periods.

A (formal) proof of *(iii)* is omitted and replaced by the following observation. When $s \in (s_a, s_2)$ the seller reacts to a marginal increase in the shutdown reward with a supply contraction to preserve his ex-post incentives to stay on the market. In other words the seller maximizes his discounted profits subject to an intertemporal constraint. Ceteris paribus, the (shadow) cost of shifting buyers and thus revenues from today to future periods is higher the *lower* the discount factor as part of this revenues come back under the form of discounted profits[46]. Given this observation it is straightforward to show that there always exists a low enough δ such that profits decrease with the shutdown reward.

E. Proof of Theorem 1

A history in period t is a sequence of prices, a sequence of innovations and a sequence of purchases by consumers. Since the equilibrium profiles of each cycle are uniquely[47] determined, through a dominance argument, by the (expected) continuation value of the game after an innovation takes place (proposition 2) then one can Wlog restrict attention to continuation profiles that depend only on events that occurred since this particular cycle begun. Given any sequence of continuation values $\{s^e_\tau\}_\tau$ such that (a) and (b) are satisfied, where $\tau = 1, 2, ..,$ indexes the innovations, it is always possible to construct at least an equilibrium. Consider the first order autonomous system:

$$s^e_\tau + c = \pi(\delta, s^e_{\tau+1}) \tag{14}$$

That any fixed point of $\pi(\delta, s) - c$ is a stationary equilibrium follows by definition. Since π is continuous, $\pi(\delta, 0) > 0$ and $\partial\pi(\delta, s^e)/\partial s^e < 1$ (whenever it exists) then a fixed point s^* necessarily exists and is unique for any $c \in [0, \pi(\delta, 0)]$ by Brouwer's theorem. For $c < \pi(\delta, 0)$, $s^* > 0$ and entry occurs with probability one.
In addition to fixed points, such dynamic system may make it possible the emergence of some phenomena, such as cyclical or converging patterns, that maybe rooted into equilibrium behavior. Consider the boundary-value problem given by (14) and any initial condition s^e_1 and define a (particular) solution trajectory as any sequence $\{s^e_i\}_i$ generated through (14). We are interested both in the asymptotic behavior of such trajectories (convergence towards a limit point or a limit orbit) and in the convergence process, if any, because any such trajectory that satisfies (a) and (b) constitutes another (non-stationary) equilibrium of the game. Because π maybe not monotonically increasing then the inverse mapping π^{-1} may well be a correspondence, so solution trajectories obtained through (14) may not be unique. However if $|\partial\pi(\delta, s^e_{\tau+1})/\partial s| < 1$ then $\widetilde{\pi}$, defined as the unique continuous selection of π, is a contraction mapping (and hence $(\widetilde{\pi} - c)^{-1}$ is not). Therefore any solution trajectory generated through (14) from some $s^e_1 \neq s^*$ either diverges away or is mapped to zero. $c < \pi(\delta, 0)$ guarantees that any trajectory that passes trough zero is mapped outside the domain. [Author cite here] extends corollary 3 to the case where no

[46] Formally the seller program is given by: $\max_x a(x) + b(x)\delta + \delta^2 s$ s.t. $b(x) + \delta s = s$. with $b' > 0$ and $a' < 0$ at the optimum.

[47] Recall that the equilibrium is unique "up to the seller initial choice" . Yet conditioning such initial choice over history won't change anything since the value of the game is constant over any such initial choice.

restrictions are imposed to deal with the $s = 0$ case.

F. Proof of Proposition 4 and 5

If $c = 0$, (4) becomes $\pi(\delta, s^*) = s^*$ which is true iff $s^* = \frac{[1-F(r^*)]r^*}{1-\delta}$ where r^* is the rental solution (see the proof of proposition 1 for details). Uniqueness comes from the fact that for $s \in (s_2, s^*]$ (i.e. for c low enough) the seller innovates every period hence $\pi'_s = \delta > 0$.
Differentiating (4) yields $\partial\Pi(\delta, c)/\partial c = -[1 - \pi'_s(\delta, \Pi(\delta, c))]^{-1} < 0$ since π'_s is bounded from above by 1. (ii),(iii),(iv) and (v) follow straightforwardly from proposition 3.

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