The Long-Run Growth Effects of R&D Subsidies*

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Abstract: This paper presents a model of R&D-driven growth without scale effects where firms can engage in both horizontal and vertical R&D activities. Unlike in earlier models of R&D-driven growth without scale effects by Jones (1995), Segerstrom (1998) and Young (1998), R&D subsidies can have long-run growth effects. Indeed, for a wide range of parameter values, a permanent increase in the R&D subsidy rate decreases the long-run rate of economic growth. An intuitive explanation for why R&D subsidies can retard growth is provided.

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1 Introduction

Perhaps the main conclusion that emerges from the R&D-based endogenous growth literature is that public policies can have long-run growth effects by influencing the incentives firms have to innovate. Romer (1990), Segerstrom et al. (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) all find that R&D subsidies encourage firms to devote more resources to R&D activities and as a result increase the long-run rate of economic growth. Furthermore, because R&D subsidies promote growth, many other public policies that indirectly affect the R&D incentives of firms can also have long-run growth effects. For example, Rivera-Batiz and Romer (1991) show that lower tariffs between countries lead to a permanent increase in the world economic growth rate.

This literature has been challenged in an important paper by Jones (1995), who points out that all of the above-mentioned endogenous growth models have an undesirable "scale effect" property, namely, that economic growth is faster when firms devote more resources to R&D. Since 1950, the number of scientist and engineers engaged in R&D in advanced countries has increased dramatically without generating any upward trend in per capita growth rates. Furthermore, when Jones modifies Romer's (1990) endogenous growth model to eliminate the prediction of scale effects, he finds that doing so also eliminates the long-run growth effects of R&D subsidies.[1] In the Jones (1995) model, higher R&D subsidies increase the relative size of the R&D sector, but have no effect on the long-run rate of economic growth, which only depends on the population growth rate and other exogenous parameters.

In Jones (1995), firms engage in horizontal R&D to increase the number of industries in the economy (create entirely new products). Segerstrom (1997) shows that the same results apply in a model where firms engage in vertical R&D to improve

[1] In Romer's (1990) model, as the stock of knowledge in the economy increases over time, the productivity of researchers increases proportionately. Jones (1995) makes a less optimistic assumption about how the productivity of researchers changes over time; he assumes that new knowledge contributes to the productivity of researchers at a decreasing (instead of constant) rate.

the quality of existing products.[2] In this paper, I examine the robustness of their conclusion that R&D subsidies do not have long-run growth effects by analyzing a model where firms can engage in both horizontal and vertical R&D activities.

This model does not have the scale effect property; positive population growth does not lead to any upward trend in the rate of economic growth over time. Instead, the model has a unique balanced growth equilibrium where the economy grows at a constant rate over time. In the polar extreme cases where firms choose to only engage in horizontal R&D activities or choose to only engage in vertical R&D activities, the model has the same qualitative properties as in Jones (1995) and Segerstrom (1997); R&D subsidies do not have long-run growth effects but merely increase the fraction of the labor force that engages in R&D activities. However, in the more interesting inbetween cases where firms engage in both horizontal and vertical R&D activities, I find that R&D subsidies almost always have long-run growth effects (except for a knife-edge set of parameter values). Thus, the seemingly innocuous assumption in Jones (1995) and Segerstrom (1997) that there is only one type of R&D activity plays, in fact, a critical role in driving their policy conclusions.

The most surprising result in this paper, though, is that the long-run growth effects of R&D subsidies can go both ways. This is remarkable given the model's other properties; at each point in time, (i) economic growth is entirely driven by technological change, (ii) technological change is entirely driven by the R&D activities of firms, (iii) higher R&D subsidies induce firms to devote more resources to both horizontal and vertical R&D, and (iv) the economy's economic growth rate is an increasing function of the horizontal and vertical R&D expenditures of firms. Under these circumstances, it is natural to expect that R&D subsidies increase the economy's long-run economic growth rate, if they have any growth effects. But I find that for a wide range of parameter values, higher R&D subsidies actually lead to a lower long-run rate of economic growth. After deriving simple parameter conditions

[2] In Grossman and Helpman (1991), the productivity of researchers does not change over time. To rule out scale effects, Segerstrom (1997) modifies this model by assuming that the productivity of researchers steadily declines over time.

which completely characterize when R&D subsidies promote economic growth, have no growth effects, and retard economic growth, I present an intuitive explanation for why R&D subsidies have long-run growth effects. Even in the more "normal" case where R&D subsidies promote growth, the reasons why are quite different from those discussed in the earlier endogenous growth literature, indicating that the properties of "endogenous" growth models fundamentally change when the prediction of scale effects is eliminated.

This paper builds directly on an earlier model by Howitt (1997). Unlike in other models of horizontal and vertical innovation developed by Young (1998) and Dinopoulos and Thompson (1998), where general R&D subsidies do not have long-run growth effects, Howitt's model has the interesting property that R&D subsidies unambiguously promote long-run economic growth. I find that R&D subsidies can also retard long-run economic growth because I analyze a model with considerably more general assumptions about the returns to R&D expenditure (and a slightly different final goods production function). Whereas Howitt assumes that there are constant returns to vertical R&D expenditure, I allow for an arbitrary degree of decreasing returns, motivated by the empirical evidence of significant decreasing returns to firm R&D expenditure reported in Kortum (1993) and Thompson (1996). Also Howitt's model has the equilibrium property that the patents-per-researcher ratio is constant over time. By making more general assumptions about how the productivity of R&D workers changes over time, I allow for the possibility that the patents-per-researcher ratio decreases over time, motivated by the empirical evidence reported in Kortum (1997) that this ratio has declined significantly over time in many countries.

The rest of this paper is organized as follows: The model is presented in section 2 and the balanced growth properties of the model are derived in section 3. Both the effects of general and targeted R&D subsidies are studied in this section. Also a new intuitive explanation is presented for why R&D subsidies sometimes promote economic growth and sometimes retard economic growth. Section 4 offers some concluding comments.

2 The Model

2.1 Production

Consumption goods and R&D services are both produced by firms under conditions of perfect competition using the same constant returns to scale production function. The inputs in the production process are labor and a continuum of intermediate products. Specifically, the total output of the economy at any date t is

$$Y_t = C_t + H_t + V_t = L_{yt}^{1-\alpha} \int_0^{N_t} A_{it} x_{it}^{\alpha} \, di \tag{1}$$

where Y_t is gross output, C_t is consumption, H_t is horizontal R&D expenditure, H_t is vertical R&D expenditure, L_{yt} is the labor devoted to producing output, N_t is the measure of how many different intermediate products exist at time t, x_{it} is the flow output of intermediate product i used throughout the economy, A_{it} is a productivity parameter attached to the latest version of intermediate product i, and $\alpha \in (0,1)$ is a parameter which determines the elasticity of demand for intermediate products.[3] Each intermediate product is in turn produced using labor only, according to the production function $x_{it} = L_{it}$, where L_{it} is the input of labor in industry i.

Consider a typical firm j that produces final goods, that is, either consumption goods or R&D services. At time t, this firm solves the profit maximization problem

$$\max_{L_{yjt}, x_{ijt}} L_{yjt}^{1-\alpha} \int_0^{N_t} A_{it} x_{ijt}^{\alpha} \, di - \int_0^{N_t} p_{it} x_{ijt} \, di - w_t L_{yjt}$$

where L_{yjt} is the labor employed by firm j to produce final output, x_{ijt} is the flow of intermediate input i used by firm j, p_{it} is the price of intermediate input i and w_t is the wage rate for labor, measured in units of final output (the numeraire for all prices). Solving for profit maximizing behavior yields the first order condition $p_{it} = A_{it}\alpha(L_{yjt}/x_{ijt})^{1-\alpha}$. However, since all firms face the same prices p_{it}, all firms must choose the same input ratios ($x_{ijt}/L_{yjt} = x_{it}/L_{yt}$ for all j) and this first order

[3] The production function (1) differs from the production function used by Howitt (1997) in that labor is an input in the production of final goods. Howitt assumes that final goods are produced using intermediate inputs only.

condition can be written more simply as

$$p_{it} = A_{it}\alpha(L_{yt}/x_{it})^{1-\alpha}. \tag{2}$$

This is the inverse demand function for intermediate input i by the producers of final goods and it implies that the elasticity of demand for each intermediate input is $\frac{-1}{1-\alpha}$. The second first order condition for profit maximization is

$$w_t = (1-\alpha)\int_0^{N_t} A_{it}\left(\frac{x_{it}}{L_{yt}}\right)^{\alpha} di, \tag{3}$$

which helps pin down the equilibrium real wage rate for labor at time t. It is easily verified that when (2) and (3) hold, the firms that produce final goods all earn zero economic profits.

2.2 Innovation

Firms engage in vertical R&D activities with the goal of developing higher quality intermediate products. Each vertical innovation is associated with a higher value of A_{it} for some industry i. Let $A_{mt} \equiv \max\{A_{it}; i \in [0, N_t]\}$ denote the leading-edge productivity parameter at time t. I assume that a vertical innovation at time t in industry $i \in [0, N_t]$ results in a new intermediate product which embodies the leading-edge productivity parameter A_{mt}. The Poisson arrival rate of vertical innovations in each industry $i \in [0, N_t]$ at time t is denoted by ϕ_t. This Poisson arrival rate does not vary across industries at time t because the reward for innovating is the same in all industries (is proportional to A_{mt}, as will be established later) and R&D costs are also assumed to be the same across industries.

Following Caballero and Jaffe (1993) and Howitt (1997), I assume that the leading-edge productivity parameter A_{mt} grows over time as a result of knowledge spillovers produced by vertical innovations:

$$g_{At} \equiv \frac{\dot{A}_{mt}}{A_{mt}} = \left(\frac{\sigma}{N_t}\right)(\phi_t N_t) = \sigma\phi_t, \tag{4}$$

where $\sigma > 0$ is a given R&D spillover parameter. Equation (4) has a natural interpretation. The size of the knowledge spillovers is proportional to the aggregate flow of vertical innovations $\phi_t N_t$ in the economy. The factor of proportionality is given by σ/N_t and can be interpreted as the marginal impact of each vertical innovation on the stock of public knowledge which researchers use. I divide by N_t in the factor of proportionality to capture the idea that as the economy develops an increasing number of intermediate products, each vertical innovation has a smaller impact on the aggregate economy.

Firms engage in horizontal R&D activities with the goal of developing different intermediate products, that is, creating entirely new industries. Horizontal innovation is associated with increases in N_t over time. At time t, each horizontal innovation results in a new intermediate product i whose productivity parameter A_{it} is drawn randomly from the existing distribution of productivity parameters across industries.

2.3 Intermediate Product Markets

I assume that any firm that innovates immediately receives a patent on its innovation and that patent rights are strictly enforced. Thus, a firm that innovates does not have to worry about other firms copying its product.[4] Given the absence of copying, a firm that horizontally innovates does not have to deal with competitors in its newly created industry and thus, this firm can earn positive profit flows until the next vertical innovation in its industry occurs. A firm that vertically innovates enters into Bertrand price competition with the previous incumbent in its industry, a firm that produces a lower quality product. It is either in the interest of the new industry leader to practice limit pricing (as in Grossman and Helpman (1991)) or to charge an unconstrained monopoly price, depending on whether the quality difference between the two competing firms is small or large. In either case, the previous incumbent does not ever sell any output or earn any profits in equilibrium. Faced with these grim future prospects, I assume that the previous incumbent immediately exits and then cannot threaten to re-enter the industry. Thus, a firm that vertically innovates also

[4] For a quality ladders endogenous growth model where patent protection is not perfect and firms copy products developed by other firms, see Davidson and Segerstrom (1998).

earns monopoly profits until the next vertical innovation in its industry occurs.[5]

The incumbent monopolist of intermediate product i has total cost of production $w_t x_{it}$ and the (inverse) demand for its product is given by (2). Solving this firm's profit maximization problem $\max_{x_{it}} \pi_{it} = (p_{it} - w_t)x_{it}$ yields the standard monopoly markup over marginal cost $p_{it} = w_t/\alpha$, the quantity of intermediate product i that is supplied:

$$x_{it} = L_{yt} \left(\frac{A_{it}\alpha^2}{w_t} \right)^{1/(1-\alpha)}, \tag{5}$$

and the monopoly profit flow in industry i:

$$\pi_{it} = L_{yt}\alpha(1-\alpha)A_{it} \left(\frac{A_{it}\alpha^2}{w_t} \right)^{\alpha/(1-\alpha)}. \tag{6}$$

Equation (6) implies that the profits earned by an innovative firm can either rise or fall over time. On the one hand, population growth (operating through increases in L_{yt}) leads to increases in the demand for the innovative firm's product over time, and these demand increases contribute to increasing the innovative firm's profits over time. On the other hand, increases in the real wage w_t associated with economic growth mean that the firm's production costs increase over time and these production cost increases contribute to decreasing the innovative firm's profits over time.

2.4 The Distribution of Relative Productivities

Let $G(\cdot, t)$ denote the cumulative distribution of (absolute) productivity parameters A_{it} at time t. Pick any $A > 0$ that was the leading-edge productivity parameter at some time $t_0 \geq 0$ and define $\Phi(t) \equiv G(A, t)$. Then $\Phi(t_0) = 1$ since no industry can have a productivity parameter larger than that of the leading-edge productivity parameter at time t_0, which by construction is A. It also follows that

$$\dot{\Phi}(t) + \phi_t \Phi(t) = 0 \tag{7}$$

[5] As in Grossman and Helpman (1991) and Aghion and Howitt (1992), since current industry leaders have less to gain from vertically innovating than other firms, they do not participate in vertical R&D races and vertical innovation always results in the previous leader firm being driven out of business. For a model where industry leaders have R&D cost advantages and as a result participate in vertical R&D races, see Segerstrom and Zolnierek (1998).

holds for all $t \geq t_0$. To understand this differential equation, first note that since horizontal innovations represent random draws from the distribution of productivity parameters, they do not change the distribution of productivity parameters and thus can be ignored when characterizing the time path of Φ. Next note that after time t_0, the rate at which vertical innovations cause the mass of industries behind A to fall is the overall flow of vertical innovations occurring in industries currently behind A. There are $\Phi(t)$ such industries and the Poisson arrival rate of vertical innovations in each one of these industries is ϕ_t.

Taking into acount the initial value condition $\Phi(t_0) = 1$, the unique solution to the first order linear differential equation (7) is

$$\Phi(t) = e^{-\int_{t_0}^{t} \phi_s ds} \tag{8}$$

for all $t \geq t_0$. Equation (4) represents another first order linear differential equation which, taking into account the initial condition $A_{mt_0} = A$, has a unique solution

$$A_{mt} = Ae^{\sigma \int_{t_0}^{t} \phi_s ds}. \tag{9}$$

for all $t \geq t_0$.

Let $a_{it} \equiv A_{it}/A_{mt}$ denote the relative productivity of the incumbent monopolist in industry i at time t and correspondingly, let $a \equiv A/A_{mt}$. Then (8) and (9) together imply that $\Phi(t) \equiv \text{Prob}(A_{it} \leq A) = (A/A_{mt})^{1/\sigma}$ for all $A \geq A_{m0}$, which can be alternatively expressed as

$$\text{Prob}(a_{it} \leq a) = F(a) \equiv a^{1/\sigma} \tag{10}$$

for all $a \geq A_{m0}/A_{mt}$. As t converges to $+\infty$, A_{m0}/A_{mt} converges to zero. Thus, the distribution of relative productivities converges monotonically over time to the invariant distribution $F(\cdot)$. Since the focus in this paper is on the steady-state equilibrium properties of the model, I will assume that the distribution of relative productivities is $F(\cdot)$ at time $t = 0$, that is, equation (10) holds for all $a \geq 0$. Then the distribution of relative productivities does not change over time and differentiating yields the time-invariant density function of relative productivities

$$f(a) \equiv \frac{1}{\sigma} a^{(1-\sigma)/\sigma}. \tag{11}$$

2.5 Real Wage Dynamics

Each incumbent monopolist charges the standard monopoly markup over marginal cost $p_{it} = w_t/\alpha$. Taking into account (2), this implies that all final good producers choose the same input ratio $x_{it}/L_{yt} = (A_{it}\alpha^2/w_t)^{1/(1-\alpha)}$, which when substituted into (3) yields

$$w_t^{1/(1-\alpha)} = (1-\alpha)\alpha^{2\alpha/(1-\alpha)} \int_0^{N_t} A_{it}^{1/(1-\alpha)} di. \tag{12}$$

The integral in (12) can now be solved using the information about the invariant distribution of relative productivities: $\int_0^{N_t} A_{it}^{1/(1-\alpha)} di = A_{mt}^{1/(1-\alpha)} N_t \int_0^1 a^{1/(1-\alpha)} f(a)\, da = A_{mt}^{1/(1-\alpha)} N_t/\Gamma$ where $\Gamma \equiv \frac{\sigma}{1-\alpha} + 1$. Substituting the value of this integral back into (12) and simplifying yields the real wage dynamics condition

$$w_t = \frac{(1-\alpha)^{1-\alpha}\alpha^{2\alpha}}{\Gamma^{1-\alpha}} A_{mt} N_t^{1-\alpha} \tag{13}$$

The real wage w_t rises over time as vertical R&D increases the leading-edge productivity parameter A_{mt} and horizontal R&D increases the measure of industries N_t in the economy.

2.6 The Labor Market

The total supply of labor L_t is fixed inelastically at each time t by the population, which grows over time at the constant exogenous rate $g_L > 0$, that is, $L_t = L_0 e^{g_L t}$. Workers are either employed producing intermediate products or final goods. Thus, the full employment of labor condition is

$$\int_0^{N_t} x_{it}\, di + L_{yt} = L_t.$$

Substituting into this expression using (5) and (12) yields

$$L_{yt} = \frac{L_t}{1 + \frac{\alpha^2}{1-\alpha}}, \tag{14}$$

which pins down the employment of labor in the final goods sector throughout time.

Substituting into (1) using (5), (13) and $\int_0^{N_t} A_{it}^{1/(1-\alpha)} di = A_{mt}^{1/(1-\alpha)} N_t/\Gamma$ yields the output dynamics condition

$$Y_t = \frac{\alpha^{2\alpha}}{(1-\alpha)^{\alpha}\Gamma^{1-\alpha}} L_{yt} A_{mt} N_t^{1-\alpha}. \tag{15}$$

Equations (13), (14) and (15) together imply that the real wage rate of labor (w_t) grows over time at the same rate as GDP per worker (Y_t/L_t).

2.7 Consumers

For simplicity, I assume that each consumer lives forever, has linear additive preferences over consumption at each point in time and a constant rate of time preference $\rho > 0$. Thus, a consumer born at time t_0 maximizes the discounted utility function

$$\int_{t_0}^{\infty} e^{-\rho t} c(t)\, dt$$

subject to the usual intertemporal budget constraint, where $c(t)$ is the consumer's expenditure at time t. Then the market interest rate must equal ρ throughout time.

2.8 Vertical R&D

The Poisson arrival rate ϕ_{ijt} of vertical innovations in industry i by firm j at time t is given by

$$\phi_{ijt} = \frac{\lambda_v V_{ijt}^{\delta} K_{ijt}^{1-\delta}}{A_{mt}^{d}}$$

where $\lambda_v > 0$ is a vertical R&D productivity parameter, V_{ijt} is firm j's vertical R&D expenditure flow, K_{ijt} is the firm-specific knowledge possessed by firm j that is useful for vertical R&D, the exponent $\delta \in (0, 1]$ measures the degree of decreasing returns to vertical R&D expenditure, and the exponent $d > 0$ determines the rate at which research problems become more complex and harder to solve as the leading-edge productivity parameter A_{mt} increases over time. This vertical R&D technology is more general than in Howitt (1997), who restricts attention to the special case $d = \delta = 1$.

At each point in time t, the profit-maximizing vertical R&D firm j in industry i solves the problem $\max_{V_{ijt}} \phi_{ijt}\Pi_{vt} - V_{ijt}(1-\beta_v)$, where Π_{vt} is the expected discounted profit earned from winning a vertical R&D race at time t and β_v is the vertical R&D subsidy rate. The first order condition for this problem is

$$\frac{\delta\, \lambda_v \Pi_{vt}}{A_{mt}^{d}} \left(\frac{V_{ijt}}{K_{ijt}}\right)^{\delta-1} = 1 - \beta_v, \qquad (16)$$

which is the usual requirement that the marginal expected benefit of an extra unit of vertical R&D equals its marginal cost. When $\delta < 1$, (16) implies that an increase in the reward for innovating Π_{vt} induces each firm j to increase its R&D effort V_{ijt}. Equation (16) also implies that $V_{ijt}/K_{ijt} = V_{it}/K_{it}$ for all j where $V_{it} \equiv \sum_j V_{ijt}$ and $K_{it} \equiv \sum_j K_{ijt}$, that is, each firm devotes resources to vertical R&D is proportion to the firm-specific knowledge that it possesses. I assume that there is symmetry across R&D firms (K_{ijt} is the same for all j) and that vertical R&D races are perfectly competitive (K_{ijt} is infinitesimally small). The latter assumption implies that the likelihood of any one firm winning a vertical R&D race is negligable.

As the economy grows over time and the stock of knowledge increases, researchers have more ideas to work with in developing new ideas, which by itself makes them more productive. I capture this basic insight in Romer (1990) by assuming that $K_{it} \equiv \sum_j K_{ijt} = Y_t/N_t$ for all i, that is, the total amount of firm-specific knowledge in each industry equals per industry ouput, which will grow over time in equilibrium. Then the first order condition for vertical R&D profit maximization (16) can be written more simply (using $V_{it} = V_t/N_t$) as

$$\left(\frac{\delta\,\lambda_v \Pi_{vt}}{A_{mt}^d}\right) v_t^{\delta-1} = 1 - \beta_v, \tag{17}$$

where $v_t \equiv V_t/Y_t$ is the fraction of GDP that is allocated to vertical R&D.

The returns to engaging in vertical R&D are assumed to be independently distributed across firms and over time. Thus, it is easily verified that the Poisson arrival rate of vertical innovations in each industry is

$$\phi_t = \sum_j \phi_{ijt} = \lambda_v v_t^\delta y_t \tag{18}$$

where $y_t \equiv Y_t/(N_t A_{mt}^d)$. The Poisson arrival rate of vertical innovations ϕ_t is an increasing function of the fraction of GDP that is allocated to vertical R&D v_t.

2.9 Horizontal R&D

The discovery rate of new industries by firm j at time t is given by

$$\dot{N}_{jt} = \frac{\lambda_h H_{jt}^\gamma \mathcal{K}_{jt}^{1-\gamma}}{A_{mt}^d}$$

where $\lambda_h > 0$ is a horizontal R&D productivity parameter, H_{jt} is firm j's horizontal R&D expenditure flow, $\mathcal{K}_{jt}$ is the firm-specific knowledge possessed by firm j that is useful for horizontal R&D, the exponent $\gamma \in (0, 1]$ measures the degree of decreasing returns to horizontal R&D expenditure and as with vertical R&D, the exponent $d > 0$ determines the rate at which research problems become harder to solve as the leading-edge productivity parameter A_{mt} increases over time.

At each point in time t, the profit-maximizing horizontal R&D firm j solves the problem $\max_{H_{jt}} \dot{N}_{jt}\Pi_{ht} - H_{jt}(1 - \beta_h)$, where Π_{ht} is the expected discounted profits earned from a horizontal innovation at time t and β_h is the horizontal R&D subsidy rate. The first order condition for this problem is

$$\frac{\gamma\,\lambda_h \Pi_{ht}}{A_{mt}^d}\left(\frac{H_{jt}}{\mathcal{K}_{jt}}\right)^{\gamma-1} = 1 - \beta_h, \tag{19}$$

that is, the marginal expected benefit of an extra unit of horizontal R&D equals its marginal cost. Equation (19) implies that $H_{jt}/\mathcal{K}_{jt} = H_t/\mathcal{K}_t$ for all j where $H_t \equiv \sum_j H_{jt}$ and $\mathcal{K}_t \equiv \sum_j \mathcal{K}_{jt}$, that is, each firm devotes resources to horizontal R&D is proportion to the firm-specific knowledge that it possesses. I assume that $\mathcal{K}_t \equiv \sum_j \mathcal{K}_{jt} = Y_t$, so the total stock of firm-specific knowledge useful for horizontal R&D grows over time at the same rate as the economy's gross output. Then the first order condition for horizontal R&D profit maximization (19) can be written more simply as

$$\left(\frac{\gamma\,\lambda_h \Pi_{ht}}{A_{mt}^d}\right) h_t^{\gamma-1} = 1 - \beta_h, \tag{20}$$

where $h_t \equiv H_t/Y_t$ is the proportion of gross output devoted to horizontal R&D.

The growth rate of the measure of industries can now be determined by summing up the discovery rates for all the individual firms that engage in horizontal R&D:

$$g_{Nt} \equiv \frac{\dot{N}_t}{N_t} = \sum_j \frac{\lambda_h (H_{jt}/\mathcal{K}_{jt})^\gamma \mathcal{K}_{jt}}{N_t A_{mt}^d} = \lambda_h h_t^\gamma y_t \tag{21}$$

The rate at which the measure of industries grows over time g_{Nt} is an increasing function of the fraction of GDP that is allocated to horizontal R&D h_t.

2.10 The Rewards for Innovating

The reward for innovating is the expected discounted value of profit flows earned by the innovative firm before being replaced by the next innovator in its industry. Thus, for a firm that innovates in industry i at time t and whose new intermediate product embodies the productivity parameter A_{it}, the reward for innovating is

$$\Pi_t(A_{it}) = \int_t^{\infty} e^{-\int_t^{\tau}(\rho+\phi_s)ds}\hat{\pi}_{\tau}(A_{it})d\tau, \tag{22}$$

where $\hat{\pi}_{\tau}(A_{it})$ is the monopoly profit flow at time τ for a firm whose technology embodies the productivity parameter A_{it}. In (22), the instantaneous discount rate applied to the profits earned by an innovative firm is the market interest rate ρ plus the rate of creative destruction ϕ_s; the latter being the instantaneous probability of further innovation in the industry under consideration. It follows from (6) that

$$\hat{\pi}_{\tau}(A_{it}) = L_{y\tau}\alpha(1-\alpha)A_{it}\left(\frac{A_{it}\alpha^2}{w_{\tau}}\right)^{\alpha/(1-\alpha)}. \tag{23}$$

Since a vertical innovation at time t results in a new intermediate product which embodies the leading-edge productivity parameter A_{mt}, the reward for a vertical innovation at time t is simply $\Pi_{vt} = \Pi_t(A_{mt})$. Thus, as was claimed earlier, the reward for a vertical innovation does not vary across industries at time t.

Calculating the reward for a horizontal innovation at time t is slightly more complicated since the productivity parameter A_{it} for a new intermediate product is drawn randomly from the existing distribution of productivity parameters across industries. Given that the density function for the relative productivity parameter $a_{it} \equiv A_{it}/A_{mt}$ is $f(a_{it})$, the expected discounted reward for a horizontal innovation at time t is

$$\Pi_{ht} = \int_0^1 \Pi_t(a_{it}A_{mt})f(a_{it})\,da_{it}. \tag{24}$$

This completes the description of the model.

3 Balanced Growth Properties

In this section, the balanced growth equilibrium properties of the model are analyzed. After establishing that the model has a unique balanced growth equilibrium where all endogenous variables grow at constant (not necessarily identical) rates over time, the circumstances under which R&D subsidies promote economic growth and retard economic growth are characterized. Intuitive explanations are provided for why both outcomes occur. The effects of subsidizing horizontal and vertical R&D at the same rate $\beta = \beta_h = \beta_v$ are studied first, followed by the more complicated case where horizontal and vertical R&D are subsidized at different rates.

Equation (1) implies that in any balanced growth equilibrium, both the fraction of GDP allocated to horizontal R&D and the fraction of GDP allocated to vertical R&D must be constant over time ($h_t = h$ and $v_t = v$ for all t). Since g_{At} must be constant over time in a balanced growth equilibrium by definition, (4) implies that the Poisson arrival rate of vertical innovations must be constant over time also ($\phi_t = \phi$ for all t). It then follows from (18) that y_t must also be constant over time in any balanced growth equilibrium ($y_t = y$ for all t). Thus, the quality and variety growth rates can be written more simply as:

$$g_A = \sigma \lambda_v v^\delta y, \tag{25}$$

and

$$g_N = \lambda_h h^\gamma y. \tag{26}$$

To reduce to a managable level the number of cases that need to be studied, in the rest of this section I analyze the model's properties assuming the parameter restrictions $\gamma < 1$, $\delta < 1$ and $d > 1$. The first two parameter restrictions are easy to justify. $\gamma < 1$ means that there are decreasing returns to horizontal R&D expenditure and $\delta < 1$ means that there are decreasing returns to vertical R&D expenditure. Kortum (1993) and Thompson (1996) both report evidence of significant decreasing returns to firm R&D expenditure. The justification for the parameter restriction $d > 1$ is less transparent and comes from thinking about the model's implications for patenting behavior.

3.1 Patenting Behavior

In a balanced growth equilibrium, the rate of patenting for horizontal innovations is $\dot{N}_t$ and the labor devoted to horizontal R&D is hL_{yt}. It follows from (15), (21) and the definition of y that the patents-per-researcher ratio associated with horizontal R&D at time t is given by

$$P_{ht} \equiv \frac{\dot{N}_t}{hL_{yt}} = \frac{\lambda_h \alpha^{2\alpha}}{(1-\alpha)^{\alpha}\Gamma^{1-\alpha}} \left(\frac{N_t^{1-\alpha}}{A_{mt}^{d-1}} \right) h^{\gamma-1}.$$

The rate of patenting for vertical innovations is ϕN_t and the labor devoted to vertical R&D is vL_{yt}. It follows from (15), (18) and the definition of y that the patents-per-researcher ratio associated with vertical R&D at time t is given by

$$P_{vt} \equiv \frac{\phi N_t}{vL_{yt}} = \frac{\lambda_v \alpha^{2\alpha}}{(1-\alpha)^{\alpha}\Gamma^{1-\alpha}} \left(\frac{N_t^{1-\alpha}}{A_{mt}^{d-1}} \right) v^{\delta-1}.$$

Both patents-per-researcher ratios decline over time if and only if $(d-1)g_A > (1-\alpha)g_N$. Thus $d > 1$ is a necessary condition for the overall patents-per-researcher ratio to decline over time.

Table 1 illustrates what has happened to the patents-per-researcher ratio in five advanced countries: the United States, France, Japan, Sweden and the United Kingdom. In calculating the patents-per-researcher ratio, the rate of patenting is mea-

Table 1: Average Annual Growth Rate in the Patents-Per-Researcher Ratio

Country	Time Period	Growth Rate
United States	1965-1993	-2.18%
France	1965-1993	-6.07%
Japan	1965-1993	-0.11%
Sweden	1971-1993	-6.26%
United Kingdom	1969-1993	-5.74%

sured by the annual total number of patents granted to residents in WIPO (1983) and WIPO (various issues), and R&D employment is measured by the total number of R&D scientists and engineers in National Science Board (1998). As Table

1 shows, the patents-per-researcher ratio has significantly declined over time in the United States, France, Sweden and the United Kingdom. The only exception is Japan, where the patents-per-researcher ratio has been roughly constant over time. Given this evidence, it makes sense to focus on the model's properties when $d > 1$, since the patents-per-researcher ratio necessarily increases over time when $d \leq 1$.

3.2 Economic Growth

Let g denote the growth rate of the real wage w_t, that is, the economic growth rate for the economy. Differentiating (13) with respect to time yields

$$g = g_A + (1-\alpha)g_N = \sigma\lambda_v v^\delta y + (1-\alpha)\lambda_h h^\gamma y. \tag{27}$$

The economic growth rate g is a increasing function of both the share of GDP devoted to horizontal R&D h and the share of GDP devoted to vertical R&D v.

3.3 The Population Growth Condition

In any balanced growth equilibrium, y_t must be constant over time. This has a strong implication. Substituting into the definition of y_t using (14) and (15), it follows that

$$y_t \equiv \frac{Y_t}{N_t A_{mt}^d} = \left(\frac{L_t}{1+\frac{\alpha^2}{1-\alpha}}\right)\left(\frac{\alpha^{2\alpha} A_{mt}^{1-d} N_t^{-\alpha}}{\Gamma^{1-\alpha}(1-\alpha)^\alpha}\right).$$

Taking logs of both sides and then differentiating respect to time yields the population growth condition:

$$g_L = (d-1)g_A + \alpha g_N = (d-1)\sigma\lambda_v v^\delta y + \alpha\lambda_h h^\gamma y. \tag{28}$$

The rates at which both the leading-edge productivity parameter A_{mt} and the measure of industries N_t can grow over time in a balanced growth equilibrium is constrained by the growth rate of the labor force g_L. With the productivity of researchers falling over time as the problems they face become more complex and harder to solve, the resources devoted to R&D must increase over time just to maintain a constant rate of economic growth. The growth rate of the labor force determines the rate at which the resources devoted to both horizontal and vertical R&D can increase over time in a balanced growth equilibrium and thus the growth rate of the labor force also helps determine how fast the economy can grow over time.

3.4 The Vertical R&D Condition

In a balanced growth equilibrium where ϕ_t is constant over time, $w_\tau = w_t e^{g(\tau - t)}$ is implied by (13), and $L_{y\tau} = L_{yt} e^{g_L(\tau - t)}$ is implied by (14). Substituting these expressions into (23) and then evaluating the integral in (22) using (13) and $A_{it} = A_{mt}$ yields the expected discounted reward for a vertical innovation at time t

$$\Pi_{vt} = \frac{L_{yt}\alpha^{1+2\alpha}(1-\alpha)^{1-\alpha}\Gamma^\alpha A_{mt} N_t^{-\alpha}}{\rho - g_L + \phi + \frac{g\alpha}{1-\alpha}}.$$

Substituting this reward Π_{vt} back into (17) and simplifying using (15) and (27), I obtain the vertical R&D condition

$$\frac{\delta \lambda_v \Gamma \alpha (1-\alpha) v^{\delta - 1} y}{(\rho - g_L) + \lambda_v \left(1 + \frac{\alpha\sigma}{1-\alpha}\right) v^\delta y + \alpha \lambda_h h^\gamma y} = 1 - \beta_v, \tag{29}$$

which specifies the values of h, v and y that are consistent with vertical R&D profit-maximization in a balanced growth equilibrium. I assume that $\rho > g_L$ to guarantee that the denominator in (29) is always positive.

3.5 The Horizontal R&D Condition

Following the same procedure as was used to derive Π_{vt}, I find that the discounted reward for a horizontal innovation with productivity parameter A_{it} at time t is

$$\Pi_t(A_{it}) = \Pi_t(a_{it}A_{mt}) = \frac{L_{yt}\alpha^{1+2\alpha}(1-\alpha)^{1-\alpha}\Gamma^\alpha A_{mt} N_t^{-\alpha} a_{it}^{1/(1-\alpha)}}{\rho - g_L + \phi + \frac{g\alpha}{1-\alpha}}.$$

Substituting this expression into (24) and integrating using (11), I obtain the expected discounted reward for a horizontal innovation at time t:

$$\Pi_{ht} = \frac{L_{yt}\alpha^{1+2\alpha}(1-\alpha)^{1-\alpha}\Gamma^{\alpha-1} A_{mt} N_t^{-\alpha}}{\rho - g_L + \phi + \frac{g\alpha}{1-\alpha}}.$$

Substituting this reward Π_{ht} back into (20) and simplifying using (15) and (27), I obtain the horizontal R&D condition

$$\frac{\gamma\lambda_h\alpha(1-\alpha)h^{\gamma-1}y}{(\rho-g_L)+\lambda_v\left(1+\frac{\alpha\sigma}{1-\alpha}\right)v^{\delta}y+\alpha\lambda_h h^{\gamma}y}=1-\beta_h, \tag{30}$$

which specifies the values of h, v and y that are consistent with horizontal R&D profit-maximization in a balanced growth equilibrium.

3.6 General R&D Subsidies

The population growth condition (28), the vertical R&D condition (29) and the horizontal R&D condition (30) represent a system of 3 equations in 3 unknowns (h, v and y) that must be simultaneously satisfied by any balanced growth equilibrium. An analysis of these 3 equations yields

Lemma 1 *The model has a unique balanced growth equilibrium. Focusing on this equilibrium, a permanent increase in the general R&D subsidy rate* β
(i) permanently increases the fraction of GDP allocated to vertical R&D v *but decreases the long-run product quality growth rate* g_A *if* $\gamma > \delta$,
(ii) permanently increases the fraction of GDP allocated to vertical R&D v *but does not change the long-run product quality growth rate* g_A *if* $\gamma = \delta$, *and*
(iii) permanently increases the fraction of GDP allocated to horizontal R&D h *but decreases the long-run product variety growth rate* g_N *if* $\gamma < \delta$.

Proof: See the appendix.

Given Lemma 1, equation (27) implies that general R&D subsidies have the following long-run growth effects:

Theorem 1 *A permanent increase in the general R&D subsidy rate* β
(i) decreases the long-run economic growth rate g *if* $d > 1/(1-\alpha)$ *and* $\delta > \gamma$,
(ii) increases the long-run economic growth rate g *if* $d > 1/(1-\alpha)$ *and* $\gamma > \delta$,
(iii) decreases the long-run economic growth rate g *if* $1/(1-\alpha) > d$ *and* $\gamma > \delta$,
(iv) increases the long-run economic growth rate g *if* $1/(1-\alpha) > d$ *and* $\delta > \gamma$, *and*

(v) has no effect on g if either $d = 1/(1-\alpha)$ or $\delta = \gamma$.

Proof: See the appendix.

To understand what is driving the counter-intuitive results reported in Lemma 1 and Theorem 1, it is useful to examine two polar extreme cases. First, I explore the model's properties when firms only engage in vertical R&D activities ($H_t = 0$) and then when firms only engage in horizontal R&D activities ($V_t = 0$).

The equation $H_t = 0$ holds if it is not profitable for firms to engage in any horizontal R&D activities ($\gamma = 1$ and λ_h is sufficiently small). Then (28) and (27) together imply that the long-run rate of economic growth with only vertical R&D g_v is given by

$$g_v \equiv g_A = \frac{g_L}{d-1}. \tag{31}$$

Note that the R&D subsidy rate β does not appear in (31). Thus, a permanent increase in the R&D subsidy rate β has no effect on the long-run rate of economic growth, which is is completely determined by exogenous parameters like the population growth rate g_L. In the special case where $H_t = 0$, the model has the same qualitative properties as Segerstrom's (1997) model of growth driven by vertical innovation.

The equation $V_t = 0$ holds if it is not profitable for firms to engage in any vertical R&D activities ($\delta = 1$ and λ_v is sufficiently small). Then (28) and (27) together imply that the long-run rate of economic growth with only horizontal R&D g_h is given by

$$g_h \equiv (1-\alpha)g_N = \frac{(1-\alpha)g_L}{\alpha}. \tag{32}$$

Note that the R&D subsidy rate β does not appear in (32) either. Thus, a permanent increase in the R&D subsidy rate β has no effect on the long-run rate of economic growth, which is is completely determined by exogenous parameters like the population growth rate g_L. In the special case where $V_t = 0$, the model has the same qualitative properties as Jones's (1995) model of growth driven by horizontal innovation.

Comparing (31) and (32), it immediately follows that $g_h > g_v$ if and only if $d > 1/(1-\alpha)$. Thus the d-parameter inequalities in Theorem 1 have simple economic interpretations. The condition $d > 1/(1-\alpha)$ means that economic growth is faster when firms only engage in horizontal R&D activities and the condition $1/(1-\alpha) > d$ means that economic growth is faster when firms only engage in vertical R&D activities.

The next step in understanding Lemma 1 and Theorem 1 is to develop an economic interpretation of the $\gamma\delta$-parameter inequalities. Suppose that we start off in a balanced growth equilibrium where $h = v$ and $\delta > \gamma$ holds. The marginal cost and marginal benefit curves associated with both horizontal and vertical R&D are illustrated in Figure 1. The initial marginal cost of both horizontal and vertical R&D

Figure 1: The effect of a general R&D subsidy when $\delta > \gamma$

is given by $1-\beta$ and is illustrated in Figure 1 by the horizontal line. An increase in the general R&D subsidy rate β causes the marginal cost line to shift down (as is illustrated by the dashed line). Referring back to (17) and (20), both marginal benefit curves are downward sloping and given $\delta > \gamma$, the marginal benefit of horizontal R&D curve is steeper than the marginal benefit of vertical R&D curve. Thus, an increase in the R&D subsidy rate β leads to a smaller increase in h (the movement from point A to point B) than in v (the movement from point A to point C). The parameter condition $\delta > \gamma$ means that there are greater decreasing returns to horizontal R&D effort than to vertical R&D effort and under these circumstances, R&D subsidies promote vertical R&D effort to a greater extent than horizontal R&D effort. The parameter condition $\delta < \gamma$ has the opposite economic interpretation: this condition means that there are greater decreasing returns to vertical R&D effort than to horizontal R&D effort and under these circumstances, R&D subsidies promote horizontal R&D effort to a greater extent than vertical R&D effort.

It is now time to put the pieces of the puzzle together and develop an intuitive understanding of the results in Lemma 1 and Theorem 1. The effects of a permanent increase in the general R&D subsidy rate β are illustrated in Figure 2. In (g_A, g_N)

Figure 2: Two adjustment processes leading to new balanced growth equilibria

space, it follows from (27) that each iso-growth curve is a downward sloping line with slope $\frac{-1}{1-\alpha}$ and it follows from (28) that the population growth condition is a downward sloping line with slope $\frac{-(d-1)}{\alpha}$. Thus, the slope of the population growth growth condition exceeds the slope of each iso-growth line (in absolute value) if and only if $d > \frac{1}{1-\alpha}$. Figure 2 illustrates the $d > \frac{1}{1-\alpha}$ case.

Starting from a balanced growth equilibrium path, the initial effect of a general R&D subsidy increase is to encourage firms to devote more resources to both horizontal and vertical R&D. Greater R&D effort in turn leads to faster rates of both horizontal and vertical innovation. When $\gamma < \delta$, that is, there are greater decreasing returns to horizontal R&D, the R&D subsidy increase encourages vertical R&D effort to a greater extent than horizontal R&D effort and the quality growth rate g_A

jumps up more than the variety growth rate g_N. On the other hand, when $\delta > \gamma$ and there are greater decreasing returns to vertical R&D, the R&D subsidy increase encourages horizontal R&D effort to a greater extent than vertical R&D effort. Then the variety growth rate g_N jumps up more than the quality growth rate g_A. In either case though, the increase in R&D effort and the corresponding increase in the rate of technological change means that the complexity of the problems researchers are trying to solve grows over time at a faster than usual rate. Thus, one should expect the overall productivity of researchers in discovering new products to gradually fall over time. The process of gradually declining innovation rates continues until the economy reaches an outcome that is sustainable in the long-run, that is, until the population growth condition has been reached.

When $\gamma < \delta$, the entire adjustment process in response to an increase in the R&D subsidy rate β is illustrated in Figure 2 by the initial jump from equilibrium point A to point B and then by the gradual fall in innovation rates from point B to the final equilibrium point C. Because horizontal R&D is subject to greater decreasing returns, the R&D subsidy increase initially leads to a larger increase in vertical R&D effort and the quality growth rate g_A jumps up more than the variety growth rate g_N. Then, with the complexity of both horizontal and vertical R&D problems increasing over time at a faster than usual rate, the horizontal and vertical innovation rates gradually fall to point C on the population growth condition. The long-run effect of a permanent R&D subsidy increase is to promote vertical innovation g_A at the expense of horizontal innovation g_N. In the illustrated case $d > 1/(1-\alpha)$, the vertical R&D-only equilibrium is associated with a lower economic growth rate than the horizontal R&D-only equilibrium. Thus, by promoting vertical innovation g_A at the expense of horizontal innovation g_N, the R&D subsidy increase has the long-run effect of moving the economy closer to the vertical R&D-only equilibrium and thus serves to retard economic growth. As illustrated, the final equilibrium point C is on a lower iso-growth line than the initial equilibrium point A.

When $\delta < \gamma$, the above-mentioned story about how the economy adjusts over time gets reversed. In this case, the adjustment process in response to an increase in the R&D subsidy rate β is illustrated by the initial jump from equilibrium point A to point D and then by the gradual fall in innovation rates from point D to the final equilibrium point E. Because vertical R&D is subject to greater decreasing returns, the R&D subsidy increase initially leads to a larger increase in horizontal R&D effort and the variety growth rate g_N jumps up more than the quality growth rate g_A. Then, with the complexity of both horizontal and vertical R&D problems increasing over time at a faster than usual rate, the horizontal and vertical innovation rates gradually fall to point E on the population growth condition. The long-run effect of a permanent R&D subsidy increase is to promote horizontal innovation g_N at the expense of vertical innovation g_A . In the illustrated case $d > 1/(1-\alpha)$, the vertical R&D-only equilibrium is associated with a lower economic growth rate than the horizontal R&D-only equilibrium. Thus, by promoting horizontal innovation g_N at the expense of vertical innovation g_A, the R&D subsidy increase has the long-run effect of moving the economy closer to the horizontal R&D-only equilibrium and thus serves to promote economic growth. As illustrated, the final equilibrium point E is on a higher iso-growth line than the initial equilibrium point A.

The above-mentioned intuition only needs to be slightly modified to deal with the $1/(1-\alpha) > d$ case. Then the vertical R&D-only equilibrium is associated with a higher economic growth rate than the horizontal R&D-only equilibrium. When the R&D subsidy increase has the effect of promoting vertical innovation g_A at the expense of horizontal innovation g_N ($\delta > \gamma$), the R&D subsidy increase has the long-run effect of moving the economy closer to the vertical R&D-only equilibrium and thus serves to promote economic growth. On the other hand, when the R&D subsidy increase has the effect of promoting horizontal innovation g_N at the expense of vertical innovation g_A ($\delta < \gamma$), the R&D subsidy increase has the long-run effect of moving the economy closer to the horizontal R&D-only equilibrium and thus serves to retard economic growth.

It is now clear why Howitt (1997) reaches the unambigous conclusion that general R&D subsidies promote long-run economic growth. Howitt assumes that vertical R&D is subject to constant returns whereas horizontal R&D is subject to decreasing returns ($\gamma < \delta = 1$), which implies that general R&D subsidies have the long-run effect of promoting vertical innovation at the expense of horizontal innovation. Furthermore, Howitt assumes that $d = 1 < 1/(1-\alpha)$, which implies that the horizontal R&D-only equilibrium is associated with a finite economic growth rate and the vertical R&D-only "equilibrium" is associated with a infinitely high economic growth rate (substitute $d = 1$ into (31)). Under these circumstances, general R&D subsidies promote long-run economic growth by encouraging vertical innovation at the expense of horizontal innovation and moving the economy closer to the vertical R&D-only "equilibrium" with the infinitely high economic growth rate.

3.7 Targeted R&D Subsidies

Given the above-mentioned intuition, it would appear that the government can promote economic growth even when $\gamma = \delta$ by using appropriately chosen targeted R&D subsidies to guarantee that the right type of innovation increases in the long-run. For example, when $d < 1/(1-\alpha)$ and the vertical R&D-only equilibrium is associated with a higher growth rate than the horizontal R&D-only equilibrium, the government could promote economic growth by only subsidizing vertical R&D activities. This is indeed the case, as the following results establish:

Lemma 2 *A permanent increase in the horizontal R&D subsidy rate* β_h
(i) permanently increases the fraction of GDP allocated to horizontal R&D h *and increases the long-run product variety growth rate* g_N *if* $\delta \geq \gamma$,
(ii) permanently decreases the fraction of GDP allocated to vertical R&D v *and decreases the long-run product quality growth rate* g_A *if* $\delta \leq \gamma$.

A permanent increase in the vertical R&D subsidy rate β_v
(iii) permanently decreases the fraction of GDP allocated to horizontal R&D h *and decreases the long-run product variety growth rate* g_N *if* $\delta \geq \gamma$,
(iv) permanently increases the fraction of GDP allocated to vertical R&D v *and increases the long-run product quality growth rate* g_A *if* $\delta \leq \gamma$.

Theorem 2 *A permanent increase in the horizontal R&D subsidy rate* β_h

(i) decreases the long-run economic growth g if $d < 1/(1-\alpha)$,

(ii) increases the long-run economic growth g if $d > 1/(1-\alpha)$, and

(iii) has no effect on g if $d = 1/(1-\alpha)$.

A permanent increase in the vertical R&D subsidy rate β_v

(iv) increases the long-run economic growth g if $d < 1/(1-\alpha)$,

(v) decreases the long-run economic growth g if $d > 1/(1-\alpha)$, and

(vi) has no effect on g if $d = 1/(1-\alpha)$.

Proofs of Lemma 2 and Theorem 2: See the appendix.

Theorem 1 states that if either $d = 1/(1-\alpha)$ or $\delta = \gamma$, then general R&D subsidies do not have long-run growth effects. Given Theorem 2, the more fundamental of these two parameter conditions is $d = 1/(1-\alpha)$. If $d = 1/(1-\alpha)$, then no R&D subsidies (general or targeted) have long-run growth effects, whereas if $\delta = \gamma$ and $d \neq 1/(1-\alpha)$, then targeted R&D subsidies have long-run growth effects.

4 Conclusions

Building on earlier work by Howitt (1997), this paper presents an endogenous growth model with both horizontal and vertical R&D. Firms engage in vertical R&D to improve the quality of existing products and firms engage in horizontal R&D to increase the number of industries in the economy (create entirely new products). Firms that innovate and become industry leaders earn temporary monopoly profits as a reward for their R&D efforts. Thus, the "process of creative destruction" originally described by Schumpeter (1942) drives economic growth in this model.

The model has a unique balanced growth equilibrium in which the fraction of the labor force that engages in R&D is constant. Due to positive population growth, the expected discounted profits generated by both horizontal and vertical innovations grow over time. However counterbalancing these trends are the forces of increasing complexity; as technology advances, the resource costs of further advances also increase. In the balanced growth equilibrium, the rising rewards for innovating induces firms to hire more R&D workers but these effects are exactly balanced by the falling productivity of researchers, resulting in constant rates of both horizontal and vertical innovation. Thus, the model's properties are roughly consistent with the evidence presented in Jones (1995) to refute R&D-driven endogenous growth theory. Also the model's prediction of a constant share of GDP allocated to R&D is roughly consistent with the U.S. postwar data presented in Howitt (1997) and the model can account for the declining patents-per-researcher ratio evidence reported in Kortum (1997).

The focus of the paper is on understanding the long-run effects of R&D subsidies. Starting from a balanced growth equilibrium, when there is a permanent increase in the R&D subsidy rate, firms immediately respond by increasing their horizontal and vertical R&D expenditures. However, with firms devoted more resources to R&D, technological complexity also increases more rapidly. Researchers exhaust the supply of simplier problems more quickly and find themselves wrestling with more complicated research problems. Both horizontal and vertical innovation rates gradually fall over time in response to the steady decline in the productivity of R&D workers. Since the rates of horizontal and vertical innovation are ultimately constrained by the growth rate in the labor force and the R&D subsidy increase does not change the labor force growth rate, horizontal and vertical innovation rates continue to fall over time as long as they are both higher than in the initial (pre-R&D subsidy increase) balanced growth equilibrium. Thus, in the long-run, any change in the horizontal innovation rate is matched by a corresponding opposite change in the vertical innovation rate. R&D subsidies never permanently increase both the horizontal and vertical innovation rates in the economy. This is the fundamental new insight in the paper and the key to understanding the long-run growth effects of R&D subsidies.

Given that R&D subsidies have long-run growth effects by promoting one type of R&D (horizontal or vertical) at the expense of the other type of R&D, it only remains to be determined which type of R&D is associated with the higher economic growth rate and which type of R&D is promoted by R&D subsidies in the long-run.

If R&D difficulty increases rapidly (slowly) as product quality improves in the typical industry, then the economic growth rate that can be sustained in the long-run when firms only do vertical R&D is low (high), as is formally shown in Segerstrom

(1997). On the other hand, the rate at which R&D difficulty increases as product quality improves is irrelevant to determining the economy's long-run economic growth rate when firms only do horizontal R&D. Thus if R&D difficulty increases rapidly, the long-run economic growth rate is higher when firms only do horizontal R&D and if R&D difficulty increases slowly, the long-run economic growth rate is higher when firms only do vertical R&D.

The type of R&D that is promoted by general R&D subsidies in the long-run only depends on how the static returns to horizontal and vertical R&D expenditure differ. If horizontal R&D expenditure is subject to greater static decreasing returns than vertical R&D expenditure, then general R&D subsidies promote vertical R&D at the expense of horizontal R&D in the long-run. The reverse holds if vertical R&D expenditure is subject to greater static decreasing returns. With targeted R&D subsidies, things are simplier: regardless of how the degrees of static decreasing returns to horizontal and vertical R&D expenditure differ, horizontal R&D subsidies promote horizontal R&D at the expense of vertical R&D and vertical R&D subsidies promote vertical R&D at the expense of horizontal R&D in the long-run.

With the above-mentioned insights in hand, the long-run growth effects of R&D subsidies follow rather straightforwardly. General R&D subsidies retard growth if they promote vertical R&D effort (horizontal R&D expenditure is subject to greater decreasing returns) and vertical R&D is associated with slower growth (R&D difficulty increases rapidly as product quality improves). General R&D subsidies also retard growth if they promote horizontal R&D effort (vertical R&D expenditure is subject to greater decreasing returns) and horizontal R&D is associated with slower growth (R&D difficulty increases slowly as product quality improves). And of course, general R&D subsidies promote long-run economic growth in the opposite cases.

The long-run growth effects of targeted R&D subsidies are even easier to characterize. Since horizontal R&D subsidies unambiguously promote horizontal R&D at the expense of vertical R&D in the long-run, they retard growth if and only if horizontal R&D is associated with slower growth (R&D difficulty increases slowly as product quality improves). Likewise, since vertical R&D subsidies unambiguously promote vertical R&D at the expense of horizontal R&D in the long-run, they retard growth if and only if vertical R&D is associated with slower growth (R&D difficulty increases rapidly as product quality improves). Of course, targeted R&D subsidies promote long-run economic growth in the opposite cases.

Thus, as was claimed in the introduction, R&D subsidies can either retard or promote long-run economic growth, and even when they promote growth, the reasons why are quite different from those discussed in the earlier "endogenous growth with scale effects" literature. The properties of endogenous growth models fundamentally change when scale effects are removed by making less optimistic assumptions about the returns to R&D activities.

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Appendix

Proof of Lemma 1: The vertical R&D condition (29) and the horizontal R&D condition (30) can only simultaneously hold if

$$\frac{\delta\lambda_v v^{\delta-1}\Gamma}{1-\beta_v} = \frac{\gamma\lambda_h h^{\gamma-1}}{1-\beta_h}. \tag{33}$$

This "mutual R&D" condition has a natural economic interpretation. Changes in the economic environment that increase the relative reward for innovating contribute to increasing both the fraction of GDP devoted to horizontal R&D h and the fraction of GDP devoted to vertical R&D v, that is, h and v tend to rise or fall together.

Using (25) and (26) , (33) can be written more compactly as

$$g_N = c_1 g_A v^{\epsilon}, \tag{34}$$

where $c_1 \equiv \frac{\lambda_h}{\sigma\lambda_v}\left(\frac{\lambda_h\gamma(1-\beta_v)}{\lambda_v\Gamma\delta(1-\beta_h)}\right)^{\gamma/(1-\gamma)}$ and $\epsilon \equiv \frac{1-\delta}{1-\gamma}\gamma - \delta$. Substituting (25), (26) and (34) into (28) and (29), the population growth condition becomes

$$g_L = g_A\left(d - 1 + \alpha c_1 v^{\epsilon}\right) \tag{35}$$

and the (now general) R&D condition becomes

$$\rho - g_L = g_A\left[\frac{\delta\Gamma\alpha(1-\alpha)}{\sigma(1-\beta_v)v} - \frac{1}{\sigma} - \frac{\alpha}{1-\alpha} - \alpha c_1 v^{\epsilon}\right]. \tag{36}$$

Equations (35) and (36) represent a system of two equations in two unknowns (v and g_A) that can be solved for a balanced growth equilibrium. These equations are graphed in Figure 3 assuming that $\gamma > \delta$ (which implies that $\epsilon > 0$). Then the population growth condition (35) is unambiguously downward sloping and has a strictly positive vertical intercept, whereas the R&D condition (36) is unambigously upward sloping and goes through the origin. As illustrated in Figure 3, there is a unique intersection of these two curves at point A, which pins down the balanced growth equilibrium values of v and g_A. With these values determined, (34) pins down g_N, (25) pins down y, , and then (26) pins down h. Thus the model has a unique balanced growth equilibrium when $\gamma > \delta$. In the special case of $\gamma = \delta$ (not illustrated in Figure 3), the population growth condition (35) is a horizontal line with a strictly

Figure 3: The effect of a general R&D subsidy when $\gamma > \delta$

positive vertical intercept and the above-mentioned argument continues to imply that the model has a unique balanced growth equilibrium.

The effect of permanently increasing the general R&D subsidy rate $\beta = \beta_h = \beta_v$ is illustrated in Figure 3 by the movement from point A to point B. An increase in β unambiguously causes the R&D condition (36) to shift down, while having no effect on the population growth condition (35). Thus a higher general R&D subsidy increases v (the fraction of GDP devoted to vertical R&D) but decreases g_A (the quality growth rate in the typical industry) if $\gamma > \delta$. If $\gamma = \delta$, then a higher general R&D subsidy increases v but has no effect on g_A, as the population growth condition is a horizontal line and does not shift in response to the increase in β.

If $\gamma < \delta$, then the above-mentioned arguments do not go through smoothly, so I solve the model somewhat differently in this case. Using (25) and (26), the mutual R&D condition (33) can be alternatively expressed as

$$g_A = c_2 g_N h^{\mu}, \tag{37}$$

where $c_2 \equiv \frac{\sigma\lambda_v}{\lambda_h}\left(\frac{\lambda_v\delta\Gamma(1-\beta_h)}{\lambda_h\gamma(1-\beta_v)}\right)^{\delta/(1-\delta)}$ and $\mu \equiv \frac{1-\gamma}{1-\delta}\delta - \gamma$. Substituting (25), (26) and (37) into (28) and (29), the population growth condition becomes

$$g_L = g_N\left[(d-1)c_2 h^{\mu} + \alpha\right] \tag{38}$$

and the (now general) R&D condition becomes

$$\rho - g_L = g_N \left[\frac{\gamma\alpha(1-\alpha)}{(1-\beta_h)h} - c_2 h^{\mu} \left(\frac{1}{\sigma} + \frac{\alpha}{1-\alpha} \right) - \alpha \right]. \tag{39}$$

Equations (38) and (39) represent a system of two equations in two unknowns (h and g_N) that can be solved for a balanced growth equilibrium. These equations are graphed in Figure 4. Given $\gamma < \delta$ and $\mu > 0$, the population growth condition

Figure 4: The effect of a general R&D subsidy when $\gamma < \delta$

(38) is unambiguously downward sloping and has a strictly positive vertical intercept, whereas the R&D condition (39) is unambigously upward sloping and goes through the origin. As illustrated in Figure 4, there is a unique intersection of these two curves at point A, which pins down the balanced growth equilibrium values of h and g_N. With these values determined, (37) pins down g_A, (26) pins down y, and then (25) pins down v. Thus the model has a unique balanced growth equilibrium when $\gamma < \delta$ as well.

The effect of permanently increasing the general R&D subsidy rate $\beta = \beta_h = \beta_v$ is illustrated in Figure 4 by the movement from point A to point B. An increase in β unambiguously causes the R&D condition (39) to shift down, while having no effect on the population growth condition (38). Thus a higher general R&D subsidy increases h (the fraction of GDP devoted to horizontal R&D) but decreases g_N (the growth rate of the measure of industries) if $\gamma < \delta$. *Q. E. D.*

Proof of Theorem 1: In (g_A, g_N) space, it follows from (27) that each iso-growth curve is a downward sloping line with slope $\frac{-1}{1-\alpha}$ and it follows from (28) that the population growth condition is a downward sloping line with slope $\frac{-(d-1)}{\alpha}$. Thus, the slope of each iso-growth line exceeds the slope of the population growth growth condition (in absolute value) if and only if $\frac{1}{1-\alpha} > d$. The cases $\frac{1}{1-\alpha} > d$ and $d > \frac{1}{1-\alpha}$ are illustrated in Figures 5 and 6, respectively. The mutual R&D condition (given by either (34) or

Figure 5: The effect of a general R&D subsidy when $1/(1-\alpha) > d$

Figure 6: The effect of a general R&D subsidy when $d > 1/(1-\alpha)$

(37)) is also illustrated and is an upward sloping line that goes through the origin in (g_A, g_N) space, when h and v are fixed at their initial equilibrium values. An increase in the R&D subsidy rate β causes the slope of the mutual R&D condition to increase if $\delta < \gamma$ (since the equilibrium value of v increases in (34)) and causes the slope of the mutual R&D condition to decrease if $\delta > \gamma$ (since the equilibrium value of h increases in (37)). In the $1/(1-\alpha) > d$ case illustrated in Figure 5, an increase in the R&D subsidy rate β decreases the long-run growth rate g if $\delta < \gamma$ (the movement from A to B) and increases the long-run growth rate if $\delta > \gamma$ (the movement from A to C). In the $d > 1/(1-\alpha)$ case illustrated in Figure 6, an increase in the R&D subsidy rate β increases the long-run growth rate g if $\delta < \gamma$ (the movement from A to B) and decreases the long-run growth rate if $\delta > \gamma$ (the movement from A to C). R&D subsidies have no growth effects in the $d = 1/(1-\alpha)$ case (not illustrated), since the slope of each iso-growth line coincides with the slope of the population growth condition and R&D subsidies also have no growth effects in the $\delta = \gamma$ case (not illustrated), since the slope of the mutual R&D condition does not change. *Q. E. D.*

Proof of Lemma 2: If $\delta = \gamma$ or $\epsilon = 0$, then (35) is a horizontal line in (v, g_A) space which shifts up when β_v increases and (36) is a upward-sloping curve in (v, g_A) space which shifts down when β_v increases. Thus a permanent increase in β_v (holding β_h fixed) increases both v and g_A. Also (35) shifts down and (36) shifts up when β_h increases, implying that a permanent increase in β_h (holding β_v fixed) decreases both v and g_A.

If $\gamma > \delta$ or $\epsilon > 0$, then (35) is a downward sloping curve in (v, g_A) space which shifts up when β_v increases and (36) is a upward-sloping line in (v, g_A) space which shifts down when β_v increases. Thus a permanent increase in β_v (holding β_h fixed) definitely increases v but more work needs to be done to determine the effect on g_A. Suppose that for some $\gamma > \delta$, an increase in β_v has no effect on g_A. Then (35) implies that $c_1 v^\epsilon$ does not change when β_v increases, from which it follows that $[(1-\beta_v)v]v^{-\delta/\gamma}$ does not change when β_v increases. But we have already established that v increases in response to an increase in β_v and (36) implies that $[(1-\beta_v)v]$ does not change when β_v increases. This yields a contradiction. By the continuity of the model, it follows that g_A must always increase in response to an increase in β_v when $\gamma > \delta$.

Now consider the effects of a permanent increase in β_h when $\gamma > \delta$. Since (35) shifts down and (36) shifts up when β_h increases, v definitely decreases but more work needs to be done to determine the effect on g_A. Suppose that for some $\gamma > \delta$, an increase in β_h has no effect on g_A. Then (35) implies that $c_1 v^\epsilon$ does not change when β_h increases, and (36) implies that v does not change, contradicting our earlier finding that v definitely decreases. By the continuity of the model, it follows that g_A must always decrease in response to an increase in β_h when $\gamma > \delta$.

Returning to the case where $\delta = \gamma$ or $\mu = 0$, (38) is a horizontal line in (h, g_N) space which shifts up when β_h increases and (39) is a upward-sloping curve in (h, g_N) space which shifts down when β_h increases. Thus a permanent increase in β_h (holding β_v fixed) increases both h and g_N. Also (38) shifts down and (39) shifts up when β_v increases, implying that a permanent increase in β_v (holding β_h fixed) decreases both h and g_N.

If $\delta > \gamma$ or $\mu > 0$, then (38) is a downward sloping curve in (h, g_N) space which shifts up when β_h increases and (39) is a upward-sloping line in (h, g_N) space which shifts down when β_h increases. Thus a permanent increase in β_h (holding β_v fixed) definitely increases h but more work needs to be done to determine the effect on g_N. Suppose that for some $\delta > \gamma$, an increase in β_h has no effect on g_N. Then (38) implies that $c_2 h^\mu$ does not change when β_h increases, from which it follows that $[(1-\beta_h)h]h^{-\gamma/\delta}$ does not change when β_h increases. But we have already established that h increases in response to an increase in β_h and (39) implies that $[(1-\beta_h)h]$ does not change when β_h increases. This yields a contradiction. By the continuity of the model, it follows that g_N must always increase in response to an increase in β_h when $\delta > \gamma$.

Finally, consider the effects of a permanent increase in β_v when $\delta > \gamma$. Since (38) shifts down and (39) shifts up when β_v increases, h definitely decreases but more work needs to be done to determine the effect on g_N. Suppose that for some $\delta > \gamma$, an increase in β_v has no effect on g_N. Then (38) implies that $c_2 h^\mu$ does not change when β_v increases, and (39) implies that h does not change, contradicting our earlier finding that h definitely decreases. By the continuity of the model, it follows that g_N must always decrease in response to an increase in β_v when $\delta > \gamma$. *Q. E. D.*

Proof of Theorem 2: Changes in R&D subsidy rates do not shift the population growth condition (28) but instead induce movements along this downward-sloping line. Comparing (28) and (27), a movement on the population growth condition in the northwest direction (g_N increases and g_A decreases) is growth-promoting if $d > 1/(1-\alpha)$ and a movement on the population growth condition in the southeast direction (g_N decreases and g_A increases) is growth-promoting if $d < 1/(1-\alpha)$. Since Lemma 2 completely determines the direction of movement on the population growth condition implied by any targeted R&D subsidy increase, Theorem 2 immediately follows. *Q. E. D.*